**Introduction to Probability Theory and Stochastic Processes**
**Prof. S. Dharmaraja**
**Department of Mathematics**
**Indian Institute of Technology, Delhi**

**Lecture – 23**

So, till now we have discuss the problems on common distributions of discrete type. We have discussed one problem in binomial distribution and other two problems in the Poisson distribution. Now, we are moving into the problems in common distributions of a continuous type random variables.

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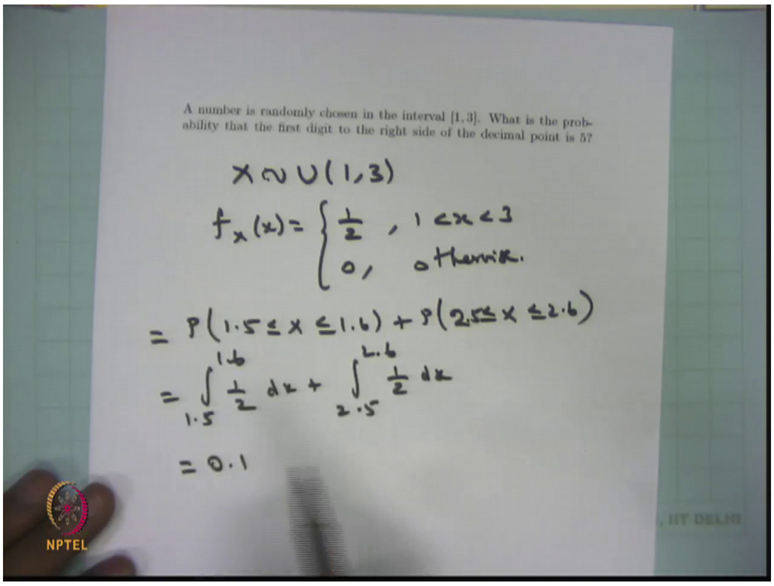


So, let us start with the first problem. a number is a randomly chosen in the interval 1 to 3. What is the probability that the first digit to the right side of the decimal point is 5. Note that whenever we choose a number randomly; that means, the possible sample points has a equiprobable; whenever we use a word a number is randomly chosen; that means, it is a real number.

The real number is a randomly chosen for example, in the interval 0 to 1 that is basically a random number generation. That means, a each possible real numbers can have a equiprobable. Whenever it has equiprobable or equilikely of that events; that means, the underlying distribution is uniform distribution. Since we are randomly choosing a real number therefore, it is going to be of the common distribution in particular it is of the uniform distribution of continuous type.

So, in this problem the real number is randomly chosen in the interval 1 to 3, the random number generation the default random number generation that is between the interval 0 to 1. But here in the interval 1 to 3 therefore, we can conclude x is a random variable which denotes the getting the random getting the real number between the interval 1 to 3 that is going to be a random variable.

So, since X is a random variable which denotes number obtaining in the interval 1 to 3 this follows uniform distribution between the interval 1 to 3 between the interval 1 to 3; that means, the probability density function of this random variable is; since it is a random number is randomly chosen; that means, it is equiprobable therefore, the density is going to be constant then only it is going to be equiprobable.

Since the interval is 1 to 3. So, the length of the interval is 2 therefore, the probability density function in the constant that is 1 by 2. Between x when x lies between 1 to 3 0 otherwise since and the random number is randomly chosen; that means, it is uniform the interval length is 2 therefore, it is 1 by 2 between the interval 1 to 3.

So, this is a probability density function of a uniform distribution between the interval 1 to 3. So, now, the question is the probability that first digit to the right of a decimal point is 5; that means, this is possible when x lies between 1.5 to 1.6 as well as the x lies between 2.5 to 2.6. The required property is probability of x lies between 1.5 to 1.6 plus probability of x lies between 2.5 to 2.6 that is same as.

Since it is a continuous type random variable integration between 1.5 to 1.6, the probability density function is 1 by 2 t x plus 2.5 to 2.6 probability density function is 1 by 2 d x. You simplify get the answer that is a point 1; that means, the probability that the first digit to the right side of the decimal 0.5 is 0.1.

