Statistical Methods for Scientists and Engineers
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Lecture – 29
Non parametric Methods - II

Yesterday, I introduced you to the topic of distribution free methods are what we call as nonparametric statistics; nonparametric statistical inference. I mention that a primary quantity or you can say primary variable or primary statistic of interest in nonparametric since the ordinary statistics. That means when we have the observations from the sample, then we consider their ordered versions, so they are called ordered statistics.

Yesterday, we have seen how to derive the distribution; the joint distribution of the ordered statistic distribution of one ordered statistics or the joined distribution of a couple of ordered statistics and a general methodology of taking any subset of the collection of ordered statistics, how to derive the distribution of that? So, certainly as I mentioned yesterday, that in various problems we are interested in different kind of ordered statistics.

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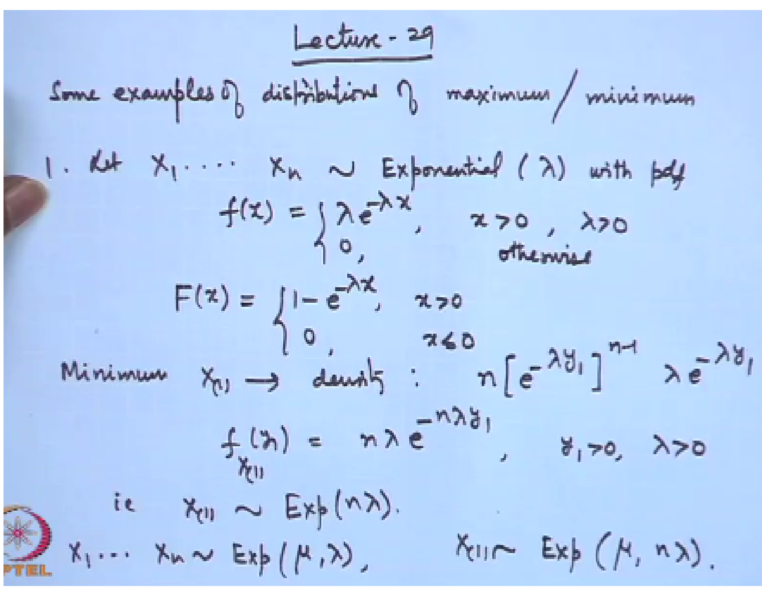

In particular, we may be interested in the minimum of the observations or the maximum of the observations or the middle of the observations or a particular position. To give you an example some examples of distributions of say, maximum, minimum etc., okay. Let us consider say first example, let us take say, X1, X2, Xn follow an exponential distribution with parameters; let me take the standard exponential distribution lambda.

That means, I am considering the density function; fx = lambda e to the power –lambda x, where x is > 0, lambda is > 0, 0 otherwise; let us consider the distribution of say, here let us consider say, CDF. So, what is the CDF here? CDF here is = 1 - e to the power –lambda x, for x positive and of course, it is 0 for x; <= 0. If I consider say, minimum of the observations that is X1, then yesterday, we have seen the distribution of the minimum can be written separately.

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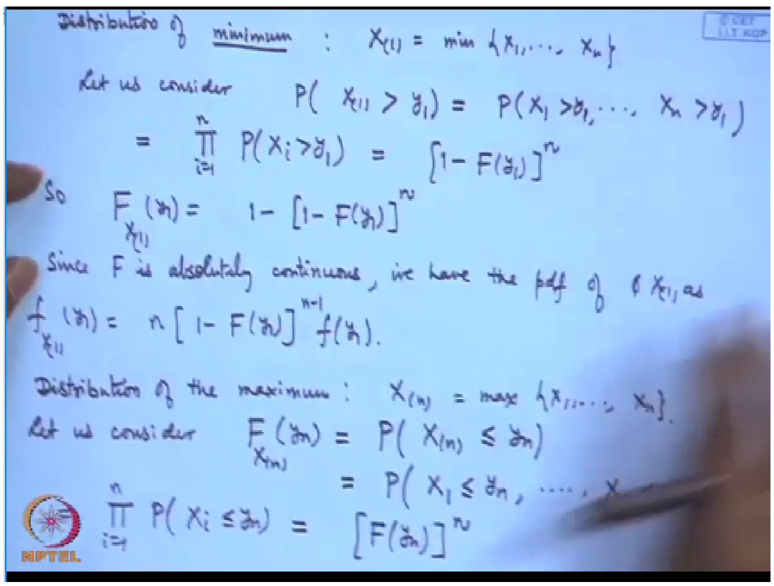

It can be obtained in a simple way as n times 1 – F y1 to be power n-1, fy1, if the common density is small f, where F is the CDF. So, if I apply this formula here, then this will have density, it will be = n times 1- Fx, so this will become e to the power - lambda y1 to the power n -1 * lambda e to the power - lambda y1. Now, you can see here this power will get added up, so you are getting the density of this minimum as n lambda e to the power –n lambda y1.

Now, this is interesting because it is of the same form, so what you are getting that is if I am having excise exponential lambda and then this is following exponential n lambda, so a generalisation of this could be that if I consider say, X1, X2, Xn following exponential Mu lambda, then you will have X1 following exponential Mu n lambda, so this is interesting. The distribution of the minimum when we are considering the observation from an exponential distribution is again and exponentially distributed random variable.

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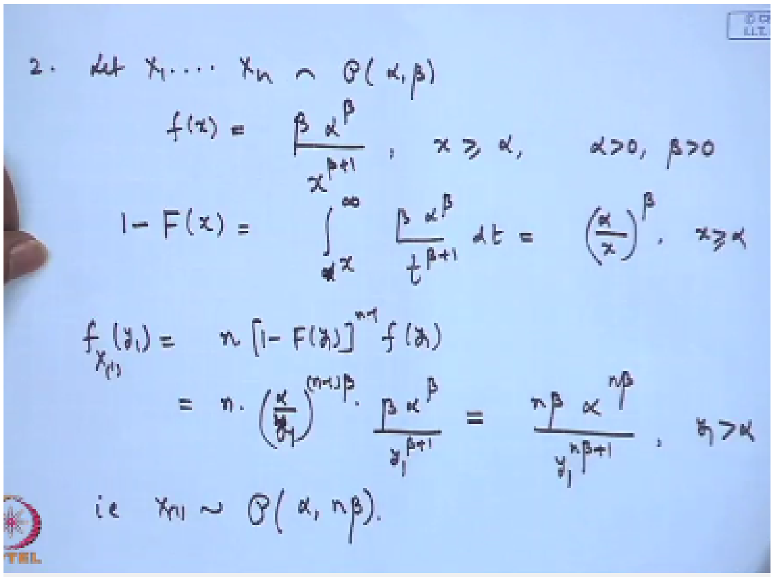

Let us take are some more interesting example; let us take say, X1, X2, Xn follow pareto distribution with parameters say, alpha and beta that means I am considering the density as say beta; alpha to the power beta / x to the power beta + 1, x >= alpha, where alpha and beta are positive, so I am considering this density as the pareto density and let us consider here CDF, so CDF here is =; so let me take 1 – CDF here.

