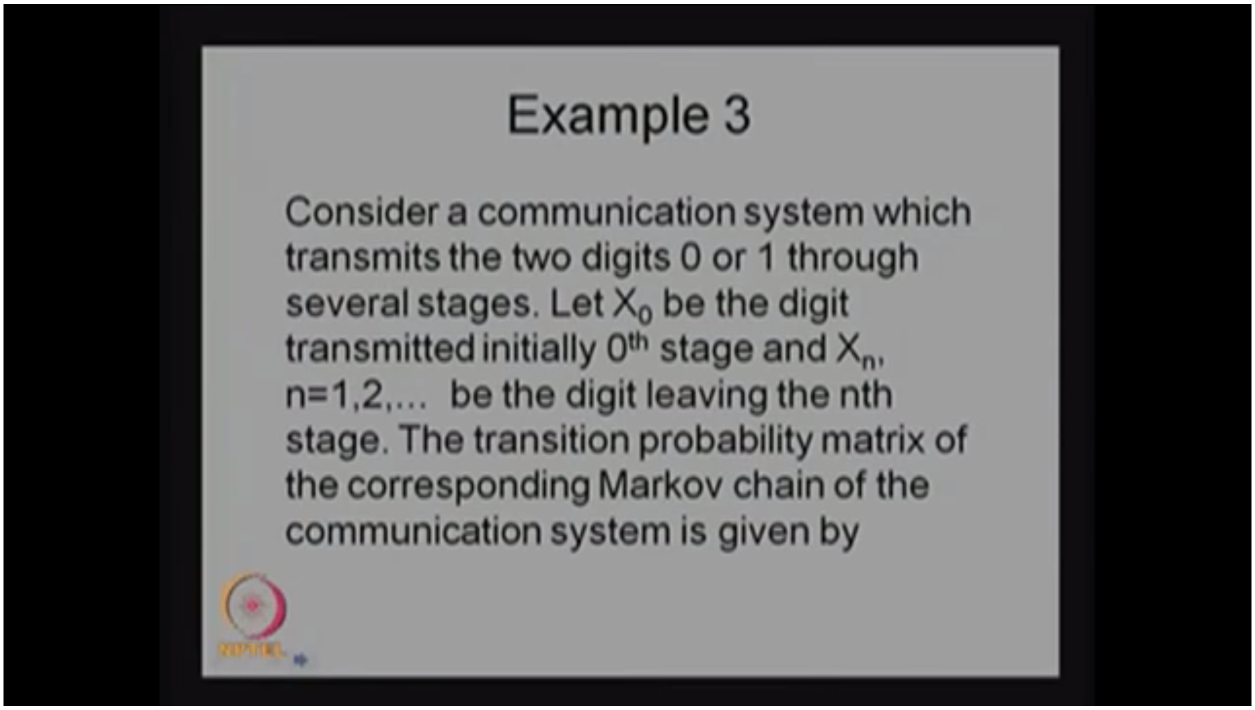