The probability can be always a can be represented in the form of proportion also so; that means, you can go for proportion urban percentage; that means, it is a 10 percentage that it is going to be having first digit to the right of the decimal point is 5, like this we can create a many more problems with this setup.

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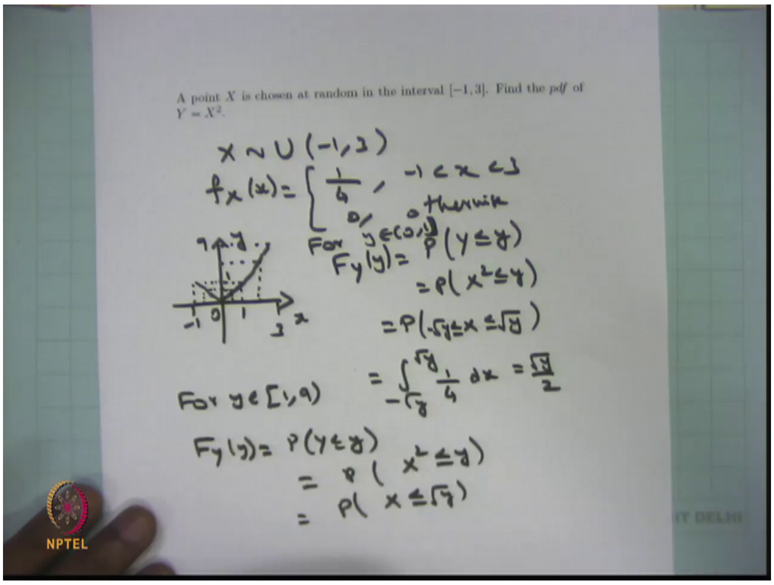


We will move into the second problem; in this problem a point X is chosen at random in the interval, find the probability density function of Y is equal to X square. That means, we can use the concept of the previous problem, that is a point X is chosen at random a point means it is a real number randomly chosen between the interval minus 1 to 3, whenever it is random insets a uniform distribution. Therefore, we can conclude X follows uniform distribution between the interval minus 1 to 3 from the previous problem in the same way one can conclude X follows. Continuous uniform distribution between the interval minus 1 to 3 therefore, the probability density function is going to be 1 divided by length of the interval that is 4 and x lies between minus 1 to 3 0 otherwise.

We will keep the probability density function of x because of the question is find the probability density function of y is equal to x square. That means, with the help of the distribution of x here to find out the probability density function of y. There are two ways you can use the CDF method to get the CDF of y then by differentiating, you can get the probability density function of y, or you can use the continuous type random variable result finding the probability density function, if it satisfies the function is a monotonic and differentiable and get the probability density function using the probability density function of x.

But here the interval is minus when x takes a value, minus 1 to 3 y is equal to x square; that means, you just to draw the parabola. So, between minus 1 to 0 it is a decreasing, 0 to 3 it is increasing therefore, you cannot apply the theorem of continuous type random variable satisfying the monotonic function and so on. You cannot apply therefore, we will go for finding the probability density function of y by finding the CDF of y first, then we go for the probability density function of y.

That is by seeing the diagram you can make out y is equal to x square where x takes a value minus 1 to 3 therefore, y is going to take the value from 0 to 9, the y going to take the value from 0 to 9. Therefore, the CDF is going to be 0 and 1 here to find out what is the CDF between the interval 0 to 9 from 9 onwards at the CDF is going to be 1.

Because of x takes a value from minus 1 to 3 and y is equal to x square therefore, the CDF of y till 0 it is 0 from 9 onwards it is going to be 1. So, the question is a what is a CDF between 0 to 9. One more observation you see the diagram when x takes a value from minus 1 to 1 y takes a value 0 to 1 whereas x takes a value from 1 to 3 y takes a value from 1 to 9 therefore, the CDF is going to be different form from 0 to 1 for the random variable y, whereas 1 to 9 the CDF is going to be of the different from.

