Statistical Inference
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Lecture No. # 15
UMVU Estimation, Ancillarity

In the last lecture I introduced the concept of minimal sufficiency and completeness of certain statistics or again these are also the properties of the family of distributions.

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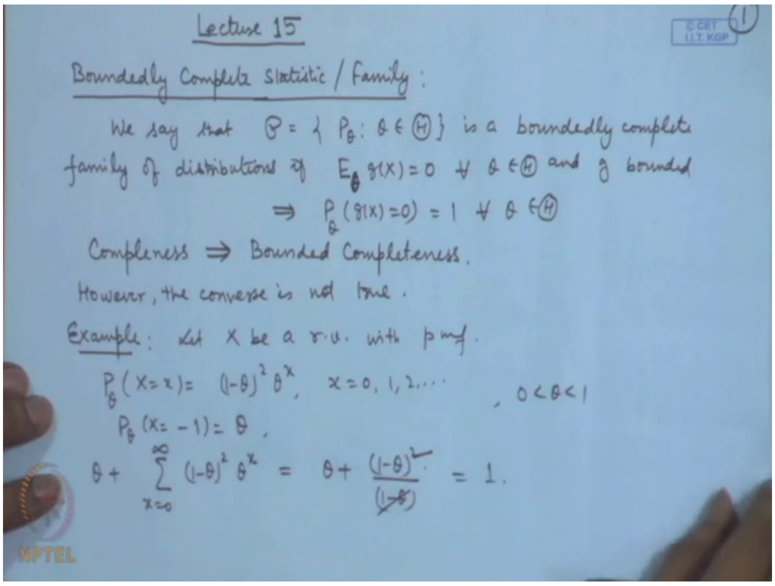

Now, before we proceed further I will define a related concept that is called Boundedly Complete. Boundedly Complete statistic or Boundedly Complete family of distributions. So, we say that P is equal to P theta is a Boundedly Complete family of distributions if expectation of g x is equal to 0 for all theta and g bounded implies that probability of g x is equal to 0 is 1 for all theta.

So, the difference from the definition of completeness is that there we wrote any function g. So, expectation g is equal to 0 for all theta and for any function g if that implied that the probability that the function is 0 with probability 1 then it was complete. If I impose the condition that g is bounded then it will imply that probability of g x equal to 0 is 1 then it will be called a boundedly complete family of distributions. So, we can say that completeness implies Bounded Completeness. However, the converse is not true.

I will give an example here. Let, X be a random variable with probability mass function given by P theta X is equal to X is equal to 1 minus theta square theta to the power X for X equal to 0, 1, 2 and so on. And P theta X is equal to minus 1 is equal to theta. Here theta is between 0 to 1. Now, you can easily see that theta plus sigma 1 minus theta square theta to the power X, X equal to 0 to infinity that is equal to theta plus 1 minus theta square divided by 1 minus theta because this is infinite geometric series with common ratio theta. So, this cancels out you get 1.

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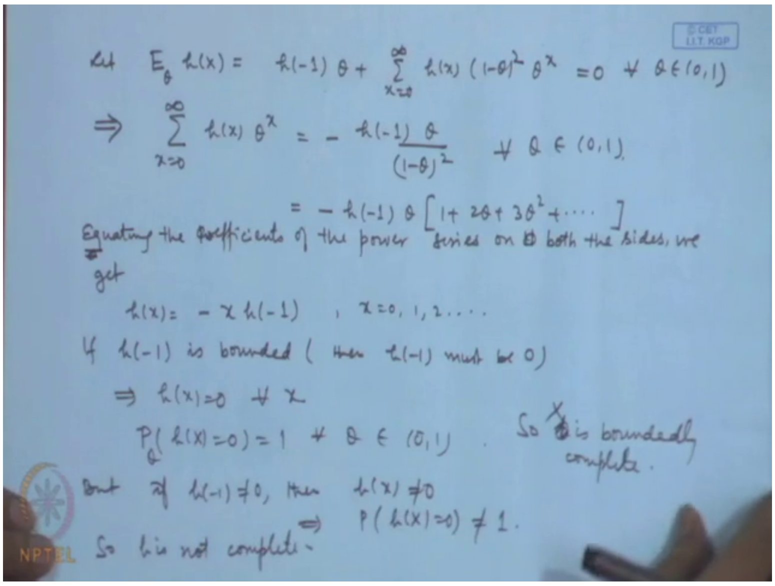

So, this is a proper probability distribution. You can say it is a shifted geometric kind of distribution. Let us show whether it is complete or not. So, consider a function h X then its expectation can be written as h of minus 1 into theta plus sigma h X into 1 minus theta square theta to the power X, X equal to 0 to infinity. Now, suppose we equate it to 0 for all theta in the interval 0 to 1. Now, this term I take to the right hand side and then we divide by 1 minus theta square. So, it is reducing to h X into theta to the power X. It is equal to minus h of minus 1 theta divided by 1 minus theta square this is for all theta in the interval 0 to 1.

Further, this 1 minus theta square in the denominator. So, if I bring it to the numerator it becomes 1 minus theta to the power minus 2 and I can expand because theta is in the interval

0 to 1. So, this we can write as minus h of minus 1 into theta and this expansion can be written as 1 plus 2 theta plus 3 theta square and so on. Now, if I consider these 2 terms the left hand side is a power series in theta and the right hand side is also a power series in theta. So, if I equate the terms we get equating the coefficients of the power series on both the sides. We get h X is equal to minus X into h of minus 1 for X equal to 0, 1, 2 and so on.

Now, if h of minus 1 is bounded then h of minus 1 must be 0 because if h of minus 1 is not 0 then this function is unbounded because it will be X into some constant. So, for boundedness the only possibilities that h of minus 1 is 0 which will imply h of X is equal to 0 for all X. That means probability of h X is equal to 0 will be 1 for all theta in the interval 0 to 1. So, h is boundedly complete not X is boundedly complete.

But if h of minus 1 is not 0 then h of X is also not 0 this implies probability that h X is 0 cannot be 1. So, h is not complete because expectation of h X is 0 but, h X will not be 0 with probability 1. So, this is an example of a boundedly complete family of distributions which is not complete.