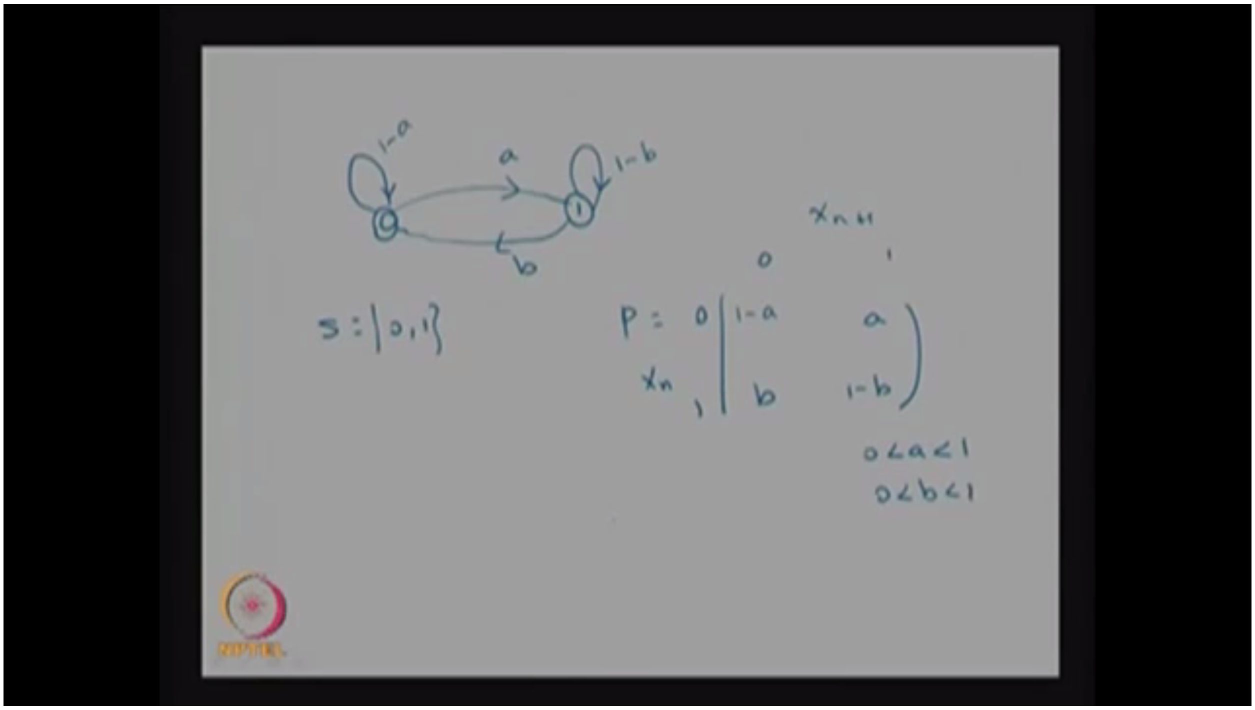


