Statistical Inference

Prof. Somesh Kumar

Department of Mathematics

Indian Institute of Technology, Kharagpur

Lecture No. # 24

UMP Tests

So far the testing procedure that we have discussed was based on the Neyman Pearson fundamental lemma. The main assumption that we made in deriving the test procedure was that the null hypothesis and the alternative hypothesis both were considered to be simple; and in this case, when we fix the probability of type one error, then we were able to derive the test which is having the minimum probability of type two error or the maximum power, and we called it the most powerful test. However, in most of the real life situations, we do not come across the simple hypothesis verses simple hypothesis problems. In most of the complex situations, we have composite hypothesis.

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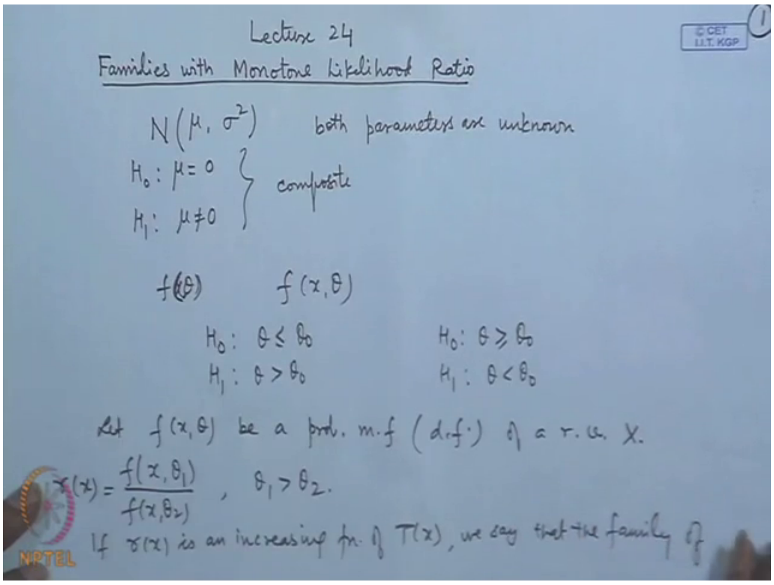

As a very simple case, we may have the family of distributions as normal mu sigma square distributions, and we may like to even now we may like to test something like, whether mu naught mu is equal to zero or mu is not equal to zero. Note here that, now this H naught is not a simple hypothesis; this is composite, because sigma is square is unknown. Here we have assumed both parameters to be unknown; both parameters are unknown. Therefore these are now composite both hypothesis are composite hypothesis. And therefore, the Neyman Pearson lemma does not help us to give a solution in this particular problem; that means, does not give a most powerful test.

The simplest composite hypothesis are of this nature, that we may have a one parameter family, say family of distribution with one parameter theta say effects theta. And we may like to test about, say h naught theta less than or equal to theta naught against say theta greater than theta naught, or alternative we may have say h naught theta greater than or equal to theta naught against h 1 theta less than theta naught.

Now, let us remember our cases that some examples we consider for the Neyman Pearson lemma, where we had considered theta is equal to theta naught against theta is equal to theta 1. I had consider two cases; one was theta naught less than theta 1, and another was theta naught greater than theta 1. When theta naught was less than theta 1, we got a one sided testing region; that is the rejection region, that is for larger values of expire we were rejecting h naught.

Now, in that problem, in place of theta 1 suppose we replaced by another value theta 2, suppose we replace by another value theta 3, the testing procedure remains the same as longer this second value in the alternative hypothesis remains larger than theta naught. In a similar way if we are considering the reverse case theta naught greater than theta 1, then the rejection region was for is smaller values of x bar, and once again if we replace this alternative hypothesis theta 1 in the same direction; that means, value which is greater than theta naught or less than theta naught, then the rejection region does not get effected. What does it mean? It means, that for those values we are getting the most powerful tests; that means, this normal distribution with one parameter, the second parameter sigma was considered to be known has certain property.

Now, in these situations for the changing values we get the maximum power at each of the values, this is called uniformly most powerful test. Now, this family distributions which will satisfy this property; that means, where we will get such test; it is having some particular name, it is call the family's with monotone likelihood ratio property. In particular, for the one sided testing of hypothesis problems like theta less than are equal to theta naught against theta greater than theta naught, or theta greater than or equal to theta naught against theta less than theta naught etcetera, For such cases we are actually getting the uniformly most powerful test. The result that are proved, their actually you can say they are extension of the Neyman Pearson fundamental lemma.

So Firstly, let me define this family's, so let f x theta be a probability mass function or density function of a random variable say x, let us write down the ratio f x theta 1 divided by f x theta 2, let us call this name; this ratio let me call it r x. And let us takes a theta 1 greater than theta 2, if r x is an increasing function of some variables say T x, then we say that the family of densities.

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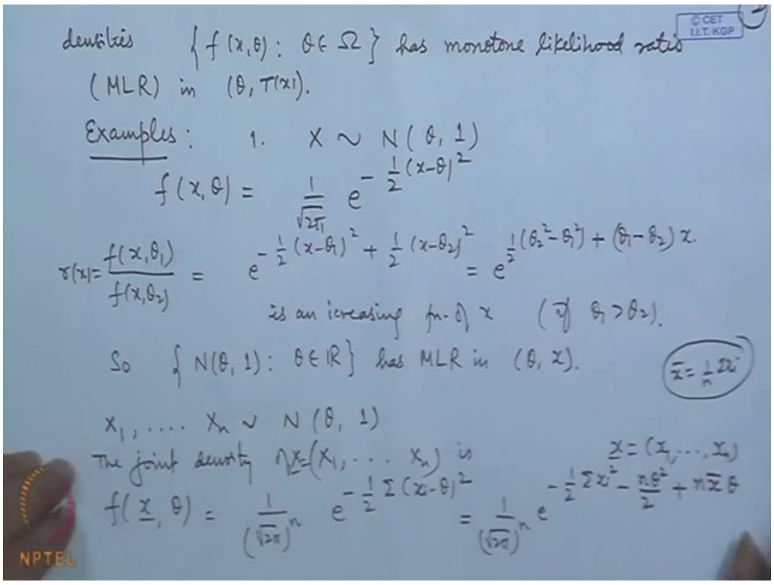

The word densities means, it in includes the probability mass functions. So, that is f x theta, theta belonging to the parameter is phase has monotone likelihood ratio, that we call MLR in theta T x. Let me given example here, let us consider say x following a normal distribution with mean theta and known variants 1, let us write down the distribution f x theta is 1 by root two pi e to the power minus half x minus theta square. Let us consider this ratio r x, that is f x theta 1 divided by f x theta 2, now when we write this ratio this gets cancelled out, and you have e to the power minus half x minus theta 1 square plus half x minus theta 2 square, that is equal to e to the power half theta 2 square minus theta 1 square, and then you will have plus theta 1 minus theta 2 x.

