Introduction to Probability Theory and Stochastic Processes
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Lecture – 09

We are in week 2 and in the last class we have discussed the definition of a Random Variable and then we have discussed the cumulative distribution function of the random variable. And we have given the one example for the CDF of the random variable also we have given 5 different CDFs in graphical way.

Now, we are going to discuss the types of random variable based on the CDF of the random variable. If you recall that 5 different diagrams which we have made it for the CDF out of those 5 different examples; we can classify those 5 examples into 3 types of random variable. First, we discuss the discrete type random variable and second we discuss continuous type random variable and third we discuss mixed type random variable. So, any random variable can be classified into any one of these 3 types of random variable; namely discrete type random variable, continuous type random variable or mixed type random variable.

So, in this class I am going to explain the types of random variable with the form of first the definition of the discrete type random variable, then continuous type random variable, then mixed type random variable. And followed by the definition I am going to give one or two examples for each type; so that is a plan for today's lecture. So, let me start with the first type; so, it is called types of the random variable.

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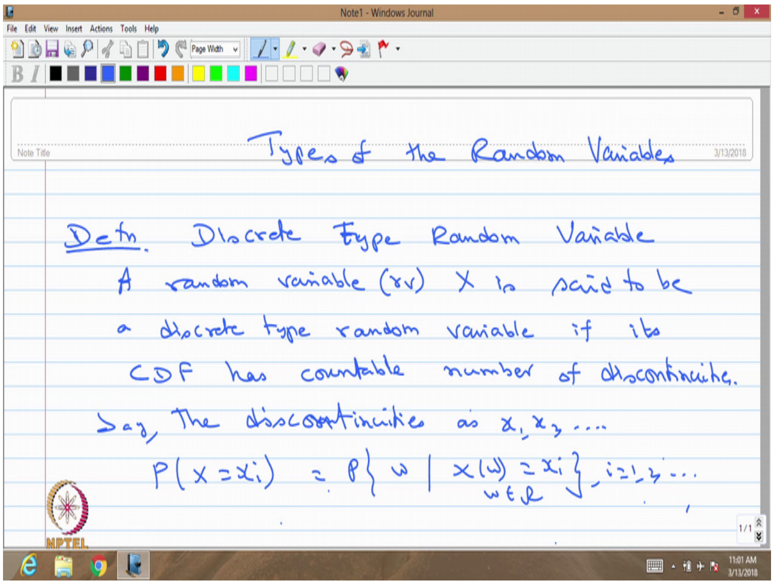

Types of the random variables; in the first definition is discrete type random variable; when we say the random variable is discrete type random variable. A random variable in short we will be keep writing rv; a random variable X is said to be a discrete type random variable if its CDF cumulative distribution function has countable number of discontinuities.

The countable number of discontinuities means the discontinuities are jumps can be finite or countable infinite; that means, a random variable is X is said be a discrete type random variable; if its CDF has a countable number of a discontinuities or it has a countable number of jumps. Say the discontinuities are jumps as x 1, x 2 and so, on initially I am writing countable infinite it could be finite also.

Then one can define the probability of X takes a value xi's that is nothing, but P of collection of w such that X of w gives the value x where w is belonging to omega. So, this is for i is equal to 1, 2 and so, on; you have a probability space omega F P; P is a probability measure. So, the probability of X equal to small x that is nothing, but collection of possible outcomes gives the probability in which the X of w gives the value X. So, you collect those possible outcomes, which gives the values xi under the operation capital X; so, that is nothing, but the event.

So, find out the probability of event that event is going nothing, but the probability of X equal to X.

