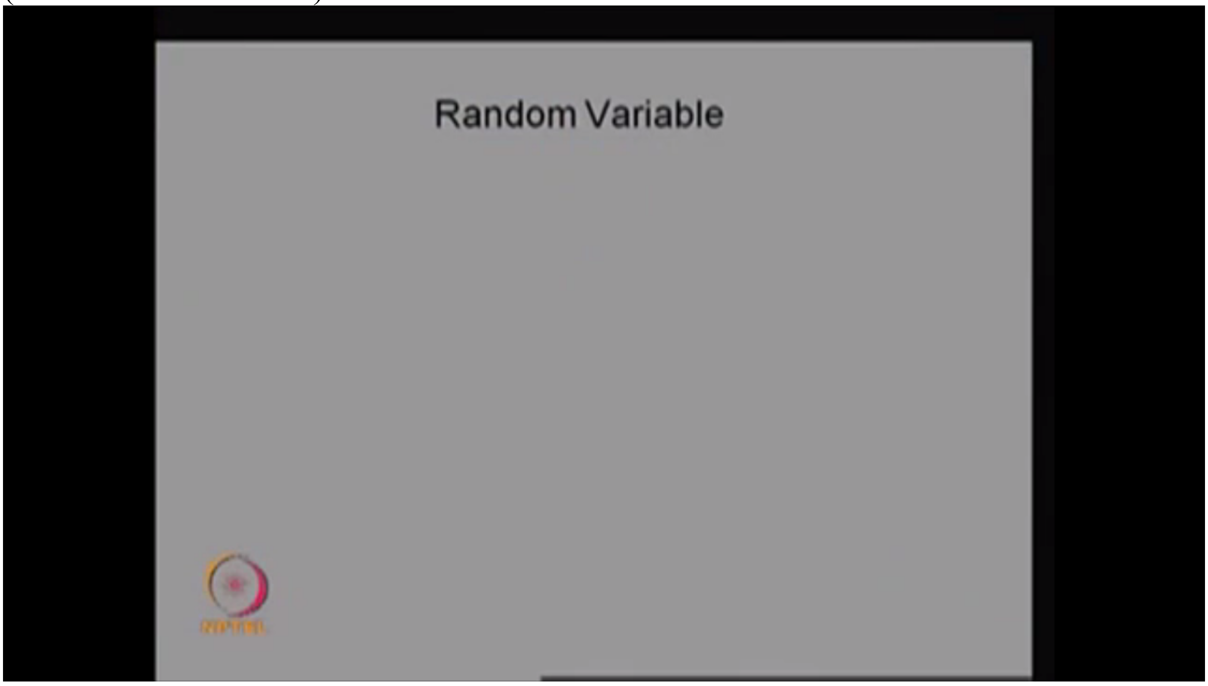
Random Variable and cumulative distribution function

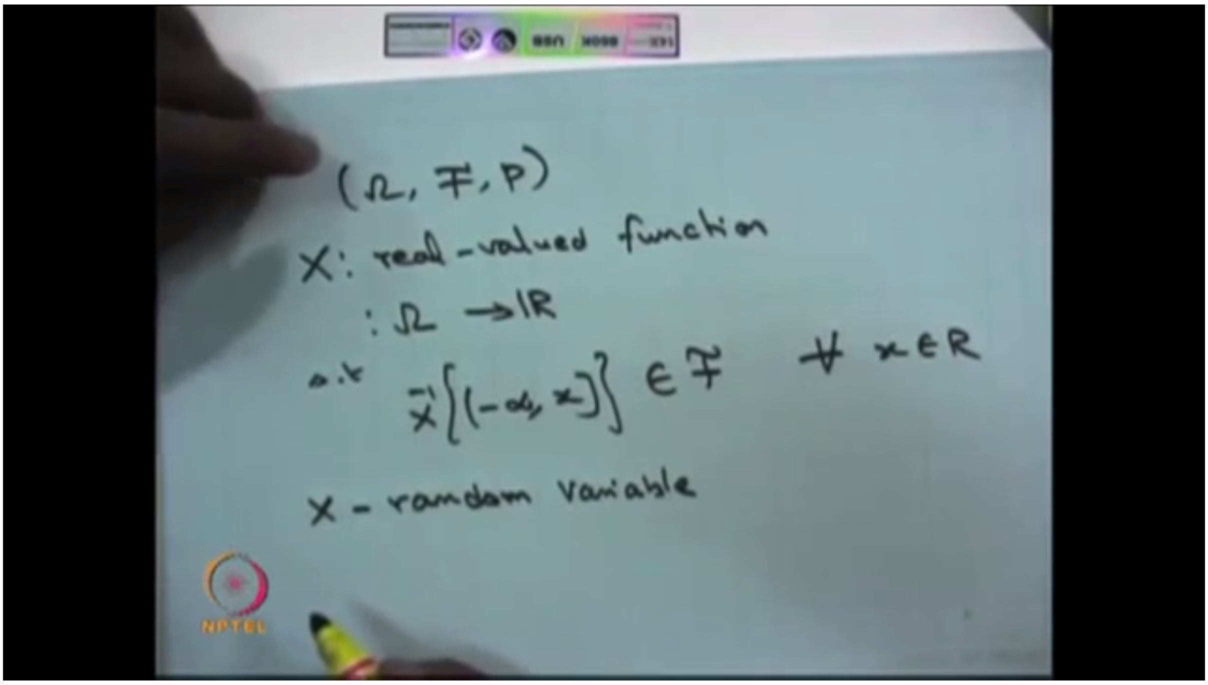
Dr. S Dharmaraja
Department of Mathematics, IIT Delhi

(Refer Slide Time 00:01)



Now we are moving into the next concept that is called a random variable. That means you have a probability space. You started with the probability space and you are defining a real valued function, which maps omega to R such that if you find out the inverse image of any x in the real line, that inverse image between minus infinity to x belonging to F. If this condition is satisfied by any real valued function, which maps from omega to R, then that is going to be call it as a random variable.