So, you can look at this, this is an increasing function if i am taking increasing function of x, if theta 1 is greater than theta 2 because this is constant, and if theta 1 is greater than theta 2 e to the power this becomes an increasing a function of x. So, this family of distributions normal theta 1; where theta belongs to real line, this has monotone likelihood ratio in theta and x.

Now, i have return here the distribution of one observation, suppose in place of x i have x 1 x 2 x n. suppose i have x 1 x 2 x n In this case, f x theta we have to write the joint distribution of x 1 x 2 x n, so the joint density of x 1 x 2 x n. So, let me give the notation f x, where x is standing for the values x 1 x 2 x n of capital X 1 capital X 2 capital X N. So, this becomes 1 by root 2 pi to the power n e to the power minus 1 by 2 sigma x i minus theta is square.

Let us simplify this, we can write it as 1 by root 2 pi to the power n e to power minus half sigma x i square minus n theta square by 2 plus, now you have the cross product term twice x i theta with minus sign, and minus minus will become plus and 2 will cancel out. So, you get twice n x bar theta, where x bar is the 1 by n sigma x i.

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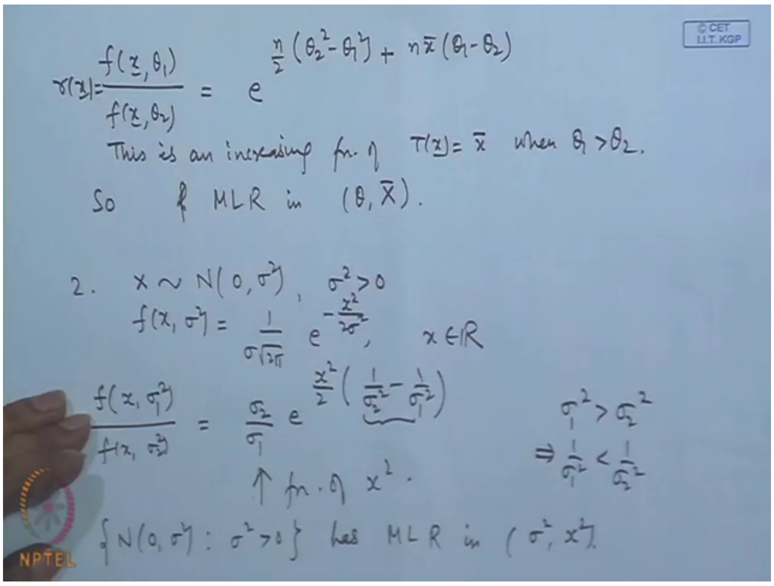

So, now you write down this ratio f x theta 1 divided by f x theta 2, that is turning out to be, now, when you write the ratio, this constant determinate get cancelled out e to the power minus half sigma x i square will get cancelled out, we will be left with e to the power n by 2 theta 2 square minus theta 1 square plus n x bar theta 1 minus theta 2. Now, this is constant, for theta 1 greater than theta 2 this becomes an increasing function of x bar, so this ratio and increasing function of T x is equal to x bar when theta 1 is greater than theta 2. So, this family of distribution normal theta 1, when we are having n observations, so we have MLR in theta and x bar we can say. Now, the similar thing we can observe for various distributions; let me give a couple of more examples, here i have considered the normal distribution when the variances assume to be known. Now, there can be a other case where mean may be known and the variance may be unknown, let us state that case, let me again consider say 1 observation and then I will consider n observations, generally we are dealing with the sample.

So, let me taking this case, here sigma square is positive parameter, if we consider the density function here; it is 1 by sigma root 2 pi into the power minus x square by 2 sigma is square, where x is any real number. Therefore, if I consider the ratio f x sigma 1 square divided by f x sigma 2 square, now this when will give me sigma 2 by sigma 1, this 1 by root two pi will get cancelled out e to the power x square by 2 1 by sigma 2 square minus 1 by sigma 1 square. Now, let us takes a sigma 1 square greater than sigma 2 square; that means, 1 by sigma 1 square is less than 1 by sigma 2 square. So, this term because positive and therefore, this is increasing function of x square, so this family of normal zero sigma square distributions, this has monotone likelihood ratio in sigma square x square.

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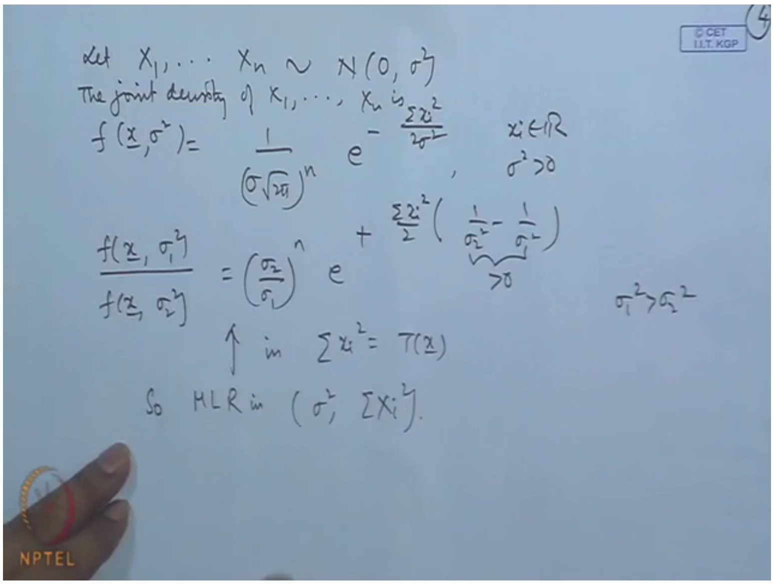