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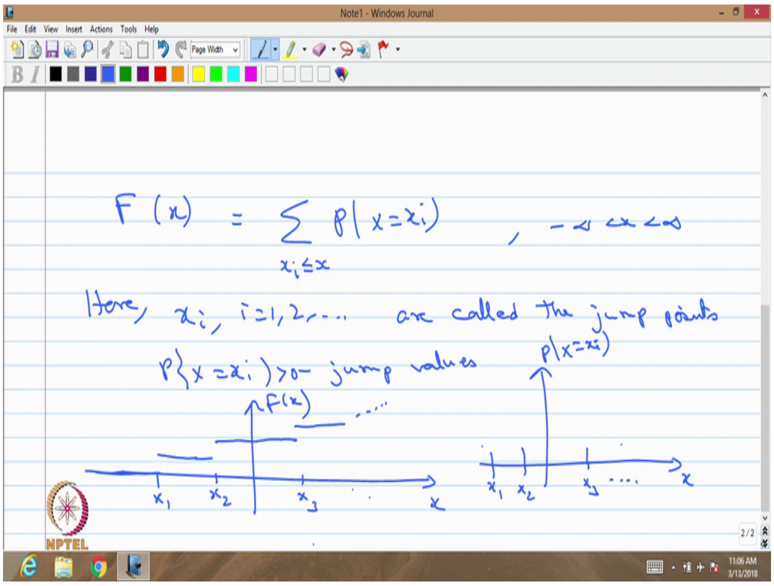

Therefore when you write the CDF of the random variable x; I can go for suffix x or suffix I can go with the suffix x; that means, I am talking about the CDF of the random variable x that is nothing, but collection of all the P of X takes a value xi's such that all the xi's has to be less than or equal to small x.

So, here x is a between minus infinity to infinity. So, whenever you have a discrete type random variable the CDF is nothing, but collection of adding a P of X equal to xi's where xi's is less than or equal to small x. Here the xi's where i is equal to 1 2 and so, on; it could be finite or it could be countably infinite or called the jump points why it is called the jump points? The CDF of the, this random variable has the jump only at this points and the probability of X takes the value x i that is nothing, but the jump values.

So, this jump values are strictly greater than 0. So, wherever the X equal to it X equal to xi which the P is equal to greater than 0 P of X equal to xi is greater than 0. So, these values are called a jump values at those jump values the CDF has the jump. And wherever there is a jump and those points are called the jump points that means, the P is equal to P of i; X equal to x i for different X it could be either 0 or greater than 0; if it is greater than 0 then those xi's are called jump points. And the P of X equal to xi in which it is greater than 0 and those are called the jump values.

Therefore, I can draw the CDF of in general CDF of a discrete type random variable as suppose x 1 is here then the CDF is 0 till x 1 at the point x 1 there is a jump. So, that height is P of X equal to x 1 that is a jump value and this x 1 is jump point till the next point x 2 the value same and the CDF has a jump at the point x 2 suppose at x 2; the next jump is some value. So, P of X equal to x 2 there is a jump.

So, this difference height that is a jump value till x 3 it has the same value then there is a next jump like that it may keep going. And you know that it is a CDF therefore, it always start from 0 it will end 1. And this CDF has only jumps that is very important then only it is going to be call it as a discrete type random variable and other points the values is 0.

I can equivalently draw the another diagram for P of X equal to xi ; that means, at point x 1 it has some height and x 2 it has another height and x 3 it has another height; suppose this height need not be same, it can be a different values. So, this heights are nothing, but the jump values and these are all the jump points like that I may have many more. So, whenever you have a CDF of some random variable; which has only jumps the jumps could be finite or countably infinite then that random variable is called it as a discrete type random variable.

The way I have written P is equal to P of X equal to xi which has the jump values and jump points; I can create another I can define a another function that is called probability mass function.

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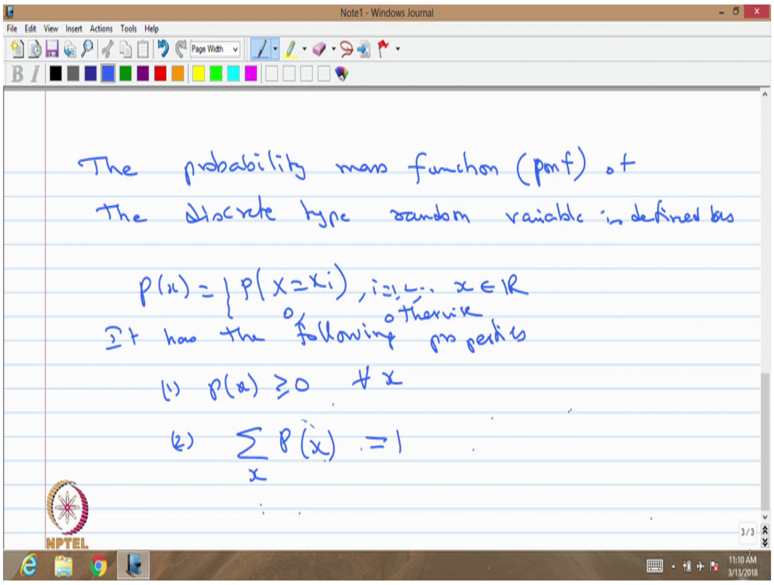