So, that will become = x to infinity beta alpha to the power beta/ t to be power beta +1 dt, so this then simply becomes. if I integrate t to the power - beta – 1, so I will get t to the power – beta/ - beta; - beta, - beta will cancel out and then t to the power – beta will give me x, so that means, I will get alpha/x to the power beta for x >= alpha. Therefore, if I consider now the density function of the smallest, then it is n times 1 – F of y1 to the power n -1 * f of y1.

So, this is = n times and you consider alpha/x to the power; sorry, y1 to the power n -1 beta and then you have the density here that is beta alpha to the power beta/ y1 to the power beta + 1. So, if you combine the terms, you will get n beta alpha to the power n beta / y1 to be power n beta +1, for y1 > alpha. So, you compare this with this density, beta has changed to n beta, alpha remains the same.

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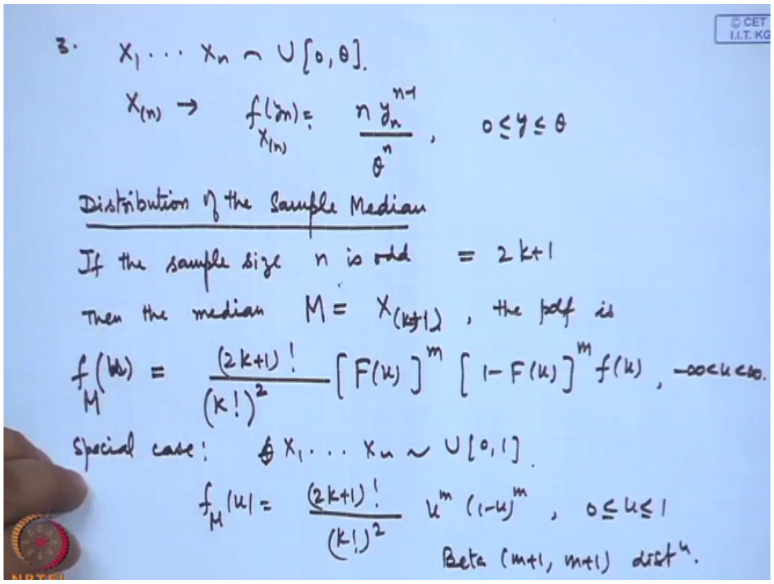

That is we are saying X1 follows pareto alpha n beta, so this is another interesting example, where the distinction of the ordered statistics is in the same family of distributions. Let me give 1 example of say maximum here, say X1, X2, Xn follow say uniform 0 theta and let us consider the distribution of Xn, then by the same logic here the density function of X will be 1/theta between 0 to theta, so the CDF will be X/ theta.

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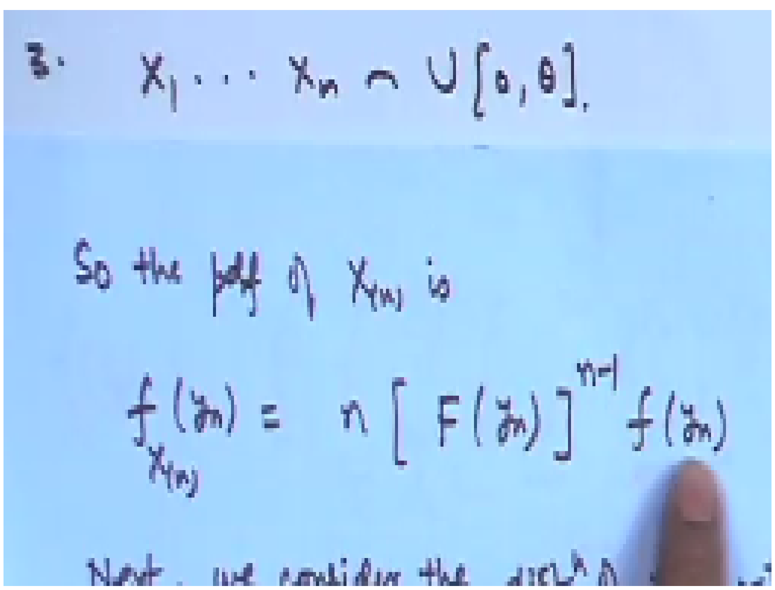

And if I apply this formula for the maximum here which is derived yesterday as; n F to the power n -1 * f of yn, if I apply this formula, then I will get the density of this as; n yn the power n -1/ theta to be power n; 0 < y <= theta. So, these are some of the densities; here the forms of this distribution of the ordered statistics can be derived in a simple fashion or you can say in a closed form.

Sometimes, they are recognisable sometimes; they may be in a different form also. Now, I mentioned yesterday that one of the particular ordered statistics that will be in use or useful is the median; median of the sampled observations. So, we also may be interested in the distribution of the sample median, let us look at that. Distribution of the sample median, so in fact, I should also mention that what is the further use of these properties.

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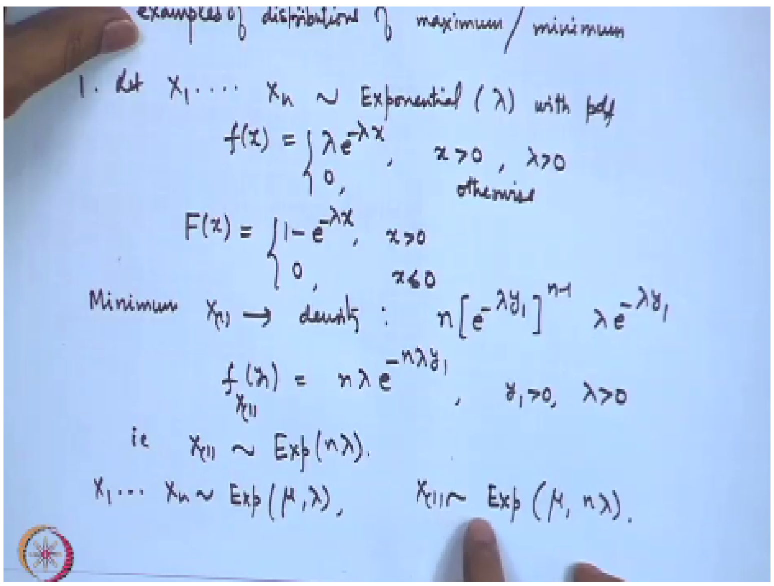

For example, here if I have shown that if X1, X2, Xn follow exponential lambda, then X1 follows exponential n lambda. Now, if you are interested in the averages here, then the average or you can say if they are denoting the lifetimes, then the average life here is 1/ lambda for each of the observations but if I am looking at the minimum, then the average life is becoming 1/ n times lambda that means, it is 1/ nth of the original lifetime.

Now, this can be an eye opener because in the; say in a certain industrial situation, if we are interested to use any of the components, then it is all right, we can say that the average life is say, 1/lambda. Suppose, I am saying lambda is 6 months, so if I put say, 1/6; say if put; for example, lambda is = 1/2, suppose we are measuring in the years, then it is becoming 2 years but if I have say, 10 observations.

And then I consider the one which fails the first for example, it may be a series system. If it is a series system, then the life of the minimum will determine the whole thing and here it will become 1/ n lambda, that means it will become 2/10 that means, it is only; so it is coming a few months only because 24/ 10 if I put, it is turning out to be only 2 months; 2.4 months, which is much smaller, which you are thinking that it will be 2 years but it is not so.