Now, not here, that suppose I take a sample here in place of x, let us take sample x 1 x 2 x n following normal zero sigma x square, and let us write down the same thing of once again. The joint distribution; the joint density of x 1 x 2 x n, that will become 1 by sigma root 2 pi to the power then e to the power minus sigma x i square by 2 x square, where sigma x square is positive and each x i's on the real line. So, when we write down the ratio, now this term get cancel out will get sigma 2 by sigma 1 to the power n e to the power minus sigma x i square by 2 1 by sigma 2 square minus 1 by sigma 1 square, so this i will put plus here. Once again, you not here this is positive if sigma 1 square is less than sigma 2 square Sorry. Now, if sigma 1 square is greater than sigma 2 is square, then this term becomes positive, so this is increasing in sigma x i square, that we will call T x. So, this family has monotone likelihood ratio in sigma square and sigma x i square. Now, this T x has a special role, then we will derive the uniformly most powerful test; you will see that the test will depend upon this itself. So, I will discuss a few more applications little later; let us look at the main result of this section of now, that is as an application of the monotone likelihood ratio property. How the uniformly most powerful test exist?

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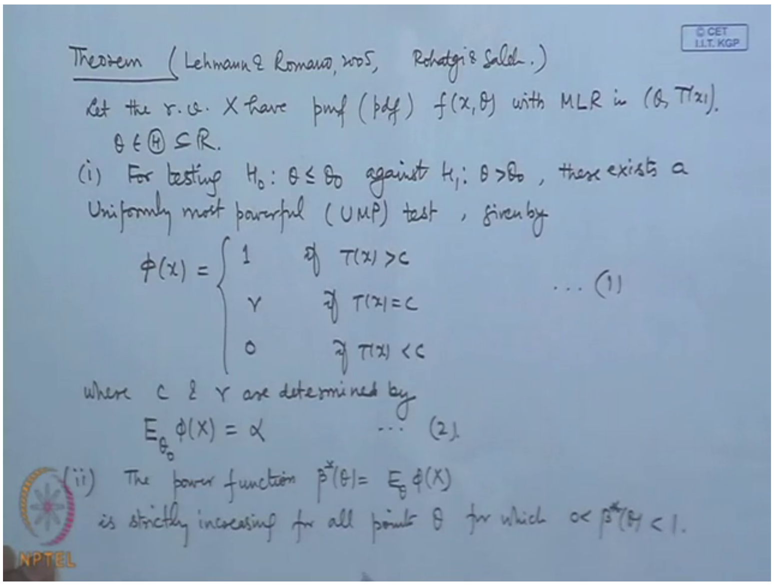

So, I state the theorem; for a proper statement of this theorem you may look at the books of Lehmann and Romano 2005, or you may look at Rohatgi and Selah, the proofs are also given there. So, I am not discussing the proof here.

So, let us consider, let the random variable x have probability mass function or probability density function, say f x theta with monotone likelihood ratio in theta T x, And of course, here theta is a real parameter; theta belongs to say theta which is subset of the real line.

So, the result that we are having here is that, for testing h naught theta less than or equal to theta naught against h one theta greater than theta naught, there exists a uniformly most powerful test; that is, u m p test given by as before we will use phi notation for the test function. So, you reject if T x is greater than c, you reject with probability gamma if T x is equal to c, and you accept if T x less than c. Where c and this gamma r determined by expectation of theta naught phi x is equal to alpha, let me call this conditions one and two, note here that similarity with Neyman Pearson lemma, in the Neyman Pearson lemma we had return f 1 by f naught greater than k. Now, if f 1 by f naught is an increasing function of T x, then that region is transformed to T x greater than c. So, it is as I mentioned, it is direct extension of the Neyman Pearson fundamental lemma only, the result is coming from there. The power function; that is, we have use the notation say beta is star theta, that is equal to expectation theta phi x is strictly increasing for all points theta for which it lies between zero and 1.

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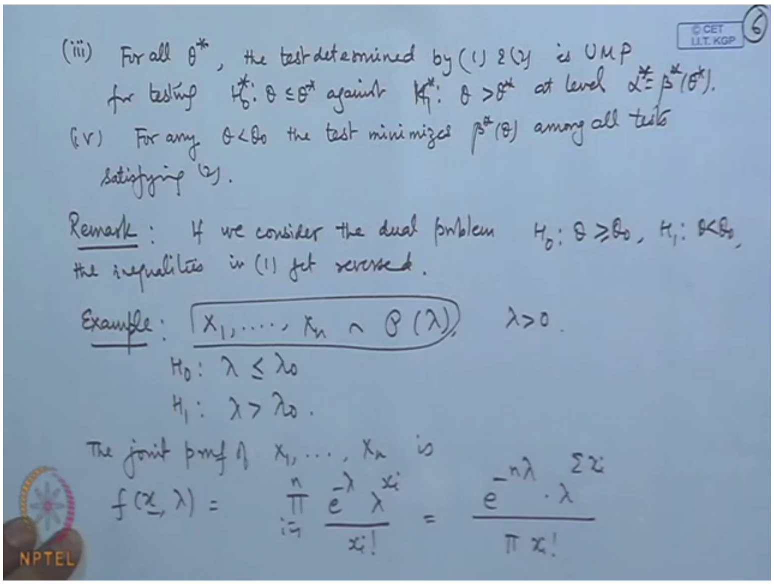

For all theta star, the test determined by 1 and 2 is uniformly most powerful for testing h prime theta less than or equal to theta prime against k prime theta greater than theta prime at level say alpha prime is equal to, let me put star here, this mu hypothesis I am calling h naught and h 1 star at alpha star is equal to beta star of theta star. And for n e theta less than theta naught, the test minimizes beta star theta among all tests satisfying the condition 2. I will skip the proof here; one can look at the book of Lehmann for the detailed proof of this is statements.

