Statistical Inference
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Lecture No. # 23
Application of NP Lemma

In the last lecture, I have introduced the concept of most powerful test of a statistical hypothesis. And we were developing the Neyman Pearson theory; in that first result was the so called Neyman Pearson fundamental lemma. And this test gives the most powerful test for testing simple null hypotheses against a simple alternative hypothesis. As an example, I had given the normal distribution testing for the mean. Today, I will discuss various other applications of this Neyman Pearson lemma, and how then it can be extended to cover the cases, when we may have composite hypothesis in the null hypothesis or in the alternative hypothesis. So, we will consider these applications today.

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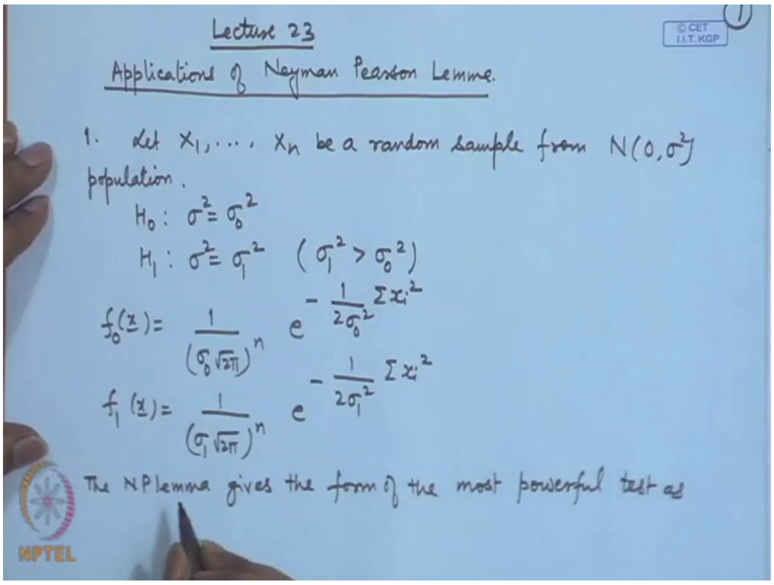

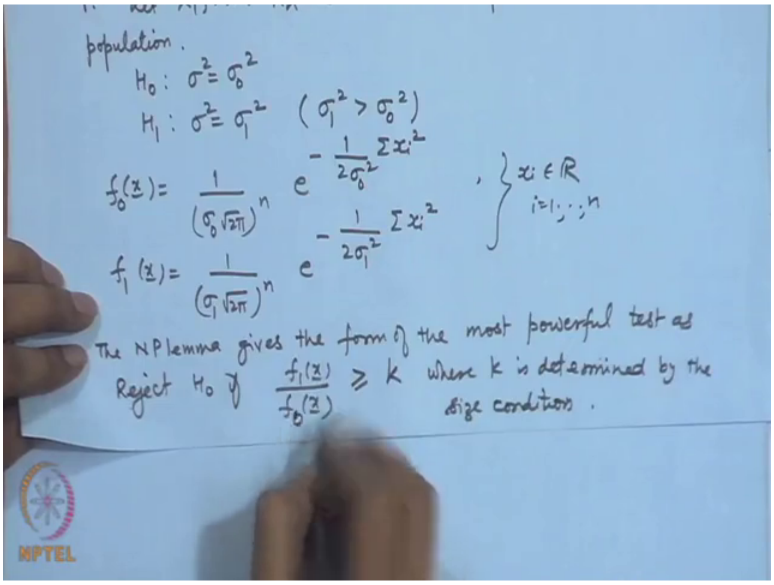

So, let me start with suppose we have a say x 1, x 2, x n. Let x 1, x 2, x n be a random sample from say normal 0 sigma square population. So, we were interested in testing the say a null hypothesis sigma square is equal to say sigma naught square against say sigma is square is equal to sigma 1 square. Now sigma 1 is square is not equal to sigma naught square. So, let us consider say the case sigma 1 square is greater than sigma not square.

So, in order to consider the application of the Neyman Pearson fundamental lemma, we should write down the distribution, which is the joint density of a x 1, x 2, x n under the null hypothesis and the alternative hypothesis, we call it f naught and f 1. So, f naught x that is equal to 1 by sigma naught root 2 pi to the power n, e to the power minus 1 by 2 sigma naught square sigma x i square. So, f 1 x will then be equal to 1 by sigma 1 root 2 pi to the power n e to the power minus 1 by 2 sigma 1 square sigma x i a square.

Now, the Nyman Pearson lemma gives the form of the most powerful test as. So, we will consider the rejection region this is continuous distribution, if we remember the form of the Nyman Pearson lemma, the form of the test function I will recollect here. It is given in this particular fashion. The form of the test in the Nyman Pearson lemma is given by reject h naught when f 1 x is greater than k times f naught x. And accept when f 1 is less than k f naught and we are considering the rejection with probability gamma there is a constant here when f 1 is equal to constant times f naught x.

Now, in the case of continuous distribution this probability will be 0. The probability of this occurrence therefore, we do not have to write this thing, rather we can include the equality at

1 of the places either at the rejection are in the acceptance. So, for convenience I will include in the rejection region. So, the test is reject h naught if f 1 x by f naught x is greater than or equal to k, where k is determined by the size condition. So, let us write down this f 1 by f naught x greater than or equal to k. Since these densities are valid for whole real line that is this x i is belong to r for phi is equal to 1 to n. So, this ratio is defied for all values of x 1, x 2, x n on the real line.

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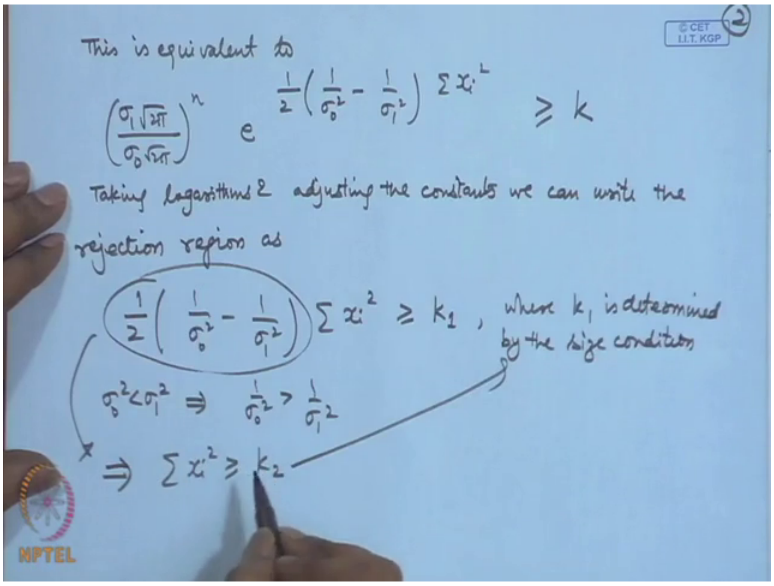

So, we write the region as this is equivalent to. So, you will have sigma 1 by root 2 pi divided by sigma naught root 2 pi to the power n, e to the power 1 by 2, 1 by sigma naught square minus 1 by sigma 1 square sigma x i square greater than or equal to k. Now in this problem, sigma naught sigma 1 or known-constants; so I can adjust this here. I can also take log taking logarithms and adjusting the constants, we can write the rejection region as 1 by 2, 1 by sigma naught square by minus 1 by sigma 1 square sigma x i square greater than or equal to say k 1. I have changed the name of the constant here, because I will be adjusting this here and then I have to take the log here some other constant is coming.

