# Modeling Transport Phenomena of Microparticles
Prof. G. P. Raja Sekhar
Department of Mathematics
Indian Institute of Technology - Kharagpur

## Lecture – 10
## Hydrodynamics of Squeeze Flow

Hello! Welcome back. So in the last class we discussed about lubrication approximation. So today we are going to discuss a similar application which is popularly known as a squeeze flow. So as the word indicates I am sure you would have experienced a squeeze flow in a variety of applications in real life. So typically squeeze flow has applications in devices where you have such configurations.

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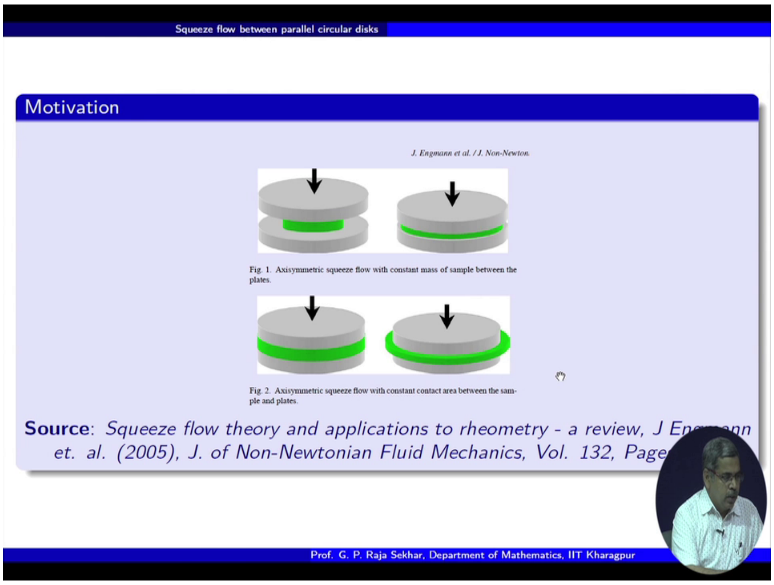


So there is some material and then you apply some force so then it gets squeezed it and then you get due to the corresponding flow you get the corresponding liquid like this okay. So there are a lot of applications so in particular filters and then some industrial dampers and where typically lubrication plays a vital role okay.

So today we are going to discuss about the squeeze flow okay. So as the configuration indicates so we are going to consider such configuration where so we consider a Newtonian flow within this okay. Of course this example I have taken from something non-Newtonian flow but today we are going to discuss about Newtonian flow bounded between two circular disk.

And then the lower disc will be stationary and the upper disk is exerted by some force okay. So let us formally understand the problem.

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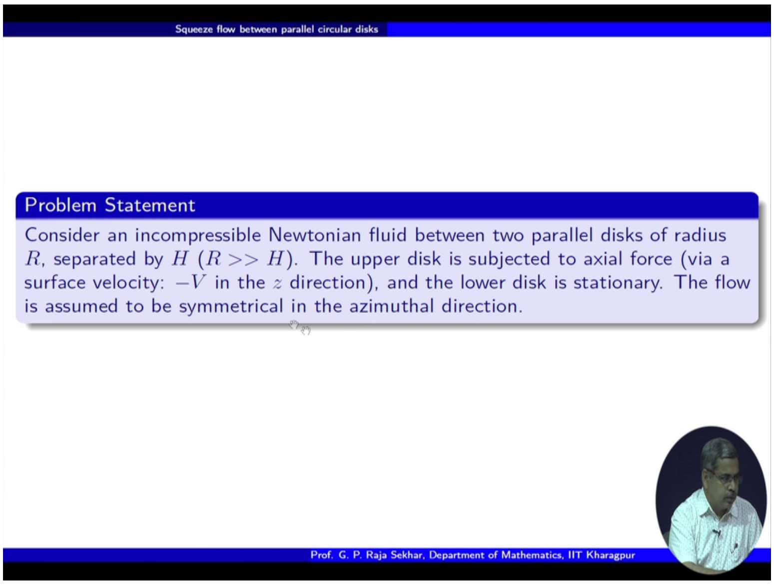


So the problem is we consider incompressible Newtonian fluid between two parallel disk of radius R separated by a distance H and we are assuming that R is much larger than H and as already indicated the upper disk is subjected to axial force. So typically when we say vertical so it is positive direction since we are pressing the disc so it is a taken away and negative okay.

So this is negative direction and the lower disk is stationary and the flow is assumed to be symmetrical in the azimuthal direction. So this is nothing but axisymmetry okay.