Now, one may not here, I have considered theta less than or equal to theta naught against theta greater than theta naught. As I gave the heuristic argument, that in the Neyman Pearson lemma, and as I also I give the normal distribution example when we were testing for the mean, the rejection region for the larger value of x bar, and here it is for the larger value of T x. So, if we reverse, like for the null hypothesis region we consider greater, and for the null alternative hypothesis we consider less than or equal to, then the rejection region will also get the reverse. So, what I just give it as a comment here, if we considered the say dual problem h naught theta greater than or equal to theta naught against h 1 theta less than theta naught, the inequalities in 1 get reversed. So, you have the solution in a similar manner way.

Let me take an application here, say we have a random sample say x 1 x 2 x n from say poison lambda distribution, and we consider say hypothesis lambda less than or equal to say lambda naught against say lambda greater than lambda naught. Now, let us look at this family of poison distributions, whether it as monotone likelihood ratio or not. So, the joint probability mass function of x 1 x 2 x n, so we write it as f x lambda product i is equal to 1 to n e to the power minus lambda lambda to the power x i by x i factorial, that is equal to e to the power minus n lambda lambda to the power sigma x i divided by product x i factorial.

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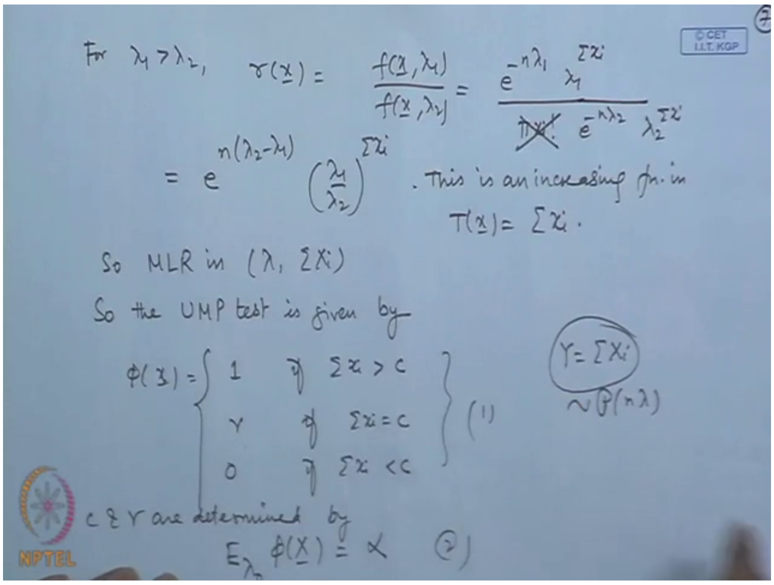

So, if we consider the ratio for lambda 1 greater than lambda 2, let us consider the ratio f x lambda 1 by f x lambda 2. So, it is becoming e to the power minus n lambda 1 lambda 1 to the power sigma x i divided by product x i factorial divided by e to the power minus n lambda 2 lambda 2 to the power sigma x i, and this term will get cancelled out.

So, we can write it in a simplified fashion as, lambda 2 minus lambda 1 lambda 1 by lambda 2 the power sigma x i. This lambda 1 by lambda 2 is greater than 1, because lambda 1 is greater than lambda 2 therefore, this will become an increasing function, this is an increasing function in T x is equal to sigma x i. So, we have monotone likelihood ratio in lambda and sigma x i.

So, we can apply the theorem that i gave, if the family has monotone likelihood ratio and theta and T x, then for one sided null hypothesis verses one sided alternative hypothesis the uniformly most powerful test is obtained here. So, let me write it here, so the UMP test is given by phi x; this here x means x 1 x 2 x n, it is rejecting if sigma x i is greater than c, it is rejecting with probability gamma if sigma x i is equal to c, it is zero if sigma x i is less than c.

Now, the sigma x i is actually, let me write say it has equal to y then that will follow poison distribution n lambda. Now, c and gamma r determined by the condition expectation of the lambda naught phi x is equal to alpha, now this is reducing to, let me write it as a one and this as two, so this condition to let me simplify.

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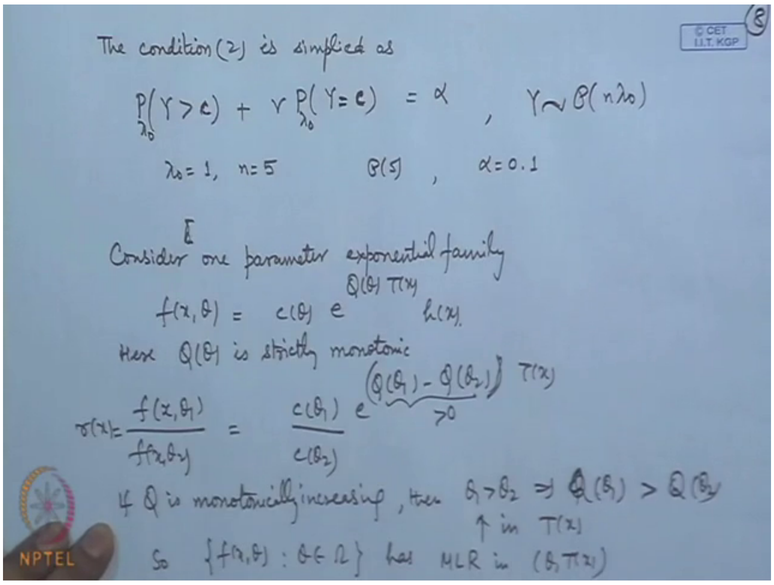

The condition two .So, expectation of lambda naught phi x, since this phi x is completely dependent upon sigma x i that is y, so it is becoming probability of y greater than c plus gamma times probability of y is equal to c, this is equal to alpha when the true parameter value is lambda naught. Now, another point which I would like to explain here, in the case of simple verses simple hypothesis we had the probability of type one error as a single value, but when we have composite hypothesis for the null hypothesis, then the probability of type one error is a function.