Now, earlier we mention that k is determined by the size condition. So, we will say that k 1 is determined by the size condition. Now, note here we had sigma not square less than sigma 1 square. So, this means that 1 by sigma naught square is greater than 1 by sigma 1 square. Now again this is a constant. So, I adjust this here; so this is equivalent to saying sigma x i square is greater than or equal to some constant say k 2.

Now, let us look at the determination of k 2. So, if k 1 is determinant by size condition then k 2 is also determined by the size condition. Now in order to determine this k 2 we need the probability of rejecting h naught when it is true and we will put it is equal to alpha. So, let us look at this. So, basically what we need here is the distribution of sigma x i square, because when we consider the probabilitity statement here, this will involve the distribution of sigma x i.

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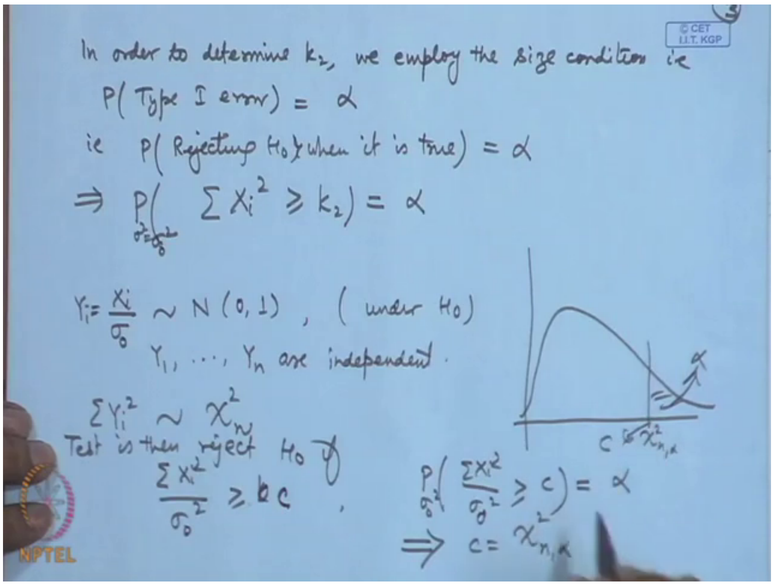

So, we write it like this in order to determine k 2 we employ the size condition that is probability of type 1 error is equal to alpha that is probability of a rejecting h naught when h naught is true. So, when it is true that is equal to alpha. Now let us look at this, here we are saying sigma x i square greater than or equal to k 2. When the distribution is sigma naught square that is sigma square is equal to sigma naught square this should be equal to alpha. Consider here the original random variables x i's we had considered a random sample from normal 0 sigma is square.

So, if you consider x i by sigma that follows normal 0 1 and they are independent let me call it y i. So, if we consider sigma naught here, then under h naught x i by sigma naught that is y i this will follow normal 0 1 and y 1, y 2, y n are independent. So, if we consider sigma y i square that will follow chi square distribution on n degrees of freedom. So, this test then we can consider as sigma x i square by sigma naught square greater than or equal to some c for example, test is the n reject h naught if sigma x i square by sigma naught square is greater than are equal to k 2 by sigma naught square which I write as c here.

Now, if we want probability of this sigma x i square by sigma naught square greater than or equal to c. When sigma naught square is the 2 parameter value, if we want this probability to be alpha then this implies that c should be chi square n alpha; that means, if we consider the curve of chi square distribution, then chi square n alpha that is this probability should be equal to alpha. So, c is here this point will be c.

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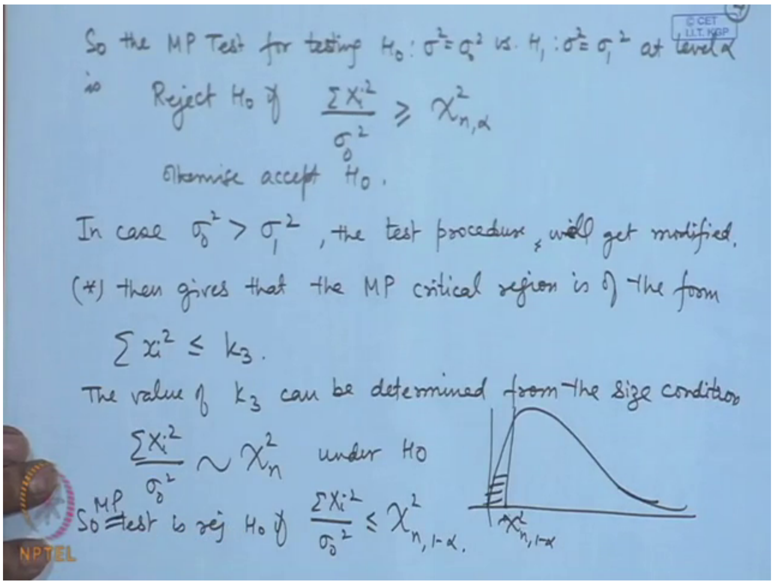

So, the test is then becoming. So, the most powerful test for testing h naught sigma is square is equal to sigma naught square against h 1 sigma is square is equal to sigma 1 is square, at level alpha is reject h naught if sigma x i square by sigma naught square is greater than or equal to chi square n alpha. Otherwise accept h naught that is we do not reject h naught here.

Now, I will consider one variation in this problem here. Here I have considered sigma one square greater than sigma naught square. Accordingly our test is rejecting for larger values of sigma x i square. On the other hand, suppose I change here, in place of sigma 1 is square i takes sigma 1 is square less than sigma naught square, if I do that then you look at the derivation of the test procedure, this quantity will become negative. If sigma not square is greater than sigma 1 square then 1 by sigma naught square will become less than 1 by sigma 1 square; that means, this quantity will be become negative. Then the test procedure will get reversed, we will get sigma x i square less than or equal to some constant say k 3. And therefore, incase sigma naught square is greater than sigma 1 square the test procedure will get modified.

So, for example, you may consider, let me call this condition as a star. A star then gives that the most powerful critical region is of the form sigma x i square less than or equal to say k 3. And as before the way we have derived the probability of type one error is equal to alpha that will give me the value of k 3.

So, in that case what will happen? The value of k 3 can be determined from the size condition. Now once again we will have sigma x i square by sigma naught square that will follow chi square n under h naught. So, now, what is happening is that we need this less than or equal to quantity. So, this will become chi square n 1 minus alpha.