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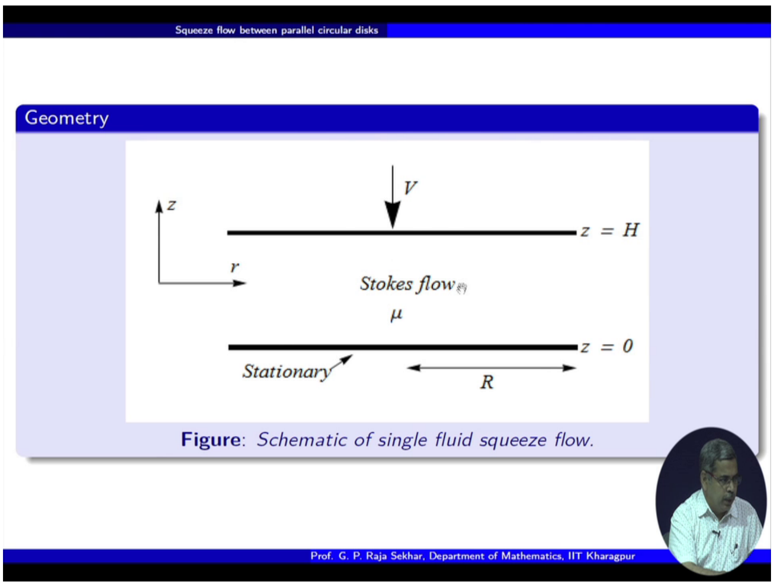

So let us look at the configuration. So since we are discussing as two plates so these are assumed to be circular but for configuration wise 0they are indicated like this okay. So we are assuming that the plates are located at z equal to 0 and equal to H. So we have the corresponding geometry here so this is r and then this z  okay.

So with this configuration so this plate is stationary and then disk so this is a velocity is given in this direction okay and we are assuming Stokes flow with viscosity Mu. So since we are considering steady Stokes flow so we have corresponding equation of continuity and then momentum balance. So for the cylindrical configuration with the axisymmetry we write down the equations in component form okay.

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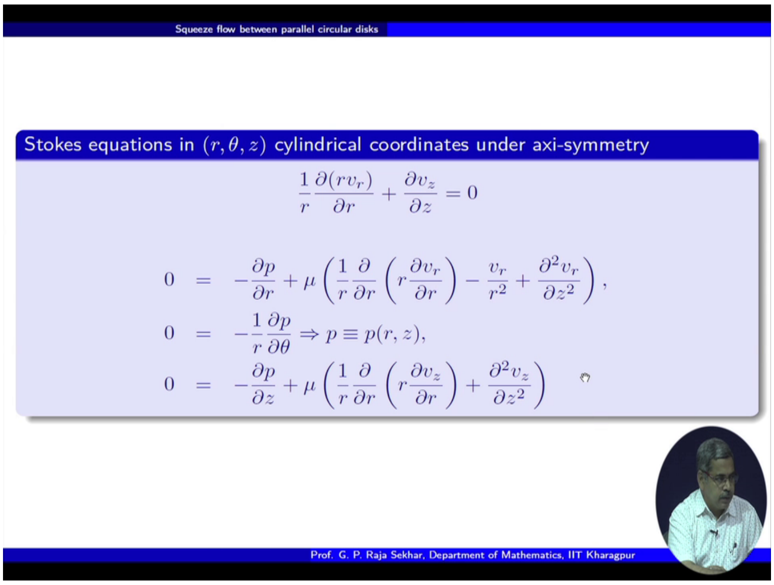


So this is the equation of continuity as you can see there is no v Theta term because of the axis symmetry assumption. Now let us look at the r momentum so r momentum you can see there are no derivative terms with respect to Theta that is again due to the axisymmetry assumption okay. So already we have discussed these equations in a some of our previous lectures.

So one can refer for the full system of Navier-stokes equations in cylindrical coordinates. From there if you apply access symmetry we get this is the r momentum and Theta momentum this is redundant because all the variations with respect to Theta are 0. So we get the corresponding u Theta is 0. So therefore we get this corresponding pressure derivative is 0.

So this immediately indicates that pressure is function of r and z only. And finally we are left with the z momentum equation so which is given by this okay. So this is the basic assumption so now if you see we have radial velocity and we have actual velocity. So we would like to know for this configuration since we are assuming r is much larger than distance between the plates disks so what kind of competition between vr and vz and which is more prevailing compared to the other.

So this one has to analyze. So this is possible via the corresponding order analysis. So correspondingly, we introduce some scaling arguments.