However, this is an increasing function which i mentioned in the statement of the theorem also, that the power function is strictly increasing function, so the probability of type one error is increasing. So, when you are getting theta is equal to theta naught, then at that point the maximum value is adoptant. So, effectively this condition is actually the size condition that is expectation of phi x equal to alpha lambda naught, this is the maximum probability of type one error here, that we are fixing to be equal to alpha. So, the size condition now, it is reduced to a condition which is involving the distribution poison n lambda naught therefore, from the tables of the poison distribution one can calculate this. Suppose, I say lambda naught is equal to 1, and n is equal to say phi, then basically we are looking at the tables of poison phi distribution. Suppose, I say alpha is equal to 0.1, then basically what we are seeing here is that what is the point from where… Now, the c could be; need not be an integer actually, we may fix that thing in such a way, if it is an integer then this value may be positive, if it is naught integer then this may become zero.

Now, you may see from the tables that whether this randomization with probability gamma is required or not, if it is not required then this probability can be taken to be zero, there will be a point where after you will have the probability alpha. In case, that is naught possible, then we suitably choose a value where we lot of probability and then we may give some value of gamma also.

So, now this can be calculated from the tables of the from the tables of the poison distribution. We can also see like a binomial distribution, we suppose we are hyper geometric distribution, suppose we have negative binomial distribution, in all of these distributions we are able to find out the uniformly most powerful tests. I will consider this derivation of the test in the following lecture; let me further develop this theory of the UMP tests here.

So, let us consider one parameter exponential family, so we are considering the form of the probability mass function or the probability density function as c theta e to the power Q theta T x n two h x. Here Q function is strictly monotonic function; that means, it could be monotonically increasing or monotonically decreasing. Let us write down the ratio f x theta 1 by f x theta 2, then this is becoming c theta 1 e to the power Q theta 1 minus Q theta 2, T x and h x will get cancelled out by c theta 2.

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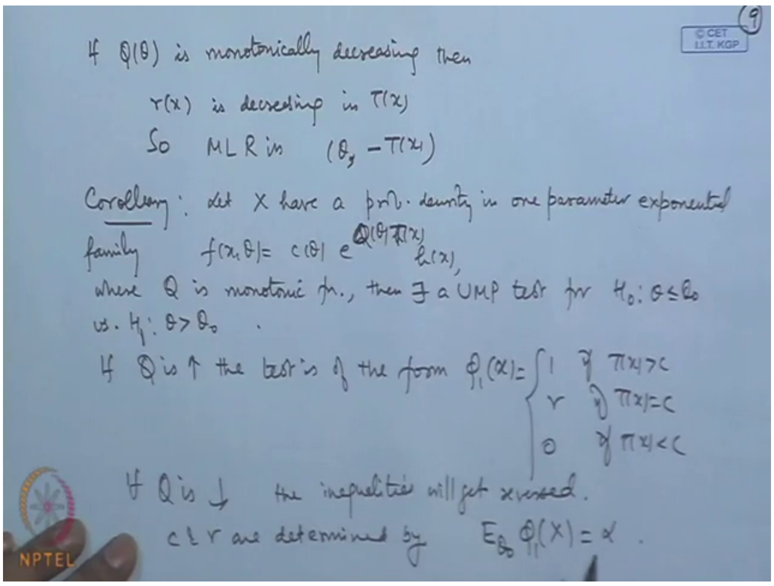

If Q is monotonically increasing, then theta 1 greater than theta 2 will imply Q theta 1 greater than Q theta 2; that means, this term will become positive, and you will have this as increasing function of increasing function of T x. So, this ratio becomes an increasing function of T x, so the family f x theta; this will have monotone likelihood ratio in theta T x. On the other hand, if I consider say Q theta 2 be monotonically decreasing, if Q theta is monotonically decreasing then this r x term, what will happen here? That Q theta 1 will become less than Q theta 2, if theta 1 is greater than theta 2 therefore, this term will become decreasing function of T x, and therefore, monotone likelihood ratio will be in minus T x; decreasing in T x, so MLR will be in theta and minus T x; that means, the test function will get reversed, inequalities like here we are T x greater than c you will it will become T x less than c.

So, as a corollary of the previous theorem, we can write then let x have a probability density in one parameter exponential family, that is f x theta is equal to c theta e to the power Q theta T x into h x, where Q is monotonic function then there exist a UMP test for h naught theta less than or equal to theta naught against theta greater than theta naught. If Q is increasing, the test is of the form phi 1 x is equal to 1, if T x is greater than c if T x is equal to c it is zero if T x is less than c, if Q is decreasing the inequalities will get reversed, and here c and gamma are determined by expectation of theta naught phi 1 x is equal to f 1.

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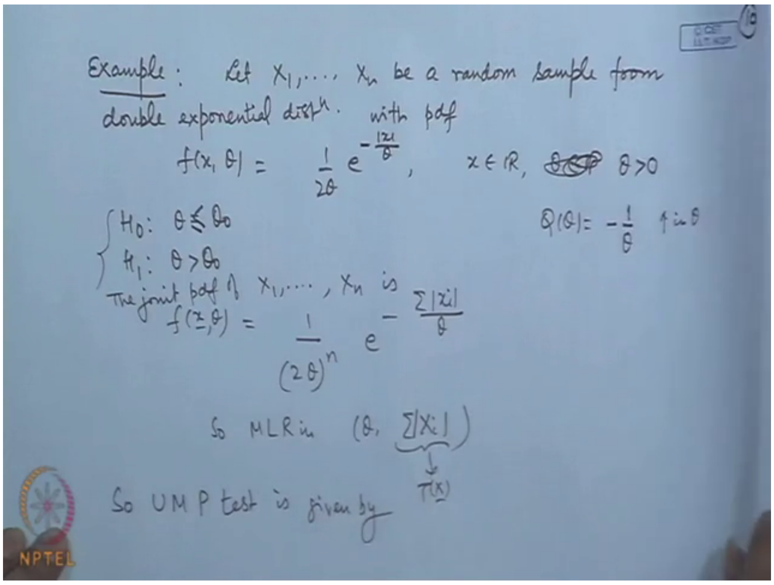

Let me consider one example, let x 1 x 2 x n be a random sample from double exponential distribution, with p d f given by say f x theta is equal to half e to the power minus modulus x by theta and here 1 by 2 theta. Here x is a real number, and theta has to be a positive parameter here, let us consider say, theta is less than or equal to theta naught against theta greater than theta naught. You can easily see that this is a one parameter exponential family, and the monotone likelihood ratio; here you may consider Q theta as equal to minus 1 by theta.

