Probability and Statistics
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Module No.#01
Lecture No. #13
Special Distributions-IV

In the previous lecture, we have introduced negative exponential distribution andwe gave its origin as the distribution of the waiting time for the first occurrence in aPoisson distribution.

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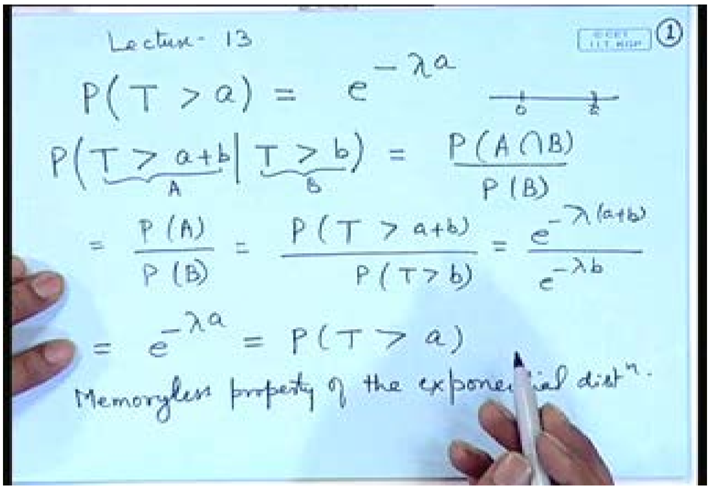

So, we considered probability of T greater than T, if T is having an exponential distribution. So, if we consider probability of, say T greater than a number a, then it is equal to e to the power minus lambda a.

Now, if we consider,so this means that the first occurrence has not taken place till time a.That means, in the interval 0 to a, it has not taken place. So, if we consider probability of, say T greater than saya plus b, given that T is greater than b. So, if we consider this aseventA, this as event B, then it is equal to probability of A intersection B divided by probability of B.Now,A intersection B,so here,A is a subset of B.Therefore, this becomes probability of A divided by probability of B, that is probability of T greater than a plus b divided by probability of T greater than B. So, by the formula that we have developed, this one will become equal to e to the power minus lambda a plus b divided by e to the power minus lambda b. So, after simplification, it becomese to the power minus lambda a, which is nothing, but the probability of T greater than a.

So, this phenomena represents that, if the event has not taken place till time b, then it will not take place till an additional time a, the probability of that is same as, that the event will take place after time a starting from time 0.That means,if we are considering the waiting time for certain thing,then the probability of waiting for an additional time is same irrespective of the starting point.

So, like geometric distribution, this is called the memoryless property of the exponential distribution.You can see thatexponential distribution is analogous to the geometric distribution of the discrete case.In the geometric distribution,we were considering Bernoullian trials and we were waiting for the first success or first occurrence. So, here also, it saysPoisson process and we are waiting for the first occurrence. So, in a sense,thisexponential distribution is a continuous analog of the geometric distribution.

So, if we consider, say occurrence to be failure of certain component in a mechanical system, and if we know, that the system has not failed till that time, then the probability of failure after a certain time is same irrespective of the starting point where we have taken. So, this type ofthing is interestingbecause many times we buy, say second hand items, second hand transistor, second handcomputer or calculator because if it is still working, then the probability of its failure after a certain time will remain the same. So, this is because of the memoryless property of the exponential distribution.

Let us also consider some modifications of this exponential distribution because here, we are starting from the time 0, but many times, for example, you consider certain item which we are purchasing from the market, then the market,whenyou are purchasing, then the shopkeeper gives you a guarantee for certain time like 1 year or 2 year etcetera.That means, the item is not supposed to fail before that because if it is failing, he is taking it back. So, it is again as if you are starting afresh.

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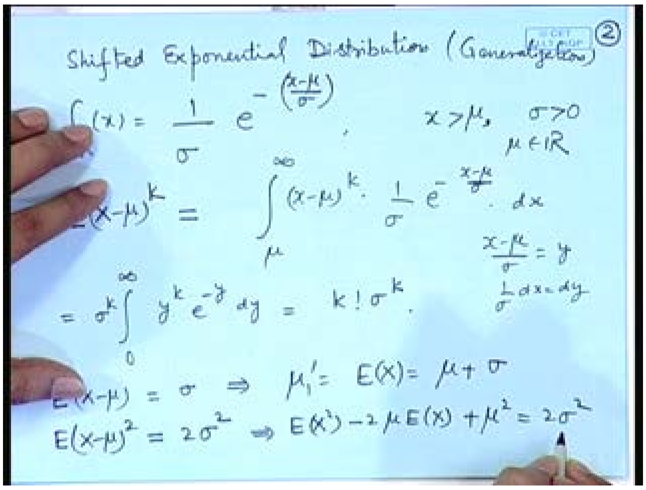

So, this is considered as a shifted exponential distribution by certain constant.Another thing we notice here is that, when we considered theform lambda e to the power minus lambda t, where lambda was a rate of the Poisson distribution, then the mean of this turnout to be 1 by lambda. Since, there is a single parameter,if we write, say 1 by lambda, say something like sigma, then this will become 1 by sigma e to the power minus t by sigma.

So, in the popular form with the shifted origin, we can consider the form of exponential distribution as, so we call it shifted exponential distribution.The density, we can write as 1 by sigma e to the power minus x minus mu by sigma. So, here, x is greater than mu and of course, sigma is positive.The advantage of this form is that,in place of time, if we are considering some otherrepresentation for x;x need not be time all the time,so if it is something else, then mu can be negative also and then also, this distribution remains valid. So, you can consider, this is a generalization of the original exponential distribution.