This example will talk about the communication system in which whenever the transmission takes place with the digits 0 and 1 in the several stages. Now you are going to define the random variable X naught be the digit transmitted initially, that's a 0th step. Either the transmission digit will be 0 or 1, therefore only two possibilities can take place at any Nth step transmission, either 0 or 1, like that we are making the transmission over the different stages. Therefore, this Xn over the n will form a stochastic process, because you never know which digit is a transmitted in the Nth stage. So each stage is going to be a 1 random variable and you have a collection of random variable over the stages, therefore it is a sequence of random variables. So this is going to form a stochastic process, and this stochastic process is nothing but the discrete time, discrete state stochastic process, because the possible values of Xn is going to be 0 or 1, therefore the state space is 0 or 1, and it's a discrete time discrete state stochastic process.

The way the subsequent transmission takes place depends only on the last transmission not the previous stages, therefore you can assume that this follows a Markov property. Therefore, this stochastic process is going to be called as a discrete time Markov chain.



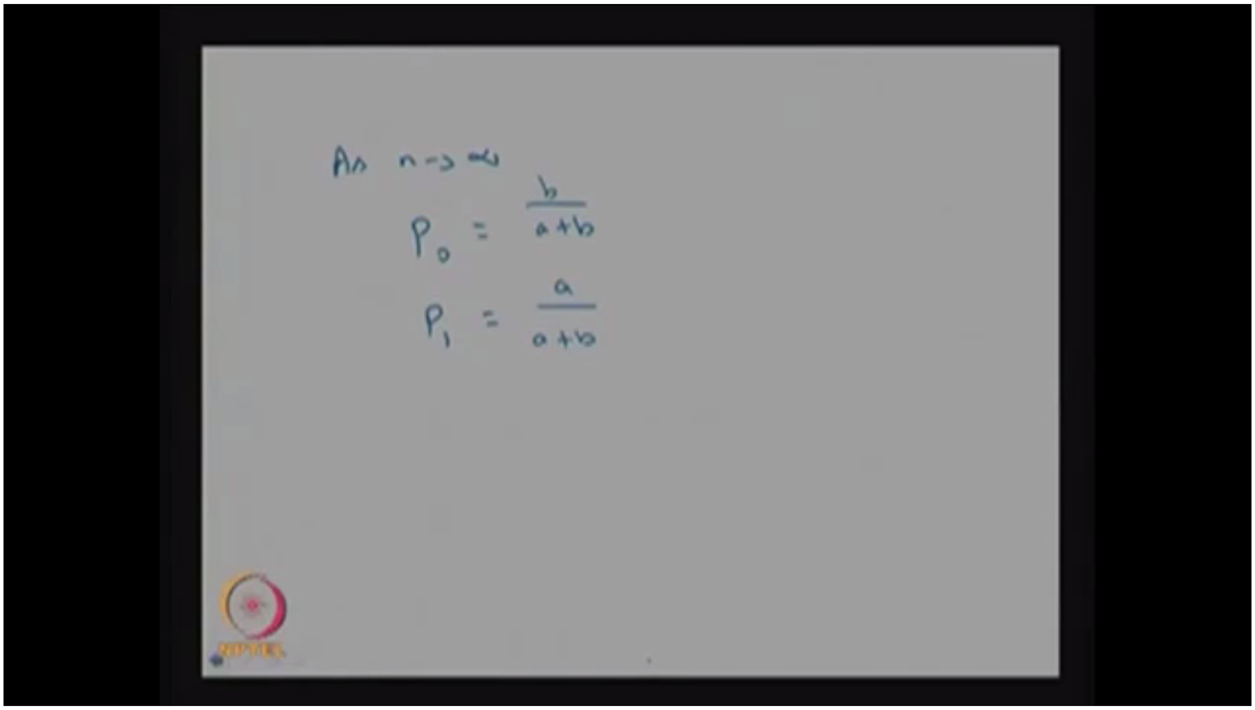
Now our interest is to find out so now I will provide what is the one-step transition probability for the Markov chain or let me give the transition diagram for that. So state transition diagram, the possible states are 0 or 1 because the state space is 0 and 1, and the probability that in the next step also, the transmission is 0 with the probability 1-a. This is a conditional probability. This is a conditional probability of the Nth stage, the transmission was 0. The n+1th stage is also the transmission is 0 with the probability 1-a. The one-step transition probability of system is moving from 0 to 1 that probability is a. That means the Nth stage the transmission was the digit 0, the n+1th stage the transmission will be the digit 1 with the probability a. Similarly, I am going to supply the one-step transition probability of 1 to 1 that is 1-b, and this is b. That means that this 1 to 0, that probability is 1 to 0 is b and 1 to 1 is 1-b. Obviously, this a lies between 0 to 1 and b also lies between 1.

$$P^{(n)} = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} \quad 0 \quad & \quad 1 \quad \end{array} \\ \begin{bmatrix} \dfrac{b+a(1-a-b)^n}{a+b} & \dfrac{a-a(1-a-b)^n}{a+b} \\ \dfrac{b-b(1-a-b)^n}{a+b} & \dfrac{a+b(1-a-b)^n}{a+b} \end{bmatrix} \end{array}$$