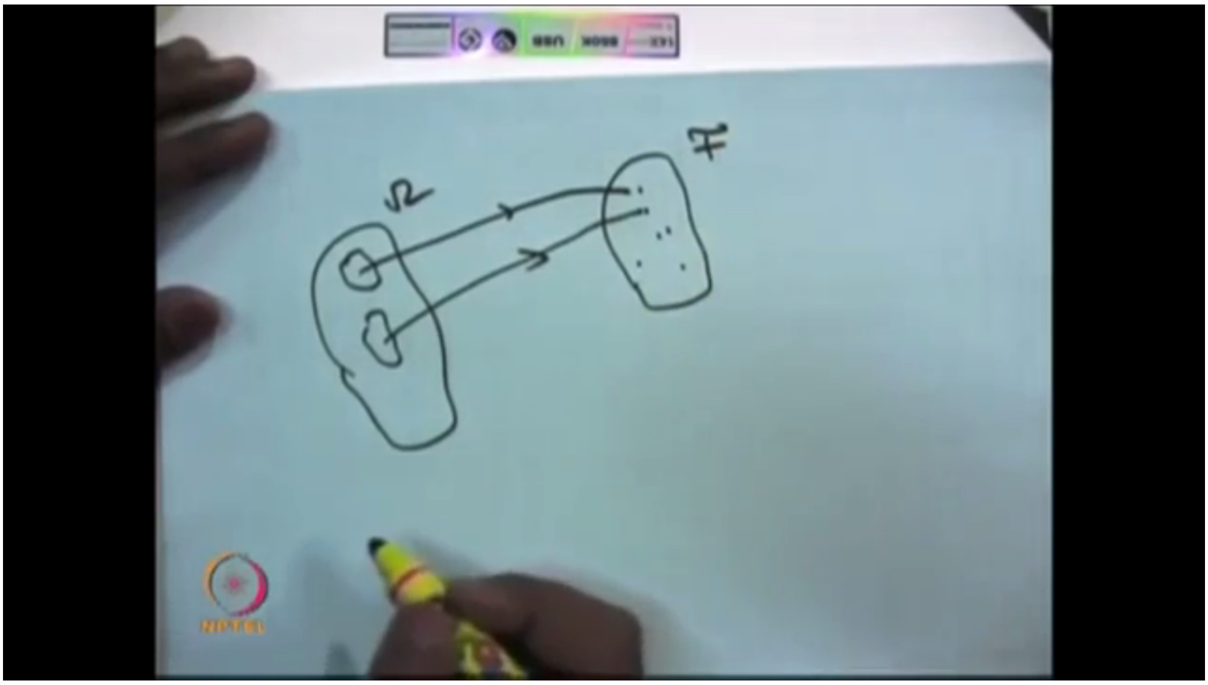
(Refer Slide Time 01:07)



That means after you have a collection of possible outcomes, you are finding one sigma-algebra. You can make more than one sigma-algebra over the omega. So you have a one fixed sigma-algebra. It could be a trivial one or the non trivial one and so on. So you have a fixed F. After fixing the F, you have a probability measure and the probability measure is nothing to do with the random variable at all. Still you have a probability space and from the probability space, you are defining a real valued function such that the inverse image is belonging to F.

That I can make out with the simple diagram. This is the omega and from the omega, you have created the F. F means it has the events and the events are nothing but the few possible outcomes. That means this possible outcomes you land up with the one element and this possible outcomes you land up with the another event and so on. So like that the different few possible outcomes that is going to be one of the elements in the F.

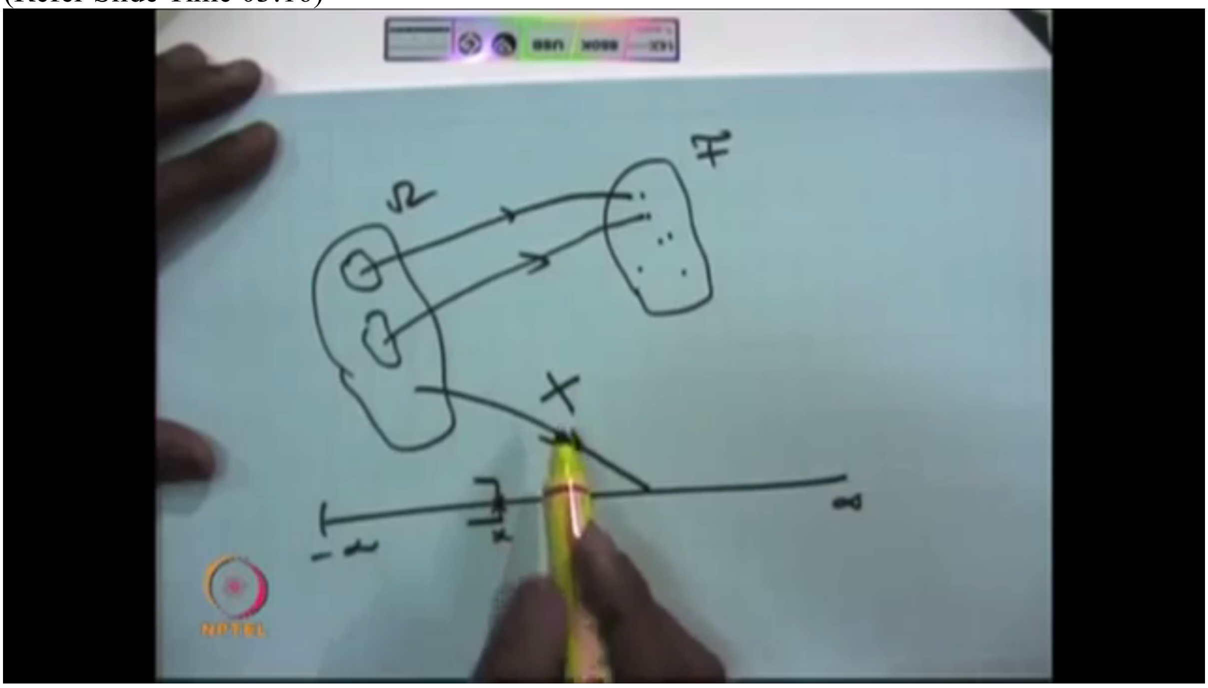
(Refer Slide Time 02:18)



You have created another real valued function that is X from omega to R. This is a real valued function and you take any point some x in the real line, and if you find out what is the inverse image from minus infinity to till x, you collect what is the inverse image.