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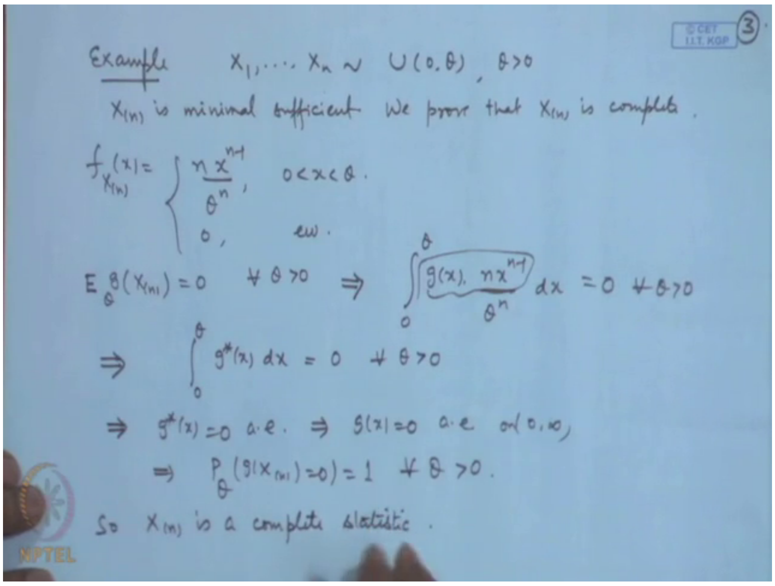

Now, there are relationships between sufficiency and completeness also. Also, there is a general way of determining complete statistics for example, if the distributions are in the exponential family I have already given the example of binomial distribution, poisson distribution.

So, in the poisson distribution family is complete. If I consider sufficient statistics or minimal sufficient statistics that is turning out to be sigma X i which is again having poisson distribution with parameter n lambda. So, if poisson lambda is complete poisson n lambda is also complete. So, sigma X i is complete. So, we can conclude that in most of the standard examples that we have discussed the corresponding sufficient or minimal sufficient statistics will also be complete. Let me just take the example of non regular family. Say, let me consider say X 1, X 2, X n following uniform 0 theta distribution.

Then X n is minimal sufficient. We prove that X n is complete. Let us consider the distribution of X n, that is n X to the power n minus 1 by theta to the power N 0 less than X less than theta, it is 0 elsewhere. So, if I consider expectation of say g of X n is equal to 0 for all theta, then this statement is equivalent to g X n X to the power n minus 1 by theta to the power n d X from 0 to theta it equal to 0 for all theta. Now, this is equivalent to saying a function of x over all the intervals 0 to theta is integrated to 0. Again, by the (( )) integration theory it implies that g star must be 0 almost everywhere. This g star function I have taken to be this.

So, this implies that g X is equal to 0 almost everywhere on 0 to infinity this implies that probability that g X n is equal to 0 is 1 for all theta. So, X n is a complete statistic. So, there is a relation between minimal sufficiency and complete sufficiency. In fact we have the following theorem.

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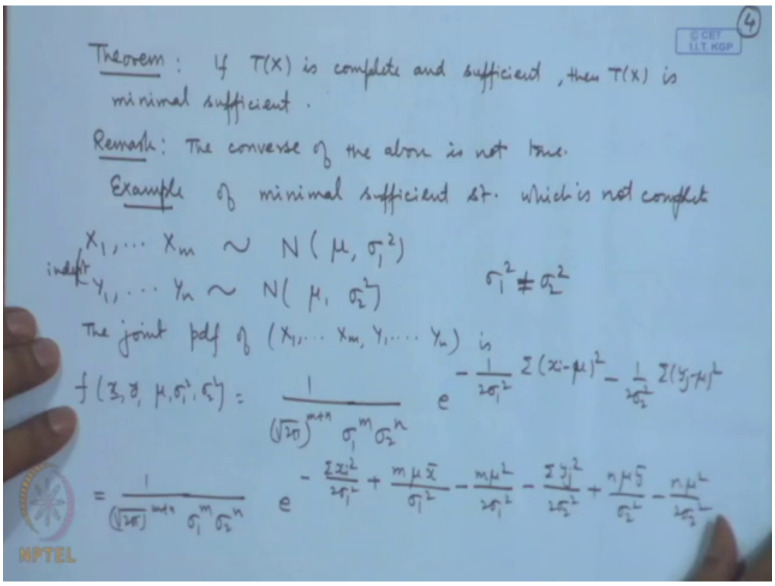

If T x is complete and sufficient then T x is minimal sufficient. However, the converse of the above statement is not true that is we may have example of say minimal sufficient statistic which is not complete. Let us take say X 1, X 2, X M, a random sample from normal with mean mu and variance sigma 1 square and y 1, y 2, y N this is another independent sample from normal with mean mu and variance sigma 2 square. Here, sigma 1 square and sigma 2 square are different.

Let us consider the joint distribution of X 1, X 2, x M and y 1, y 2, y N. The joint P d f of X 1, X 2, X M, y 1, y 2, y N that is equal to 1 by root 2 pi to the power M plus N sigma 1 to the power M sigma 2 to the power N, E to the power minus 1 by 2 sigma 1 square sigma X i minus mu square minus 1 by 2 sigma 2 square sigma y j minus mu square. This we can simplify as 1 by root 2 pi to the power M plus N sigma 1 to the power M, sigma 2 to the power N, E to the power minus sigma X i square by 2 sigma 1 square plus M mu x bar by sigma 1 square minus M mu square by 2 sigma 1 square minus sigma y j square by 2 sigma 2 square plus N mu y bar by sigma 2 square minus N mu square by 2 sigma 2 square.

So, if we apply the ratio by writing down this joint P d f at 2 points x y and say X prime y prime then these terms will get canceled out and we will be left with sigma X i square minus sigma X i prime square into parametric function, X bar minus y bar into the parametric function, X bar minus X bar prime, y bar minus y bar prime and sigma y j square minus sigma y j prime square. So, if we write down this function here.

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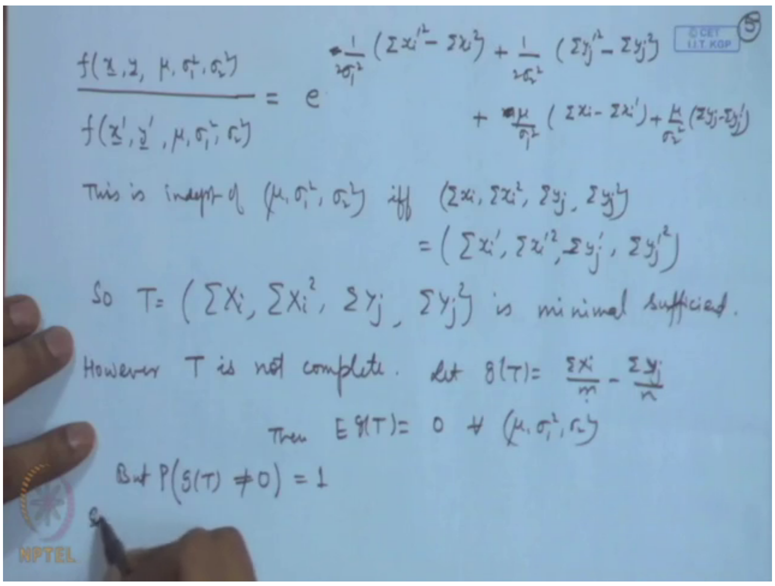

Say, f X y mu sigma 1 square sigma 2 square divided by say f X prime y prime mu sigma 1 square sigma 2 square then that is equal to E to the power 1 by 2 sigma 1 square sigma X i prime square minus sigma X i square plus 1 by 2 sigma 2 square sigma y j prime square minus sigma y j square.