$$for \ |1-a-b| < 1$$

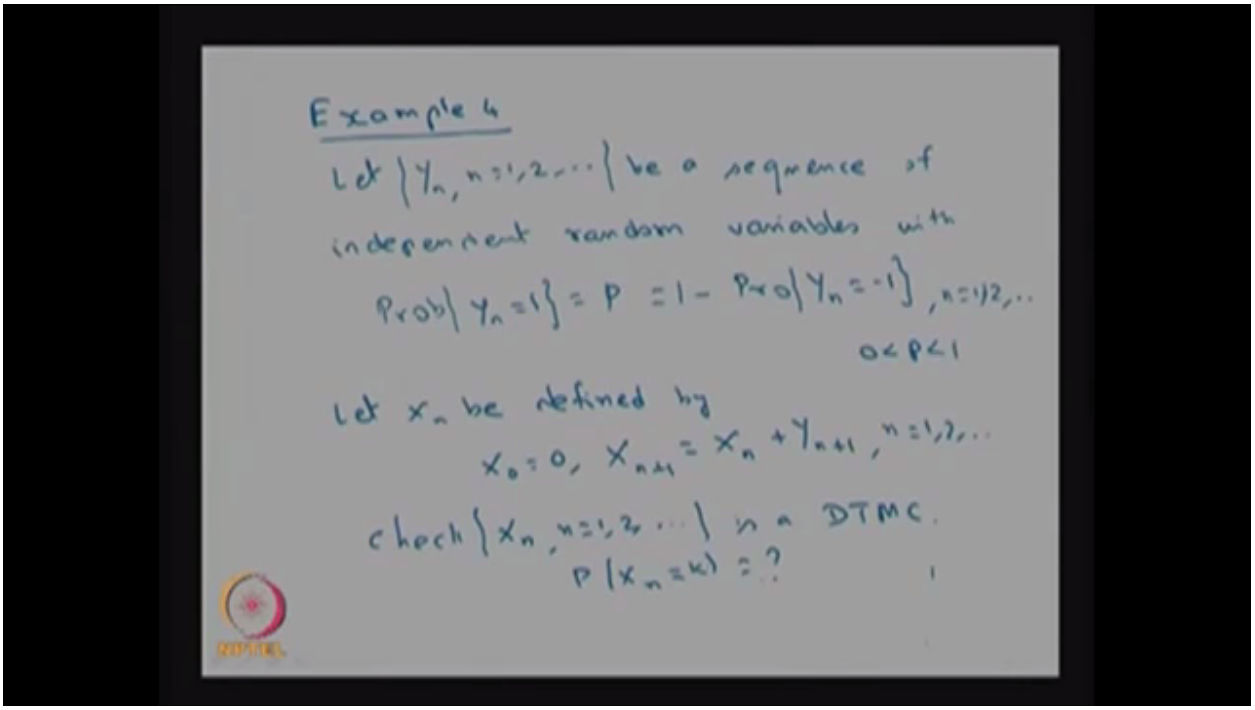
So this is -- the a is the probability that the system is transmitting from 0 to step -- sorry nth stage with the digit 0 and the n+1th with the digit 1, that probability is a, therefore the negation is 1-a, because the system can transmit either 0 or 1. So once you say that the one step transition probability of 0 to 1 is a that 0 to 0 will be 1-a. Similarly, 1 to 0 is given as the probability b and the other digit transmission will be 1, therefore it is going 1 to 1 will 1-b. So this is the state transition diagram and this is a one-step transition probability for a given time homogeneous discrete time Markov chain.

Our interest is to find out what is the distribution of the Xn. For that, you need what is the Nth step transition probability matrix. Since the one-step transition probability matrix is given, you can find out the P square, P power 3 and so on. By induction method, you can find out the P power m, but using the P power m, you can find out the P m+n, therefore you can come to the conclusion, what is the N step transition probability of system is moving from 0 to 1, and do 0 to 0 and so on. So this is nothing but -- I am just giving only the result, b+a x 1-a-b power n divided by a+b and this is nothing but a-a x 1-a-b power n divided by a+b. Similarly, if we find out the N step transition probability of system moving from 1 to 0 that is b-b 1-a-b power n divided by a+b, and this is nothing but a+b x 1-a-b power n divided by a+b. So here, I am just giving the N step transition probability matrix form by given P, you should find out the P square, P power q by, by induction you can find out the P power n and this is valid provided 1-a-b, which is less than 1, because you are finding the P power n matrix, so here it needs some determinant also unless otherwise the absolute of 1-a-b, which is less than 1, this result is not valid. So provided this condition, the P of n that is the matrix. So that is same as P power n also. P of n is same as P power n.