If we consider this particular form, then you can see it is easier to calculate the moments of x minus mu or x minus mu by sigma. So, let us consider expectation of x minus mu to the power k which is,of course not necessarily the central moment because we have not shown that mu is the mean.However, for convenience we are considering this. So, it is equal to x minus mu to the power k1 by sigma e to the power minusx minus mu by sigma dx from mu to infinity.

So, we may put x minus mu by sigma is equal to y, that is 1 by sigma dx is equal to dy.Then, this isequal to 0 to infinity sigma to the power k, y to the power k, e to the power minus y dy, thatisequal to gamma k plus 1 or k factorial sigma to the power k.

So, in particular, if we consider expectation of x minus mu to the power 1, this is equal to sigma.This means, mean of the shifted exponential distribution is mu plus sigma.If we had considered lambda there, then it would have been mu plus 1 by lambda. So, definitely we would be interested in the variance here. So, expectationof x minus mu square is equal to 2 sigma square, which gives us expectation of x square minus 2 mu expectation x plus mu square is equal to 2 sigma square.

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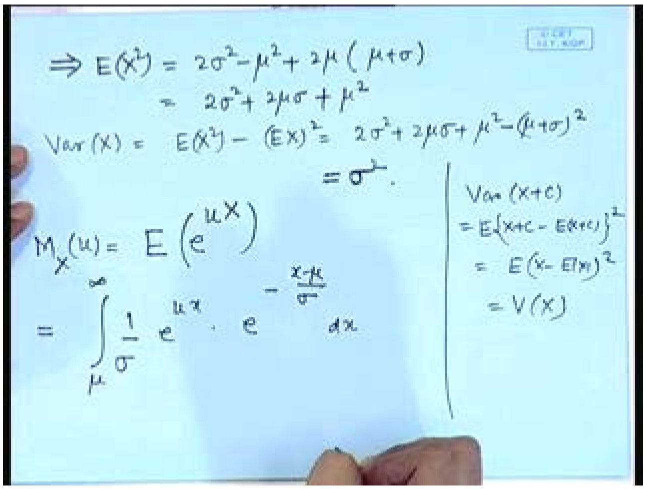

So, further simplification gives, expectation of x square is equal to 2 sigma square minus mu square plus 2 mu expectation x is mu plus sigma. So, this becomes equal to 2 sigma square plus 2 mu sigma minus mu square plus 2mu square. So, it becomes plus mu square. So, if you calculate the variance of x, that is equal to expectation x square minus expectation x wholesquare, that is equal to 2 sigma square plus 2 mu sigma plus mu square minus mu plus sigma whole square, which is simply sigma square.

So, variance has not changed by shifting.This is true because the fact that variance is independent of the shift in the origin.The variance of x plus c is expectation of x plus c minus expectation of x plus c whole square. So, this is equal to expectation of x minus expectation x whole square, which is the variance of x. So, the variance is unaffected by the change in the origin.We can consider here, the moment generating function of thisand also, the moment generating function of the original exponential distribution.

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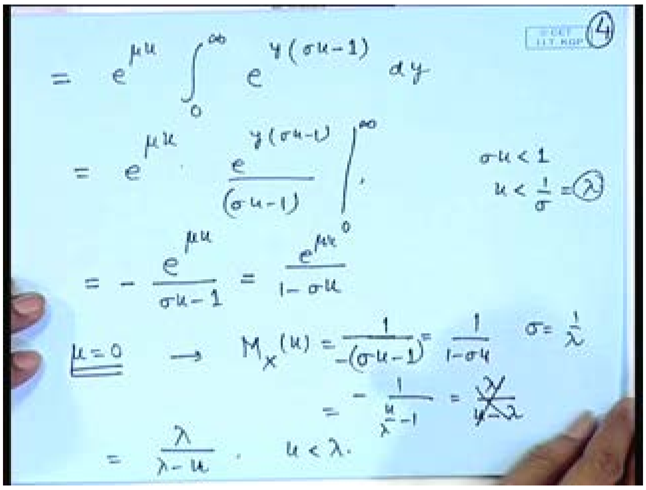

So, let us look at this.Moment generating function at a point u, it is equal to expectation of e to the power ux. So, this is equal to 1 by sigma e to the power u,xe to the power minus x minus mu by sigma dx mu to infinity.So, we can consider here, x minus mu by sigma is equal to y, the transformation that we made here. So, it is becoming 0 to infinity e to the power u and x becomes sigma y plus mu, e to the power minus y dy.

So, this can be simplified.It is equal to e to the power muu, e to the power y sigma u minus 1dy from 0 to infinity, e to the power muu and this becomes e to the power y sigma u minus 1 divided by sigma u minus 1, provided sigma u is less than 1 or u is less than 1 by sigma, which was basically lambda from 0 to infinity. So, at infinity, this becomes 0 because I am taking sigma u to be less than 1and at 0, this becomes 1. So, you are getting e to the power muu divided by sigma u minus 1.

In particular, if we had taken mu is equal to 0,then this M x u would have been 1 by sigma u minus 1.If we put sigma is equal to, say 1 by lambda, then this will become 1 by u by lambda minus 1, that is, equal to lambda by u minus lambda, where there is a minus sign here becausewhen we put 0, there is a minus sign here. So, this will become actually equal to e to the power muu 1 minus sigma u. So, this will becomewith a minus sign 1 by

1 minus sigma u. So, minus is here. So, this is equal to lambda by lambda minus u for u less than lambda.

So, the moment generating function of an exponential distribution, when we consider the original form, that is waiting time starting from the 0, then mgf is lambda by lambda minus mu or 1 by 1 minus sigma mu sigma u.If we consider the starting from mu, then it is e to the power muu by 1 minus sigma. So, here, we can see the relationship between these exponential distributions.