Then plus M mu or mu by sigma 1 square, sigma X i minus sigma X i prime plus mu by sigma 2 square sigma y j minus sigma y j prime. So, this is independent of mu sigma 1 square and sigma 2 square if and only if we have sigma X I, sigma X i square, sigma y i, sigma y i square is equal to sigma X i prime, sigma X i prime square, sigma y i y j prime and sigma y j prime square. So, T is equal to sigma X i, sigma X i square, sigma y j, sigma y j square is minimal sufficient. However, T is not complete.

Let us consider g T as a sigma X i by M minus sigma y j by n then expectation of g T is equal to 0 for all mu sigma 1 square, sigma 2 square because expectation of X i and expectation of y j is mu so it is M mu by M minus M mu by n but, g T is not 0. Actually, probability that g T is not 0 is 1 probability that g T is equal to 0 is actually 0 so T is not complete so this is an example of a minimal sufficient statistic which is not complete.

To determine complete statistics in general settings are to prove the completeness in general settings of exponential family 1 only needs to check the kind of parameter space that we are having. So, we have the following general theorem

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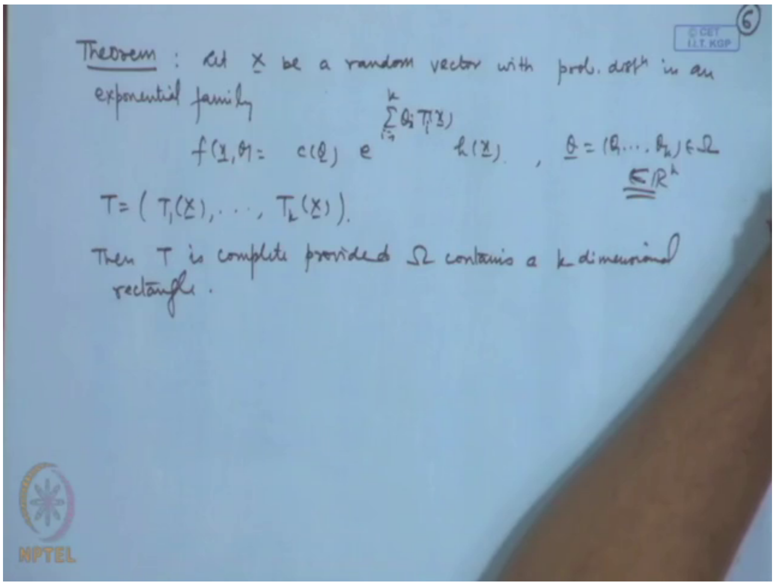

Which I will state without proof for the proof one can look at the book of Lehmann testing of hypothesis book. So, let x be a random vector with probability distribution in an exponential family say we write which in the form c theta E to the power sigma theta E T x into H x.

So, here c theta is a function of parameter h x is function free from parameter and parameter is occurring in the exponent here theta is equal to theta 1 theta 2 theta k that is it is belonging to r k. Let me say it belongs to omega and omega is a subset of r k. Let us write T as T 1 x and so on T k x then T is complete provided omega contains a k dimensional rectangle. If you look at the previous example here.

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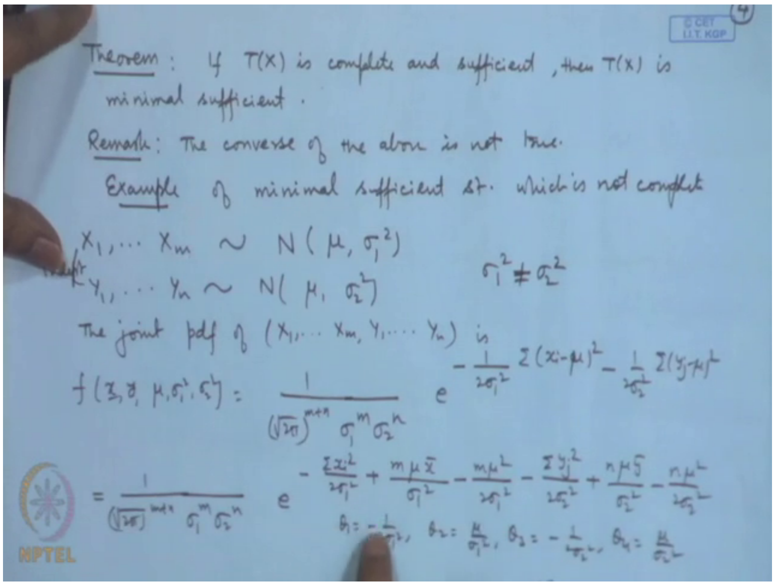

This is actually a 3 parameter distribution here. Here what we are getting is 1 by 2 sigma 1 square or you can say 1 by sigma 1 square, mu by sigma 1 square then 1 by sigma 2 square and mu by sigma 2 square.

However, they are not independent. Actually, the parameter is 4 dimensional if we write theta 1 is equal to say minus 1 by 2 sigma 1 square, theta 2 as equal to mu by sigma 1 square, theta 3 as equal to say minus 1 by 2 sigma 2 square and theta 4 is equal to say N say mu by sigma 2 square. Then this is a 4 dimensional parameter but, there is dependency upon that for example, given theta 1, theta 2 and theta 3 we can determine theta 4. So, the parameter space does not contain a 4 dimensional rectangle.

And that is why we could actually show that this is not complete T was not complete here. We have seen the application of sufficiency in estimation problems. We saw that if we have an unbiased estimator we can certainly improve upon it by conditioning upon the sufficient statistics, the result was known as the Rao-Blackwell Theorem. Now, if we couple this concept with the completeness we get a stronger result. In fact we can reduce the problem to determination of the uniformly minimum variance unbiased estimator.