So, naturally this is increasing in theta because 1 by theta is decreasing, so minus 1 by theta is increasing. So, this is strictly a monotonic function, so this suppose i write down the joint distribution of x 1 x 2 x n, half x 1 x 2 x n that is equal to 1 by 2 theta to the power n e to the power minus sigma, so monotone likelihood ratio in theta and sigma x i, this is T x sigma modulus of x i. Therefore, by an application of this corollary that i mentioned UMP test for this problem,

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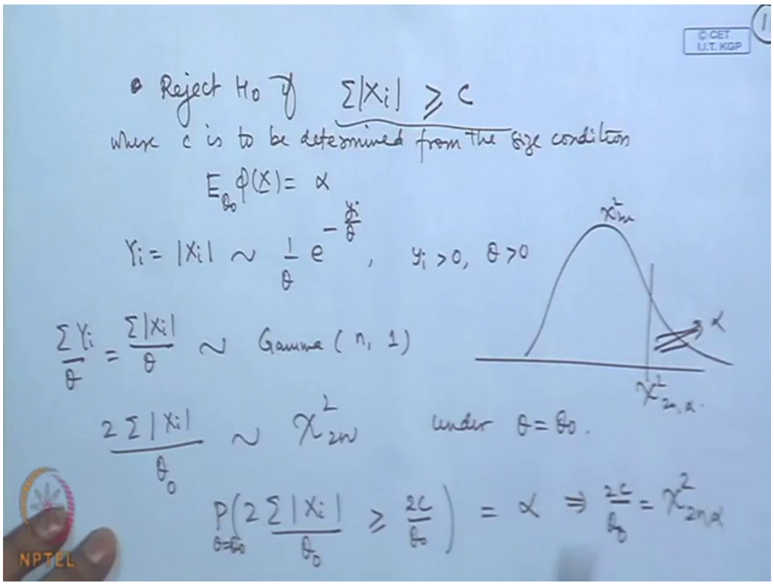

Let us say reject h naught if sigma modulus of a x i is greater than c. Now, note here that we are dealing with the continuous distributions, so I have returned this part only, this part will be the acceptance region. Now, sigma of modulus x i is equal to c, we need not write this portion here because this will have probability zero, so without lose of generality i am including the equality here.

Now, what we need to do is to determine this, where c is to be determined from the size condition, that is expectation of phi x is equal to alpha. Now, let us consider say y i is equal to modulus of x i, if x i is having double exponential distribution, then modulus of x i will have simple exponential distribution, that is 1 by theta e to the power minus y i by theta. So, this will have distribution 1 by theta e to the power minus y i by theta.

So, sigma modulus of x i by theta, that is y i sigma y i by theta that will have gamma distribution with parameters n and 1; that means, twice sigma modulus x i by theta naught, that will follows chi square distribution on 2 n degrees of freedom under theta is equal to theta naught. So, when we consider this size condition, that is probability of sigma modulus x i by theta naught twice greater than or equal to some 2 c by theta naught, this is equal to alpha when theta is equal to theta naught, this implies that this two c by theta naught should be equal to chi square 2 n alpha; that means, on the chi square curve with the 2 n degrees a freedom this probabilities equal to alpha, so this is chi square 2 n alpha phi.

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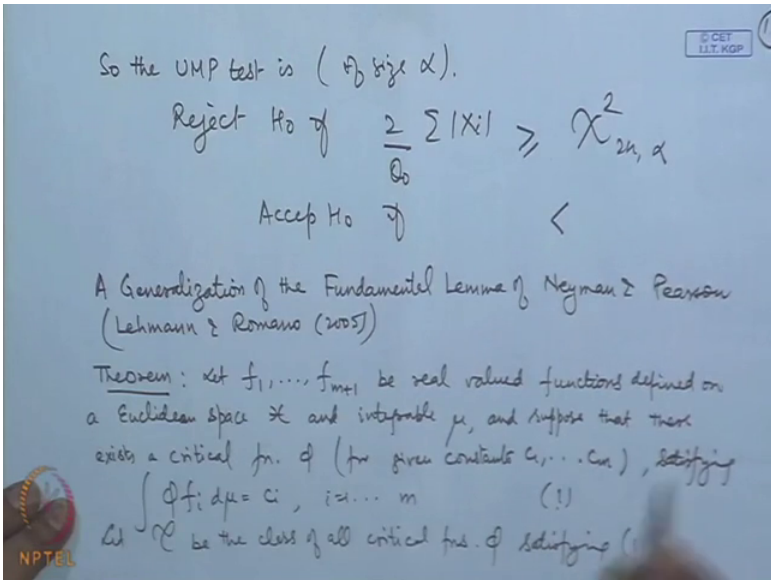

So, the test is written as, so the UMP test is reject H naught if twice sigma modulus x i by theta naught is greater than or equal to chi square 2 n alpha; this is the UMP test of size alpha, and of course, accept H naught if this is less. So, you can see this extension of the Neyman Pearson theory to the family's with the monotone likelihood ratio is helpful in providing the uniformly most powerful tests for one sided testing problems. If the families have monotonic likelihood ratio, we are able to directly use these things here, and we are having exact test here; that means, once we have the observations, and we our testing problem is clearly specified, then at a given level of significance we can provide decision, whether we should accept a null hypothesis are not. On the other hand, if you do not is specify alpha in advance, then we can find out the p value here.

Now, let me proceed further with this theory here. Now, for further extension of this theory of most powerful tests, a generalization of the Neyman Pearson fundamental lemma was done. Let me state this results without any proof here, and these results are used for solving further problem; that means, here we are considering theta less than are equal to theta naught, so it is strictly one sided.