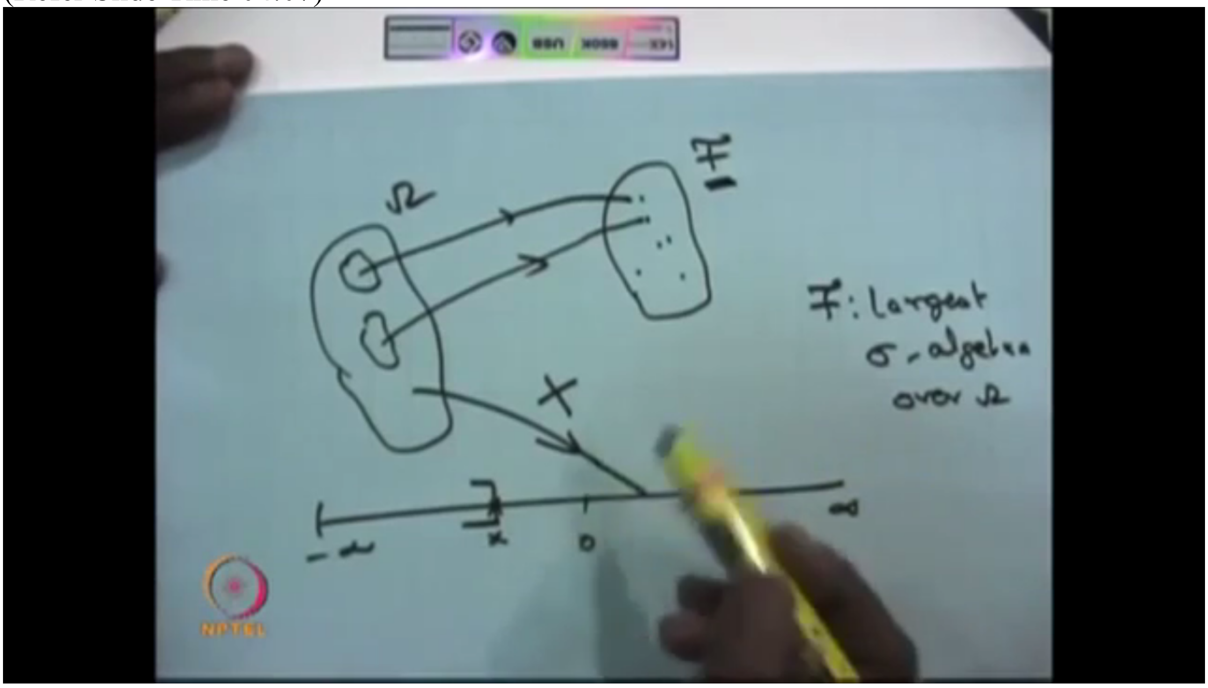
You got it under the mapping X from minus infinity to the close to interval x. You collect all the possible outcomes that is going to give the value between minus infinity to close to interval x. You collect such a possible outcomes. If you collect such a possible outcomes and that is going to be one of the elements in the F for different values of x, then the real valued function is going to be call it as a random variable.

(Refer Slide Time 03:16)



That means once you know the F, if you create a real valued function, after checking that condition, you can conclude that real valued function is going to be a random variable. That means if you have a some other F, there is a possibility some real valued function may not be the random variable. That means how you choose F, that is going to play a role of come to the conclusion the real valued function is going to be a random variable or not. If you take F is going to be the largest one, the largest sigma-algebra over F over Omega, then any real valued function is going to be satisfied this property.

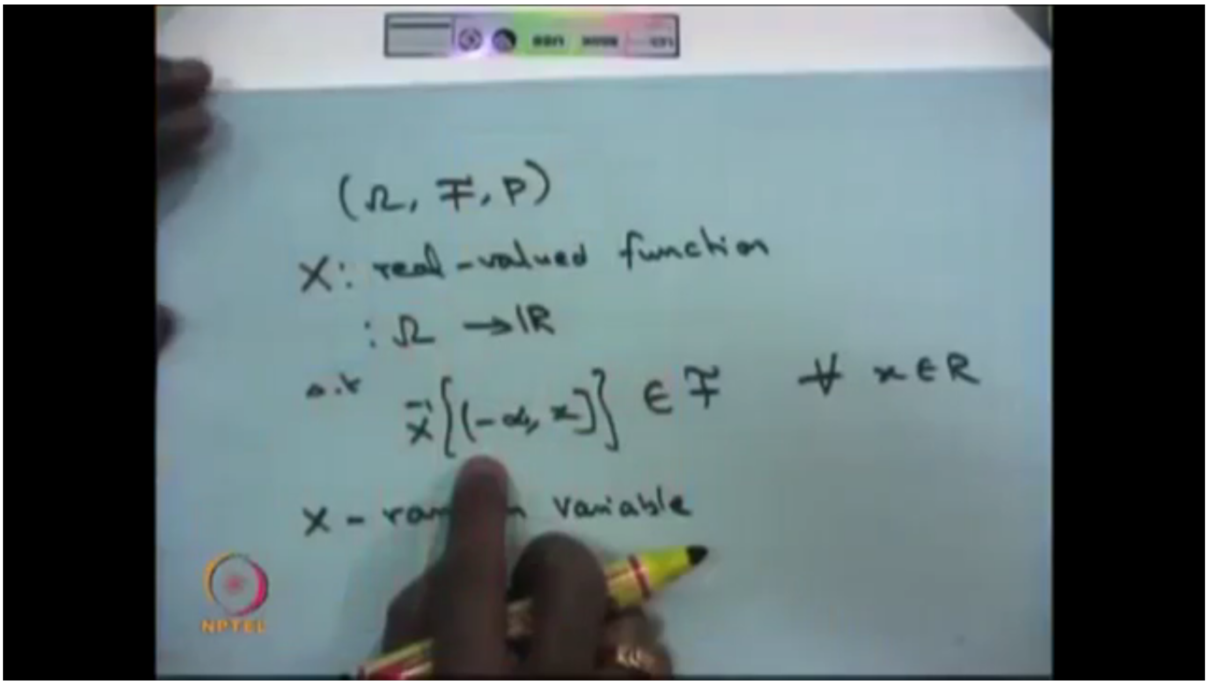
(Refer Slide Time 04:07)