The probability mass function in short it is pmf the probability mass function of the discrete type random variable; as the probability mass function of a discrete type random variable is defined as P of X is nothing, but the probability of X takes a value x i this P of X is called the probability mass function. So, this is defined for all x belonging to R real line it has it has the following properties.

The first property; since we made P of X is probability of x takes the value xi probability of X equal to x i. So, wherever there is a probability of X equal to xi it is this much otherwise it is 0; this is the way the of X is defined it has the property the P of X is always going to be greater than or equal to 0 for all X; that means, wherever there is a jump value that value is going to be strictly greater than 0 wherever there is no jump; that means, the P of X is going to be 0.

The second property if you add all the values of a P of X equal to x i you will get P of X summation will be 1. Always the probability mass function satisfies these 2 properties, it is greater than or equal to 0 and the summation over x its going to be 1. So, if you have a discrete type random variable; it has a probability mass function satisfies these 2 conditions or these 2 properties. If any real valued function satisfying these 2 properties; then one can say this is the probability mass function of some discrete type random variable. So, any probability mass any discrete type random variable has probability mass function satisfying these 2 properties or any real valued function satisfying these 2 properties is the probability mass function of some discrete type random variable.

I will go for one easy example which we have discussed earlier in the form of CDF recall.

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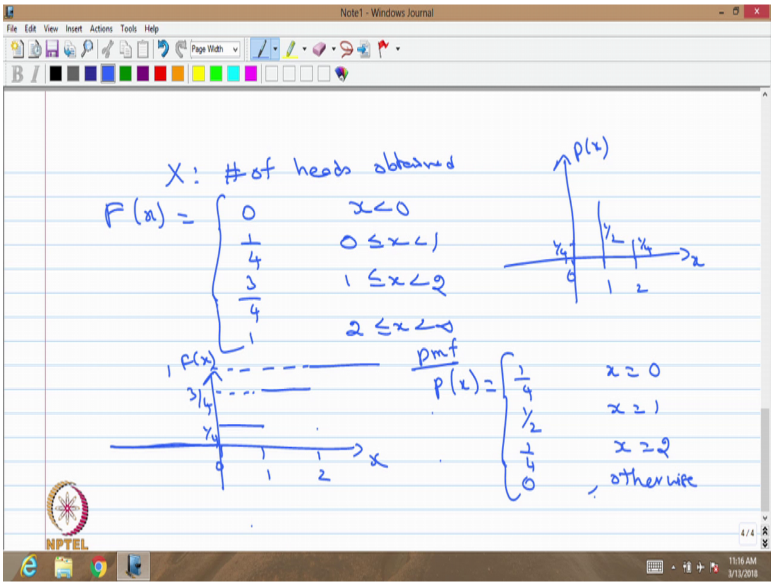

There is a one problem in which the random variable X is number of heads obtained for the random experiment of tossing 2 unbiased coins, when you toss a 2 unbiased coin. And if you define the random variable X is a number of heads obtained then the F of x is 0 till x is less than 0 it is 1 by 4 from 0 to 1; 3 by 4 it is a from 1 to 2. And the values is going to be 1 from 2 to infinity; so, this is the example we have discussed in the CDF for the random variable.

Now, we are discussing the same example if you draw the CDF of these this random variable CDF capital F of x. So, till 0; it is 0 0 there is a jump of 1 by 4 at the point 1 there is a next jump it is 3 by 4; the value at the point 1 is 3 by 4. So, the jump values 3 by 4 minus 1 by 4. So, 1 by 2 at the point 2 there is another jump of 1 by 4; so, it will be 1 from 2 onwards therefore, the P of X is at x is equal to 0 it is 1 by 4 at x is equal to 1; 1

by 2 at x is equal to 2 it is 1 by 4 and all other point it is 0 all other points. So, I write otherwise say when we write otherwise; that means, the probability mass function value is 0 at sorry it is 1 by 4 at 0; 1 by 2 at 1 and 1 by 4 at 2 and all other points it is 0. So, I can draw the probability mass function also.