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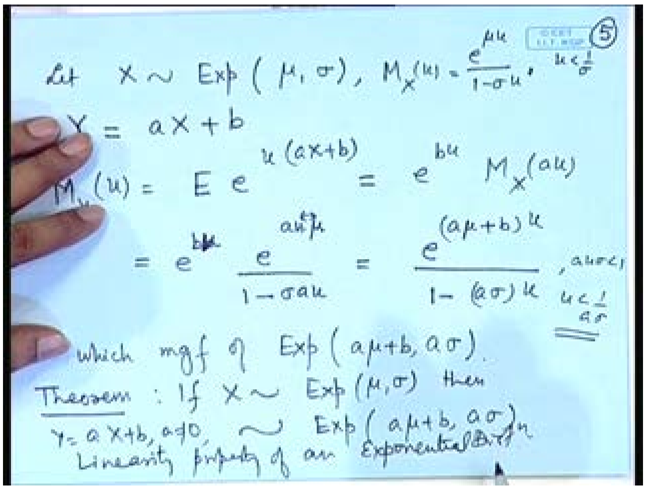

So, let X follow exponential distribution with parameters mu and sigma.Consider, the transformation Y is equal to, say ax plus b.Consider the moment generating function of this, then it is expectation of e to the power u into ax plus b.This is equal to e to the power bu and the moment generating function of x at the point au. So, what is the moment generating function of x at the point u, that we derived it as e to the power muu by 1 minus sigma u for u less than 1 by sigma.

So, if we make use of this term here, in place of u, we can put a mu here. So, it is equal to e to the power aumu divided by 1 minus sigma au. So, this we can adjust and write it asamu plus bu.See this term was buthere it is amu. So, amu, this actually u will interchange here. So, it becomes amu plus b and then, u divided by 1 minus a sigma mu.Compare thisthe term here,the moment generating function of x is e to the power muu by 1 minus sigma u. So, here, mu is replaced by amu plus b and sigma is replaced by a sigma.The form is the same.At the same time, since we have replaced au by u in theexpression for M x, this expression was valid for u sigma less than 1. So, here, au sigma must be less than 1.That means u is less than 1 by a sigma.

So, obviously, you can see by comparing this, that, this ismgf of,which is mgfof exponential distribution with parameter amu plus b and a sigma. So, this means, we have proved the following theorem.If X follows exponential mu sigma, then y is equal to ax plus b, where a, is not 0 follows exponential distributions with parameter amu plusb a sigma.This is basically linearity property of an exponential distribution.That means any linear function of an exponential distribution is again having exponential distribution. So, this form is more general.This is a 2 parameter exponential distribution and it is more useful.

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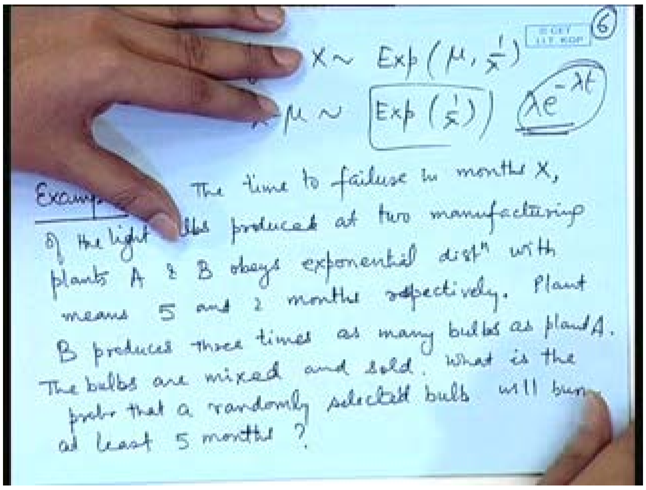

In particular, we will have, if X follows exponential, say mu 1 by lambda, then, x minus mu will follow exponential 1 by lambda.That means the standard form, that is, t lambda e to the power minus lambda t. So, both the forms are useful.Let us takeone example here for exponential distribution.The time to failure in months,so suppose, it is X of the light bulbs produced attwomanufacturing plants, say A and B obeys exponential distribution with means 5and 2 months respectively.Plant B produces 3 times as many bulbs as plant A. The bulbs are mixed,so they look indistinguishable to a naked eye and sold.What is the probability that a randomlyselected bulb will burn at least 5 months?

So, let us consider here,the distribution of X is exponential, but it is not the same for all the bulbs. For a certain proportion of bulbs, the distribution is exponential with mean 5.That means, the density will be 1 by 5 e to the power minus x by 5 and for certain bulbs, the mean time is 2 months. So, the density will be 1 by 2 e to the power minus x by 2.

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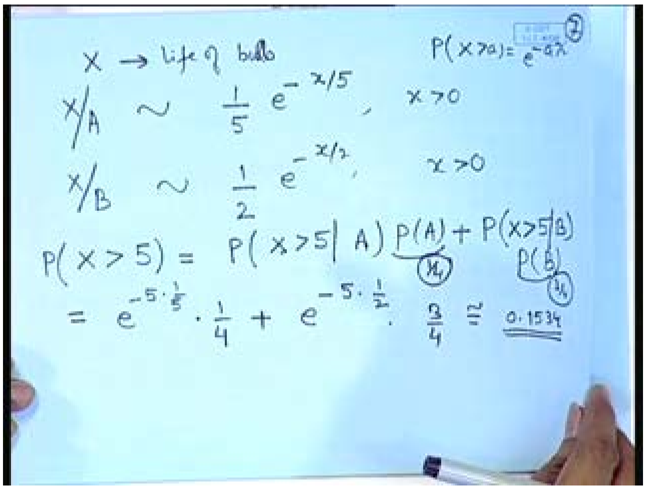

So, if you are X as the time or life of bulb,then X given, that it is produced by plant A, this has a density 1 by 5e to the power minus x by 5, for x greater than 0 andof course,0otherwise.If we are considering the bulb produced by plant B, then the density function is half e to the power minus x by 2 for x greater than 0.