So, test is reject h naught, if sigma x i square by sigma naught square less than or equal to chi square n 1 minus alpha. So, this is the most powerful test m p test. So, here you have seen that how the application of Neyman Pearson lemma is helpful in deriving the most powerful tests for a fixed size; that means, then we are fixing the probability of type one error the most powerful test is giving me the exact method of deciding whether to accept are reject a null hypothesis. In this particular example you see exactly we are getting the observations are x 1, x 2, x n.

So, given the observations you calculate sigma x i square by sigma naught square and compare it with that tabulated value of chi square n alpha. Suppose alpha is equal to say 0.05 and n say 10, then you consider the corresponding value of chi square 10 variable on 0.05. This value will be given the tables of chi square distribution and we are in a position to take an exact decision. On the other hand, we may also consider the p value; that means, what is the value of alpha for which we will be rejecting. What is a minimum value of alpha? So, in case if alpha is not a specified beforehand then we can consider the minimum value that and we can base our scientific decision on that fact, that this kind of situation occurs for example, in many medical problems are clinical trials were, we we may have to take a decision based on the given circumstances.

So, we need not fix alpha in advance this point about p value had mentioned earlier when I was giving the basic concepts here. So, that can be done for almost all the test of this nature, that we can consider actually the p values. Now a part from the normal distribution let me also give a applications to other distribution such as exponential distribution double exponential distribution or we may not even be able to write down the form in a closed fashion we may have f naught as one density f 1 as other density. So, I will consider few examples and exhibit that this Nyman Pearson lemma in each of this cases gives solution; that means, we are in a position to take a decision whether to accept are reject a null hypothesis when the cases are simple verses simple let us considers say exponential distribution.

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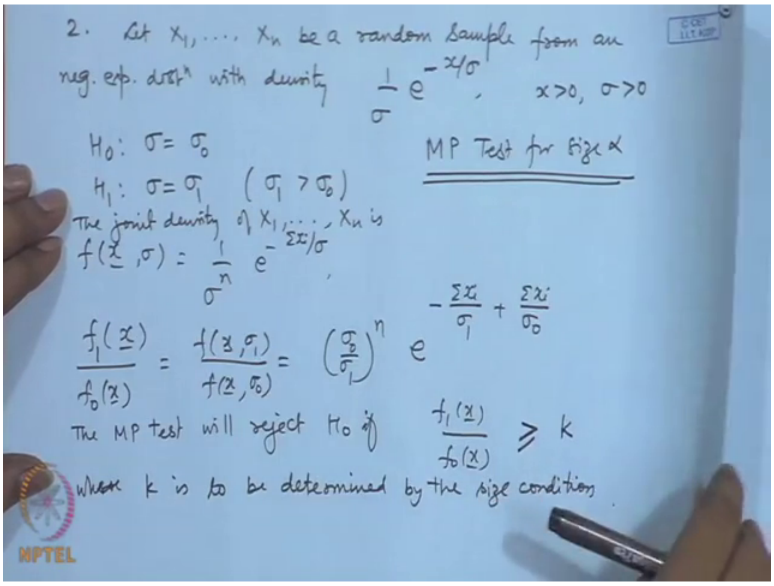

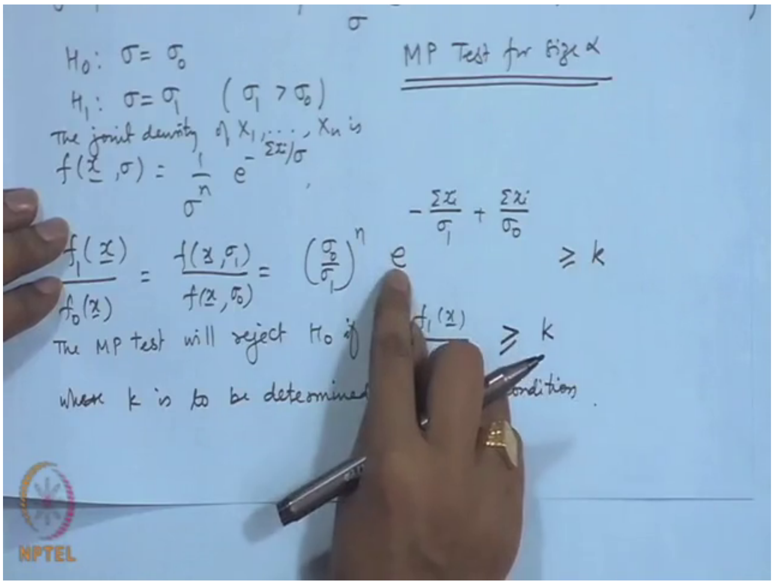

So, let x 1, x 2, x n be a random sample from an negative exponential distribution say with density function 1 by sigma e to the power minus x by sigma, x is positive sigma is positive. Let us consider say hypothesis testing problem say sigma is equal to sigma naught against, sigma is equal to sigma 1. And once again for convenience let us consider in the beginning say sigma one is greater than sigma naught. We want the most powerful test for given size alpha, we will use the Neyman Pearson lemma for determination of this.

So, let us consider the form of the joint distribution of x 1, x 2, x n joint density of x 1, x 2 ,x n is given by f x sigma, so 1 by sigma to the power n, e to the power minus sigma x i by sigma. Note here that for all x i positive this densities positive therefore, we can consider the ratio that is f 1 x by f naught x that is the densities corresponding to sigma 1 and sigma naught value of the parameter. So, when you write down the ratio you will get a constant here sigma naught by sigma 1 to the power n; and then e to the power minus sigma x i by sigma 1 plus sigma x i by sigma naught.

So, the most powerful test will reject h naught, if f 1 by f naught is greater than k. Where k as the determent from the size condition, once again a point to be noted here is that we are dealing with the continuous distributions. So, the probability of equality is 0 that is this is equal to k. Therefore, we may include rejection revision this equality point here, we may put it in the acceptance region also, and it does not makes an any difference in the nature of the test, because the probability of equality will be 0.

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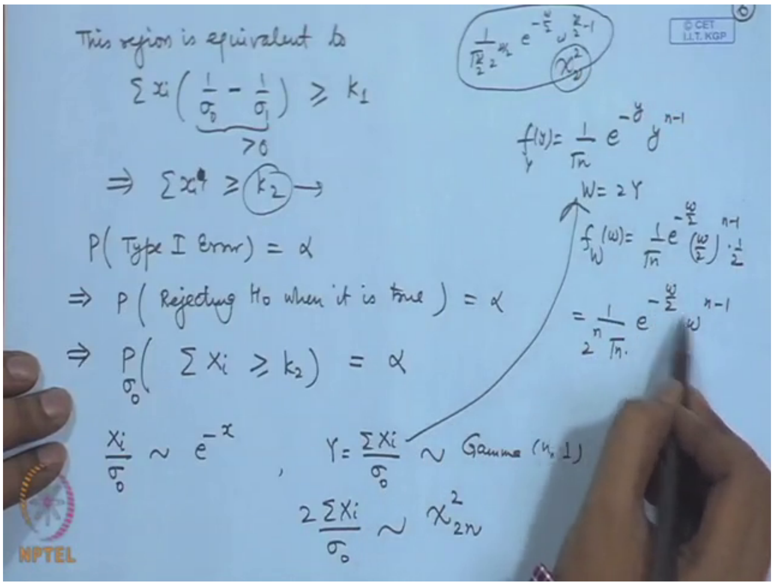