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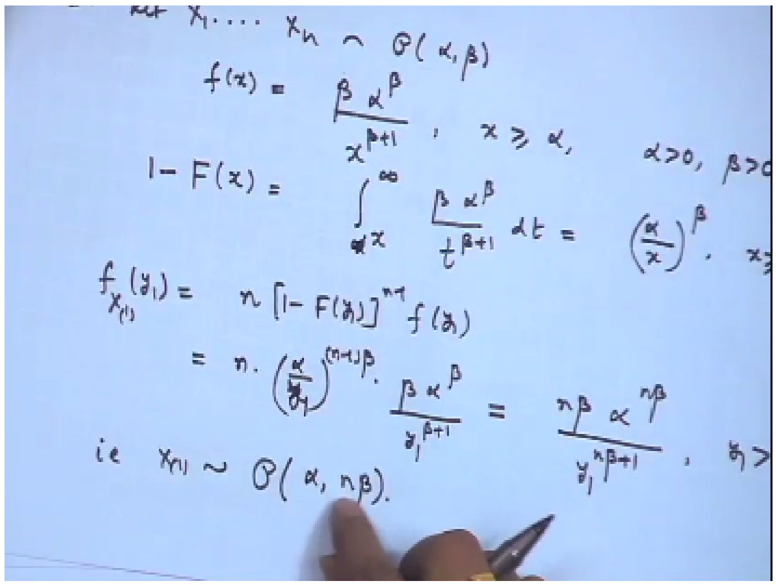

So, actual lifetime of the entire system will become very small compared to; in a similar way, you can think of the pareto here, this is beta a here and here it is become n beta, so in certain case when you are using beta as a parameter, then it is becoming actually n times, so that is a difference. If we are making use of the ordered statistics, these facts provide interesting information about our nature of the means or; so basically, various characteristics of this ordered statistics we are able to find out.

So, we will explore this further that is why the distributions and means, variances and other things, asymptotic distribution etc., will be of much interest for ordered statistics. So, let us now look at the description of the sample median. So, if the sample size n is odd, so odd means generally we put say something like 2 k + 1, then the sample median is the middle of the observation, let me call it median as M, then that is = X k+ 1.

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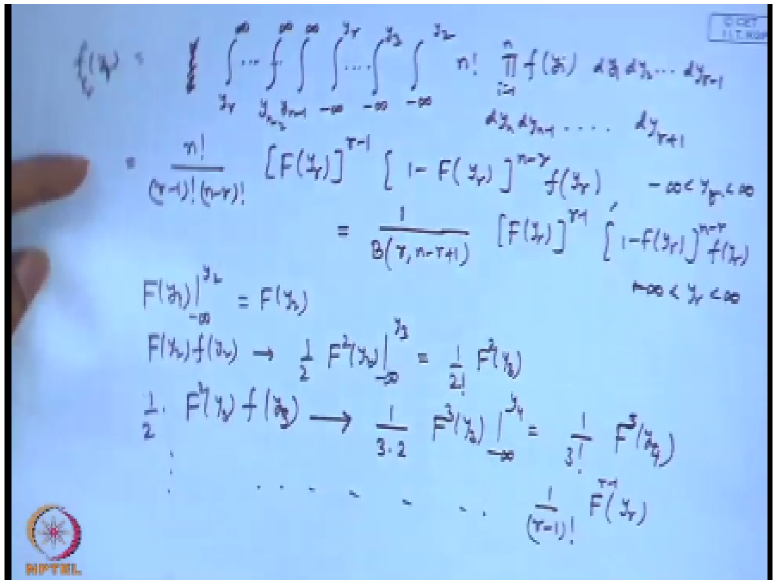

So, we have already derived the form of a general ordered statistics, which is of the R th term, so let me just show it again, where we will substitute the value. So, if you look at this term here, the distribution of the rth order statistics is obtained as n factorial/ r -1 factorial, n – r factorial F to the power r -1 and 1 - F to the power n –r multiplied by a small f, where F is the CDF. So, now if I take r is = m +1 here, where n is 2; r = k +1, where n = 2k +1.

So, let us substitute here that will give me the distribution of the sample median for this particular case. So, the pdf is; let me call it x; fm and I will use the notation small, okay m is sometimes used for the sample size etc, so let me not conclude; let me use it something like say, u here, then it is = 2 k + 1 factorial, r - 1 that will give me k factorial, then n - r will again give me k factorial, so it is k factorial square.

F of u to the power m, 1 - F to the power m, small f of u, so you are able to obtain the distribution of the sample median, if it is; if the sample size is odd, it is much simpler here. You also look at the special case that is say, F; that means original distribution are uniform distribution on the interval 0 to 1, in that case, this will become 2k + 1 factorial by k factorial square u to the power m 1 – u to the power m, 0 < u < 1.

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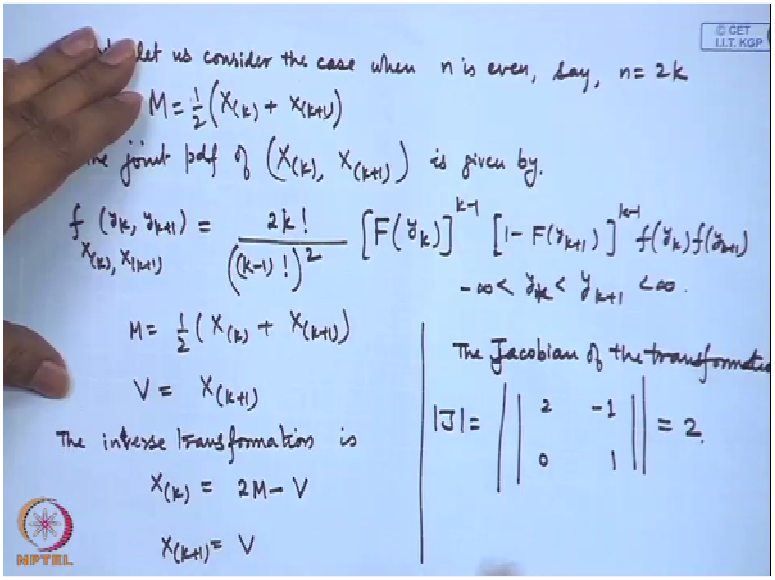

Now, if you are looking at this one, this is nothing but a beta m + 1, m + 1 distribution, so we know the moment structure and other things also for a beta distribution. For example, the mean of a; mean of this becomes simply 1/2 because it is alpha/ alpha+ beta and similarly, other moments of this can be obtained. Let us take the other case; let us consider the case, when n is even, say n is = 2k.