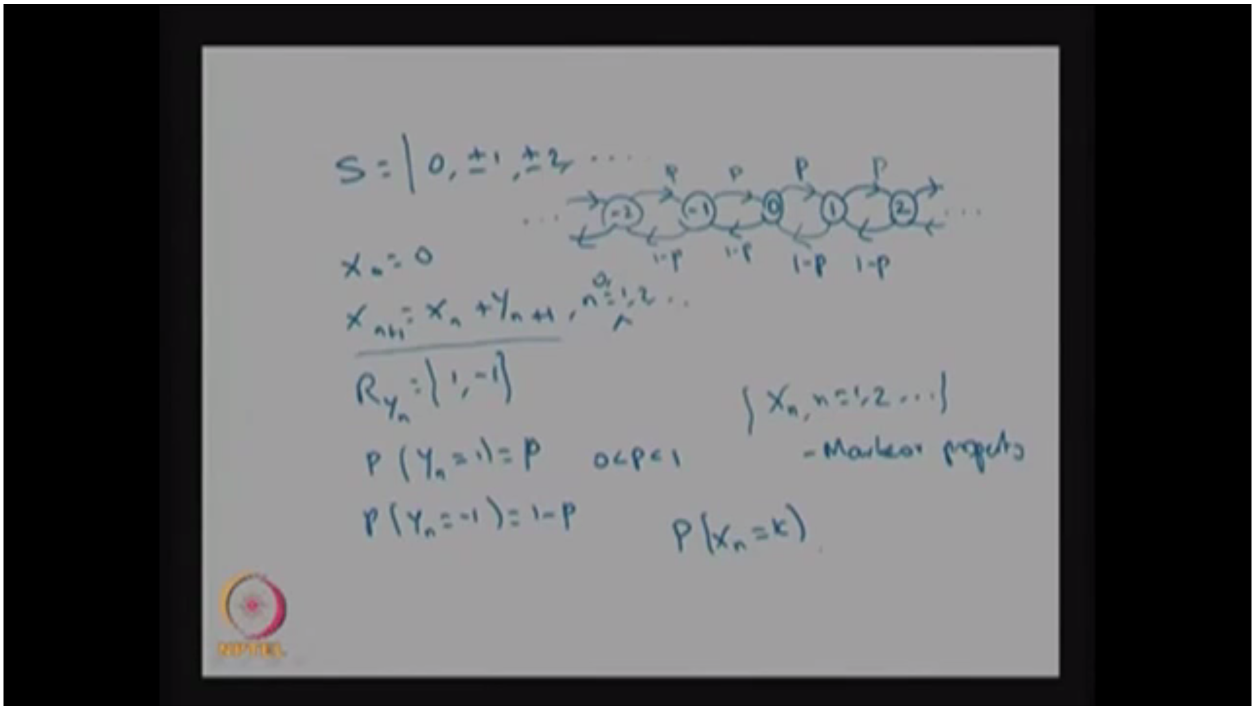


So n times to infinity, you can come to the conclusion what is the probability that the system will be in the state 0 that is same as b divided by a+b, and similarly, what is the probability that the system will be in the state 1 as n times to infinity that will be a divided by a+b. This can be visualized from the state transition diagram easily. Whenever the system is keep moving into the state 0 or 1 with the probability a, b and with the self-loop 1-a and 1-b, the subsequent stages the system will be in any one of these two states. So the proportion of b divided by a+b, the system will be in the state 1.

Similarly, with the proportion a divided by sorry a divided by a plus B the system will be in the state 1 with the proportion b divided by a+b, the system will be in the state 0 in a long run. The interpretation of as n times to infinity, this probability is nothing but in a long run, in a long run with this proportion, the system will be in the state 0 or 1. So this state transition diagram will be useful to study the long run distribution or where the system will be as an n times to infinity to study those things, the state transition will be useful.