We are interested in a randomly selected bulbs life to be more than 5months. So, here, we can apply the theorem of total probability. So, probability of X greater than 5, given that, it is produced by plant A into the probability of being produced at plant A plus probability of X greater than 5,given that it is produced at plant B into probability of plant B. So, this is equal to, now if we are considering probability of X greater than 5from this one, then consider the general formula probability of X greater than A is equal to e to the power minus a lambda.

So, here lambda is equal to 1 by 5and A is equal to 5. So, this becomes e to the power minus 5into 1 by 5 into probability of A.Now, what is probability of A?It is given that B produces 3 times as many bulbs as A. So, the probability of the bulb being selected from

A may be 1 by 4 and the probability of this may be 3 by 4.So, this is into 1 by 4 plus e to the power minus 5.Here, the parameter is 1 by 2,3 by 4. So, this can be simplified and it is 0.1535.

So, probability that a randomly selected bulb is working beyond 5 months is only 0.15approximately, which may look slightly surprising because you say that, there are 2 plants and from 1 of the plants, it is 2months and from another plant it is 5months. So, it should be nearly half, but it is not. So, because the plant B gives more supply compare to the plant A,and in the plant B, the probability of having more than 5months is much smaller because it is e to the power minus 5 by 2 and here, it is e to the power minus 1. So, this number is, obviously larger than this number.

Now, consider that we are not interested in a single occurrence or a single failure or a single happening in a Poisson process.In place of that we are considering, so consider a Poisson process.

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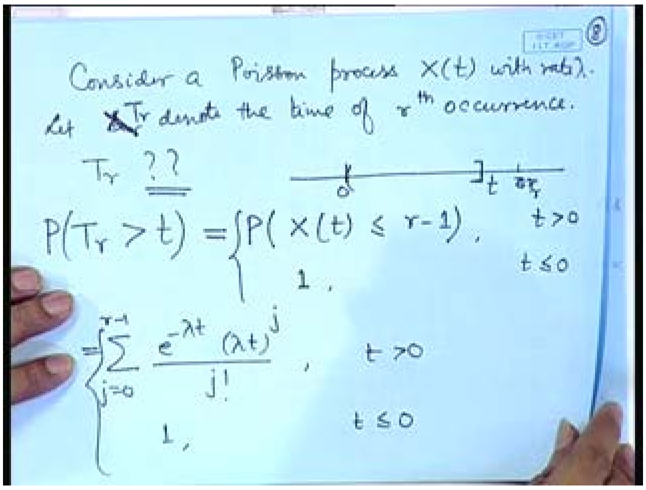

Let us denote Xt with rate lambda and let X r denote the time.Well in place of X, we will use notation, say T r because X is being used here.Let T r denote the time of rth occurrence.So, in place of the first occurrence, weare looking at a certain number of occurrences.Aswe haddiscussed yesterday also, like in the negative binomial distribution that a certain major event may occur as a consequence of certain smaller events.For example, if thousand smaller intensity earthquakes occur, then it may make the earth to crumble anda major earthquake may occur.

Asequence of a certain mishaps may close down the plant itself.A sequence ofso anysmaller kinds of events may lead to a major disaster. So, we may be interested in the waiting time for that,of course, considering the events to occur in a Poisson process.For example, a certain number of occurrences haveoccurred,say a certain number of people purchase certain tickets.Then, we may have to close down the window because the seats are full.

So, what is a distribution of T r?Once again, we can consider probability of T r greater than t. So, starting from a time 0, consider time small t.If we say that rth occurrence has not taken place till this time, suppose, this is the rth occurrence,that means, in the interval 0 to T less than or equal to r minus 1 occurrences will be there. So, this event is equivalent to probability of Xt less than or equal to r minus 1.Of course, here t is positive.It is equal to 1 for t less than or equal to 0.

Now, Xt is having a Poisson distribution with parameter lambda t. So, this isequal to e to the power minus lambda t lambda t to the power j by j factorial, summation j is equal to 0 to r minus 1.This is for t greater than 0 and it is 1 for t less than or equal to 0.Consequently, we can write down the cumulative distribution function of Tr as F of T r at the point t that is 1 minus probability of T r greater than t and this is equal to 0 for t less than or equal to 0.It is 1 minus summation j is equal to 0 to r minus 1 e to the power minus lambda t lambda t to the power j by j factorial for t greater than 0.

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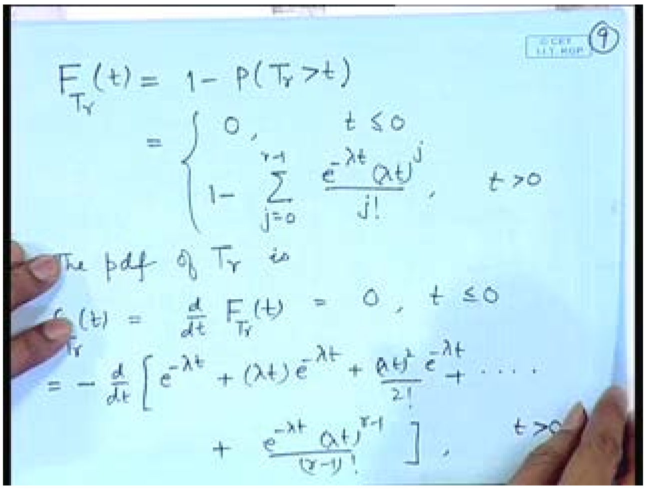

So, you can observe here, it is a time variable and this is an absolutely continuous function. So, the probability density function can be obtained by differentiation of this.Now, for T less than or equal to 0, it is 0 and in this particular portion, we are having sum of a series. So, we have to do term by term differentiation. So, this is equal to 0 for t less than or equal to 0 and in this portion, now, this is d by dt of e to the power minus lambda t plus lambda t into e to the power minus lambda t plus lambda t square e to the power minus lambda t by 2 factorial andso on, plus e to the power minuslambda t, lambda t to the power r minus 1 by r minus 1 factorial for t greater than 0.