Suppose you take the sigma-algebra over omega, which is in between the trivial one and the largest one, then the few real valued function may be a random variable and a few other real valued function may not be the random variable.

So in the usual scenario, whenever you see the random variable definition in many books, they use real valued function is going to be a random variable just like that. That means they did -- they have taken the F is going to be the largest sigma-algebra. So whenever F is going to be the largest sigma-algebra, then any real valued function is going to be a random variable.

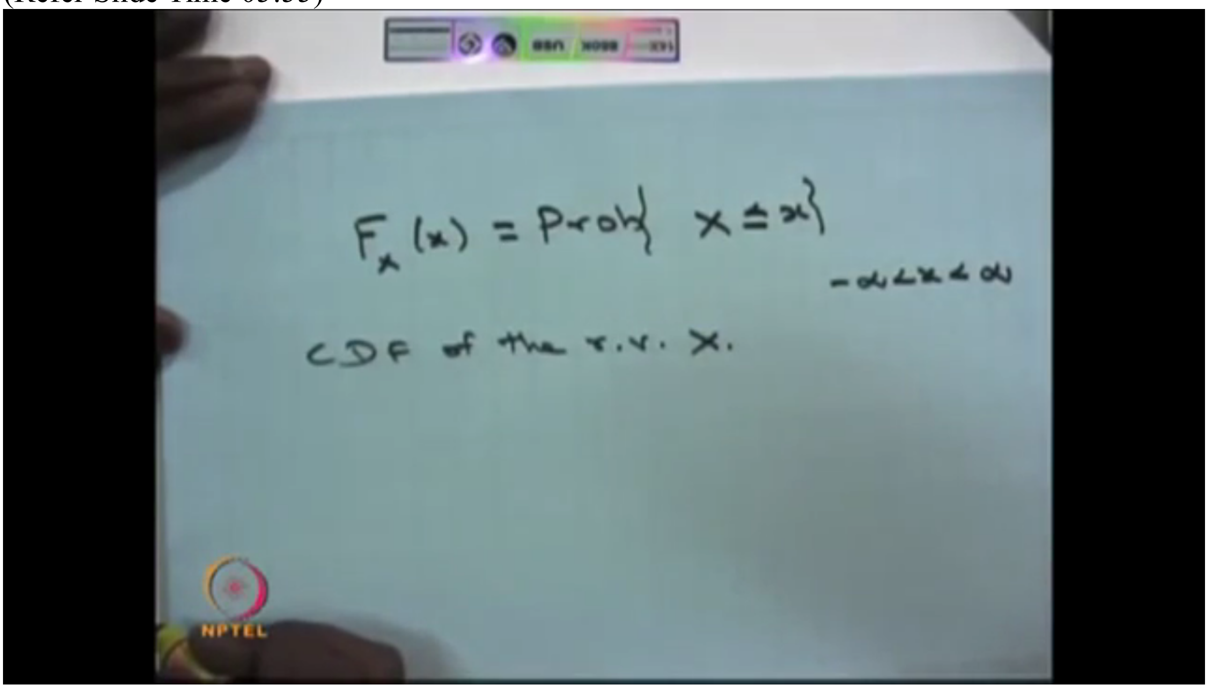
(Refer Slide Time 04:54)



And going back to the previous slide, this condition is going to be the if and only if condition is also. Suppose you have a real valued function is going to be a random variable, then this condition will be satisfied and if this condition is satisfied, then that real valued function is going to be a random variable also.

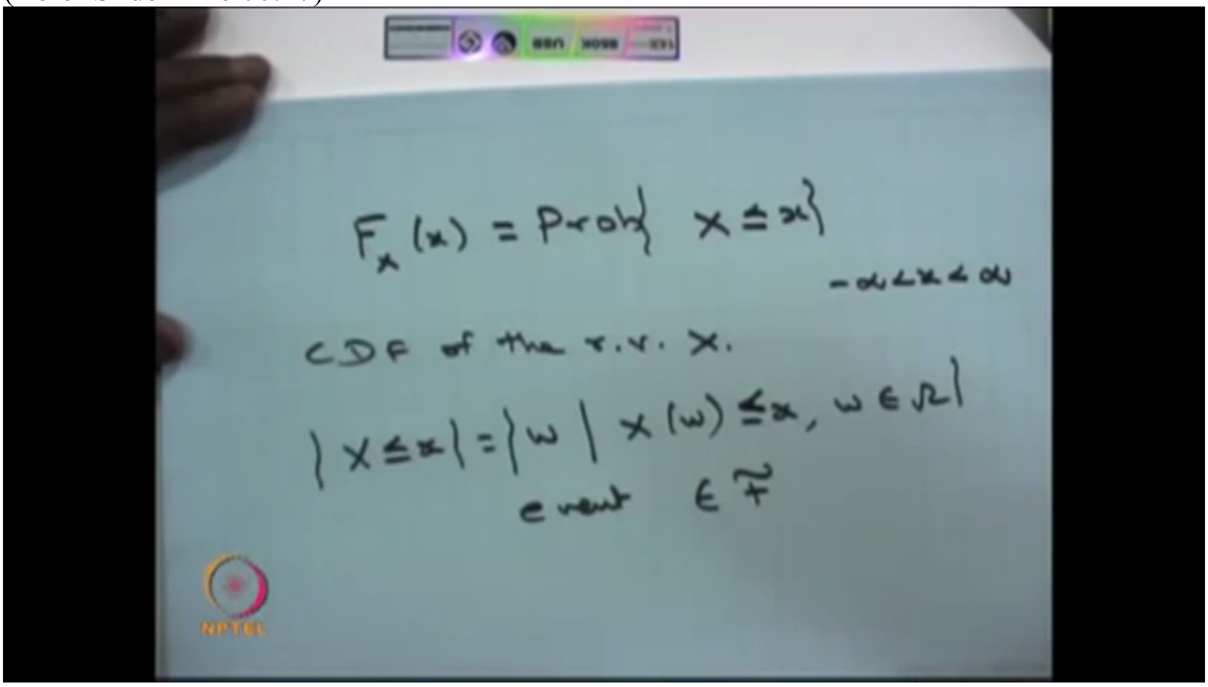
Now we are moving into the next concept called Cumulative Distribution Function. So the cumulative distribution function for the random variable x can be defined as capital F suffix X is for the random variable X and the small x is the variable x. That is going to be probability of X is less than or equal to small x and here the x lies between minus infinity to infinity. So this is going to be call it as the CDF of the random variable X.