Now, there can be cases were we may have two sided also for example, if I am considering say I binomial proportion, whether it lies in a range or it is outside a range. Now, if I say it is within a range then it is like an interval, but if you i say it is an outside a range for example, I may say it is outside the interval 1 by 4 to 3 by 4; that means, I am saying the hypothesis p less than or equal to 1 by 4 or p is greater than or equal to 3 by 4, is a two sided thing. Now, in families with the monotone likelihood ratio etcetera, this Neyman Pearson theory applicable to this also, and then there is another point; that is regarding the determination of the constant in the test. In the one sided thing, the maximum was occurring at the cut of point that is theta naught here, when we have two sided then you will have two cut of points, it will increase and then… So, what will happen, that we will consider the maximum value at both the end points, that is at both end points of the intervals.

So, these results are proved using a certain extended features of the Neyman Pearson lemma. So, the result is known as the generalization of the fundamental lemma. let me give it here first of all, a generalization of the fundamental lemma of Neyman and Pearson. This is statement and the proof one can find out in the book of Lehmann and Romano, I will be skipping the details of the proof; I will only give the statement here. Let f 1 f 2 f m plus 1 be real valued functions defined on a Euclidean space x and integrable mu, and suppose that there exists a critical function phi for given constants c 1 c 2 c n satisfying integral phi f i d mu is equal to c i i is equal to 1 to m. Let us say c is the class of all critical functions phi satisfying 1.

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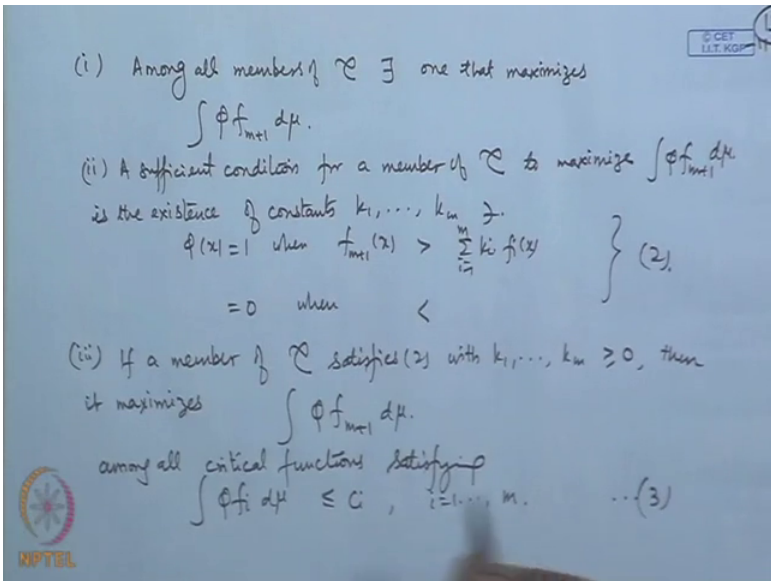

Then among all members of c there exists 1 that maximizes integral phi f m plus 1 d mu. A sufficient condition for a member of c to maximize integral phi f m plus 1 d mu is the existence of constants k 1 k 2 k n, such that phi x is equal to 1 when f m plus 1 x is greater than sigma k i f i x i is equal to 1 to n, and it is equal to zero when it is less. Thirdly, if a member of c satisfies 2 with k 1 k 2 k n greater than or equal to zero, then it maximizes integral phi f m plus 1 d mu among all critical functions satisfying phi f i d mu less than or equal to c i for i is equal to 1 to m.

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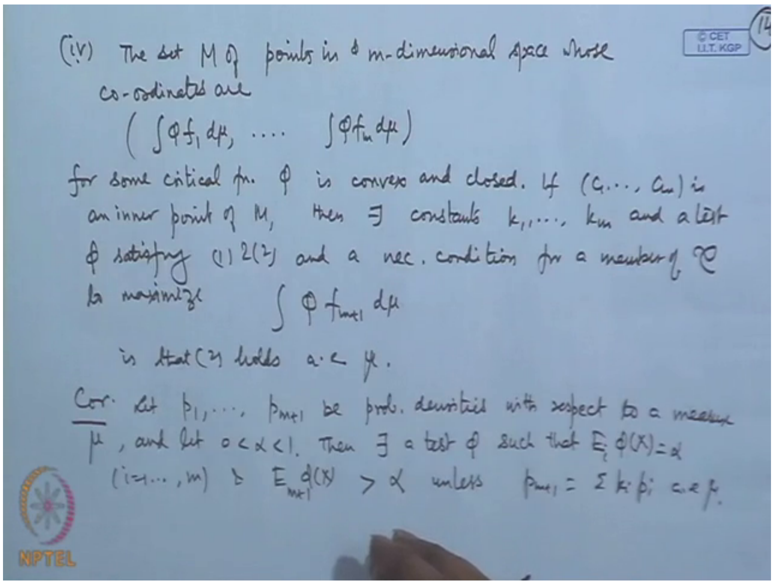

And then lastly, the set m of points in the m dimensional space whose co-ordinates are say phi f 1 d mu and so on, phi f m d mu for some critical function phi this is convex and closed. If c 1 c 2 c n is the inner point of n, then there exist constants k 1 k 2 k n, and a test phi satisfying 1 and 2 and a necessary condition for a member of c 2 maximized integral phi f m plus 1 d mu is that 2 holds almost everywhere. As i mentioned i will not be giving the proof of these results, one can see the book of Lehmann.

Now, this extension is helpful for solving more general testing problems, as a corollary I state the following, let p 1 p 2 p m plus 1 be probability densities with respect to a measure mu, and let zero less than alpha less than 1, then there exists a test phi such that expectation of phi x is equal to alpha for i is equal to 1 to n, and expectation of phi x for m plus 1 it is greater than alpha unless of course, p m plus 1 is equal to sigma k i p i almost everywhere. So, this will actually give the solution to more general two sided null hypothesis testing problems.