So, here, you observe that when we differentiate hereone term is there, but there after each term is a product oftwo terms involve in t. So, when we differentiate, we have to do by applying the formula of derivative of a product.So, if we expand the terms, the derivative of the first term will give us minus lambda e to the power minus lambda t and there is a minus sign here.Consequently, we will get it as lambda e to the power minus lambda t.

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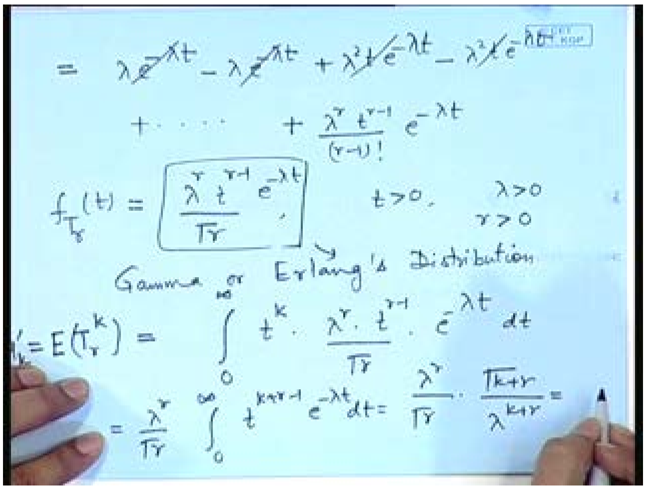

If we look at the second term and we look at the derivative of this, we will get lambda multiplied by e to the power minus lambda t and there is a minus sign here. So, we will get here, minus lambda e to the power minus lambda t and observe that, this stand cancelled out.The next term will be the derivative of this,so that will giveminus lambda and minus is here,so it becomes plus lambda square t e to the power minus lambda t.

Now, look at the third term and derivative of this. So, if you look at the derivative of thisone, here t square is coming. So, 2t and this 2 will cancel out and there will be a minus sign. So, we will getwith, because there as a minus sign outside, it becomes minus lambda square t e to the power minus lambda t. So, you can easily observe, that this is a telescopic sum.

The final term will be plus, which will be contributed by this term that will become lambda to the power rt to the power r minus 1 by r minus 1 factorial e to the power minus lambda t. So, this will be the left out term. So, we are getting the probability density function of T r as lambda to the power r t to the power r minus 1 e to the power minus lambda t by gamma r for t greater than 0.Here, lambda is a positive parameter.In the derivation,we have considered r to be an integer because of its occurrence, but after writing down this, you can see that it is valid for r greater than 0, any positive real number. So, this is known as a Gamma distribution or Erlang’s distribution.

So, this distribution arises as the distribution of thewaiting time for therth occurrence in a Poisson process, in place of the first occurrence.If you put r is equal to 1, you will get the exponential distribution because this term will vanish and you will get lambda e to the power minus lambda t. So, this is a generalization of the exponential distribution, but it has much more applicability because we are looking at a higher order of occurrences in a Poisson process.

Apart from the regular applications,one can see the characteristics of this distribution.If we consider a kth order non-central moment, that is mu k prime. So, mu k prime is equal to integral from 0 to infinity t to the power k, lambda tothe power r,t to the power r minus 1 by gamma r,e to the power minus lambda t d t.So, obviously, this is a gamma function you can write it as lambda to the power r by gamma r, t to the power k plus r minus 1, e to the power minus lambda t dt. So, it is lambda to the power r by gamma r, gamma k plus r divided by lambda to the power k plus r.

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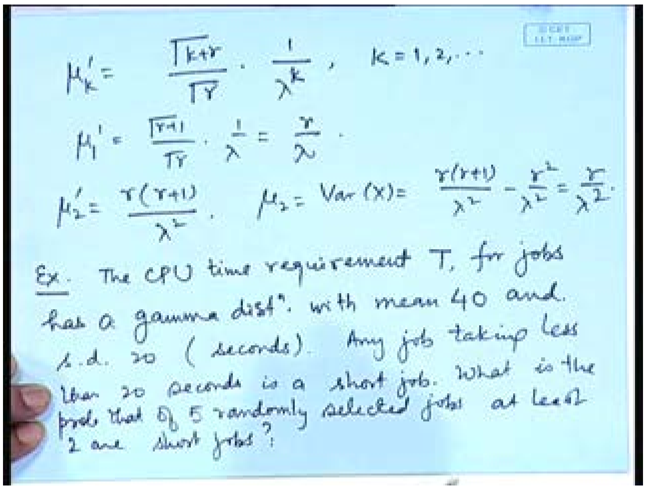

This is equal to, that is mu k prime is equal to gamma k plus r.Of course, this is valid for any k greater than 0, but we will be more concernedwith the positive integral moments. So, mu 1 prime is equal to gamma r plus 1by gamma r 1 by lambda, that is, r by lambda.So, if the waiting time for the first occurrence was average, waiting time for the first occurrence was 1 by lambda, then for the rthoccurrence, it will be r by lambda.See this is again because of the memoryless property of the exponential distribution because after first occurrence, we can again consider it as the starting of the process observing from time 0. So, again for the second occurrence, waiting time will be 1 by lambda andso on.

If we consider mu 2 prime, then this becomes r into r plus 1 by lambda square.Therefore, mu 2, that is a variance of this distribution is equal to r into r plus 1 by lambda square minus r square by lambda square, that is r by lambda square which is again, you can see in the exponential distribution,the variance was 1 by lambda squareand if you are looking at the variance of the rth occurrence, then it becomes r by lambda square.