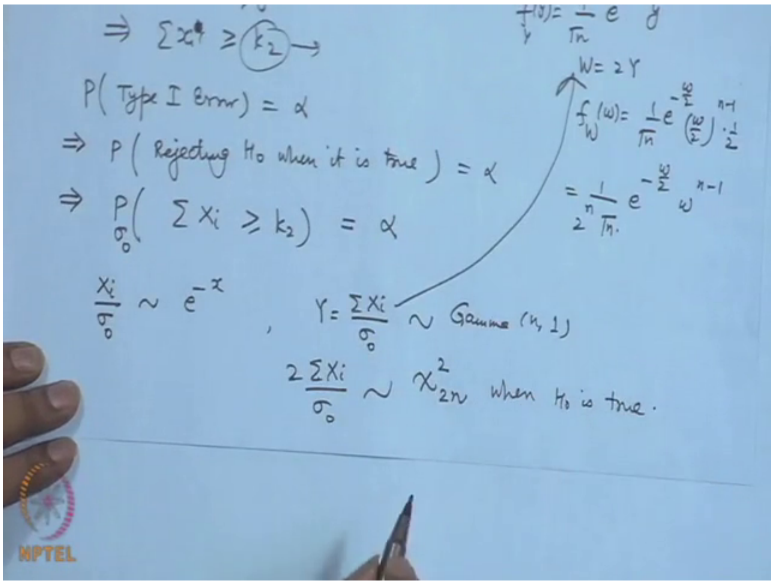

So, where k is 2 be determined by the size condition. So, if you consider this ratio here I am saying this greater than or equal to k. Now this is the constant sigma naught and sigma 1 or non. So, I can adjust this with coefficient on the right hand side, and I can also take logarithm here. If I take the logarithm here I will get sigma x i into 1 by sigma naught minus 1 by sigma 1. So, this region is equivalent to sigma x i 1 by sigma naught minus 1 by sigma one greater than or equal to some constant k 1. Now as before in the normal distribution case, this constant 1 by sigma naught minus 1 by sigma 1 the sign of this will be positive, because I am taking sigma naught to be less than sigma 1.

So, this is positive. So, this region is equivalent sigma x i greater than or equal to some k 2. And once again this k 2 is to determine from the size condition. So, if I consider probability of type one error equal to alpha; that means, probability of rejecting h naught when it is true that is equal to alpha then this is implying probability of sigma x i greater than or equal to k 2 when sigma naught is the true parameter value then it is equal to alpha; that means, I need to look at the distribution of sigma x i when sigma is equal to sigma naught.

Now, we know that the some of independent exponentials of this nature is actually a gamma. So, we can consider the derivation of the constant k 2 based on this. So, let us look at this if I consider say x i by sigma naught, then that will follow exponential with parameter simply 1. If I consider say sigma x i by sigma naught, then that will follow gamma n 1. If I consider twice sigma x i by sigma naught, then that will follow chi square distribution on 2 n degree of freedom; see we can write down the density here suppose I am considering this as say y. So, what is the distribution of y? f y is equal to 1 by gamma n e to the power minus y, y to the power n minus 1. So, if I consider say w is equal to 2 y then what we will the distribution of w. 1 by gamma n e to the power minus w by 2, w by 2 to the power n minus 1 into half that is equal to 1 by 2 to the power n gamma n e to the power minus w by 2 w to the power n minus 1.

So, if we consider the form of a chi square distribution. The chi square distribution on nu degrees of freedom is given by 1 by gamma nu by 2, to 2 the power nu by 2 e to the power minus w by 2, w to the power nu by 2 minus 1 this is the form of a chi square distribution on a nu degrees of freedom. So, if you compare this with this actually we are getting 2 n degrees the freedom. So, chi square twice sigma x i by sigma naught this will follow chi square distribution on 2 n degrees of freedom when h naught is true. Therefore, the rejection region can be return in the terms of chi square value on 12 degrees of freedom.

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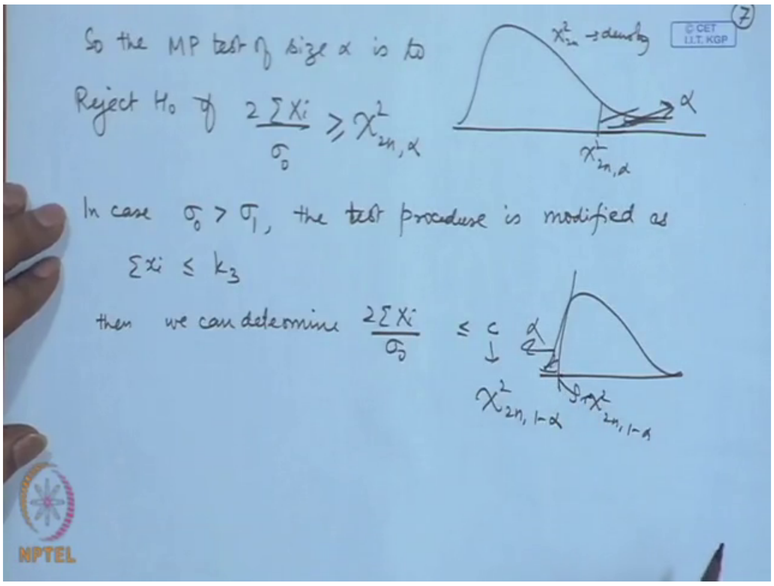

So, if we consider say chi square 2 n density. So, this point is chi square two n alpha. So, this probability is alpha say. So, the most powerful test of size alpha is to reject h naught if twice sigma x i by sigma naught is greater than or equal to chi square 2 n, accept otherwise. So, you can easily see here, that we are able to give exact decision making procedure given the level of significance. Now, if the level of significance is not a specified in the beginning, then you can look at what is the probability of this that minimum level at which this test will be rejected this null hypothesis will be rejected. So, that will be the p value. So, I have been I am considering both of this p value thing and level of significance fixed level of significance in all this situations. Once again note here that if I have a modification in my original null hypothesis, in place of sigma on being greater than sigma naught, if sigma 1 is less than sigma naught, then there will be modification here, because this coefficient will be become negative.

If this coefficient becomes negative then the region will turn out to be sigma x i less than or equal to something here. And therefore, the rejection region will then become left handed. In case sigma naught is greater than sigma 1. The test procedure is modified as sigma x i less than or equal to say k 3. Then we can determine sigma x i by sigma naught twice less than or equal to say c. So, c will become then equal to chi square 2 n 1 minus alpha, because now this is the left handed point here this probability is alpha. So, chi square 2 n 1 minus alpha here. Now, you can see here that in many of this problems we are able to work out the exact distribution here, and one interesting thing here is that the range of the random variables is the same therefore, this writing down the ratio f 1 by f naught etcetera is quite convenient, and when we write down the final test function here then we are able to derive the distribution of that.