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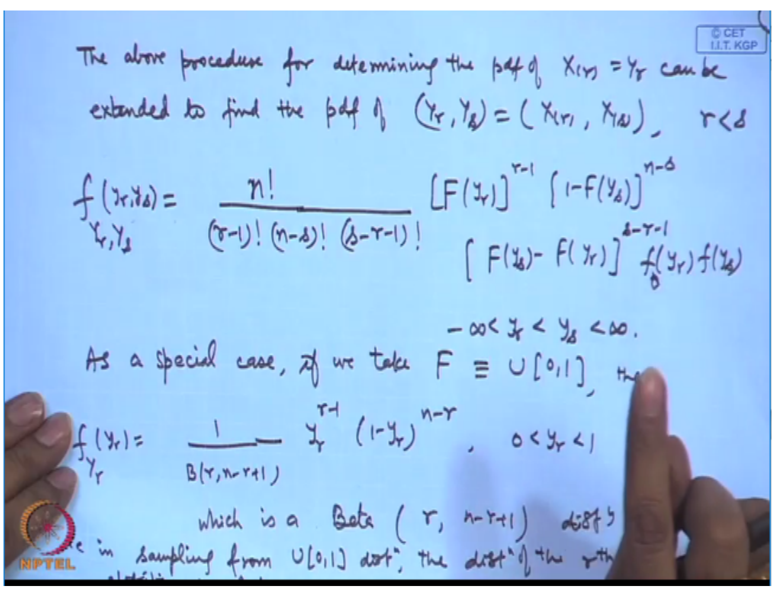

Then the median is Xk + Xk + 1/2, now if that is so; I firstly use the joint distribution of Xk and Xk + 1 and again let us go back to the formula, which I derived for the joint distribution of rth and sth order statistics that was obtained as n factorial/ r – 1 factorial, n - s factorial, s – r – 1, F to the power r -1 at yr 1 - F at ys is to the power n – s, then F of ys – F of yr to the power s – r - 1 multiplied by the densities at yr and ys.

So, now I will take r to be k and s to be k+ 1, so if we do that, then what do we get? The joint probability density function of Xk and X k + 1, this is given by f of use the notation yk, yk +1 points that is =; so it is 2k factorial, that is n, then you have r-1 that is becoming k -1 factorial, then you have k+ 1 - k -1 that is simply becoming 0, so 0 factorial that we take as 1 and then you have 2k – k + 1 that is again k -1 factorial.

So, it becomes 2k factorial/ k -1 factorial square, then I get F of yk to the power r -1, then again the term, which is corresponding to F of yk + 1- F of yk to the power s – r – 1, so since s - r -1 is becoming = 0 therefore, that power; that term will give me simply 1. So, I will get F of yk + 1 to the power k -1 and then I get, F of yk; F of yk + 1 and here of course, I have to write yk is < yk + 1, in order to derive the distribution of m, that is half of this, I have to define a transformation.

So, let me write, M is = 1/2 of Xk + X k + 1 and I define another valuable, let me call it say, V that is = X k+ 1, so the inverse transformation; we consider the inverse transformation is; this will turn out to be Xk, so basically, you are having Xk + 1 is = V, so if I put it here, I get 2M – V, so Jacobian of the transformation, so that will give me 2, -1,0,1 so that is becoming = 2. So, if I look at the joint pdf of u and v.

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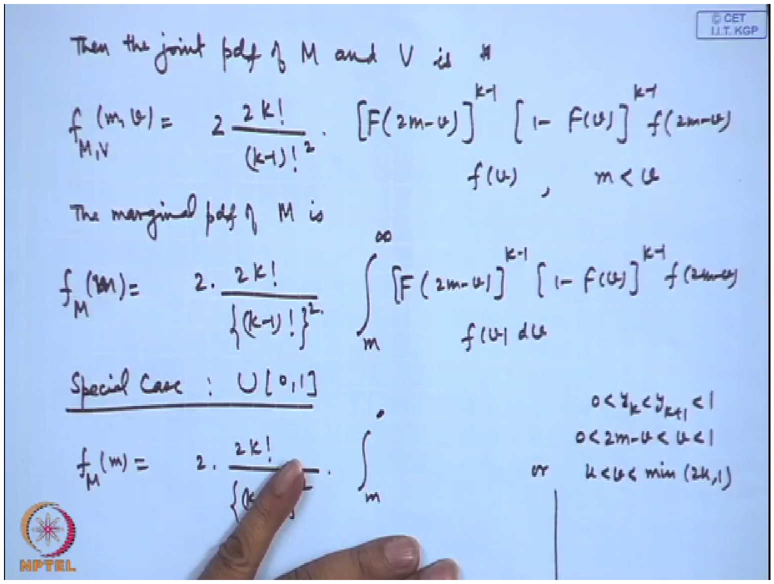

Then, the joint probability density function of M and V that is f of 2k factorial/ k -1 factorial square and it will be multiplied by 2 because the Jacobian was 2, then I get F of Yk is = 2m - v to the power k -1, 1 - F of v to the power k – 1, f of 2m – v, f of v and you are having the region here; Xk is < X k + 1 that will give me 2m – v < v e that is m is < v, so m is < v that region I will get.

So, the marginal pdf of M is obtained from integrating the joint density/ k -1 factorial square, F of 2m - v to the power k -1,1 - fv to the power k – 1, f of 2m – v, f of v, dv and v is integrated from M to infinity, so this gives me the formula for the derivation of the distribution of the sample median, when the sample size is even. Again, as I saw the case for the uniform distribution, when it was odd, we are getting simply beta distribution; let us see what we get?

The original distribution is say, uniform 0,1 then this will become = 2, 2k factorial/ k - 1 factorial square m to infinity, now this will go only up to; when I am considering the uniform distribution, the range of the random variables; original random variables is from 0 to 1. So, if I consider this, then I will get here, see 0 <; say, yk < yk + 1, this is < 1, so this is same as saying 0 < 2m – v< v < 1, which is same as saying u < v that is < minimum of 2u and 1.

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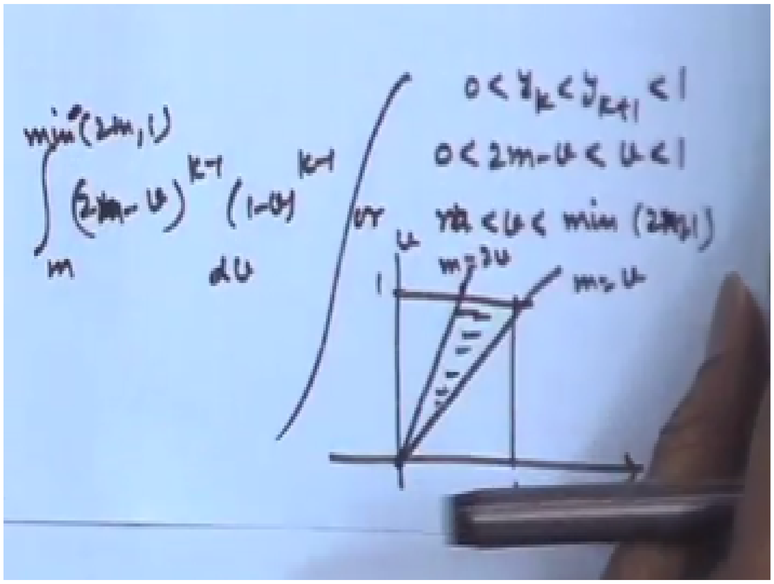

So, if I consider the region here of integration, see this is 1; so if I consider this line as m = v, this is m here and this is m, so m = v and I consider m = 2v, then we are actually getting this region here basically. On this side, I am having m and on this side I am showing v here. So, this is then becoming = m2 minimum of 2m 1, 2u -; sorry, 2m - v to the power k -1, 1- v to the power k – 1, dv.