Let us first write how the CDF going to be calculate. The CDF of the random variable y that is nothing, but the probability of y takes a value less than are equal to y, that is same as probability of x square less than are equal to y. That is same as probability of x lies between square root of minus square root of y to plus square root of y. Here I making the assumption y is greater than 0, as I said the CDF till 0 that is going to be 0.

So, our calculation goes for the CDF between the interval 0 to 9. This is same as since x is a uniformly distributed between the interval minus 1 to 3, this is same as minus square root of y to plus square root of y and the probability density function of the x is 1 by 4 d x. You can simplify and you can get the answers square root of y by 2.

Now, will go for what is the CDF between. So, this is a CDF for y lies between 0 to 1 0 to open interval 1. I am splitting the interval 0 to 9 in the form of 0 to 1, then I will go for 1 to 9. So, when y belonging to 1 to 9, now the CDF of y because only 0 to 1 you have a the probability of x lies between minus square root of y to plus square root of y because so, for every point you have a 2 inverse images. Therefore, you will get a minus square root of y to plus square root of y when y lies between 0 to 1 fine.

So, now we are going for a y belonging to 1 to 9. So, the CDF of y that is same as probability of y is less than are equal to y, that is same as now for every y when y is lies between 1 to 9, you have only 1 inverse not the 2 inverse probability of x square is less than are equal to y, that is same as the probability of x is less than or equal to square root of y.

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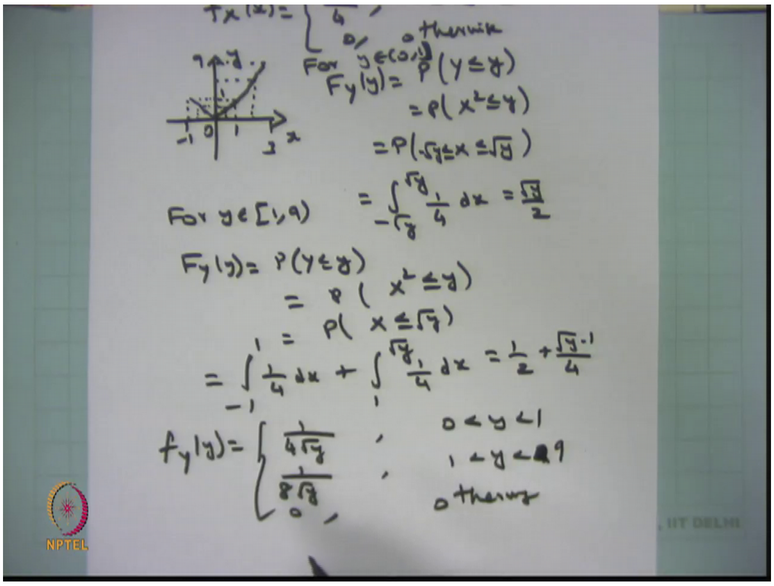


That is same as the probability of x is less than or equal to square root of y when y is lies between 1 to 9; that means, it is a from minus infinity to minus 1 the probability density function of x 0. Therefore, it is minus 1 to 1 the probability density function is 1 by 4 plus from 1 to square root of y, the probability density function is 1 by 4 d x.

You see the difference when y lies between 0 to 1 the probability of x lies between x square less than or equal to y is same as x lies between minus square root of y 2 plus square root of y because it has a 2 inverses between the interval 0 to 1. And y takes a value 0 to 1, x has a 2 inverse images. Therefore, it is a minus square root of y to plus square root of y. When y takes a value 1 to 9, the probability of x square is less than or equal to y that is same as probability of x is less than or equal to square root of y that is same as a minus infinity to minus 1 the probability density function is 0 plus minus 1 to 1 then 1 to square root of y.