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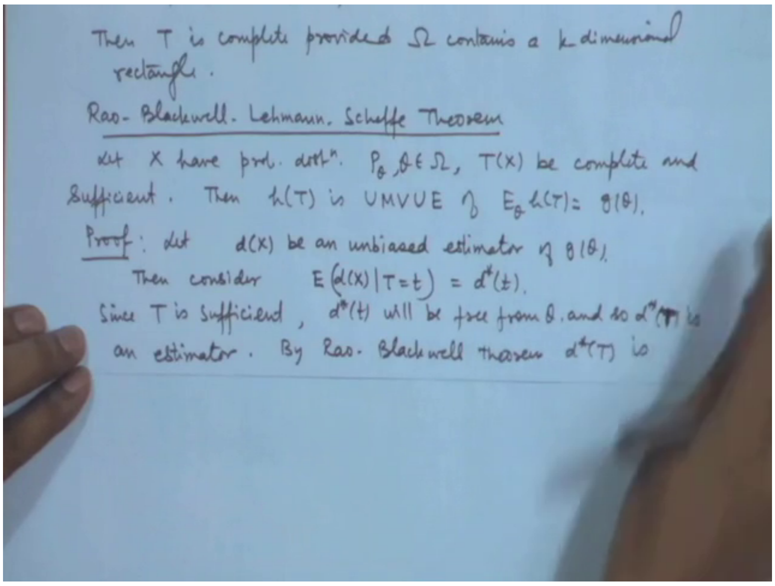

The resulting result which is actually associated with the name of Lehmann-Scheffe. So, I will couple the 2 results Rao-Blackwell and Lehmann-Scheffe and we call it Rao-Blackwell-Lehmann-Scheffe theorem. Let X have probability distribution P theta theta belonging to say omega and T X be complete and sufficient. Then h T is U M V U E of expectation of h T. Let us call it say g of theta that means for any estimable unbiased estimable function g theta if I have an unbiased estimator which is dependent upon the complete sufficient statistic then that will be actually U M V U E. Let us look at the proof of this.

Let say d X be an unbiased estimator of g theta then u will have consider expectation of d X given T. Let me denote it by say d star. Since, T is sufficient d star T will be free from theta because the conditional distribution of X given T is independent of theta therefore, this expectation will not contain any term of theta and we can call it T star T and so d star T is d star T is suppose I write capital T here this is an estimator. Now, we have already seen that by

Rao-Blackwell theorem d star T is also unbiased for g theta and variance of d star was less than or equal to the variance of d T.

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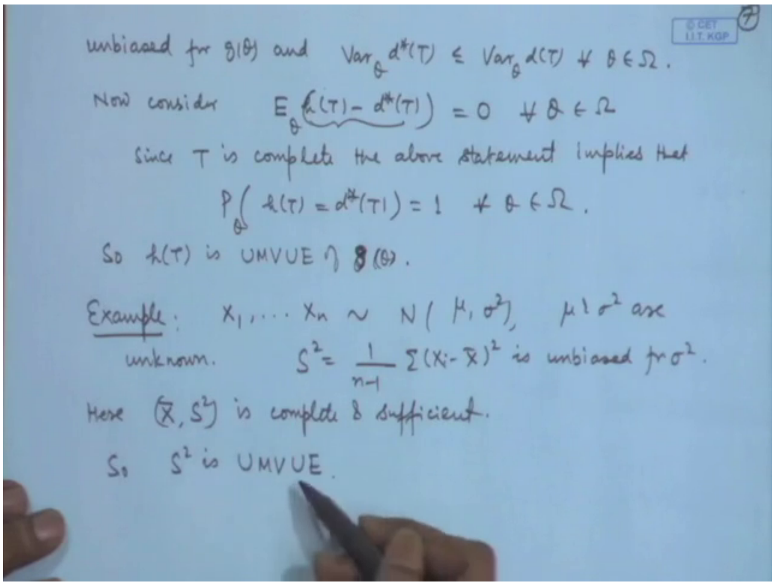

Now, consider expectation of h T minus d star T then that is 0 because both of these are unbiased for g theta. Now, this is a function of T and T is complete. Since, T is complete the above statement implies that h T must be equal to d star T with probability 1. Essentially, it proves that h T is a unique unbiased estimator of g theta. So, h T is U M V U E. Actually g star d star is also U M V U E but, these 2 U M V U E differ only on a set of measure 0.

Now, this result is extremely useful for finding out the U M V U E 's. We have seen actually in the earlier method of lower bounds that many times whatever best unbiased estimator we are able to think of the variance of that is not attaining the lower bound whether we are considering the Frechet-Rao-Cramer lower bound, Bhattacharya lower bound or Chapman-Robbins-Kiefer lower bound etcetera.

In many of the cases we saw that the variance of the unbiased estimator was bigger than the lower bound the corresponding lower bound. However, this method when we are considering a function of complete and sufficient statistic it immediately proves that the corresponding estimator will become uniformly minimum variance unbiased estimator. Essentially what is doing it will actually show that the corresponding unbiased estimator is actually the only unbiased estimator available except of course, on a set of probability 0. So, since it is unique certainly it is U M V U E.

So, if we go back to various problems where the lower bounds was not attained for example, if you consider normal mu sigma square where mu is unknown and we were considering the estimation of sigma square. So, let us consider say X 1, X 2, X n follows normal mu sigma square, mu and sigma square are unknown and we have this S square as 1 by n minus 1 sigma X i minus X bar whole square. This is unbiased for sigma square. Now, in this problem X bar and S square is complete and sufficient. So, S square is U M V U E. We had noticed here that in this particular case the lower bound that was attained by the method of Frechet-Rao-Cramer it was lower than the variance of S square.

The variance of square was 2 sigma to the power 4 by n minus 1 and the lower bound was 2 sigma to the power 4 by n but, here in this method U M V U E proving is easy because we are just looking at the expectation of S square since is equal and it is a function of the complete sufficient statistics.

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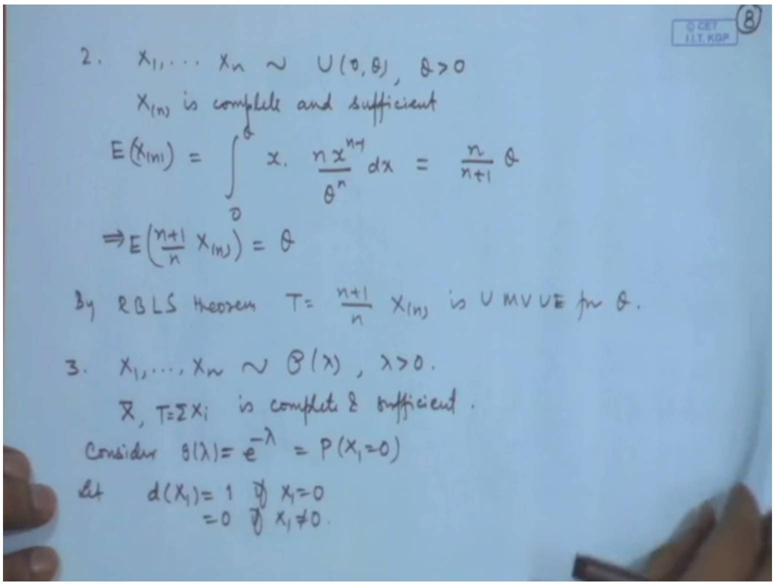

So, it becomes U M V U E. Let us take other related examples also. X 1, X 2, X n following uniform 0 theta. Here, we have shown that X n is complete and sufficient.