Let us look atone application of this.The CPU time requirement T for jobs has a gamma distribution with mean 40 and standard deviation 20.This measure is in seconds.Any job taking less than 20 seconds is a short job.What is the probability that of 5randomly selected jobs at least 2 are short jobs?

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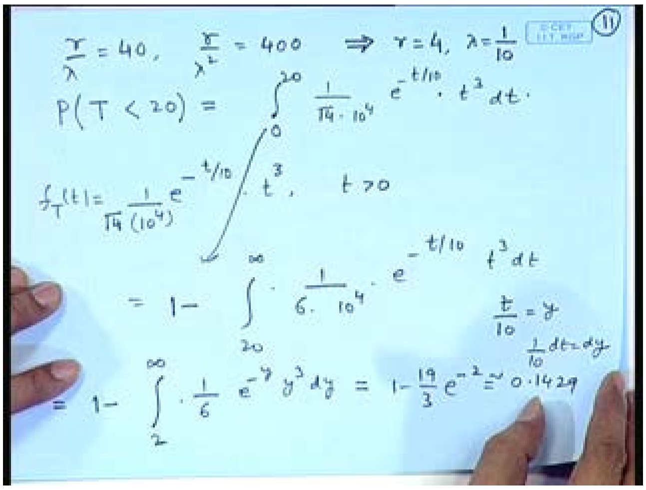

So, in order to answer this question,let us consider the setup.Here, mean is given to be 40,so that isr by lambda is 40 and standard deviation. So, here variance is r by lambda square. So, r by lambda square is 400. So, this is a 2 parameter distribution and we have now 2 equations. So, we can have r by lambda is equal to 40,r by lambda square is equal to 400. So, if we solve this, we get r is equal to4 andso lambda is equal to 1 by 10. So, that is right.What is a probability of a short job?That means T is less than 20.

So, we need to consider the density function of There.F T is equal to e to the power minus lambda t. So, that is t by 100, t to the power r minus 1, that is,3, then lambda to the power r, that is 1 by 10 to the power 4 and gamma r.This is for t greater than 0. So, we need to calculate 0 to 20the integral of this density 1 by gamma 4,10 to the power 4 e tothe power minus t by 10, t cube dt.This is the probability of a randomly selected job to be a short job.

So, if you want to evaluate this, we can consider, since exponential function is involved,the integral will be at 1 and something value 1 and at another point, some value will be there. So, it is convenient if we consider this has 1 minus 20 to infinity, 1 by 6 into 10 to the power 4, e to the power minus t by 10, t cube dt. So, from the integrals integrand, we can observe that is convenient if we put t by 10 is equal to y, that is, 1 by 10dt is equal to dy. So, this becomes equal to 1 minus, this will be 2 to infinity 1 by 6, e to the power minus y cube dy.

So, this is not a complete gamma function.This is an incomplete gamma function because integral is from 2 to infinity. So, we can do integration by parts and after doing certain simplification, this one will turn out to be 1 minus 19 by 3 e to the power minus 2, which is approximately 0.1429. So, probability of a randomly selected job being a short job is only 0.1429.That means, in the gamma distribution, if the mean is 40 and the standard deviation is 20,the probability of job being over in mu minus sigma is actually much smaller.

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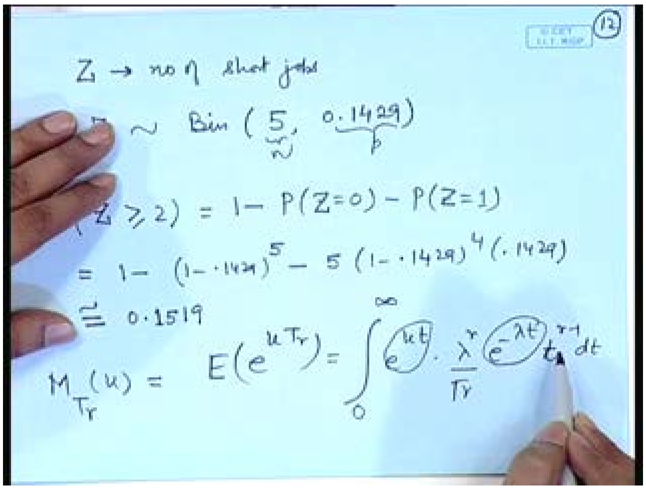

Now, we can consider the random variable, sayZ is the number of short jobs out of 5,then Z is following by nominal distribution with N is equal to 5 and P is equal to 0.1429.This is P, this is N.We are saying, what is the probability that at least 2 jobs are short jobs.That meansprobability of Z greater than or equal to 2.This we can consider as probability Z is equal to 0 and Z is equal to 1.You subtract from 1,that is, a complimentary event. So, this is equal to 0.1 minus 0.1429 to the power 5minus 5 C 1 minus 0.1429 to the power 4 into 0.1429.

So, this can be evaluated and it is approximately 0.1519. So, the probability, that at least 2 out of the 5jobs are short jobs is quite small.This is because the probability of a single job itself being short job is much smaller.So, we may look at the moment generating function etcetera of this distribution. So, moment generating function of the gamma distribution expectation of e to the power uT r, that is equal to integral e to the power ut, lambda to the power r by gamma r, e to the power minus lambda,t to the power r minus 1dt0 to infinity.

So, here, if you see this term can be combined with this. So, it becomes directly a gamma function.