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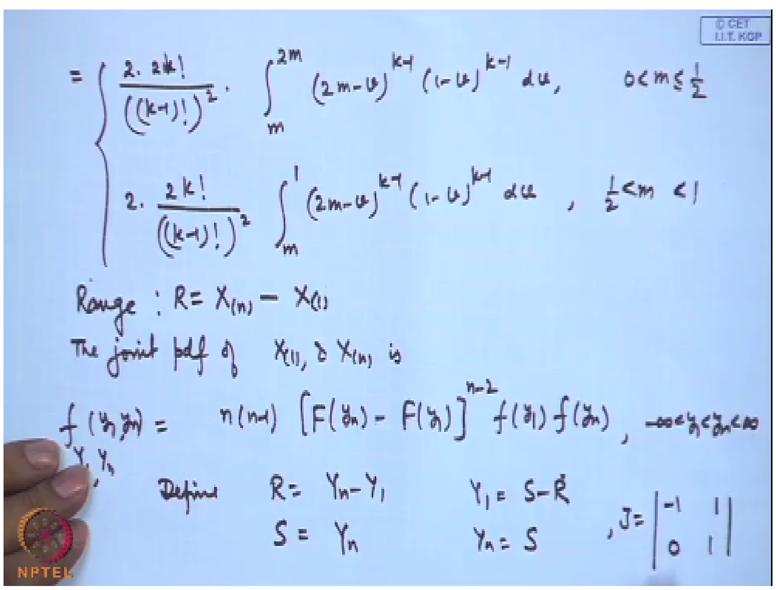

So, when you look at this integration, I will get 2 parts here, this will become = 2 times 2k factorial/ k -1 factorial square, integral from m to 2m, 2m – v to the power k -1, 1 – v to the power k -1 dv, where m is between 0 and 1/2 and it is = 2 times 2k factorial/ k -1 factorial square m to 1, 2m - v to the power k -1, 1-v to the power k-1 dv, if m is between 1/2 and 1. For various values of k, for example if I consider k = 5 or k=; here k is even.

So, suppose I take k = 4 or k =2, then these expressions can be directly evaluated, otherwise also we can be evaluated but you have to do several integration by parts here. The distribution of the sample median we are able to derive for both the cases, when the sample size is odd, then in that case it is straightforward for but in the case of even, it is in the form of an integral but that integral may be evaluated for a specific choice of the distributions.

In every case, it may not be a possible to do. For example, if I consider exponential distribution, then I can; then one of them will become a proper term and then the other one will become a finite expansion, which can be easily done and we can really evaluate. Similarly, if I consider say, pareto distribution etc., so this can be obtained but in some other distribution, say normal etc, this might be much more complicated.

Another quantity of interest in the case of nonparametric statistics is the range of the observations that means the difference between the maximum and the minimum. So, in the real life situation also, you can see the range is generally taken as an important quantity and therefore the distribution of the range is also of quite importance. When we are considering the parameter situation, then we generally consider variance or the standard deviation.

But in case of non-parametric, when we do not have exact form of the cdf or the pdf, it is difficult to study that. Therefore, a more useful quantity would be the distribution of the range. So, we look at that thing now, so range is defined as Xn, let me call it R, so Xn – X1, so we look at the distribution of that. Now, once again if you look at this, we will require the distribution of the minimum; the joined distribution of the minimum and the maximum.

We have already derived the separate distributions of the minimum and maximum but that will not be helpful. So, we again make use of the formula for the joined distribution of the ordered statistics; yr and ys. So, here you take r = 1 and s = n. Now, that has some advantage here, if you take this as 1, then this factorial will vanish, this factorial will vanish because this will be giving you simply n -1.

And here you will get; sorry, n - n that is becoming 0, 1-1 that is becoming 0, so you will get only n-2 here. So, this term can be simply find here and then we can write this in a closed form as the joint probability density function of X1 and Xn that is = n * n – 1, F of yn - F of y1 to the power n- 2, f of y1 * f of yn and certainly you will have that y1 is < yn. In order to obtain the distribution of this, we defined the transformation.

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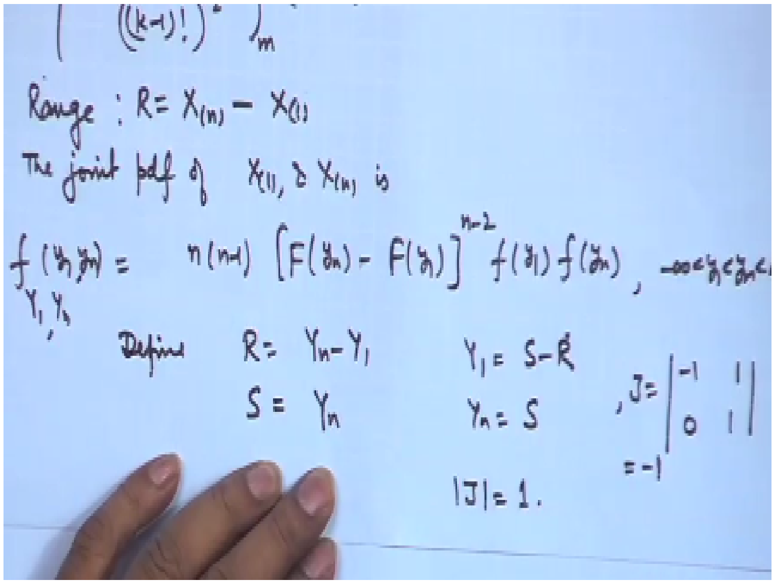

R= Yn – Y1 and then you take say, some new variable, say S is = say, Yn or Y1 for example, so if you take the reverse of this, you will get Y1 is = S - R and Yn is = S. So, if I consider the Jacobian, I will get -1, 1, 0, 1, so that is = -1. So, if I take the absolute value of this that is becoming = 1, so we can easily write down the joint distribution of R and S now.

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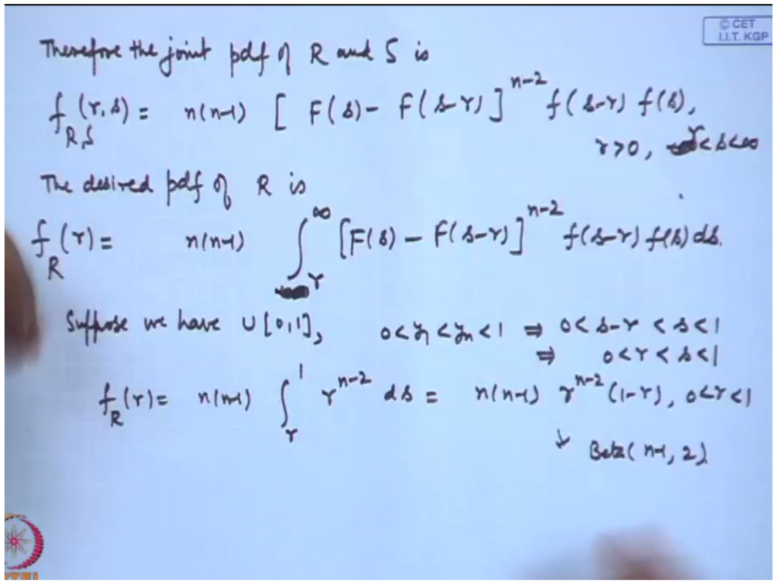

Therefore, the joint pdf of R and S; n * n - 1 F of Yn that is s - F of y1, that is s - r to the power n -2, f of s – r, f of s. Also you look at the region here; S – R is < S, so R is > basically 0, which is true, R will be > 0, so because you are subtracting the maximum - the minimum and S is anything that is means it is having the full region. So, the desired pdf of R is obtained as n * n - 1 integral and when you integrate with respect to S, this will be from - infinity to infinity.