Now, in many cases this will be dependent upon the situation we may not have state forwardly the full region divided by full region. We may have partial regions sometimes the range of the variable will be dependent upon the parameter. Therefore, the range of the 2 densities may not be exactly the same; I will explain this through a couple of examples. So, let me take case for when the full region is the same, but the distribution gets the form of the density gets modified midway; that means, for partial values of x you have form of density function for another part we may have another density function. So, let me take up this case and I will also consider one case when the range of the variable is dependent upon the parameter therefore, the 2 densities are positive not on the full region, but on partial regions.

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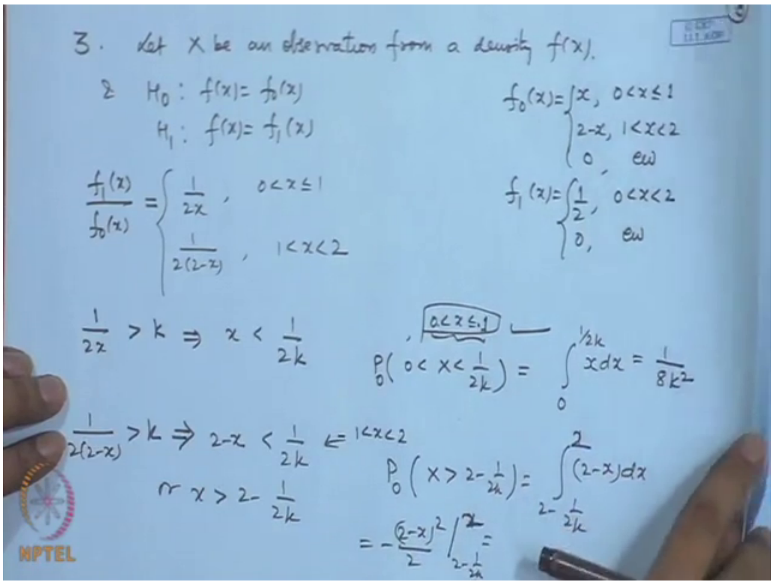

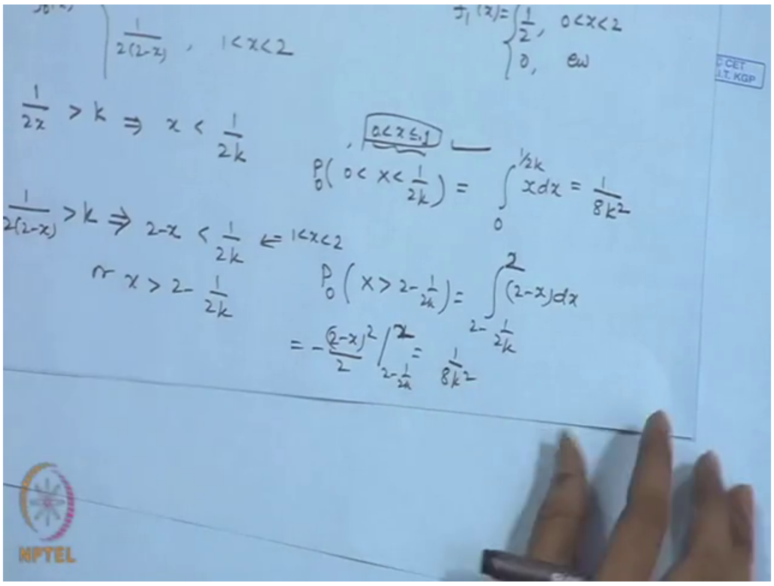

So, let us consider these cases. Let x be an observation from a density f x and h naught f x is equal to f naught x h 1 f x is equal to f 1 x. And f naught and f 1 are defined like this, f naught is the triangular distribution, it is equal to x for 0 less than x less than are equal to 1, and it is equal to 2 minus x for 1 less than x less than 2. It is actually the triangular distribution and of course, it is a 0 elsewhere. And f 1 x is half for 0 less than x less than 2. So, this is nothing, but the uniform distribution on the interval 0 to 2. Now, you note here the distribution under h not is a distribution over the range 0 to 2, but the form of the density function changes at the 0.1; whereas, the second density is having the same for throughout. So, and we write down the form of the most powerful critical region using the Nyman Pearson lemma we have to be careful in writing down the regions. So, for example, consider this f 1 by f naught. Here we assume that our decision making process based on 1 observation of course, we make consider n observation also and of course, it will increase the complex difficulty are you can say complication in the nature of the derivation.

So, this value is equal to now you look at f 1 by f naught that will be 1 by 2 x, if 0 is less than less than x less than or equal to 1. And it will be equal to 1 by twice 2 minus x, for 1 less than x less than 2. Now the question is if an a x is there which is outside this region the thing is that under h naught and h 1 that will have probability 0. So, we will naught consider that situation here. So, if I consider the rejection region 1 by 2 x greater than k. Then this is equivalent to saying x is less than one by 2 k. Now this is for the portion 0 less than x less than or equal to 1. So, if we consider probability of this region that is 0, less than x less than 1 by 2 k, this is for under h naught and here we will consider for 0 to 1 only, for 0 to 1 the densities x. So, if you integrate this it is becoming x square by 2.

So, you will get 1 by 4 k square divided by 2 that is 1 by 8 k square; if we consider 1 by twice 2 minus x greater than k, then this is equivalent to 2 minus x less than 1 by 2 k; or x is greater than 2 minus 1 by 2 k; now, these (( )) for 1 less than x less than 2. So, the probability of x greater than 2 minus 1 by 2 k, that is equal to 2 minus 1 by 2 k 2 1 2 minus x d x. So, that is equal to 2 minus x whole square by 2 with the minus sign from 2 minus 1 by 2 k 2 1. So, this is again evaluated if you put here, no sorry this is up to 2. So, if you look at the evaluate 2 this is becoming 0 and when we put 2 minus 1 by 2 k this is again 1 by 2 k whole square. So, it is again one by 8 k square. So, it is again 1 by 8 k square.