So, you do the simplification, you can get the answer that is 1 by 2 plus square root of y minus 1 divided by 4. Therefore, one can write the CDF of a the random variable that is combining minus infinity to 0 that is 0, from 0 to 1 that is square root of y by 2 between the interval 1 to 9 it is 1 by 2 plus square root of y minus 1 by 4, from 9 onwards the CDF is going to be 1. By differentiating the CDF of y you can get the probability density function of y that is the question. The question is a find the probability density function of y is equal to x square.

If the question is find the distribution of y, you can leave it with the c d f. Since the question is find the probability density function, you have to differentiate the CDF to get the probability density function of y. By differentiating the CDF you will get when y is lies between 0 to 1, you will get 1 divided by 4 times square of y by differentiating between the interval 0 to 1, you will get 1 divided by 4 square root of y.

When y is lies between 1 to 9, you will get a 1 divided by 8 times square root of y otherwise 0. So, this is a probability density function, you can verify also by integrating 0 to 1 of a 1 divided by 4 times square root of y plus integration between 1 to 9. 1 divided by 8 times square root of y you will get the answer 1 therefore, this is the probability density function of y. It is a very important problem, because of this interval minus 1 to 3, the CDF is changing between 0 to 1 and 1 to 9.

Suppose this problem would have been a point x is chosen at random in the interval minus 1 to 1, in that case the interval of y between 0 to 1 you have to inverse images you can apply the remarks of the theorem, which I have send it in the 1 dimensional random variable. You can get the probability density function of a minus 1 to 0, 1 density then 0 to 1 another density you can sum it of you can get the probability density function of y or you can apply the CDF method to get the answer.

If the question is at the point is chosen at random in the interval 1 to 3, then the interval is going to be has only 1 inverse then also the problem is going to be the different form. So, in these because of minus 1 to 3, in some portion it has a 2 inverse images in some portion it has only 1 inverse therefore, the you are partition in the interval in to piece by piece finding the CDF separately, then you are combining everything.

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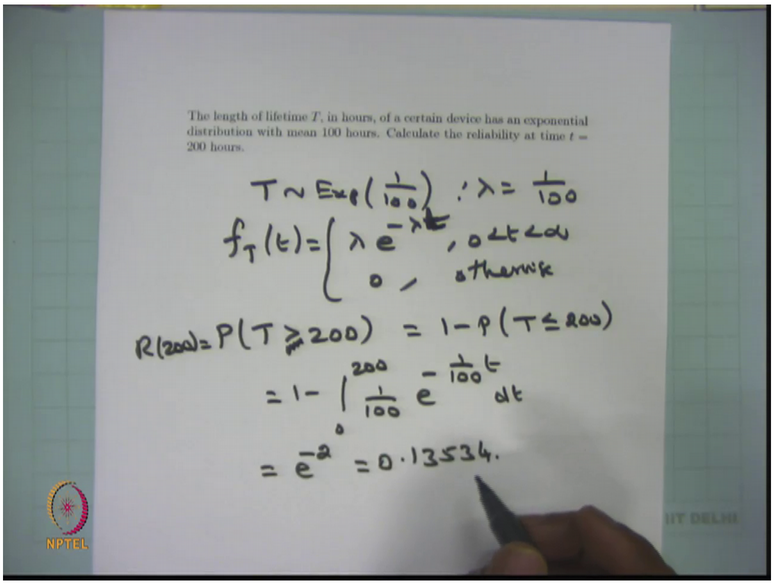


We will moving to the next problem, the problem is a length of lifetime that is denoted by capital T, unity is in hours of a certain device as a exponential distribution with the mean 100 hours. Calculate the reliability at time T is equal to 200 hours.

This is a very difficult problem in reliability analysis most of the time we make the assumption the lifetime of a electrical component follows exponential distribution. In this problem also they made already the assumption that the lifetime are the time in which this a electrical certain device is going to work, that follows exponential distribution, why it is exponential? Because the exponential distribution range is from 0 to infinity and most of the time a you never know when it is going to pay. So, it is always a time that is great now equal to 0 therefore, a the lifetime most of the time we make the assumption, that follows exponential distribution.