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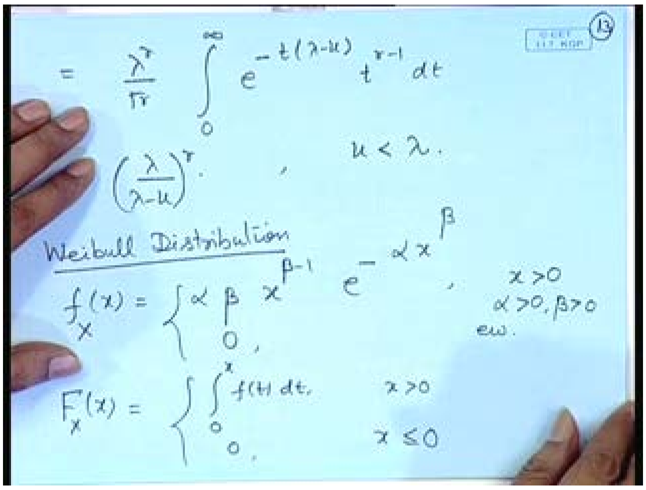

So, this is equal to lambda to the power r by gamma r, e to the power minus t lambda minus u, t to the power r minus 1,0 to infinity. So, this is just the gamma function, lambda by lambda minus uto the power r and gamma r will be cancelling out. So, this is valid for u less than lambda.You can see the similarity from the moment generating function of the exponential distribution, where r was 1. So, it was lambda by lambda minus u. So, we will show some relationshipbetween the gamma distribution and the exponential distribution later on.

The moments of the gamma distribution as we have seen, can all be calculated from the general expression for mu k prime. So, mu 3 prime, mu 4 prime can be calculated and then, mu 3 and mu 4 can also be calculated.You can see that, it will be actually positive. So, all the gamma distributions will be,in fact, positively skewed.The reason is that, if you look at density function, it is lambda to the power r, t to the power r minus 1, e to the power minus lambda t divided by,of course gamma r.

So, in the beginning, if you consider at t is equal to 0 apart from r is equal to 1, this value is going to be 0 and if you are considering, then t becoming larger.Then, in the beginning,since it is t to the power r minus 1, it may increase little bit, but there after this term will dominate.Therefore, the densities will always be positively skewed for various gamma distributions.Of course, it will depend upon, what is the value of r and what is the value of lambda for different shapes, but whatever be the shapes, they will be positively skewed.The P equal of course, depend upon the value of the r and lambda.

If we are considering the exponential distribution or the gamma distribution as the life of certain components, then another interesting distribution in the same direction isso-called Weibulldistribution.The general form of a probability density function of a Weibull distribution is given by alpha beta x to the power beta minus 1, e to the power minus alpha, x to the power beta, where x is always positive and alpha and beta are positive parameters.

So, quite naturally, one can see the form of the CDF here, because if you integrate from, since x is positive valued random variable, the integral will be from 0 to x ftdt for x positive and it is 0,of course for x less than or equal to 0. So, if you consider this integrand,it looks like a derivative of e to the power minus alpha x to the power beta.Therefore, when we calculate the CDF, this will be simply equal to 1 minus e to the power minus alpha x to the power beta for x positive and it is 0 for x less than or equal to 0.

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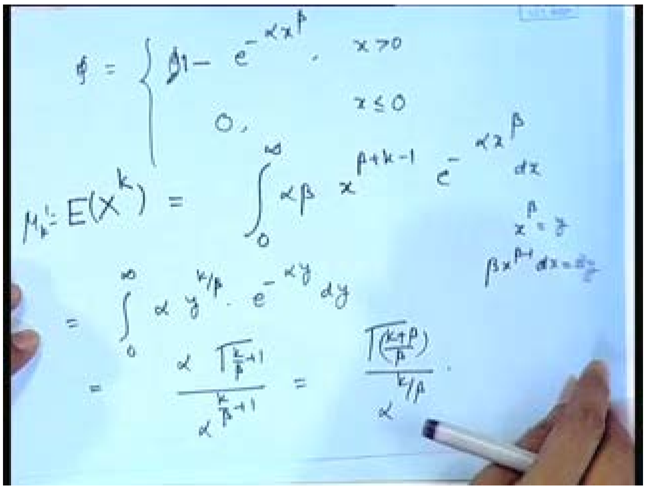

If we put beta is equal to 1, then this term vanishes this term becomes e to the power minus alpha x. So, you get alpha e to the power minus alpha x. So, that becomes exactly the density of an exponential distribution. So, this Weibull distribution is actually a generalization or extension of the exponential distribution.

So, we will see that what is the significance of making power x to the power beta here, because in the exponential we had alpha e to the power minus alpha x. So, x has been replaced byx to the power beta.

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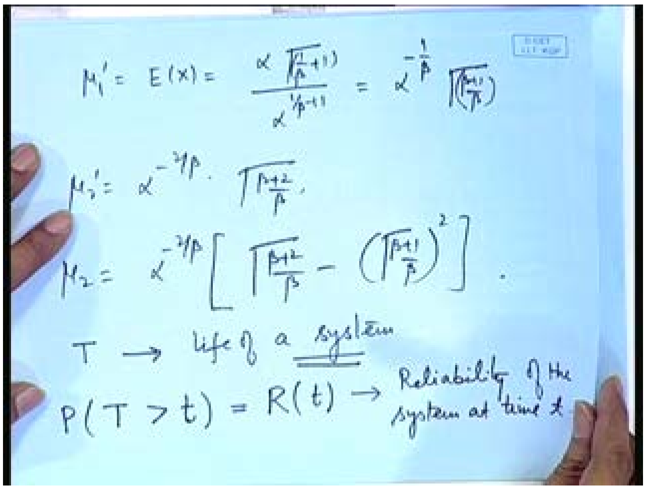

So, first is that we can look at its moment structure.So, obviously, you can make use of the gamma functions.If I consider expectation of x to the power k, it is alpha beta x to the power beta plus k minus 1. So, quite obviously, you can understand here that if we put x to the power beta is equal to y, that is beta x to the power beta minus 1 is equal to dx is equal to dy, then this becomes integral from 0 to infinity alpha andx to the power beta minusone term is combined here. So, you have x to the power k, that is y to the power kby beta, e to the power minus alpha y dy. So, this is simply a gamma function and the expression for this turns out to be alpha gamma k by beta plus 1 divided by alpha to the power k by beta plus 1.That meansgamma k by or you can write it as k plus beta by beta divided by alpha to the power k by beta.