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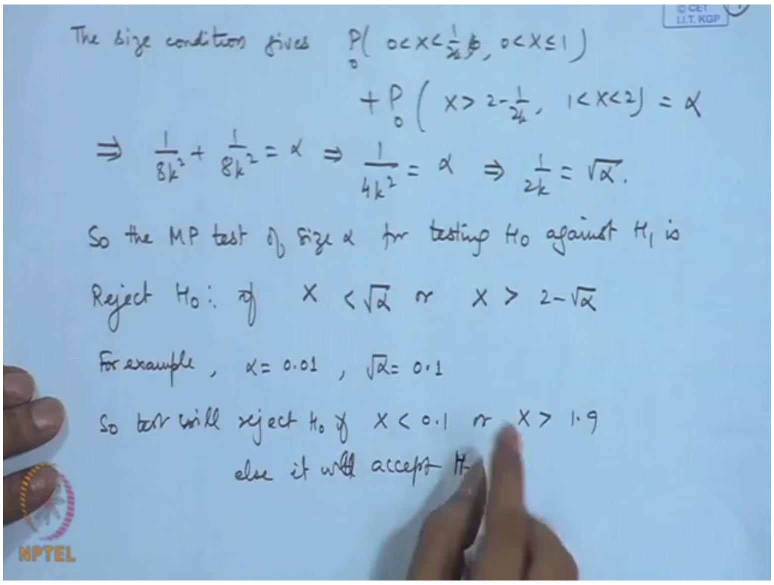

So, if we write down the size condition here, that is the probability of. So, the size condition gives probability of type 1 error that is 0 less than x less than 1 by 2 k, for 0 less than x less than or equal to 1, plus x greater than 2 minus 1 by 2 k, for 1 less than x less than 2 is equal to alpha note here that these regions are dependent upon this conditions. So, we have to consider the probability under this we have calculated both of this probability. So, it is becoming 1 by 8 k square plus 1 by 8 k square is equal to alpha, 1 by 4 k square is equal to alpha; that means, 1 by 2 k is equal to square root of alpha.

So, the region of rejection is becoming x is less than root alpha r is x is greater than 2 minus root alpha. So, the most powerful test of size alpha for testing h naught against h 1 is reject h naught, if x is less than root alpha or x is greater than 2 minus root alpha. Once again you note here that we are able to provide exact test here, that is the test, tells exactly what decision 1 has to take given a value of x.

So, for example, let us choose alpha is equal to say 0.01. Then alpha is equal to root alpha will become 0.1. So, test is then test will reject h naught if x is less than 0.1 or x is greater than 1.9, else it will accept h naught; that means, if I am having an observation between 0.1 to 1.9 then the test will accept h naught; that means, it will have no reason to reject h naught. On the other hand if x is less than 0.1 or x is greater than 1. 9 then this is not supporting h naught; that means, you will 10 to reject h naught here. In this particular example I have shown that even the form of the distribution may be changing over the range of the sample space;

however, the Neyman Pearson lemma is able to provide exact test at a given size. Let me take another example, in which the range of the variable may be dependent upon the range of the parameter and let us see in that case how the Nyman Pearson lemma works.

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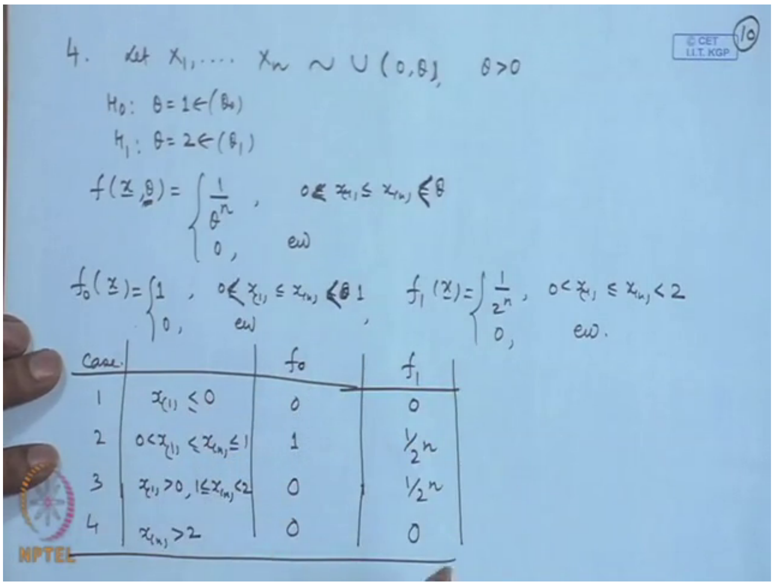

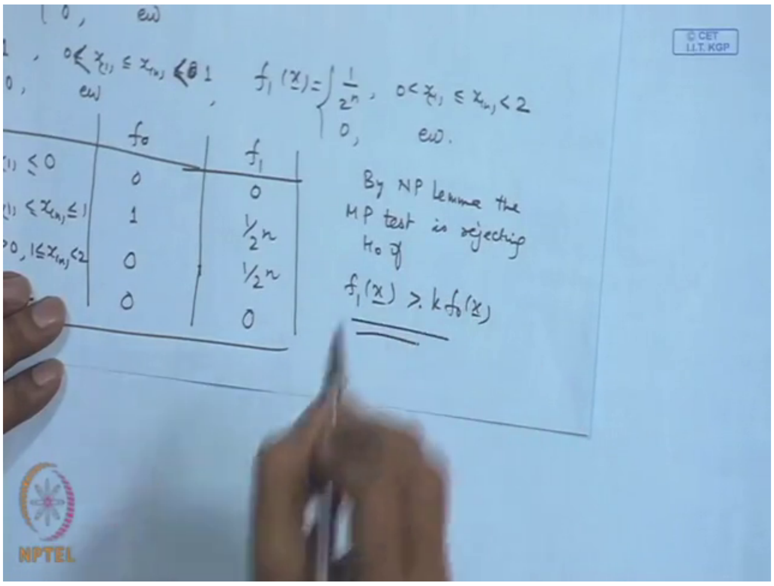

Let us consider say x 1, x 2, x n from uniform 0 theta distribution. And we consider a hypothesis testing problem say theta is equal to say 1 against say theta is equal to 2. We may also write here, say theta naught and here, I may write theta 1 and then I may consider the case theta naught less than theta 1 or theta naught greater than theta 1. So, for convenience we have considered this is special case, which theta is equal to 1 and theta is equal to 2. So, let us consider the most powerful test here. So, the joint distribution is 1 by theta 2 the power n, and here the range of the variables from 0 to theta. So, we write it in this particular form. So, when we write for f naught and f 1, for f naught this is simply 1. So, this is simply see we may if we write here open and travel we need not put equality here, we may put it like this, otherwise may out it quality the probability of those points will be 0.

So, it does not make any difference similarly, if we consider f 1 then under f 1 theta is equal to 2. So, it will become 1 by to 2 the power n 0 less than x 1, less than or equal to x n, less than 2 it is equal to 0. I was mentioning here, that the range of the densities where the 2 densities are positive is not the same. Here you can see this density is positive for 0 to 1 and this densities positive for 0 to 2.

So, let us look at the various cases of f 1 and f naught. So, we will make it in the form of a table let us consider say case 1, 2, 3, 4 like that we will write 1, 2, 3, 4. So, I will write all the cases which we may be trivial or nontrivial cases. If we observe x 1 to be less than 0; obviously, this is not possible. So, both the densities f 1 and f naught they are 0 here. If we consider the case 0 is less than. So, x 1 and x n is less than or equal to 1, in this case the first density 1, and second density 1 by 2 to the power n. Then we may have say x 1 of course, may be greater than 0, but x n is say beyond it is beyond 1 in that case what will happen that this first density becomes 0.

However the second density remains 1 by 2 to the power n. And then we may have the extreme case that is x n greater than 2, then this is 0 and this is 0. So, broadly speaking we have to consider the ray f 1 greater than k f naught from the Neyman Pearson lemma under these four cases. By n p lemma the m p test is rejecting h naught if f 1 x is greater than k times f naught x. So, here the values of k we have to choose.