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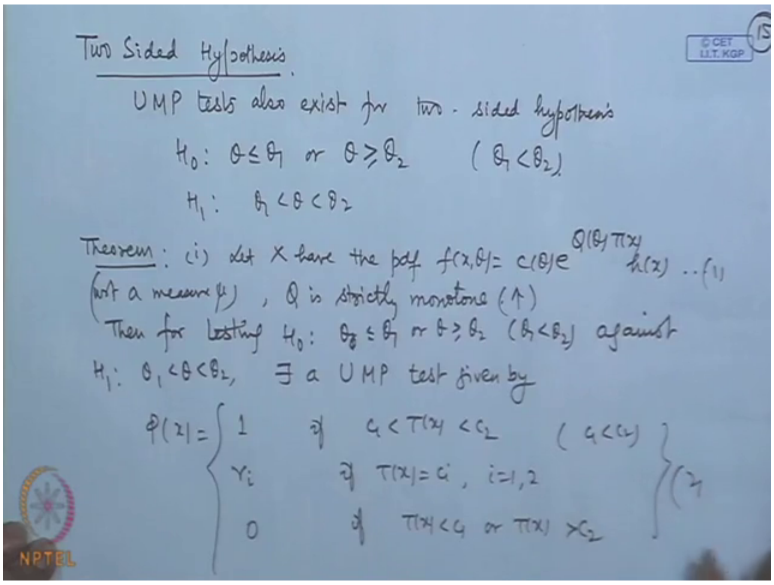

So, we have the following result than, that is if I am considering two sided hypothesis. So, we can say that UMP tests also exists for certain to sided hypothesis of this nature h naught, say theta less than or equal to theta 1 or theta greater than or equal to theta 2, where theta 1 is less than theta 2. So, we may like to test whether for example, say theta is the error measurements. So, we may like to check, whether the error measurements like within a certain range or there outside the range. It could be like, we are producing certain items and say certain ball bearings are produced, and we are looking at the diameter of the ball bearings.

So, whether the ball bearings diameters are within a range a range or it is outside the range. If it is within the range we will be excepting the product as the good item, if it is outside then we will be rejecting that. So therefore, this is a perfect case for the two sided testing hypothesis problems, we may have say H 1 as theta 1 less than theta less than theta 2.

So, the result is that, by the use of the generalization of the Neyman Pearson fundamental lemma, we can actually give the uniformly most powerful test for these situations also. So, we have the following theorem, which I will the state, let x have the probability density function with respect to a measure mu and Q is strictly monotone, then for testing theta 1 theta less than or equal to theta 1 or theta greater than or equal to theta 2, where theta 1 is less than theta 2 against the alternatives theta 1 less than theta less than theta 2, there exists a uniformly most powerful test. Off course, here against c 1 has to be less than c 2, it is gamma i if T x is equal to c i for i is equal to 1 2. So, there are two points of randomization here, and we accept if T x is less than c 1 or T x is greater than c 2. Once again, if we are considering Q to be strictly monotone here, then the familiar distribution has monotone likelihood ratio in theta T x or theta minus T x and therefore….

So, here I would take an for example, increasing say because we are writing the region in the rejection region in this one, so i am considering monotonically increasing. So, we are rejecting when the value lies between two ranges, and we are accepting for a smaller values of T x or larger values of T x. If it is decreasing, then the inequalities will get reversed, and at the boundary points of the interval we have done the randomization.

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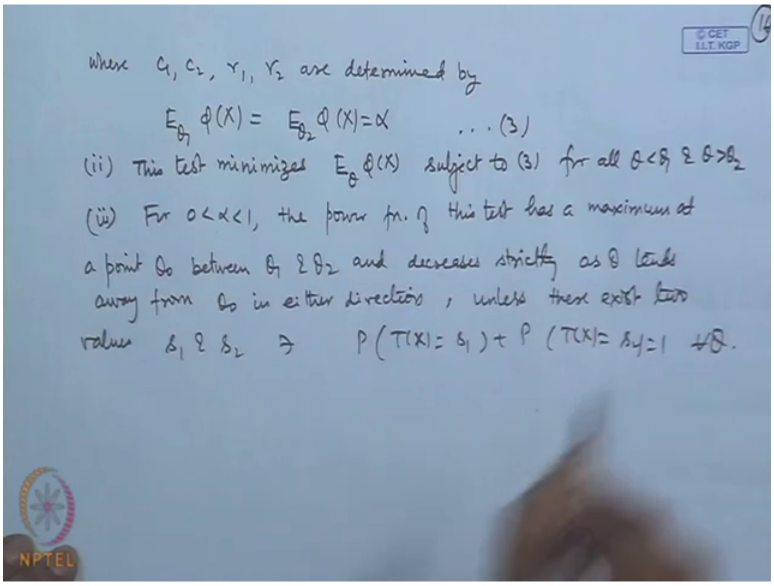

So let me consider this as 1 this as 2 say, where this constants c 1 c 2 gamma 1 gamma 2, they are determined by expectation theta 1 phi x is equal to expectation theta 2 phi x is equal to alpha. This test minimizes expectation phi x subject to 3 for all theta less than theta 1, and theta greater than theta 2. And for zero less than alpha less than 1, the power function of this test has a maximum at a point theta naught between theta 1 and theta 2, and decreases is strictly as theta tends away from theta naught in either direction, unless of course, there exists 2 values; say s 1 and s 2, such that probability of T x is equal to s 1 plus probability of T x is equal to s 2 is equal to 1 for all theta.

So, here you can see the probability of type one error will be maximized at the end point; that is, at theta 1 and theta 2, that is why we are fixing that value equal to alpha, so this is the size condition in the two sided null hypothesis problem. When we have one sided hypothesis problem, than the maximum value is occurring at the cut point; that cut of point, where the null and alternative hypothesis points are changing. But when we have two sided then will have two points; one is below and another is above, and at both the points we are having the maximum value of the probability of type one error, that value we are fixing as the alpha value. In the next lecture, I will be considering further amplification of these results; certain applications of this, and we will also consider certain properties of this power function here, which are based on actually the monotone likelihood ratio property. So, basically their properties of the expectations, I will be discussing at in the next lecture.