Now, if we look at expectation of X n that is X into n X to the power n minus 1 by theta to the power n X then this is equal to n by n plus 1 theta that means n plus 1 by n X n is unbiased for theta. Now, this is a function of complete sufficient statistics so by Rao-Blackwell-Lehmann-Scheffe theorem we conclude that n plus 1 by n X n this is U M V U E for theta. We have also seen the standard distributions like poisson distribution where for lambda we are able to derive the U M V U E but, for lambda square we are not able to derive or if I consider e to the power minus lambda then we are not able to derive the U M V U E.

But using this method we can derive. Let me explain this here. Let us consider say X 1, X 2, X n following poisson lambda distribution lambda positive. Now, here X bar or you can say sigma X i this is complete and sufficient. Suppose, I am considering g lambda is equal to e to the power minus lambda which I had explained actually this is probability of X 1 is equal to 0 that is the proportion of 0 occurrences in a given problem. Let us define say d X 1 is equal to 1 if X 1 is 0 and it is equal to 0 if X 1 is not equal to 0.

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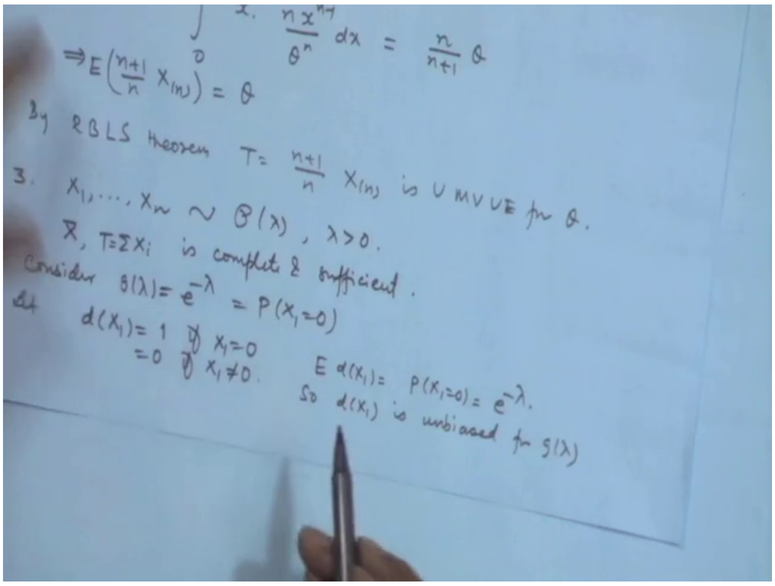

Then if I consider here expectation of d X 1 then that is equal to probability of X 1 is equal to 0 that it it is equal to E to the power minus lambda. So, d X 1 is unbiased for g lambda. However, this is not U M V U E because this is not a function of the complete sufficient statistic. So, if I apply the Rao-Blackwell-Lehmann-Scheffe theorem

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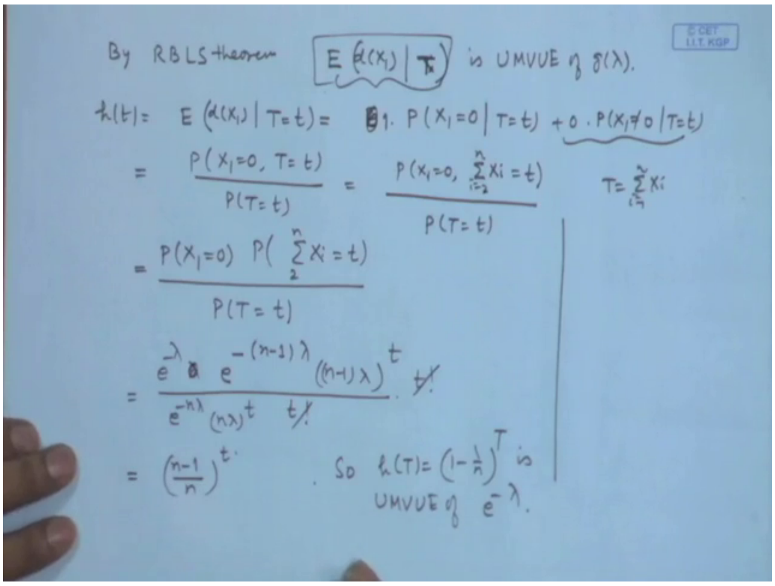

If I consider Rao-Blackwell-Lehmann-Scheffe theorem, if I consider expectation of d X 1 given T sigma x i or x square we can write then this is U M V U E of g lambda. So, the only thing remaining is that determination of this function.

We can determine it easily. Let us denote it by h T expectation of say d X 1 given T is equal to small t then this is equal to expectation of now d X 1 takes only 2 values 1 and 0. So, it is equal to probability of X 1 is equal to 0 given T is equal to t because d X 1 is equal to 0 then probability of x 1 is not equal to 0 but, value 0 multiplied then that value will not matter, x 1 not equal to 0 given T is equal to t. So, this term is vanishing. So, we need to only determine this conditional probability that is probability X 1 is equal to 0, T is equal to t divided by probability T is equal to t.

That is equal to probability X 1 is equal to 0. Now, this T is nothing but, sigma X i, i is equal to 1 to N. If I say X 1 is equal to 0 then we can say sigma of X i from 2 to N is also equal to T. Now, here you notice that the sum of independent poissons is poisson. So, the distribution of T will be poisson n lambda and distribution of sigma X I, i is equal to 2 to N that will be poisson n minus 1 lambda. So, if we use this here X 1 and sigma X i from 2 to N this will be independent so this can be written as the product of this probability. So, it becomes probability of X 1 equal to 0 into probability of sigma X i from 2 to N is equal to T divided by probability T is equal to t.

So, that is equal to E to the power minus lambda, lambda to the power 0. So, that term will not come. Then this is following poisson n minus 1 lambda so it is becoming e to the power minus n minus 1 lambda, n minus 1 lambda to the power t divided by t factorial and then probability T is equal to t. So, that is e to the power minus n lambda, n lambda to the power T into t factorial. So, these terms get cancel out and we are left with n minus 1 by n t. So, h T is equal to 1 minus 1 by n to the power T this is U M V U E of E to the power minus lambda.

So, this Rao-Blackwell-Lehmann-Scheffe theorem is extremely useful to determine the U M V U E for various functions where the method of lower bound is not applicable. Before, we discuss other examples let me also give some further relationship between the completeness and independence etcetera. Now, there is a famous result called Basu's theorem where we consier certain statistics whose distribution does not depend upon the parameter.