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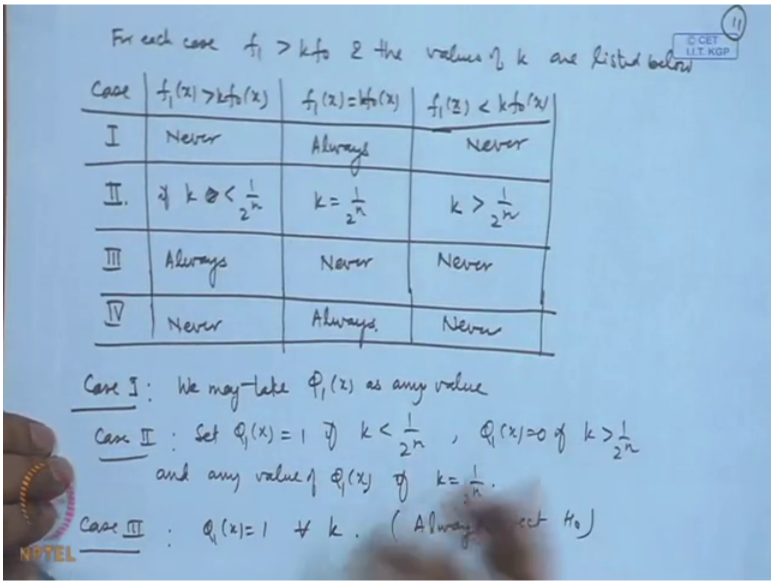

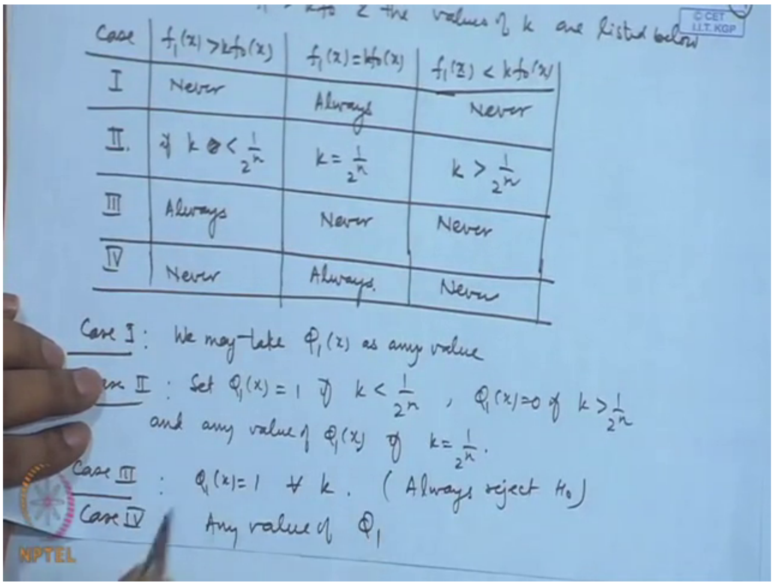

So, let us write the here, for each case f 1 greater than k f naught and the values of k are listed below. So, let us consider each case. In the case 1 if we consider f 1 x greater than f naught x. Now both the values are 0. So, this is never possible whatever be the value of k it is never possible, f 1 x is equal to f k times f naught x, that is always true. And f 1 x less than k f naught x is never possible. Let us consider second case in the second case, if we look at f 1 that is 1 by 2 to the power n, greater than k times f naught. So, this condition is true, if case greater than sorry k is less than 1 by 2 to the power n, this is true if k is equal to 1 by 2 to the power n, this is true if k is greater than 1 by 2 to the power n. Let us look at the 3rd case in the 3rd case f naught is 0.

So, f 1 greater than k f naught is always true. And therefore, this equality is less than is never possible. Let us consider a case 4 once again both of them are 0. So, inequality is never possible whereas, the equality is always true. So, now based on this we should tell when to rejected h naught and when accept h naught; that means, dependent upon these 4 cases and the choices of k, we should give what is the test function and at the same time, we should also tell that whether the probability of type 1 error is equal to alpha can be achieved for a given value of alpha. So, what we consider in case 1 since f 1 is equal to k f naught is always possible always true therefore, whatever with the value of phi 1 it does not make any difference.

So, we may take phi 1 as any value. In case 2 if k is less than 1 by 2 to the power n, in this case f 1 is greater than k f naught; that means, this is the corresponding case 2 rejecting h naught. So, if k is less than 1 by 2 to the power n we will say reject h naught and in this case we will say accept h naught; that means, phi 1 is equal to 0. However when k is equal to 1 by 2 to the power n, then we may again say a we may accept are reject depends upon we can assign something of course, a probability of this cases will be 0. So, we can say phi 1 that is the test function is equal to 1, if k is less than 1 by 2 to the power n, it is equal to 0. If k is greater than 1 by 2 to the power n, and any value of phi 1, if k is equal to 1 by 2 to the power n.

If you look at case 3 in the case 3 this condition is always true. So, we always reject that is phi 1 x is equal to 1, whatever be k that is always reject h naught. Note here that these are also heuristic, because what is happening if we are getting the 3rd case that is x n is between 1 and 2; that means, naturally the observations are from the density uniform 0 to 2, otherwise observation cannot be greater than 1, and therefore, we should definitely reject h naught and accept h 1, And in the case, four once again we may put any value any value of phi 1.

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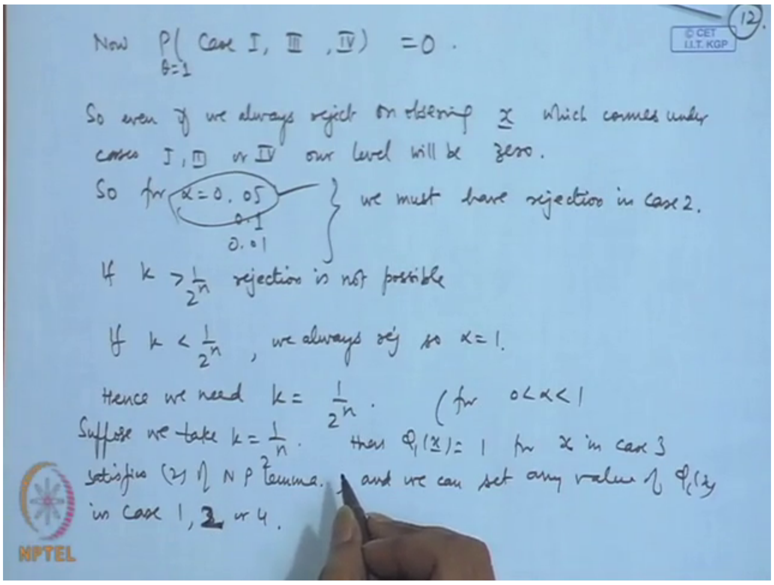

Now, probability of case 1, 3 and 4 under the null hypothesis this is 0. So, even if we always reject on observing x which comes under cases 1, 3 or 4 our level will be 0. So, for alpha is equal to 0.05 etcetera some given value of alpha are say 0.01 or 0.1 etcetera, then what we should do, we should make the probability of rejection in case 2 to be possible. So, we must have rejection in case 2. Now again if I take k to be greater than 1 by 2 to the power n rejection is not possible.