So, at 0 there is a height of a 1 by 4 and at x is equal to 1 at height of a 1 by 2 and at x is equal to 2 there is a another height 1 by 4 and all other points it is 0. And if you add all the jumps the jump values 1 by 4 plus 1 by 2 plus 1 by 4 that is 1; that is the second property and all the values are greater than or equal to 0; that means, at only these 3 jump points it has the jump values and all other place the values are 0. Therefore, this is the probability mass function of the discrete type random variable; why it is called a discrete? Because you see the CDF the CDF has only jumps and the jump points are 0 1 and 2 and the jump values are 1 by 4, 1 by 2 and 1 by 2 sorry 1 by 4.

Since the CDF has discontinuities therefore, this random variable that is number of heads obtained this random variable capital X that is a discrete type random variable. And the probability mass function of this random variable x is P of X that is 1 by 4 at x equal to 0 1 by 2 at x equal to 1 and x equal to 2 1 by 4 otherwise it is 0 since it has the 3 jump points and 3 jump values; therefore, I am just writing the probability mass function this way and 0 otherwise. There is a possibility in general you may have a CDF as a finite number of jumps or countable infinite number of jumps. And one can find out what is a probability mass function of that discrete type random variable also.

So, this is the very easiest example in which we can represent the CDF first; by seeing the CDF you can conclude it is a discrete type random variable. And from the CDF you can get the probability mass function by subtracting because the CDF is nothing, but the summation of probability mass till that point. So, from the CDF one can get the probability mass function or from the probability mass function one can get the CDF of a discrete type random variable.

Now, we will move in to the second type of a random variable that is a continuous type random variable.

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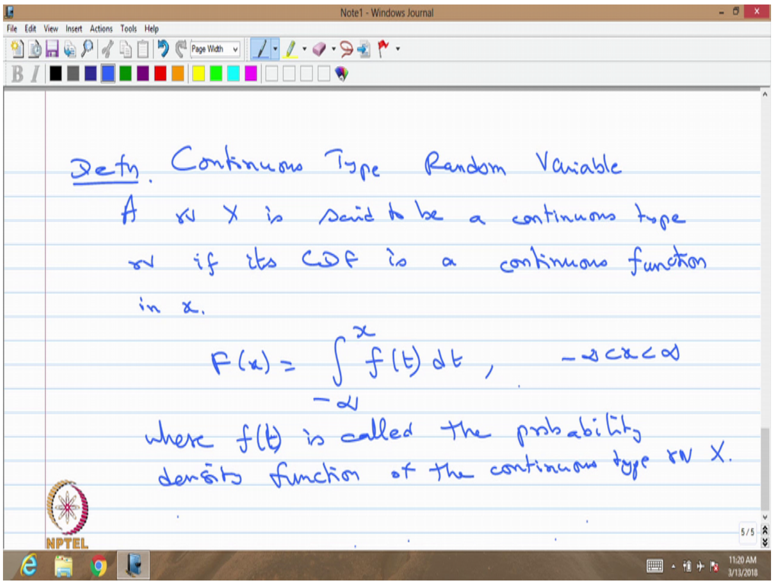

Continuous type random variable when we say the given random variable is of continuous type. A random variable X is said to be a continuous type continuous type random variable; if its CDF is the continuous function in x.

That means, whenever the CDF of random variable is a continuous function in x in the whole range from minus infinity to plus infinity. That means, there is no discontinuity in the CDF of the random variable; then you can conclude it is a continuous type. For a discrete type random variable; it has a finite or countably infinite discontinuities, but for continuous type random variable, the CDF is a continuous function in the whole range from minus infinity to infinity. That means, if you draw the CDF without lifting pen or pencil you draw the CDF then that is a continuous type random variable CDF.

Since it is a continuous function one can write the continuous function left hand side is the continuous function in the form of minus infinity to some x; some integrant dt this is possible whenever the function is a continuous function in the whole interval minus infinity to infinity. Here x is from minus infinity to plus infinity because of the CDF is the continuous function where the small f of t till now we are using the capital f of capital F; now I am start using small f, where small f of t that is called the probability density function of the continuous type random variable capital X; this is possible whenever the CDF is a continuous function. So, whenever the CDF is a continuous function we call that random variable as a continuous type random variable.