(Refer Slide Time 05:55)



So the way I relate with the probability of X is less than or equal to x, this X is less than or equal to x is nothing but you collect few possible outcomes such that under the operation X of w that gives the value less than or equal to x for all w belonging to omega. That means you collect a few possible outcomes w such that under the mapping X, X of w should give the value maximum x. So that is less than or equal to. Therefore, this is nothing but a event and this event is belonging to the capital F.

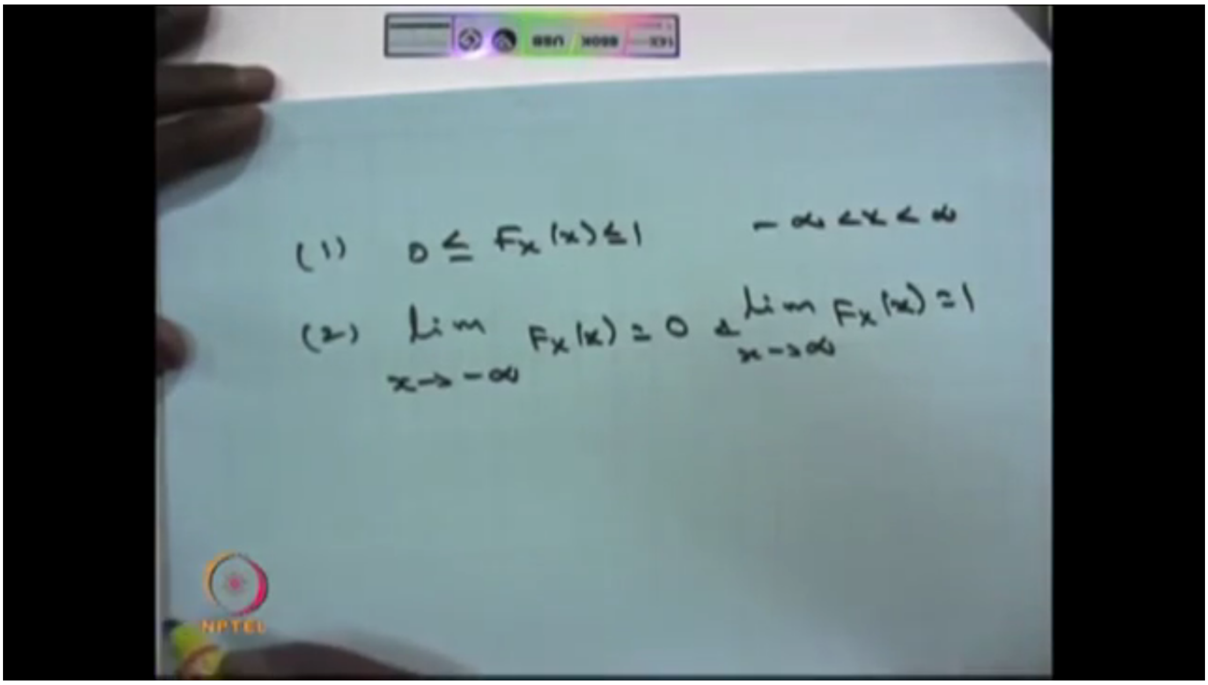
(Refer Slide Time 06:47)



Therefore, the way you have taken the probability of X is less than or equal to x, therefore, as the x moves from minus infinity to infinity, you keep on including some more possible outcomes over the x. Therefore, the probability of X is less than or equal to x varies over the x. You are going to get more probability values. Therefore, this F of x is going to satisfy few properties.