Based on the statistical history, they find the parameter value or estimate the parameter values. So, here already the parameter values given that is mean 100 hours; that means, the lifetime that is capital T that is a random variable that follows a exponential distribution with the parameter, we always a write a parameter lambda in that case the mean is going to be 1 by lambda. So, here the mean is applied that is 100 hours therefore, the parameter is going to be 1 divided by 100, because the unit of the random variable T that is in hours, and the parameter is 100 hours mean and the parameter relationship is a reciprocal. Therefore, that is exponentially distributed with a parameter 1 divided by 100 therefore, you can immediately write down what is the probability density function of a this, that is a it is lambda times e power minus lambda t, where t is lies between 0 to infinity 0 otherwise.

In this problem the lambda is 1 divided by 100 by substituting and lambda is equal to 1 by 100, you will get the probability density function of random variable capital T in this problem. So, the question is calculate the reliability at time t is equal to 200 hours; that means, a you should know what is the definition of reliability. Reliability is nothing, but the probability that the system is working till this time or equivalent of saying the probability that the component missing after this time that is called the reliability. So, at least it works this which time; that means, a here the lifetime capital t is this much time working that is a life.

So, therefore, the reliability at a time t is equal to 200 that is same as the in notation it is a reliability at 200, that is same as the probability that capital T is going to have a life more than 200 hours. The reliability at 200 that is same as probability that the lifetime of this device is going to have more than or equal to 200, that is same as sorry not more than or equal to that is going to be greater than 200. It is a continuous type random variable. So, whether you right greater than or equal to does not matter, but the reliability is define probability that a the lifetime is going to be more than that time.

So, this is same as 1 minus probability that T is less than or equal to 200 either you find out the probability of a greater than 200; that means, a you integrate the between the interval 200 to infinity the probability density function or you find out probability of T is less than or equal to 200 by integrating a 0 to 200 the probability density function then substitute here both are all the same. So, this is same as 1 minus 0 to 200, the probability density function e power minus the probability lambda times t dt. You simplify he will get the e power minus 2 that is going to be the answer. So, numerically it is going to be 0.13534. So, this is the reliability of a device at time t is equal to 200, like this you can create many more problems with this setup and you can solve.

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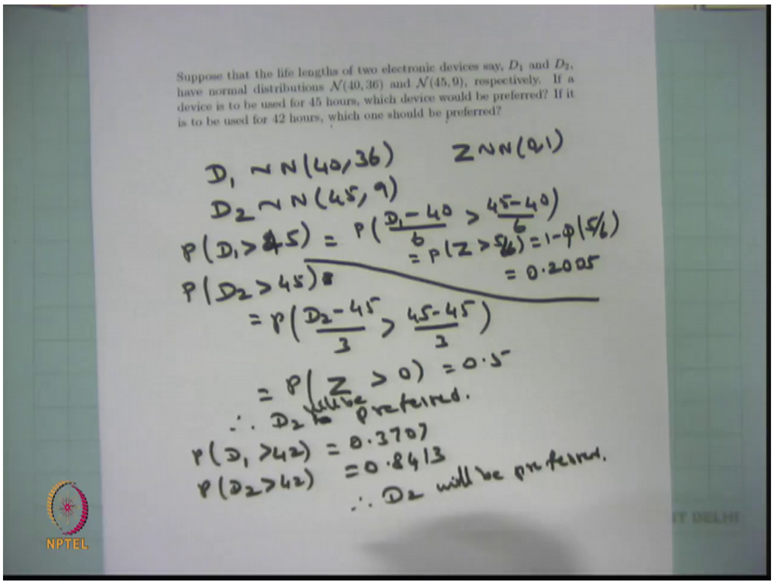


Now, will move into the next problem suppose that the life lengths of two electronic devices say D 1 D 1 and D 2, have normal distributions. I am using the notation capital N means it is a normal distribution, the first parameter 40 is a mean and the second parameter 36 that is variance of the first normal distributed random variable. The second one that is also normal distributed random variable with the mean 45 and variance 9. If a device is to be used for 45 hours, which devices would be preferred; if it use to be used for 42 hours, which want to be preferred.