F of s - F of s - r to the power n – 2, f of s – r, fs, ds, one important thing that you notice in the distribution of the sample median when the sample size was odd, it was quite difficult to write down the integral form there but in this case, since the range is the full, the derivation of the distribution of the sample range will not be that much complicated. In fact, we can see for example, if I consider say, uniform distinction suppose we have a uniform 0,1 then this is becoming s – s – r, which is nothing but simply becoming r to the power n -2.

And this is simply 1, so this is; and then the integral will become from odd to 1 because S is actually; here you will have 0 < y1 < yn < 1, so this will give me 0 < s – r < s < 1 because both are positive, so this region will come here. Now, this will give me 0 < r < s < 1; yeah there is a small error here because if I put the lower limit here, we are actually getting here R is < S that is coming here.

And at the same time, from the second one R is > 0 also, so when we do this one, this should be from r, so this was an error here. So, if you do that in the case of uniform, then this is becoming n * n – 1 r to 1 and this will become r to the power n -2 ds, so that is becoming n * n - 1 r to the power n -2, 1- r; $0 < r < 1$, which is again simply a beta distribution. Actually, this is a beta n -1, 2 distributions.

In general, the distribution of the sample range is not so straightforward however, statisticians like Hartley etc., that they tabulated the cumulative CDF of this for the normal population, for n like 10, 11, up to 20 and the asymptotic distribution has been studied by Goebel in 1944. So, what we have done is we have discussed the distribution of the ordered statistics, some special ordered statistics and certain special functions like median, or the range etc.

In a similar way, we can also study say, quantile for example, a particular position what is the distribution of X_4 are the one which is coming at; say 1/4 position; n/4 or 3n/4 or 1/5 or 4/5 and so on, different positions can be; the distributions of all of them can be obtained, sometimes they will be unique, sometimes they will be average of the 2 values but in all the cases, the distributions can be derived.

Now, next thing is that we can talk about the moments of the order statistics, like we discuss the moments of the random variables. Now, when we have the distribution of the order statistics what happens to that. Now, why is that important? As I mention in one of the examples of exponential distribution, each of the X_i had the mean 1/ lambda but the mean of X_1 that is the minimum was becoming 1/ n lambda that is 1/ nth of the origin value.

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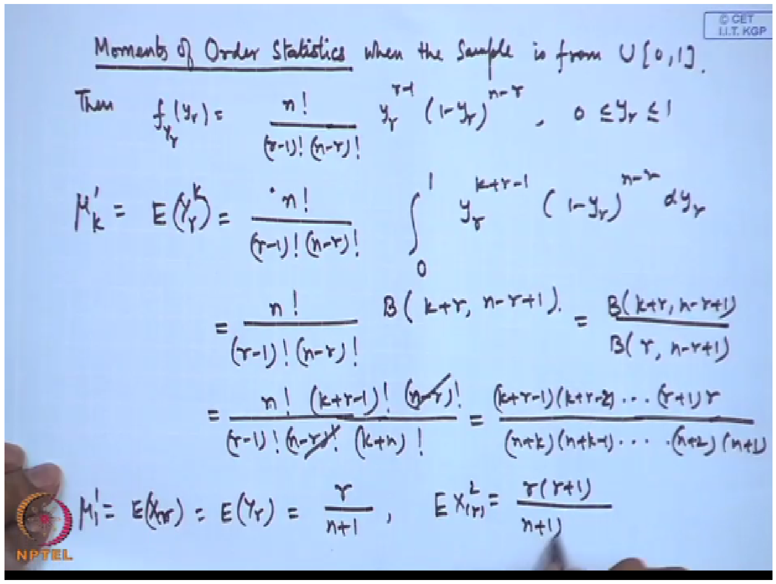

So, that is a drastic change in general, we will be interested in the distributions of the; or the moments of the ordered statistics. So, in the next session study this. So, we consider moments of order statistics. Now, you can see here that the general distribution of the order statistics is complicated as you have seen here, the form is like this. So, if I consider say for example, what is the expectation of yr are to the power k.

Then, you cannot say anything about this except that this is an integral yr to the power k * all things from - infinity to infinity. Certainly the evaluation of this integral is not possible except that you may consider some sort of special cases etc., so that part we will see that we will consider approximations of this expression and other thing is that if I actually take a specific form.

For example, if I am taking exponential distribution or a pareto distribution etc., then in some cases, the forms can be written. I am doing the simplest one, which is the uniform distribution and uniform distribution is of course, quite important as we have seen that if original random variables are from any distribution, then F of that becomes from uniform distribution, I actually started the topic of nonparametric methods with that.

So, if that is so, then and since F is an increasing function therefore, it makes sense to consider the order statistics of uniform random variables. So, if I do that, then I will show you that the forms can be easily calculated because we have derived the form of the distribution of order statistics of the uniform distribution as a beta distribution. Now, moments of beta distribution are known.

So, let us consider when the sample is from say uniform 0, 1, so in this case f of yr that is the R th order statistics, it was given by; basically it was n factorial/ divided by r -1 factorial n - r our factorial yr to the power r -1, 1 - yr to the power n - r 0 < yr < 1. So, if I consider say, Mu k prime that is expectation of yr to the power k, then it is = n factorial/ r -1 factorial n – r factorial, integral 0 to 1, yr to the power k + r -1, 1 - yr to the power n – r dyr, which you can again recognises as a beta.

So, this is n factorial/ r -1 factorial n - r factorial, so this will actually give me beta of k + r and n – r + 1, so we expand this, it becomes n factorial/ r -1 factorial. Of course, you can also say it as beta k + r n - r +1/ beta r n – r + 1, so it is simply a ratio of 2 beta but if you want an expressions is in the form of fractions, then we can write it as gamma k + r that is k+ r -1 factorial, then gamma n - r + 1, that is n - r factorial and divided by k+ n and +1.

So, it is becoming k + n factorial, now this you can expand in very systematic way, like this you can cancel out, this term you can cancel out and sorry, this will get cancelled out, so you will get here it is =; I adjust this term with this and with this and this, so I will get k+ r -1 and so on, k+ r -2 and so on up to r +, r/ n + k, n + k -1 up to n + 2, n +1. In particular, if I consider Mu 1 prime that is expectation of X1; sorry, Xr that is expectation of Yr.