The integrant in this equation that integrant is called probability density function; the way the probability mass function satisfies a few properties the 2 properties. Similarly one can identify what are all the properties going to be satisfied by the this probability density function of the discrete type random variable.

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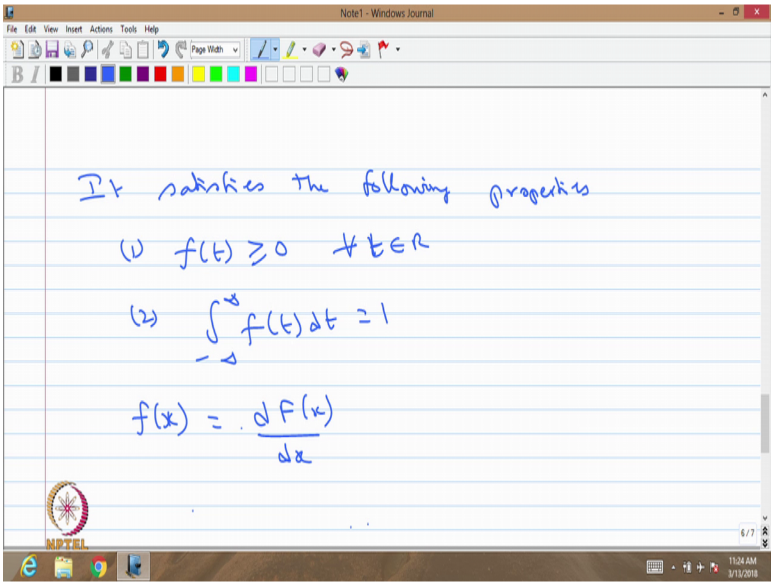

So, it satisfies it satisfies the following properties ; that means, the probability density function satisfies the following properties you know that the CDF values is lies between 0 to 1; it start from 0 it will end up 1 and monotonically increasing continuous or right continuous function.

Therefore, the integrant always greater than or equal to 0 for all t; not only that since you are writing capital F of x is the capital F of x as the minus infinity to x of integration f of t dt. So, if you go for limit x tends to infinity you know that limit x tends to infinity of capital F of x is 1 from there you can get the second property that is minus infinity to infinity f of t dt that is 1; that means, a real valued function satisfying these 2 properties will be the probability density function of some continuous type random variable.

If you have a continuous type random variable whose CDF is the continuous function in the whole real line minus infinity to infinity. Therefore, you will get the probability density function; so, we can relate suppose you know the CDF you can get the probability density function I am writing f of x either f of t or f of x does not matter by differentiating the CDF with respect to x, you will get the probability density function. From the CDF you can get the probability density function from the probability density function you can get the CDF.

We are going to do some example through that we will explain in detailed also; I am going for 2 examples through that I am going to explain this continuous type random variable.

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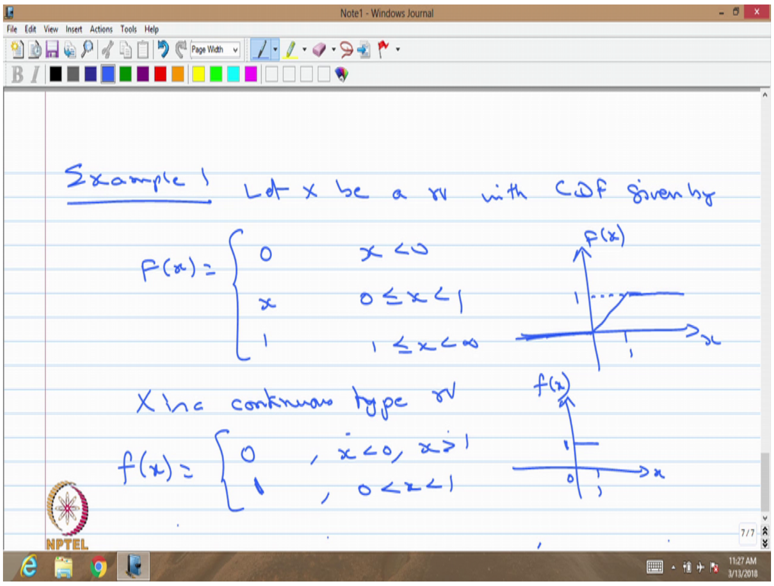