So, there are two questions, in the earlier problem we made a lifetime follows a exponential distribution. Here we made a life time follows a normal distribution with the mean 40 and 45 respectively, variance 36 and 9 respectively for two different variances. So, these are all the assumptions we make the assumption the lifetime follows normal distribution and so on and we get the results. So, what we can do we can write the problem a that is a D 1 is a random variable that is a life time length of first device, which follows a normal distribution with the mean 40 and the variance 36. Similarly the D 2 is a second random variable denotes a life length of the second electronic a device which is normal distribution with the mean 45 and variance nine.

So, the question is if the device is to be used for 45 hours which devices would be preferred. So, what you will do? We will try to find out what is the probability of if D 1 is going to be greater than 45, what is a value and similarly you will find out what is the probability of a D 2 is going to be greater than 45. So, whichever has the more probability, you will go for preparing that if a device is used for 45 hours. So, whichever has the more probability of working more than 45 hours we will prefer that electronic device ok. Let us find out the probability of D 1 greater than 45 that is same as probability of whenever you have a problem in the normal distribution, first you have to convert into the standard normal distribution then use the table 2 get the numerical value of the probabilities. So, D 1 minus mean is 40 divided by standard deviation that is 6. Here also you have to do the same thing 45 minus 40 divided by 6 which is same as.

Now, the D 1 is normal distributed normal distribution minus their mean divided by the standard deviation, that becomes a standard normal. So, I use a notation Z for standard normal Z is a standard normal distributed; that means, a mean is 0 variance is 1. So, probability of Z is greater than 45 minus 40. So, it is 5. So, it is 5 basics. So, you find out the probability of Z is greater than 5 by 6 by using the table that is a 1 minus psi of 5 by 6. I have already defined what is the meaning of psi of x; that means, a the integration from minus infinity to x, the probability density function of standard normal distribution that is going to be psi of x. So, you get the value from the table. So, this value is going to be 0.2005.

Now, we will compute the probability of a t 2 D 2 greater than 45. So, that is going to be the same way that is a probability of a D 2 minus there mean. So, the mean of second device is 45 and variance is 9 therefore, the standard deviation is 3 greater than 45 minus 45 divided by 3. D 2 minus 45 by 3 that becomes a Z standard normal distribution greater than 45 minus 45 that is 0. Probability of a standard normal distribution greater than 0, you know the date is symmetric about Z is equal to 0 therefore, the whole area is divided into 50 50 percentages the whole area is 1

So, the probability of Z is greater than 0 that is a from 0 to infinity, that is going to be 0.5. Now we got the result probability of a D 1 greater than 45, that is 0.2005, probability of D 2 greater than 45 is 0.5; that means, a the second device the probability of a second device can work for more than 45 hours is more than that of a first complement therefore, D 2 is preferred. Second question if it is used for 42 hours which one should be preferred. Now, we will go for the similar exercise for probability of D 1 greater than 42, if you do the simplification for the problem you may get the answer 0.3707, similarly probability of D 2 greater than 42 that is going to be 0.8413.

Again the probability of D 2 greater than 42 is more compare into the that of D 1. So, again D 2 will be preferred D 2 will be preferred in the first case as well as in the second case. You see the problem both are normal distributed which as the different mean different variance therefore, you cannot if suppose a the means are same and the variance are going to be different, then you can conclude something else. Suppose the variance are same the means are different then also you can have the different issue. But here the means as well as the variance are different therefore, unless otherwise you compute the probability you cannot conclude.

So, we are concluding a which one is better by getting the probabilities of a both the scenarios. So, in this lecture we have a discuss the six problems, three from a discrete type and three from the continuous type and some more problems you can see it from the assignment sheet or the problem sheet. And when you solve the different problem then you will come to know how to use the different standard distributions and getting the answer.