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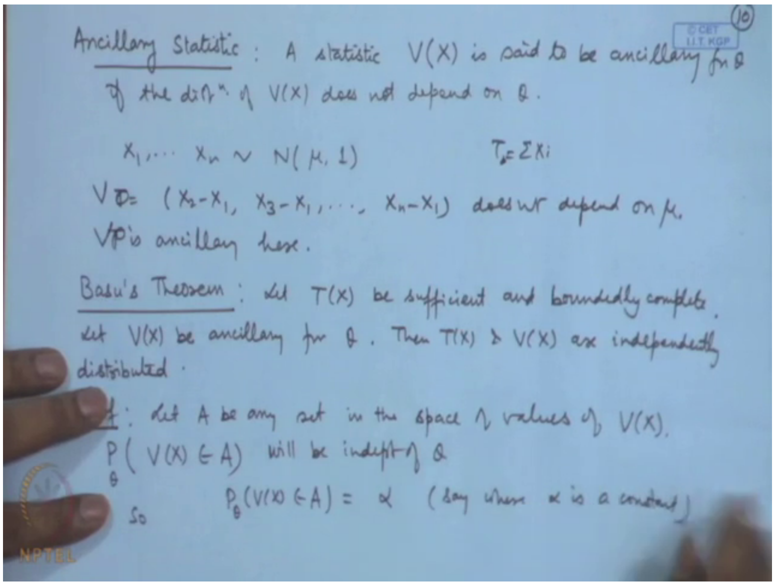

So, I define what is known as Ancillary statistic. So, a statistic let me call it V of X is said to be ancillary if the distribution of ancillary for say parameter theta if the distribution is V X does not depend on theta.

For example, if I consider say X 1, X 2, X n follows normal mu 1 and I consider T as say X 2 minus X 1, X 3 minus x 1 and so on x n minus x 1. Then the distribution of this does not depend on mu. So, T is ancillary here. Let me call it V here because T we use for the sigma X i here or X bar. Then we have the following theorem called Basu's theorem named after D

Basu. Let T be sufficient and boundedly complete. So, if it is complete or automatically bounded completeness will be true.

Let V X be ancillary for theta. Then T X and V X are independently distributed. Let us look at the proof of this. So, let a be any set in the space of values of V. So, if I consider probability of V X belonging to a then this will be independent of theta because the distribution of V X does not depend upon theta so this is going to be independent of theta. So, if we want to write a statement like this T theta V X belonging to a this is some constants say alpha, alpha is a constant.

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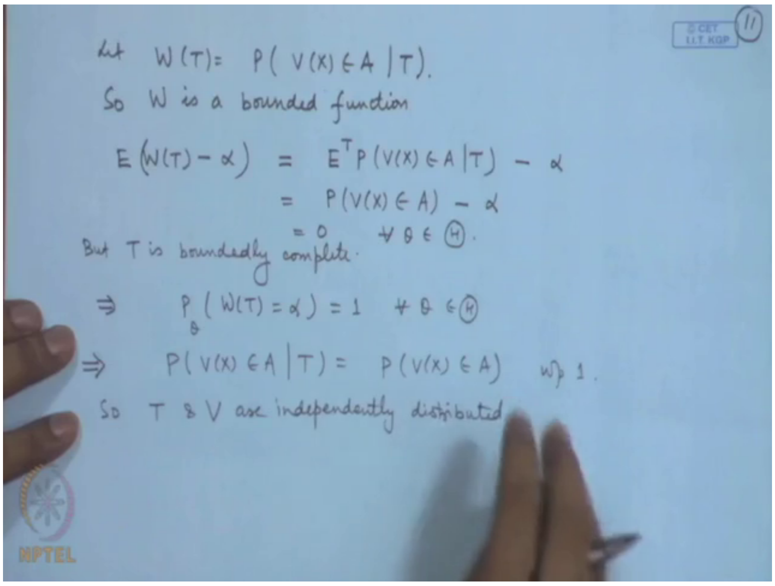

Now, let us consider a function say W of T that is equal to probability of V X belonging to A given T. Now, this is a probability so W is a bounded function, W is a bounded function. Now, let us consider expectation of W T minus alpha. Now, what this is going to be? This is expectation of probability V X belonging to a given T. Now, this expectation is over what? This conditional probability is a function of T, so this is expectation over T minus alpha. Now, this will become nothing but, probability of V X belonging to A minus alpha which is actually equal to 0 for all theta.

But T is boundedly complete, T is boundedly complete. So, this implies that probability that W T is equal to alpha must be 1 but, what is this statement? This statement is equivalent to saying probability of V X belonging to A given T is equal to alpha. What was alpha? Alpha was probability V X belonging to A that means the conditional probability of V given T is same as unconditional probability of V, this is with probability 1. So, T and V are independently distributed.

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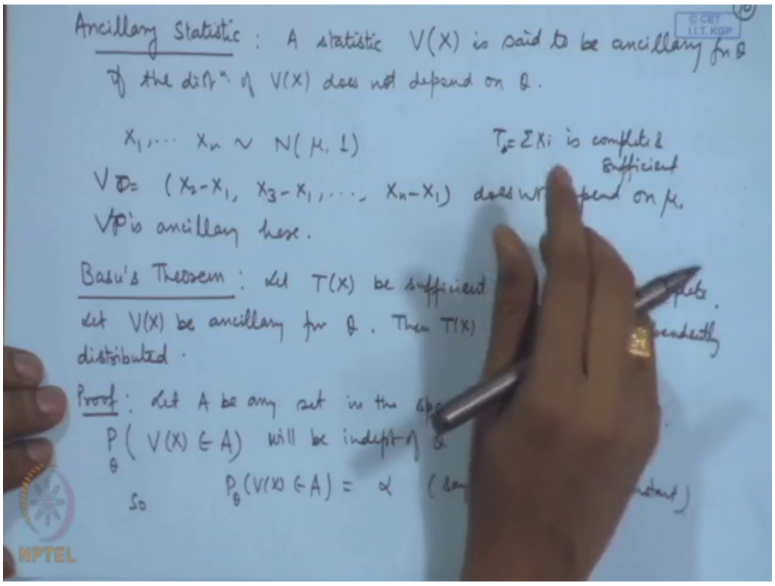

Let us look at 1 or 2 applications of this here. So, if we consider this problem here X 1, X 2, X n follows normal mu 1 and here T is equal to sigma X i this is complete and sufficient. So, this is complete and sufficient and X 2 minus X 1, X 3 minus X 1 and X n minus X 1 has a distribution which does not depend upon mu, then T and V will be independently distributed and of course, this is all also a well known result in the normal distribution theory that sigma X i and S square, X bar and S square are independently distributed.