Now we move into the next problem that is Example 4. Let, it's a sequence of random variable, be a sequence of independent random variables, independent random variables, with condition, the probability of Yn takes a value 1 that probability is P that is same as 1 minus the probability of Yn takes a value minus 1. We have a stochastic process and each random variable is an independent random variable, and the probability mass function is provided with this situation the probability of Yn takes a value 1 is P, you can assume that the P takes a value 0 to 1. That is same as 1 minus of probability of Yn takes a value minus 1 for all.



Now I am going to define another random, let Xn be defined by X naught = 0 whereas X n+a onwards that is going to be Xn + Yn + 1 for n =1, 2 and so on. So we are defining a another random variable Xn with the X naught = 0 and the X n+a = Xn + Yn + 1. Now the question is check Xn that stochastic process is the DTMC. If it is DTMC, also find out what is the probability of Xn takes a value k. We started with one stochastic process and we defined another stochastic process with earlier stochastic process, and check whether the given -- the new stochastic process is a discrete time Markov chain, that's a default one, that's a time homogeneous discrete time Markov chain. If so, then what is the probability of Xn takes a value k, that is nothing but find out the distribution of Xn. So how to find out this, the given or the Xn is going to be the DTMC?

Since Yn takes a value 1 with the probability p, and Yn takes a value minus 1 with the probability 1-p, you can make out that possible values of Yn is going to be 0 or plus or minus 1, plus or minus 2 and so on, because the relation is a X naught = 0 and the Xn is a Xn + Yn + 1, and the range of Yn, the range of Yn is 1, -1, therefore the range of Xn that inform a state space and the X naught = 0, therefore Xn, that relation is X n+1 sorry. X n+1 = Xn + Yn +1, so n takes a value 1 and so on. Therefore, the possible values of Xn will be a 0 plus or minus 1 or plus or minus 2 and so on. Therefore, that will from a state space.

Now the given clue is probability of Yn takes a value 1 of probability p, and the probability of Yn takes a value minus 1 that is 1-p and probability P lies between 0 to 1. So using this information you can make a state space of the

Yn that is going to be 1, 2 and so on -1, -2 and so on. Now you can fill up what is the one-step transition is moving from 0 to 1. That means the X 1 0 to 1, suppose you substitute 0 here then suppose it takes a value 1, then the system can move from the state zero to one in one step suppose you put the value that Xn = zero suppose you put n = 0 and the Y n+1 takes the value 1 with the probability P, then X n+1 value 1 with the probability P.