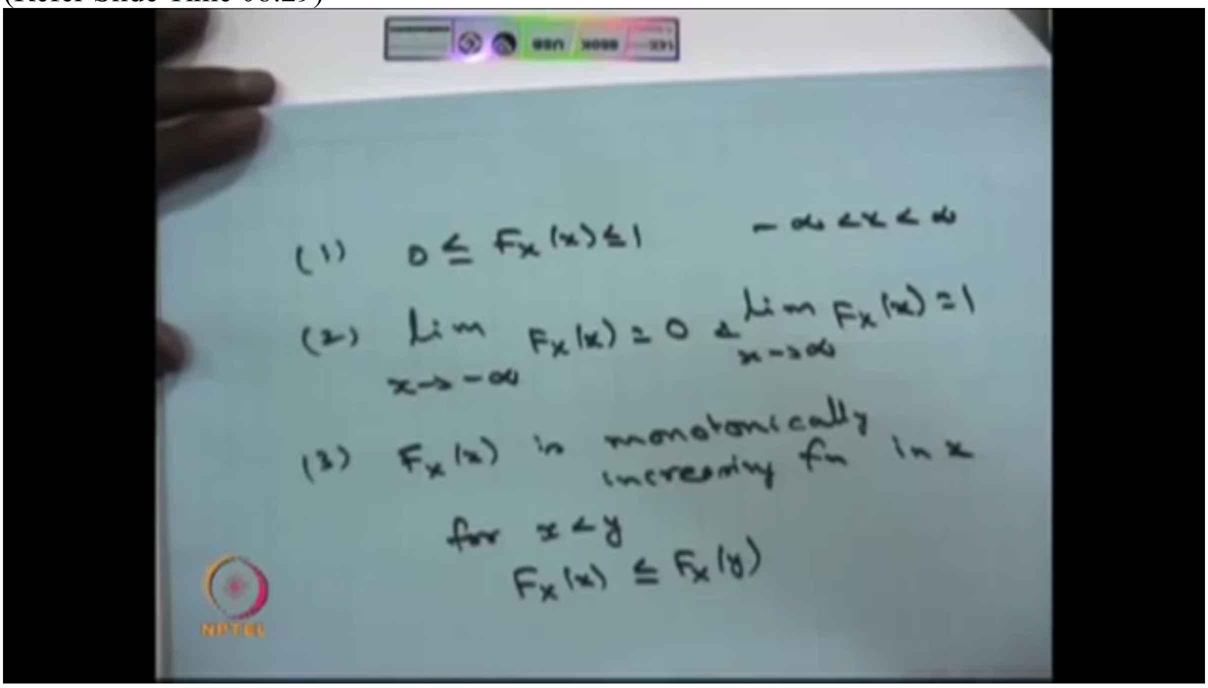
So if you see the properties of the F of x, this values is always lies between 0 to 1 for all x. Suppose you take X is almost minus infinity, then that is it is going to be 0 and if it is towards the infinity, then it is going to be 1. That means I make out a limit x tends to minus infinity, this is going to be 0 and the limit x tends to infinity, the F of x is going to be 1.

(Refer Slide Time 07:49)



The third property, the way we are keep on accumulating the possible outcomes and trying to find out the probability and that we make it as F of x, therefore, the F of x is a monotonically increasing function in x. That means over the x, if you take two values x is less than or equal to y, then the F of x value will be less than or equal to F of -- $F_x(y)$.

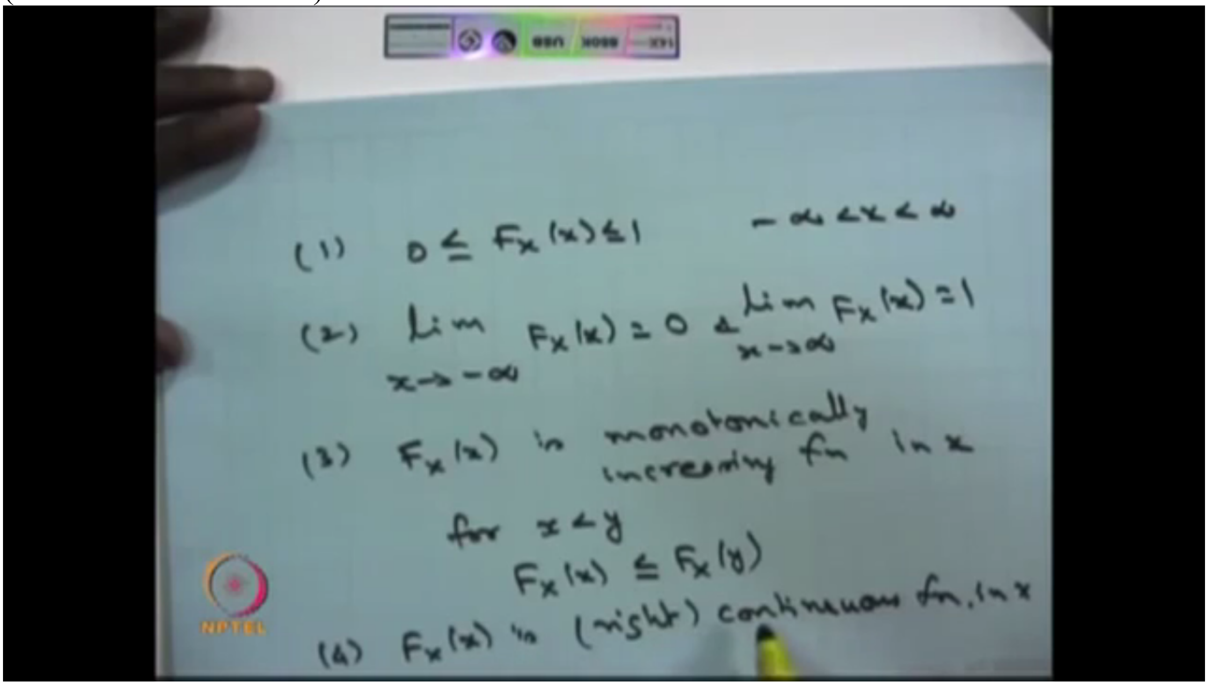
(Refer Slide Time 08:29)



That means as x is less than y, either it takes the same value or greater than value. Therefore, it is going to be in the way it is called a monotonically increasing function in x.

The fourth one it is going to be right continuous function in x. That means either it is going to be a continuous function. If it is not a continuous function, it is a right continuous. That means the left limit exists for any x as well as the right limit exists and either it is going to be a left limit is same as the right limit and value defined at that point or the left limit is different from the value defined at that point which is equal to the right value.