So that is up a the proof is actually through this only that we firstly show that X bar and X 2 minus X 1, X 3 minus X 1 etcetera are independent and therefore, since S square is directly a function of this therefore, X bar and S square are also independent. So, that is confirmed here

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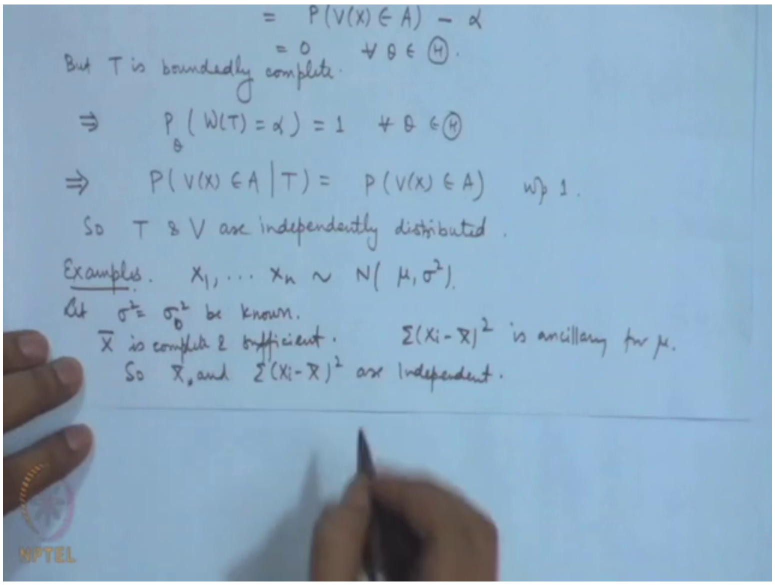

Let us generalize this example to normal mu sigma square. So, let us consider say X 1, X 2, X n follows normal mu sigma square. So, let us take say sigma square is equal to sigma naught square be known, if that is so then X bar is complete and sufficient.

And at the same time if we consider sigma X i minus X bar whole square this is ancillary for mu. Therefore, X bar and sigma X i minus X bar whole square are independent. Now, if we are writing this statement here. This sigma naught square does not play a role here because this was arbitrarily fixed so here if we say it for all sigma naught square that means X bar and sigma X i minus X bar whole square are independent in general here.

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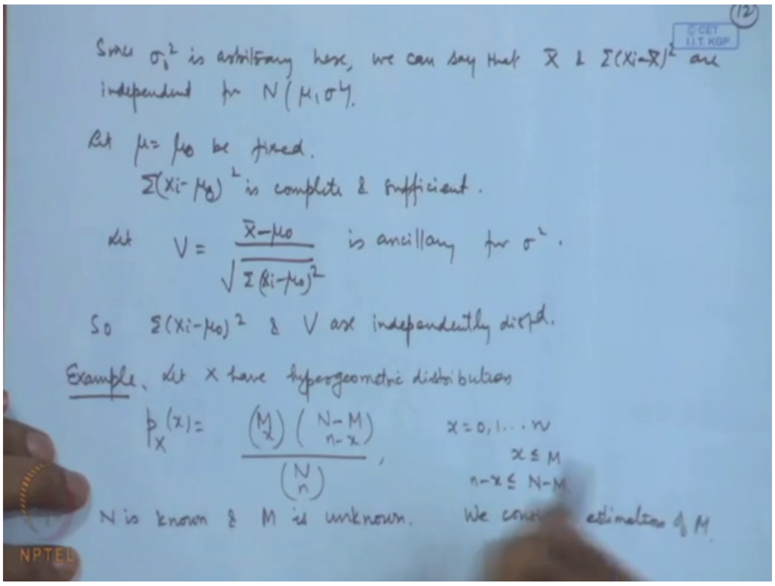

So, we can say here since sigma naught square is arbitrary we can say that X bar and sigma X i minus X bar whole square r independent for normal mu sigma square case here.

Let me take another application here. Suppose, I fix mu is equal to mu naught if we take this then sigma X i minus mu naught square is complete and sufficient. Let V be of the form say X bar minus mu naught divided by square root sigma X i minus mu naught square. You can see here if I divide by sigma here in the numerator and the denominator then the distribution will become free from the parameters here, this is ancillary here.

So, sigma X i minus mu naught square and V they are independent here. Let me consider some further applications of the minimum variance unbiased estimation. Let X have hypergeometric distribution, that is the probability mass function is given by M c X n minus M c n minus X divided by n c N. Here X is from 0, 1 to N and of course, subject to the restrictions that X is also less than or equal to M and n minus X is less than or equal to N minus m.

Here N is assume to be known and M is unknown. So, we consider estimation of M.

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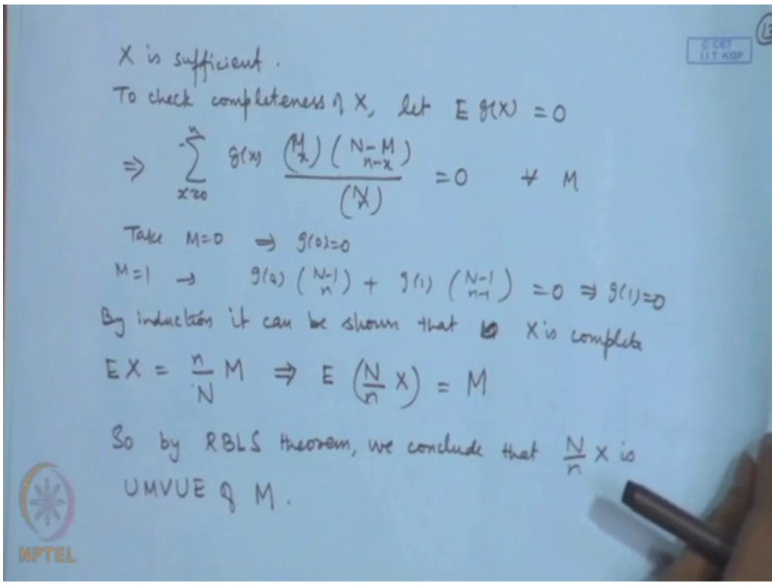

So, if we write down the distribution it is already in the factorizable form. So, X is certainly sufficient. So, X is sufficient here. Let us look at the completeness. To check completeness of X let us take expectation of a function of X is equal to 0, then that is equivalent to saying g X M c X n minus M c n minus X divided by n c N is equal to 0 for X equal to 0 to N subject to those conditions here for all m.

If I take M is equal to 0 here then this will give me g 0 is equal to 0, if I take M is equal to 1 and that will give me g 0 n minus 1 c N plus g 1 n minus 1 c n minus 1. Now, g 0 is 0 that means g 1 is also 0. So, by induction we can prove that it can be shown that M is that X is complete. Now, what is expectation of X that is equal to n by N into M so that means expectation of N by n x is equal to M so X is complete and sufficient and this is an unbiased estimator of M.