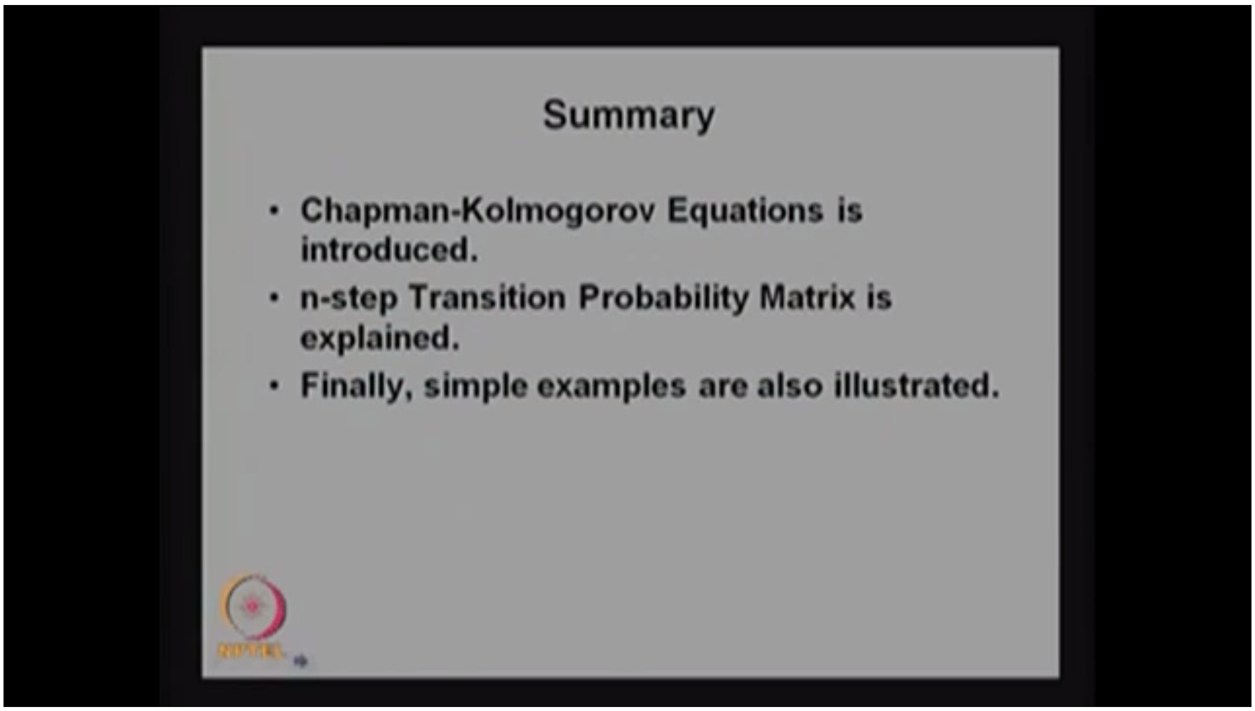
Now we can go for what is the transition state transition probability of 1 to 0. Suppose Xn value was 1, suppose Y n+1 value was minus 1, then the X n+1 value will be 0. So the one-step transitional system moving from 1 to 0 because of happening probability of Y n+1 = minus 1. That probability will 1-p, and this is up here. So whenever the system is moving from one step forward, that probability will be the probability P, and one step backward, that probably will be 1-p. So this is the way it goes forward step and this is the way it goes to the backward step, so you can fill up all other forward probability with the probability P and the backward probability with the 1-p.

Alos, we can come to the conclusion, the way we have written X n+1 = Xn +Yn +1 and the Yas are independent random variables, the X n+1 going to the value depends only on Xn not the previous X n-1 or X n-2 and so on. Therefore, the conditional distribution of X n+1 given that X n-1 till X naught, that is same as the conditional distribution of X n+1 given Xn. That means the Xn is going to satisfy the Markov property, because of this relation, because of X n+1 = Xn plus independent random variable, therefore the Xn n=1, 2, 3 and so on, this stochastic process is going to satisfy the Markov property. Therefore, this discrete time discrete state stochastic process is going to be the discrete time Markov chain, because of the Markov properties satisfied.