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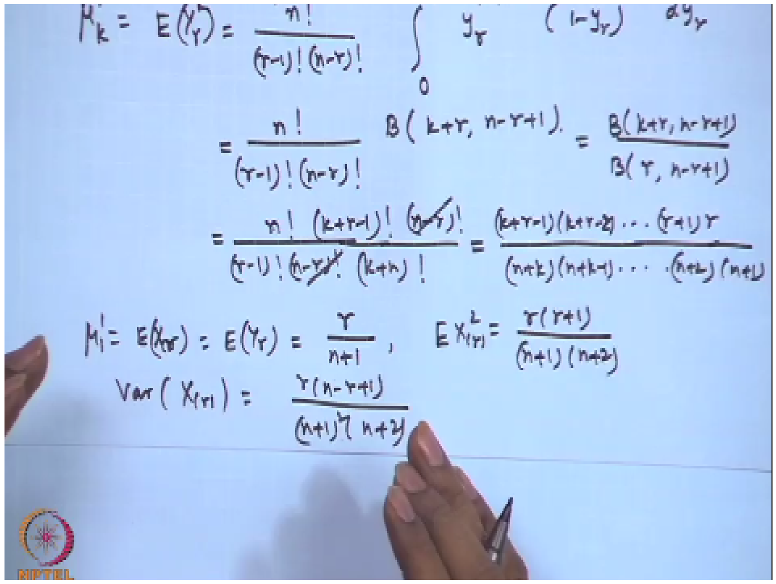

Then, that is becoming r/n +1 similarly, you can consider expectation of Xr square that will become r * r +1/ n + 1 * n + 2, so you can also talk about the variance here, the variance of Xr that is turning out to be r * n - r +1/ n + 1 square * n + 2, so this is quite interesting here. We are able to obtain the moments of the order statistics from the uniform distribution. Now, as before, if I consider the joint distribution between rth and sth, then from uniform what I will get?

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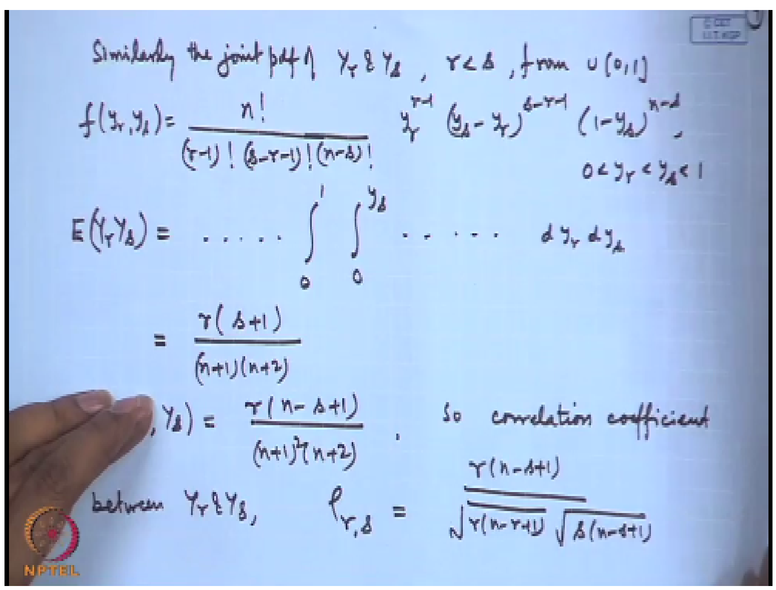

That I will get from here by substituting the Fyr as yr and 1- Fs is 1- ys and this will become ys – yr, so for uniform distribution the form that I will obtained will be; similarly the joint pdf of say, yr and ys r< s less from uniform 0, 1, then that will be F of yr, ys that is = n factorial, r -1 factorial, s - r -1 factorial, n - s factorial, yr to the power r -, ys - yr to the power s – r -1, 1 – ys to the power n – s, 0 < yr < ys < 1.

If I want to find out say, correlation between yr and ys in that case, I need the product moment also, we already obtained the form of the expectation of Xr and expectation of Xs, so similarly I can calculate the product moment, so for example what will be expectation of yr, ys. Now, in order to evaluate this kind of integrals, we can make certain transformations here, so I will not get too much into the detailed calculations, see this term will be there.

You will have actually double integral; say for example if you firstly do with respect to yr, then it will be from 0 to ys and then it will be from 0 to 1, all these terms dyr, dys, so in the first one, you make the transformation ys - yr = something, then basically you may get as tyr = t times ys then that thing will come out, you will get 1- t to the power something and here it will become t to the power something.

Then this will become; t will be from 0 to 1, then it becomes a beta function that can be evaluated. At the next stage, you can evaluate the integral, whatever term you are getting as a beta function in ys. So, I am not writing all the calculations here, by using all this transformations, it turns out that this term will be = r * s + 1/ n + 1 * n + 2. Actually, it is instructed to compare it with this term.

We had calculated expectation of Xr square as r * r + 1/ n + 1 * n + 2, here you can see this r has been replaced by s here, so based on this, if I calculate the covariance between yr and ys, then this is turning out to be r * n - s +1/ n +1 square * n + 2. So, now we can write down the formula for the correlation coefficient between yr and ys, let me call it as rho r, s. So, that will be = r * n – s + 1/ the square root of r * n – r + 1, square root s * n – s + 1.

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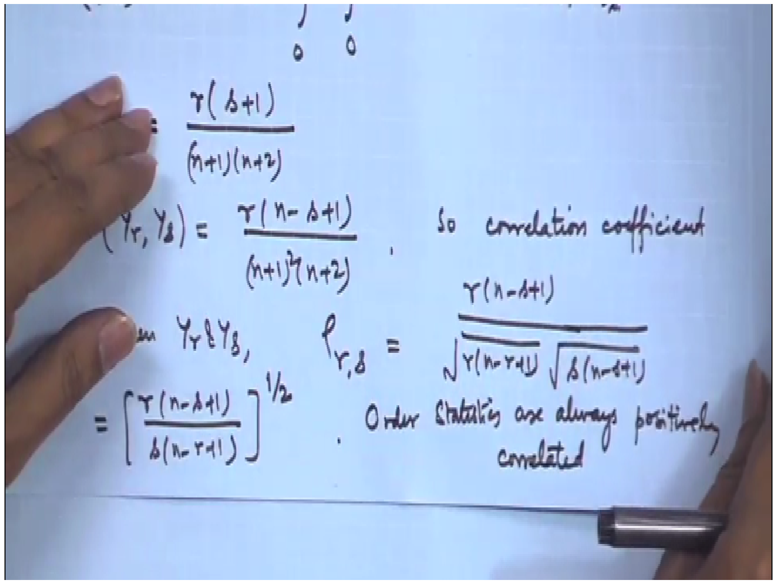

So, that will be =; this can be simplified as r * n – s +1/ s * n – r + 1 whole to the power 1/2. So, if r is <; then this is the form that you are getting; r is < s, so this is < 1 and n – s + 1 will be < n – r + 1. Another point, which you may note down that this term will be positive, so order statistics are always positively correlated, which is understandable because they are in the same direction here.