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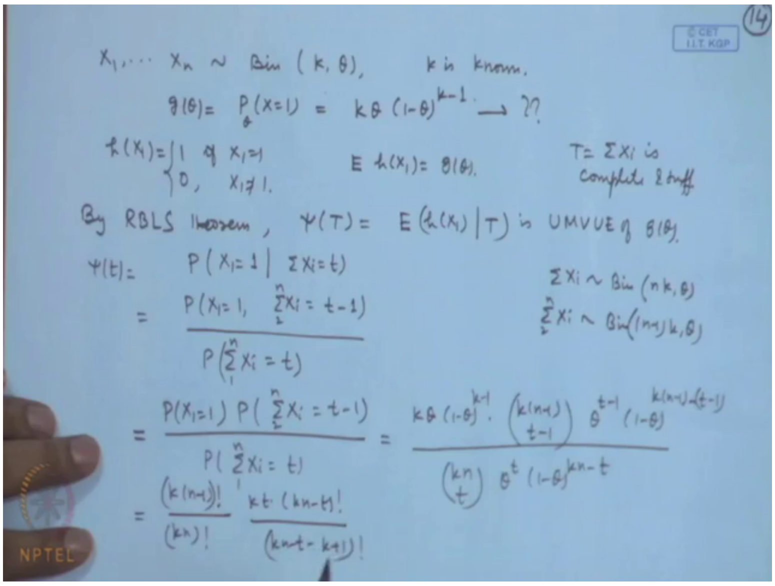

So, we conclude that by Rao-Blackwell-Lehmann-Scheffe theorem we conclude that N by n x is U M V U E of M. Let me give 1 more application. Let us consider say a random sample from a binomial distribution with parameter say k and theta where k is known. Let us define a function say g theta is equal to probability of 1 that is k theta into 1 minus theta to the power k minus 1. We want the unbiased estimator of this. Let us define a function say h X 1 is equal to 1, if X 1 is equal to 1, it is 0 if X 1 is not equal to 1.

Then expectation of h X 1 is equal to g theta. So by Rao-Blackwell-Lehmann-Scheffe theorem psi T that is here T is equal to sigma X i is complete and sufficient. So, that is equal to expectation of h X 1 given T this is U M V U E of g theta. So, we can consider here evaluation of this psi T function that will be equal to probability of X 1 is equal to 1 given sigma X i is equal to T that is equal to probability of X 1 is equal to 1 sigma X i from 2 to N is equal to T minus 1 divided by probability sigma X i 1 to n is equal to t.

Now, sigma X i will follow binomial n k theta sigma X i from 2 to n will follow binomial n minus 1 k theta. So, if we substitute these values here probability of X 1 is equal to 1 into probability of sigma X i 2 to N is equal to T minus 1, probability sigma X i is equal to T 1 to N, then that is equal to k theta into 1 minus theta to the power k minus 1 and then this is equal to k into n minus 1 c T minus 1 theta to the power T minus 1, 1 minus theta to the power k into n minus 1 minus T minus 1 divided by k n c T theta to the power T into 1 minus theta to the power k n minus t.

The terms which contain theta they get canceled out here and we are left with k into n minus 1 factorial divided by k n factorial into k T into k n minus t factorial divided by k n minus T minus k plus 1 factorial, So, if we consider this function here that is the U M V U E of g theta here. Let me end with 1 example in the exponential distribution.

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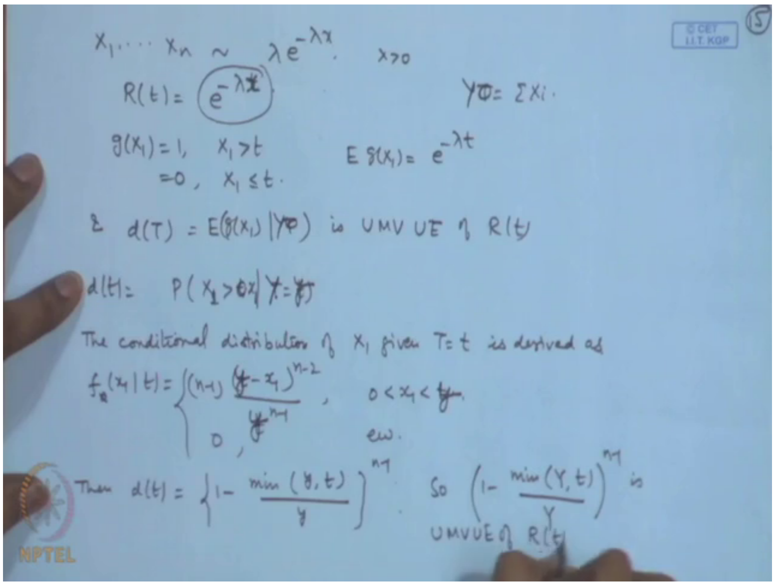

Suppose we have a random sample from exponential distribution with parameter say lambda. And we are looking at the reliability function R t is equal to e to the power minus lambda T, we want the U M V U E of this. So, define the function g X 1 is equal to 1 if X 1 is greater than T it is equal to 0, if X 1 is less than or equal to T. So, expectation of g X 1 is equal to e to the power minus lambda T and expectation of g X 1 given T that is equal to say d of T is U M V U E of R t that is for the reliability function the minimum variance unbiased estimator will turn out to be the conditional expectation of g X 1 given t.

So if I evaluate this that is nothing but, probability of X 1 greater than t given T is equal to t where T is equal to sigma X i here. Now, here we need the conditional distribution of X 1 given T. In the discrete case we are able to write down it as the joint probability divide by the probability of this term but, in the case of continuous distribution we cannot write that statement. So, what we do we divide derive the conditional distribution of X 1 given T and this distribution can be easily derived.

The conditional distribution of the conditional distribution of X 1 given T is equal to t is derived as f of X 1 given t is equal to t minus X 1 to the power n minus 2 divided by T to the power n minus 1 into n minus 1, 0 less than X 1 less than t. It is equal to 0 elsewhere. So, this probability of X 1 greater than t then turns out to be simply 1 minus minimum of X 1 greater than y so that is equal to the conditional probability of X 1 greater than T given T is equal to t turns out to be simply minimum of T and so there is a confusion here. I should have used a different notation here X 1 here so this turns out to be, there is a problem. Let us use a different notation y here and this is y, this is y is equal to say small y, so this is y here y.

So, then this will be equal to minimum of y and t divided by y to the power n minus 1. So, we conclude that 1 minus minimum of y and t divided by y to the power n minus 1 is U M V U E of reliability function in the case of exponential distribution. So, we have seen here today that the properties of sufficiency and completeness are extremely helpful in determining the problem of or solving the problem of minimum variance unbiased estimation. Essentially, it reduces the problem to find out the unique unbiased estimator which can be then easily determined.

In the next class, we consider the different approaches to the estimation. There is a approach of invariance and then and minimax estimation. I will be introducing in the next classes.