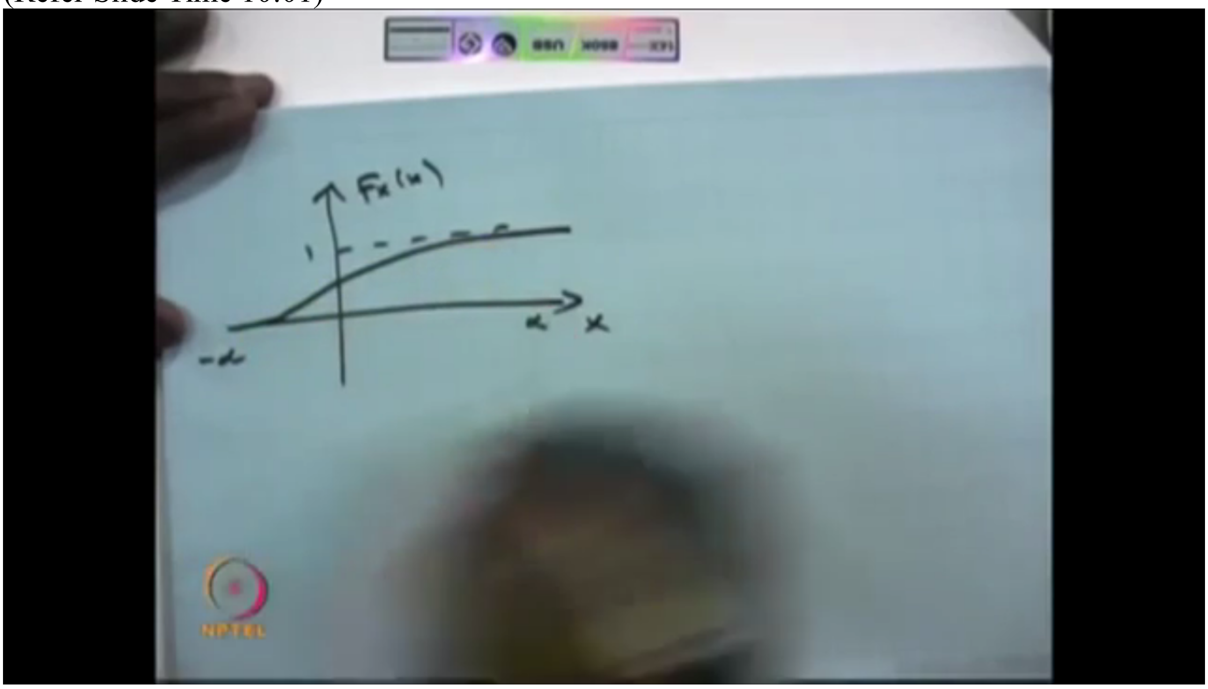
(Refer Slide Time 09:21)



Therefore, the function is going to be call it as a right continuous. So the CDF is going to be a continuous function or it is going to be a right continuous function.

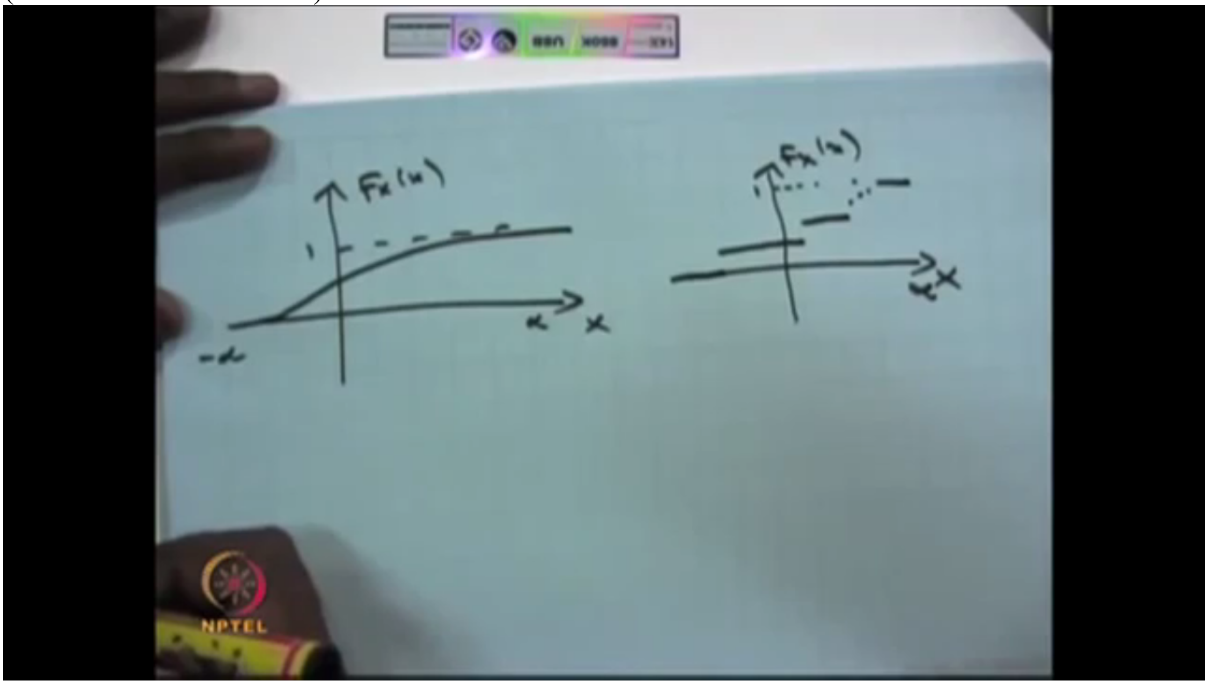
I can show few diagrams of the CDF as the x goes, the F of x will start from 0 and land up 1. So this is going to be the F(x) continuous as well as it satisfies the condition of minus infinity to infinity, it is going to be zero. Minus infinity it is 0. Infinity it is 1 and it is a monotonically increasing and continuous function.