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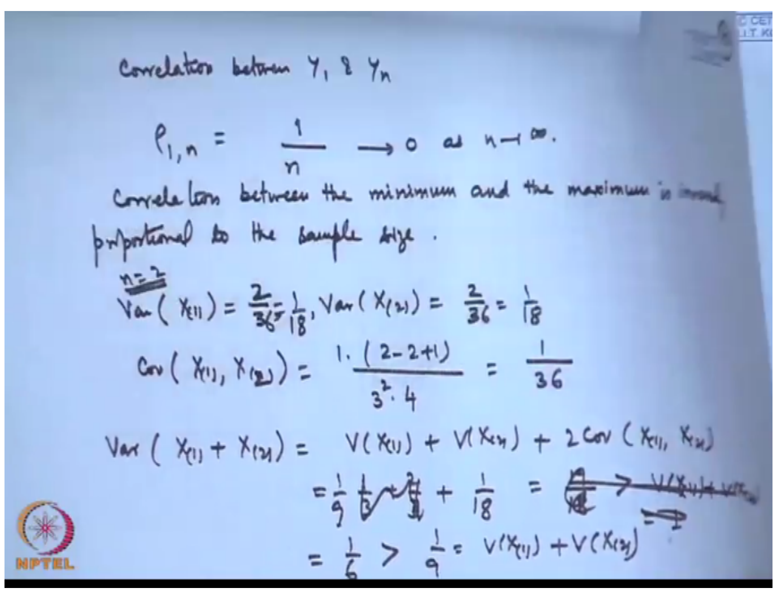

Another thing is that let us consider the minimum and the maximum. In particular, what will be; say if I consider correlation between say, minimum and the maximum that is rho 1n, what it will be equal to? So, r = 1n – n + 1 that is 1/n and n -1+1, so it is becoming 1/n square to the power 1/2, so this is very interesting. As the distance between the minimum and maximum increases, so the correlation is actually inversely proportional to that; that means it is simply inversely proportional to the length of this.

Actually, this is going to 0 as n turns to infinity, this is a very interesting observation that is the correlation between the minimum and the maximum is inversely proportional to the sample size, so you may also think like this that if I have too many observations then the relationship between the minimum and maximum will be of very mild nature that means they may become almost uncorrelated that means how they will vary.

But if the sample size is small, so suppose I say 4, 5 etc., then it is still having some significance of course, for samples of size bigger than 20 or something like that this will have very minor value because suppose, I say, n = 20, then I get only 5 or 0.05 basically. Usually, we have this like variance of summation = some of the variances if the relation is r0 that means if the random variables are uncorrelated.

Now, certainly your random variables say, which are now I am considering order statistics they are correlated, therefore if I consider say variance of say X1+ variance of X2, etc., then it will not be same as variance of X1+ variance of X2. Now, just as an example, you can consider say, variance of; suppose n = 2, then what is the variance of X1? That will be =; let us consider this; r/ n + 1, so I take n = 2 then this is simply becoming 1/3.

And what is variance of X2? That will become s/ n + 1, so that is = 2/3, because if I am taking 2, so this will become 2/3 and what will be the covariance between X1 and Xn; X2, so from the formula that we have arrived just now, here I am putting r = 1 and 2-2+1/; n = 2, so this is 3 square * 4, so this is becoming 1/ 36. So, if I look at say what is variance of say, X1 + X2 that is = variance of X1 + variance of X2 + twice covariance between X1, X2.

That is = 1/3+2/3 + 1/18 that is = say 19/18, which is certainly in this case, > variance of X1+ variance of X2 here. This is actually = 1 here because the variance of X1 is 1/3 and variance of X2 is 2/3. Another thing, which may look somewhat interesting, see here you have derived the exact expressions for; I think I made some mistakes here, this is actually not variance, this is expectation.

Variance will become = 1 and then this will be 2-1+1, so that will become 2 here and this will become 2/ and 3 square * 4 that is 36, so that is 1/ 18 here and this one will become corresponding to 2, then this is becoming 2-2+ 1 that is 2; 2/ 36 that is = 1/18 okay, so I made a mistake here, then this is becoming = 1/ 9+1/18, so that is = 1/6, which is obviously > 1/9 that is = variance of X1+ variance of X2.

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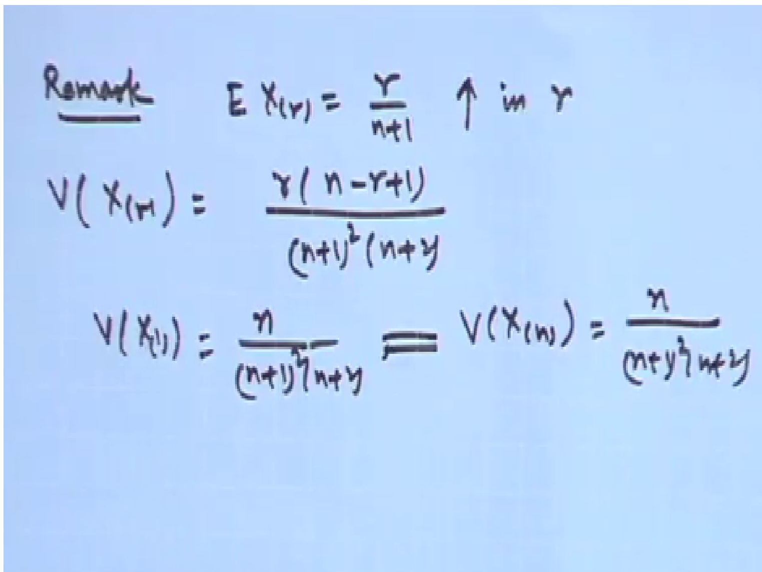

So, this is a mistake here but anyway now the fresh calculation shows the exact values here, you also look at this and the expectation of yr is directly dependent upon the positioning among the order statistics, so if I increase r, this is increasing, so this is another interesting fact here that this term is strictly increasing function of r here. So, let me just mention this remarks here; expectation of Xr that is = r/ n + 1 is increasing in r.

But if I consider the variance here that is not showing the same thing because it is r * n – r + 1/ some n + 1 square * n + 2, so this term does not show the similar behaviour actually, it will be maximised for r = say, n/2 kind of thing. Whereas, if I take minimum or the maximum that like if I considers what is a variance of X1 here, that will give me here n/ n + 1 square * n + 2 and similarly, if I consider what is a variance of Xn?

Then, this is again becoming = n/ n + 1 square * n + 2, so this is the symmetric function here, this is same, this function is becoming equal here okay, so expectation increases but the variability increases and then decreases here, so that is the way it will come here. In the next part, I will consider general order statistics right now; I have considered the order statistics from the uniform distribution, where we are able to obtain certain exact expressions for the moments of the order statistics.

In fact, if you see I can in fact, write down even the third order moment, fourth third moment and the third order and fourth order central moments, so we can actually study the measures of skewness and kurtosis also but if I am considering the general F here, in the general F none of these calculations can be done unless I have the explicit form of the F function there. Even if that is I have label in many times, for example you consider simple normal distribution.

In F is nothing but the capital 5 function, where capital 5 is the CDF of the standard normal distribution. So, if I have capital 5 to the power something and 1 - capital 5 to the power something and then a small 5, suddenly this type of integrals is not easy to evaluate because I will be multiplying by X2 the power k here. So, if I; even if I name the transmission 5x = something, the distribution part will become in the form of uniform.

But then this x will become 5 inverse of something, so I would not able to evaluate it. In such cases, it is useful to have certain tools, which can give approximate expressions are the bounds for the moments of order statistics. So, in the next lecture, I will be discussing the bounds on the expectations of order statistics