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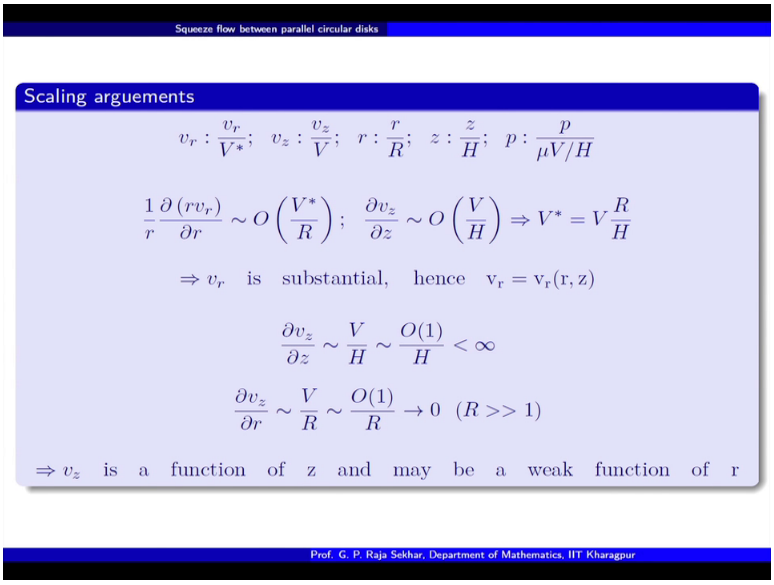


So we know that vertical velocity is of order V. So therefore we first normalize the axial velocity with V and then radial for the time being let us say it is V* which we would like to estimate and then R is radius of the disk and z is normalized by the distance between the disks and pressure as usual which we have used okay. So now, let us consider our two terms in the equation of continuity.

So you can recall so this is one term and this is another term. So correspondingly if you see this so you have an r there and the derivative so ultimately this will be of order V*/R okay. And similarly this if you introduce this scaling, so this will be of order V/H okay. So now so these are the two terms in our equation of continuity. So from this one can conclude that V* is V times R/H and R is much larger than H.

So this indicates that the radial velocity is substantial okay. So because R is a much larger than H so this indicates that the radial velocity scaling is substantial okay. So that is our conclusion. So radial velocity substantial, hence we assume to be function of r and z. So now let us get some estimates on the axial velocity. So we have one can note because already we have indicated here and so V is of order 1.

So then you can see so this is finite okay. Because this is comparable. On the other hand if you consider this, this is of order V/R and this is of this order. So now our assumption is R is much larger. So therefore this goes to 0 okay. So that means the radial variations of the actual velocity are almost negligible because R is much larger. So hence our conclusion of vr is substantial has to be supported by some conclusion on vz.

So that is nothing but vz is function of z and may be a weak function of r which is a negligible okay. So that is so in some sense the corresponding lubrication kind of approximation is directly reflected in this fashion okay. So we go with vz as a merely function of z alone and try to proceed with the analysis okay.

So if you can visualize suppose you have two disks and then pressing. So you can visualize so the flow gets like this so radial velocity has a lot of importance in this problem okay. So that is what we have seen it mathematically okay.

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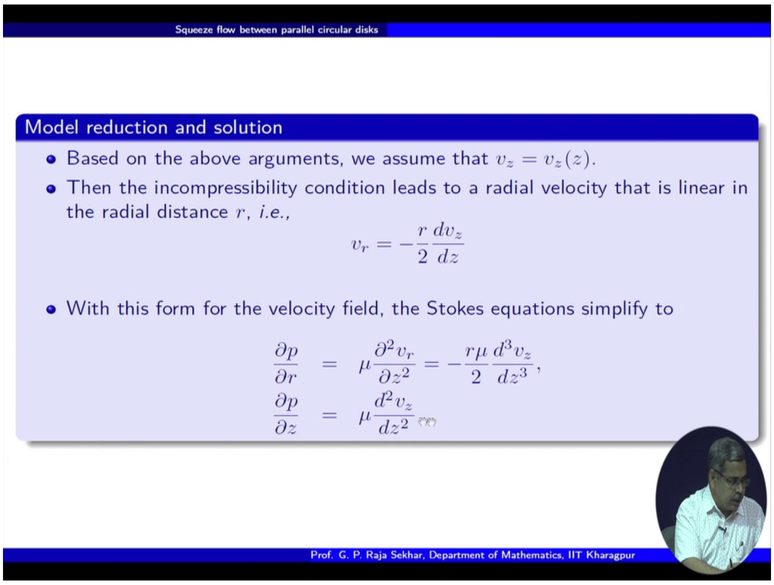


So now under this assumption how the model gets reduced? So the above arguments this is already we have concluded actual velocity is function of z alone. So then from incompressibility that is conservation of mass that is equation of continuity we can obtain this. So this is a very straightforward. So let me so we have the reduced conservation of equation under axisymmetry.