The first example let us consider the example 1; let X be a random variable with the the CDF given by capital F x that takes a value 0 till 0 and it takes a value x between 0 to 1 and it takes a value 1 from 1 onwards you can draw the CDF it is a rough diagram. CDF is 0 till 0 from 0 onwards it is x and at the point 1; onwards it becomes 1. By seeing the CDF you can conclude this is a continuous function from minus infinity to plus infinity continuous function in x; therefore, this is a continuous type random variable therefore, x is a continuous type random variable.

You can find out what is the probability density function of this continuous type random variable by differentiating the CDF. So, if you do the differentiation of CDF you will get 0 when x is less than 0. Similarly x is greater than 1 and if you differentiate x you will get 1 between the interval 0 to 1; that means, the probability density function of this continuous type random variable; the value is 1 between the interval 0 to 1.

And otherwise it is 0; that means, you can draw the this is the rough diagram the probability density function this is small f of x; the capital F of x is the CDF small f of x is the pdf. So, in short it is the pdf; so, the value is between 0 to 1; the height is 1 otherwise it is 0 therefore, no need to. So, the horizontal line 0 to 1, the height 1 that is the probability density function of these are continuous type random variable.

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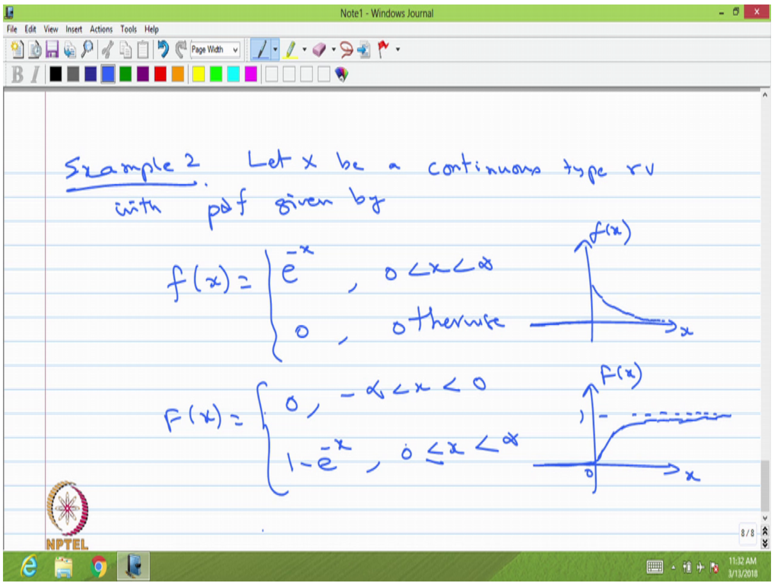

I will go for one more example; example 2 let X be a; let X be a continuous type continuous type random variable with pdf given by small f of x takes a value e power minus x between 0 to infinity; 0 otherwise whenever we say 0 otherwise or elsewhere from books they use a word elsewhere. That means, the probability density function which is positive value between the interval 0 to infinity is e power minus x and remaining intervals it is 0. So, here the remaining interval is minus infinity to infinity.

So, you can draw the probability density function. So, e power minus x; so, it takes a value it is going down asymptotically touches 0 at infinity and at 0. So, the f of x value is 0 till 0 from the probability density function, you can get the CDF by integrating minus infinity to till that point of probability density function.

So, since the probability density function is 0 from minus infinity to infinity; you can say it is 0 from minus infinity to 0. From 0 to infinity you have to integrate and find out; so, if you do the simple integration you will get 1 minus e power minus x meeting the interval 0 to infinity; you can include 0 also; so, you can make it 1 minute.

So, minus infinity till 0 and 0 onwards; it is 1 minus e power minus x; you can draw the CDF of this continuous type random variable. So, till 0 it is 0 and 0 to infinity it increases, it touches it touches 1 at infinity and this is the CDF of discrete sorry; this is the CDF of a continuous type random variable. So, from the CDF you can always get the probability density function of a continuous type random variable; from the probability density function one can get the CDF of the continuous type random variable.