(Refer Slide Time 10:01)



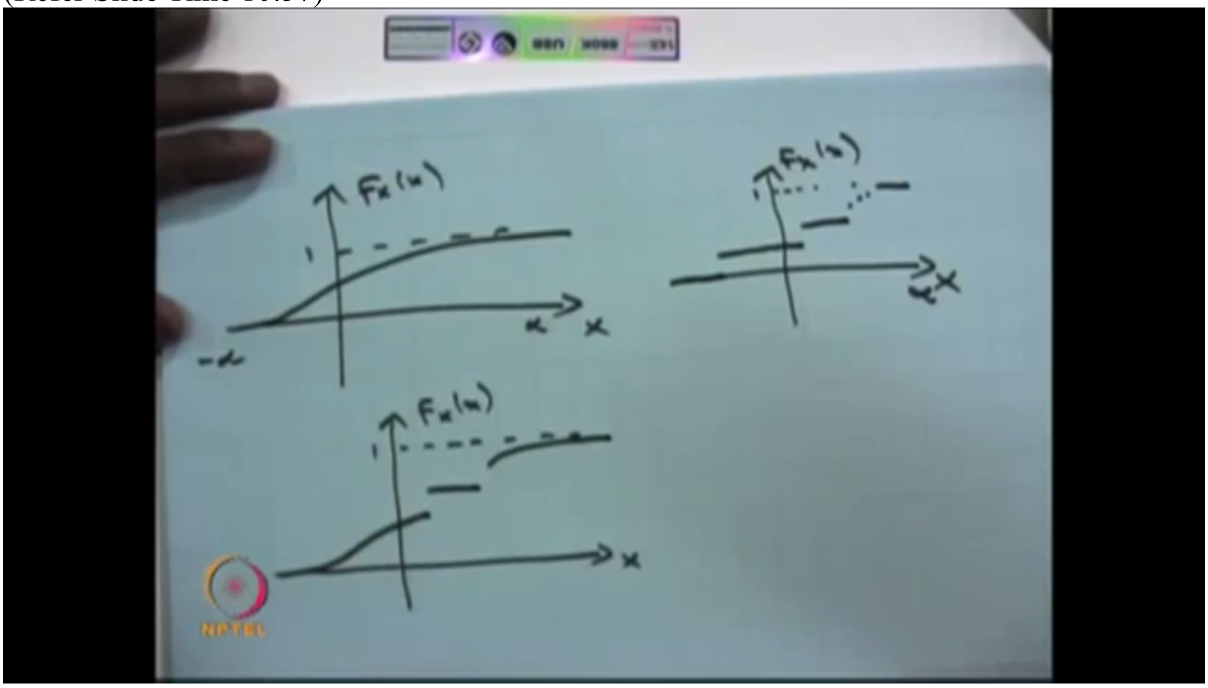
And I can give the another example of CDF. It starts from 0 and it has a discontinuity. So that means it has the -- it is -- it is a right continuous function and a monotonically increasing function, and it has the countably infinite jumps or countably infinite discontinuity and it reaches at infinity 1.

(Refer Slide Time 10:36)



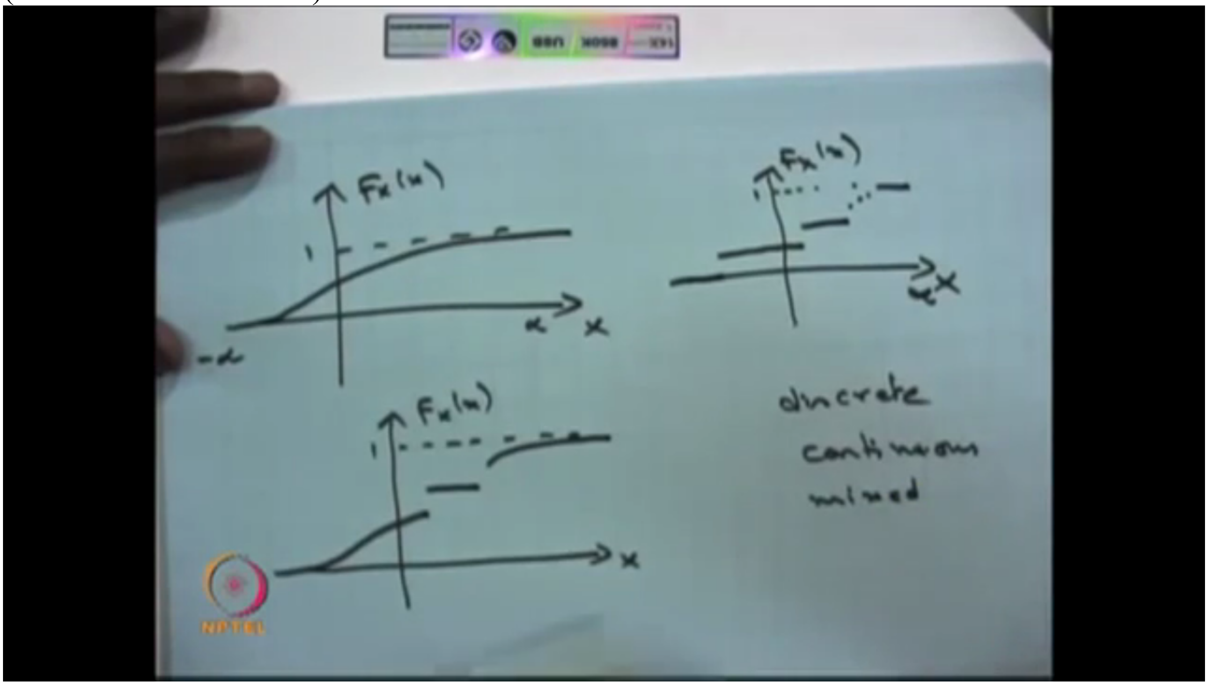
So based on the way the CDF goes, I can give one more example in which this is going to be continuous in some. Then it has the jumps. That is also possible.

(Refer Slide Time 10:57)



So the way the CDF is going to be a continuous function from minus infinity to infinity or the CDF is going to have a countably finite jumps or countably infinite jumps or it has both type, then you can classify the random variable as a discrete random variable, continuous random variable or mixed type random variable.

(Refer Slide Time 11:28)



So the random variable is going to be call it as a discrete type random variable if the CDF is going to be have a countably finite or countably infinite jumps in the CDF. Then it is called discrete random variable. If any random variable has a CDF as the continuous function from minus infinity to infinity, then that random variable is call it as a continuous random variable. If any random variable CDF has both continuous between some interval and countably finite or countably infinite jumps in some interval, then that random variable is going to be call it as a mixed type random variable.