So, in particular, if I am looking at the mean,so mu 1 prime is equal to expectation of x, that is equal to alpha 1 by beta plus 1,gamma of this divided by alpha to the power 1 by beta plus 1, that is equal to alpha to the powerminus 1 by beta, gamma of beta plus 1 by beta and mu 2 prime will become alpha to the power minus 2 by beta gamma of beta plus 2 by beta.Therefore, the variance of the Weibull distribution will be, alpha to the power minus 2 by beta, gamma beta plus 2 by beta minus gamma beta plus 1 by beta whole square, which looks slightly complicated, but nevertheless, the functions of thegamma functions can be easily calculated using tables of the gamma distribution or from calculators or computers.Nowadays, this can be easily.

However, this distribution has moreimportance in the reliability analysis of certain mechanical or electronic systems. So, let us define, what you meanby the reliability of a system. So, if T denotes the survival time or the life of a system,sowe can consider probability of T greater than t.This means the system has been functioning till time t or it has not felt till time t or the system is working at time t.

The probability of system working at time t is called the reliability of the system.

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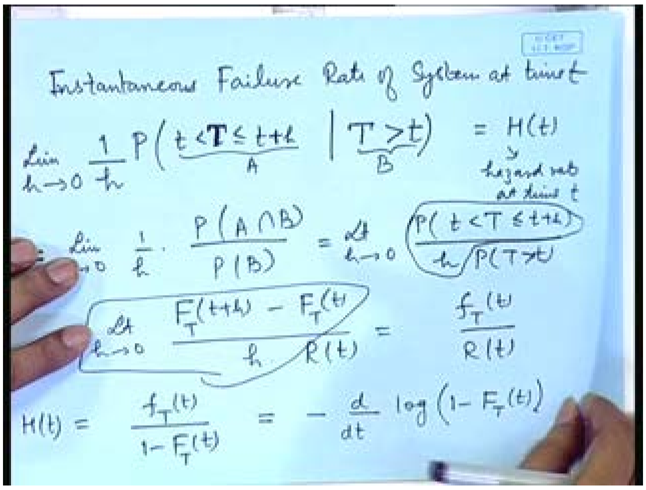

So, using this, we can define another quantity and of course, you can easily see that,this is equal to 1 minus the CDF at the point t.

We also define, what is known as Instantaneous Failure Rate of System at time t. So, let us consider the interpretation of this.The system is functioning at time t and immediately after the time t; it fails, that means in an interval from T to t plus h. So, if we are considering the rate, we consider it by divided by h and take limit as h tends to 0.Let us give some notations,say h of t; we also call it hazard rate at time t. So, this we define for any random variable which is denoting the life of a system. So, we call failure rate at time t or hazard rate at time t; that means, given that the system is functioning at time t, what is the probability that it will fail immediately after that.Therefore, we want to calculate the rate.Then, we divide by the length of the interval and take the limiters.So, let us evaluate this.

Now, if we consider this as event A and this as event B, then this is probability of A intersection B divided by probability of B.Now, once again, you can see that A is a subset of B. So, this becomes probability of T less than capital T less than or equal to t plush divided by probability T greater than t h limit as h tends to 0.

So, this term if you see, it is nothing, but the density function of the variable T because we are taking limit as h tends to 0.See you can expand it like this.The numerator becomes f of t plus h minus f of t by h and this is Rt. So, this term is nothing, but the f T by Rt or you can consider it as f T divided by 1 minus Ft. So, the hazard rate of a lifetime distribution can be denoted by the density divided by the reliability or the density divided by 1 minus the cumulativedistribution function of the random variable.

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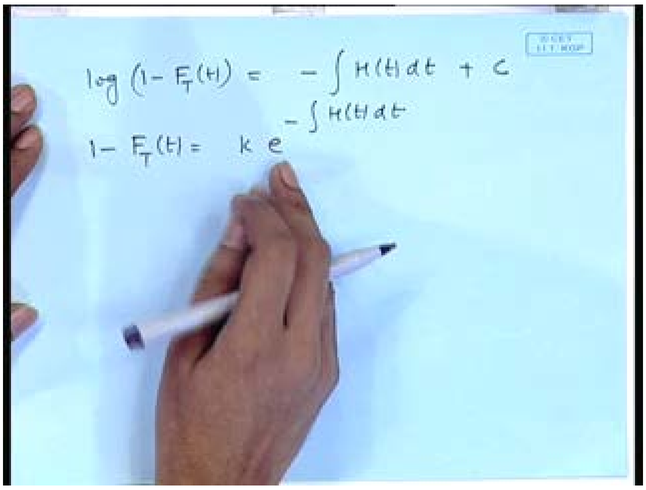

So, we are calling it as Ht.We can notice another relationship here.This is minus d by dt log of 1 minus F T. So, this looks like a first order differential equation. We can integrate it out and we will get, log of 1 minus FT as equal to and therefore, plus of course, the constant of integration. So, 1 minus FT becomes some k times into the power minus integral H tdt.This shows that given the distribution,one can calculate the hazard rate, given the hazard rate function,one can determine the CDF and hence, the probability density function of the random variable.We willlook at these quantities, that is, the reliability, the hazard rate, in context of theWeibull distribution, the exponential distribution and try to see what it signifies. So, in the next lecture, we will be covering these issues.Thank you.