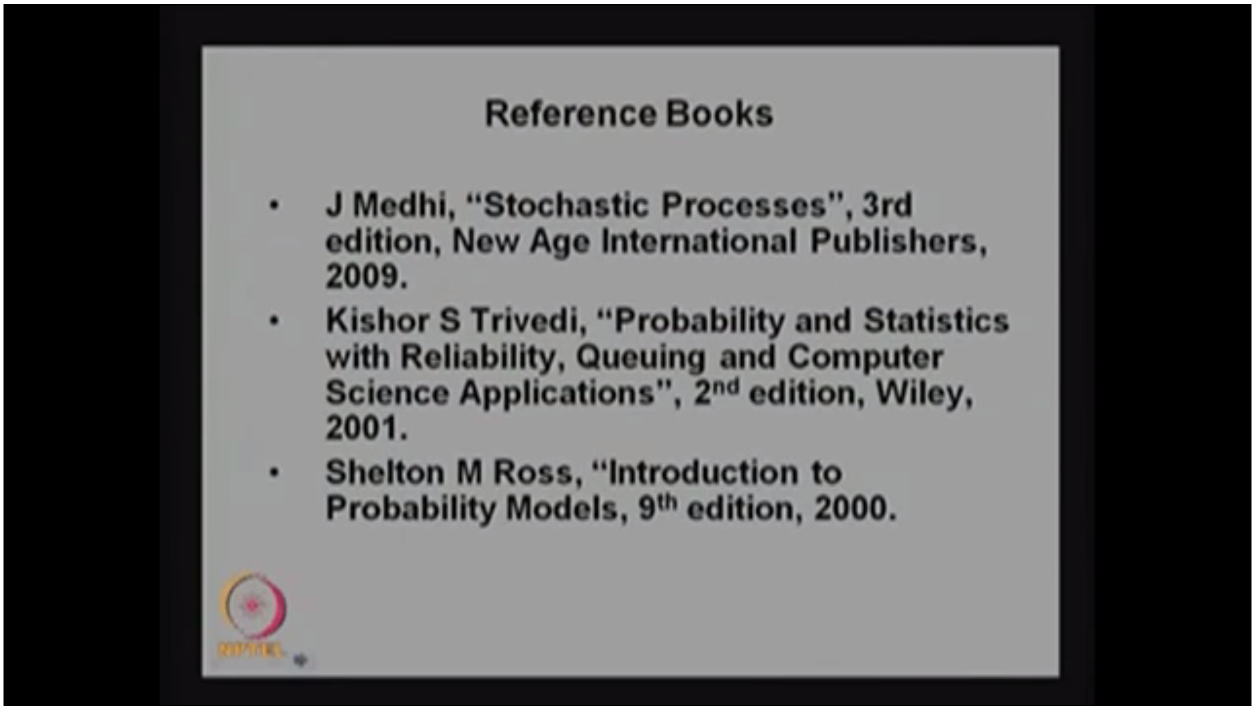
Once it is Markov properties satisfied by using the Chapman Kolmogorov equation, you can find out what is the distribution of Xn takes a value k. That is nothing but where it started at time 0 and what is the conditional distribution of the N step transition probability and the N step transitional probability is nothing but the element from the P power n, and from here you can find out the one-step transition probability matrix. From the 1 step transition probability matrix, you can find out the P square, P cube and so on and you can find out the P power n, and that element is going to be the N step transition probability. Using that you can find out the distribution. And since we don't know the value of P, where P lies between 0 to 1, it is -- I'm not going to discuss the computational aspect of finding out the distribution. This is left as an exercise and the final answer is provided.

The difference between the earlier example and this example and this example the state space is going to be a countably infinite. Therefore, the P is not going to be easy matrix, it is going to be a matrix with the many elements in it. Therefore, finding out the P square and the P power n is going to be a little complicated than the usual square matrix.

So hence, the conclusion is by knowing the initial probability vector and the one-step transition probability matrix or the state transition diagram, we can get the distribution of Xn for any n. There is a small mistake the running index for X n+1 value = Xn + Yn + 1, that is starting from 0, 1, 2. And similarly, the previous slide X n+1 = Xn + Yn + 1, and n is running from 0, 1, 2 and so on.



So in this lecture, we have discussed Chapman Kolmogorov equation and also we have discussed the N step transition probability matrix. So the N step transition probability matrix can be computed from the one-step transition probability matrix with the power of that N, and also we have discussed four simple examples for explaining the Chapman Kolmogorov equation and the N step transition probability matrix.



For the lecture 1 and 2 we have used these three books for as a reference. The first one is J Medhi's "Stochastic Processes" book, the second one is Kishore Trivedi, "Probability and Statistics with the Reliability, Queuing and Computer Science Applications", the third one is Shelton Ross, "Introduction to Probability Models". So with this, I will complete the Lecture 2 of discrete time Markov chain. Thanks.