So, the only rejection is possible for k less than 1 by 2 to the power n here rejection becoming always true. So, alpha will become 1 which is not acceptable. So, therefore we should have this k equal to 1 by 2 to the power n as a possible value. If k is greater than 1 by 2 to the power n rejection is not possible. If k is less than 1 by 2 to the power n we always reject. So, alpha is equal to 1 therefore, we should have k equal to 1 by 2 to the power n for 0 less than alpha less than 1.

So, suppose we take k equal to 1 by 2 to the power n, then phi 1 x is equal to 1 for x in case 3 satisfies 2 of n p lemma, and we can set any value of phi 1 in case 1 2 ,1 3 or 4 sorry 1, 2 or 4. Let me give complete case here, when I base our decision on x n alone. You note here that when I am considering x 1, x 2, x n random sample from you know from 0 to theta, the sufficient statistics actually x n, and x n is actually playing the role here as you have already noticed here.

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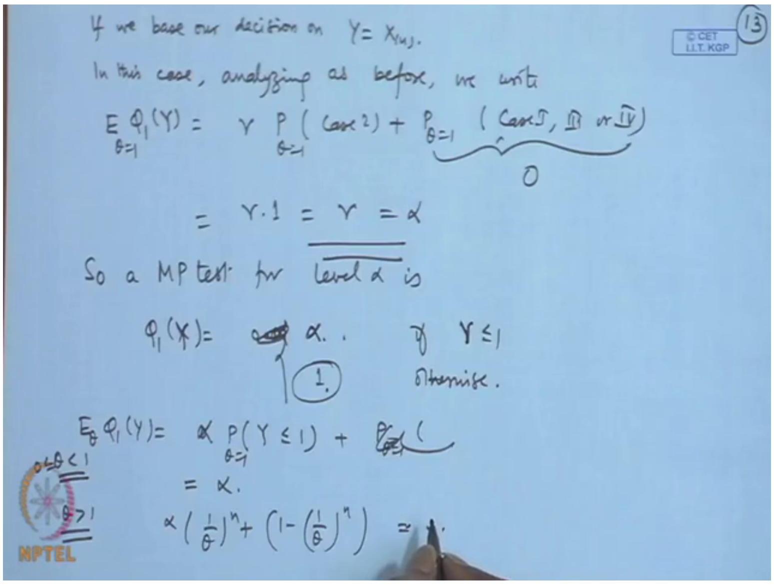

So, let me explain that part in detail. If we base our decision on say y is equal to x n. So, in this case what is happening that? Let me write here, in this case analyzing as before we write expectation of phi 1 y for theta is equal to 1, as some gamma times probability of case 2, under theta is equal to 1 plus probability of theta is equal to 1, under case 1, 3 or 4 now these are all 0. So, that is equal to gamma into 1 that is equal to gamma.

So, we should choose gamma is equal to alpha. So, a most powerful test for level alpha is phi 1 x or phi 1 y is equal to 0.0 that is alpha. If y is less than or equal to y that is equal to g 1 otherwise; that means, what we are saying reject all the time, accept for the case when y is less than or equal to 1, if y is less than or equal to 1, then you are rejecting with probability 0.05 and otherwise you are accepting. We may also consider the power function here. So, for example, power function here that is equal to alpha into probability of say y less than or equal to 1 plus probability. So, here I am taking theta is equal to 1 into. So, those cases will not occur this will have probability 0. So, this is actually equal to alpha. For this is theta less than 1; and if I take theta greater than 1 then it will become equal to alpha into 1 by theta to the power n plus 1 minus 1 by theta to the power n.

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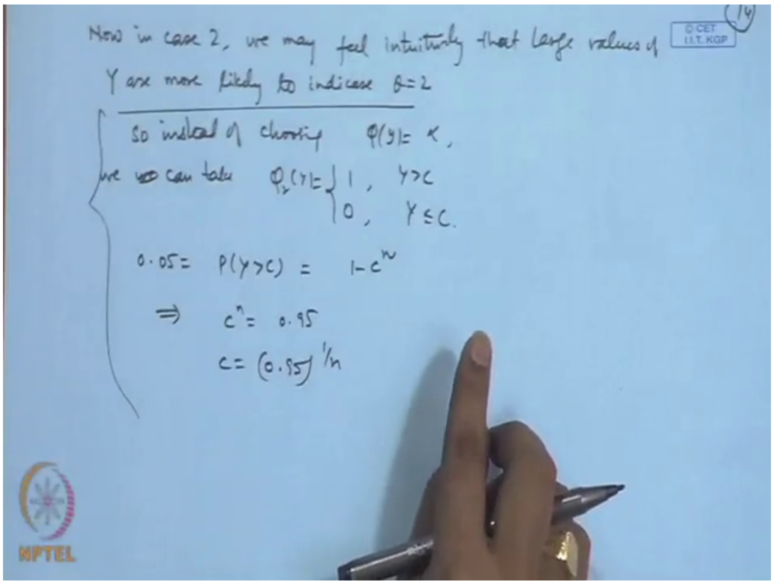

So, that is equal to well we can simplify this. So, what we are able to do is that we are able to provide an exact test here, for testing parameters in the uniform distribution. We may also consider in a slightly different fashion. Let me just explain it here, case 2 we may feel intuitively that large values of y are more likely to indicate theta is equal to 2. So, instead of choosing phi y is equal to alpha, we can take say phi 2 i is equal to 1 if y is greater than c, it is equal to 0 of y is less than or equal to c; and if we consider the probability of this. So, we are getting here say 0.05 is equal to probability of y greater than c, that is equal to 1 minus c to the power n this means, we can take c to the power n is equal to 0.95 or c is equal to 0.95 to the power 1 by n. So, this is an alternative solution here, of course, this is based on heuristic consideration that large values of alpha of y are more likely to. So, this part is not coming from n p lemma in the n p lemma if we write exactly we will take that part and the test function is of this nature, that phi 1 y is equal to say alpha if y is less than or equal to 1 and it is 1 otherwise.

So, these are the two forms that have been considering here, friends today we have considered in detail, various application of the Neyman Pearson fundamental lemma. How it gives exact tests for testing simple hypothesis verses a say simple hypothesis. The important point that you should note here, is that we need the distribution of the criteria; that means, our criteria is based on certain function of the random variable, which we call test statistic. We should be able to say something about the distribution of that under the null hypothesis then only the constant k can be determined. If we are unable to determinant that then we will not be able to provide the exact form of the test function

So, in the next lecture, as I mentioned, we will consider extension of the Neyman Pearson lemma to consider the composite hypothesis also. So, in particular, we will consider the one sided composite hypothesis testing problems, so that I will be taking of in the following lecture.