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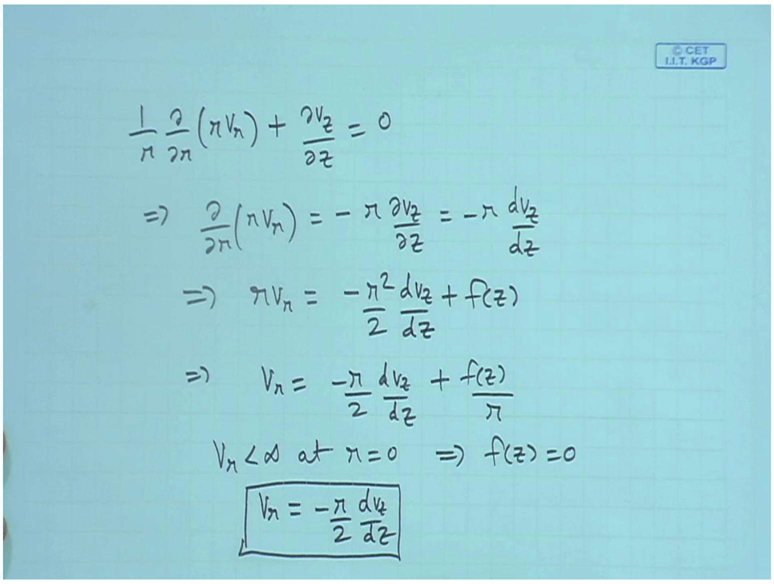


We have already seen this plus = 0 and then this vz is function of z, that already we have seen. So this implies -r so this I can write it as because its function of z alone. So now we integrate okay. So plus a function of z so this okay. But we need a boundedness condition. So that is Vr is finite at r = 0. So this indicates so therefore what we get is simply.

So the incompressibility condition has given a this is function of z. So radial velocity is linearly depending on the radial distance. So that is the inference from this relation you are getting. So any case we are going to use this for further analysis. So that is what I have indicated here so this is what the incompressibility assumption. So now we have to get the corresponding momentum equations.

So again here it is very easy to observe, please pay attention to the corresponding r momentum equation. So we have obtained vr is some constant that is function of z times r okay. So therefore this is r in the numerator and r power 2. So this is 1 over r okay. Similarly here whatever r is left with this derivative that is nullified. So then this interior is of order r and again that is nullified so this is 1 over r and here 1 over r the constants remain.

So therefore this complete combination is 0. It is very easy to calculate okay. So correspondingly vz is already we have seen its function of z. So therefore this term is also 0.

So that is what we have written here. So this is r momentum reduced to Dow p/ Dow r is this, then on using this relation one can write okay. Then the corresponding z momentum equation is a correspondingly given by this.

So now let us say if you take derivative with respect to r, so then the right hand side will be 0 because vz is function of z alone right. So what we do we differentiate this with respect to r and this with respect to z and write down here.

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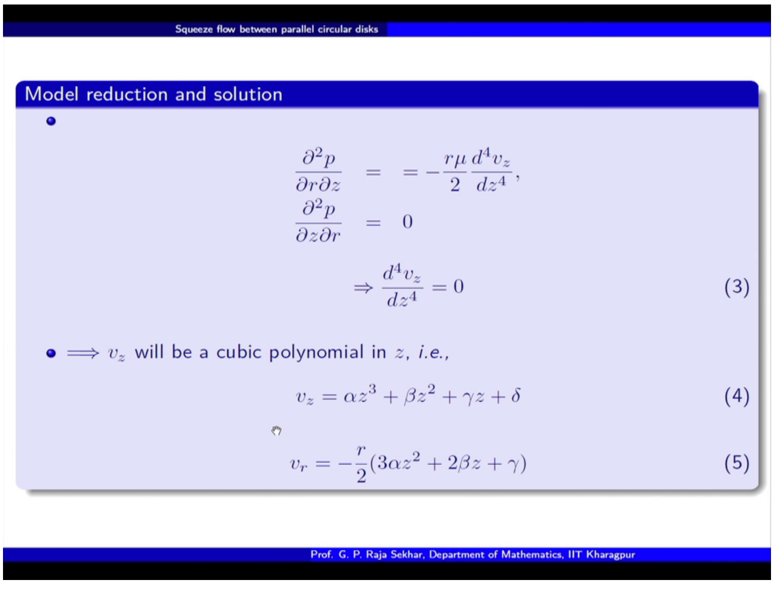


So from the z momentum equation we get this and for r momentum equation we get this? So in combining these two one can infer that the ODE for the vz is this. So this indicates the velocity structure will be a cubic in z. So where Alpha, Beta, Gamma, Delta, are some arbitrary constants that are to be determined. And once we have vz we can write down vr using the relation that we have okay.

So that means at this stage we have obtained the general structure of the velocity component okay. So once we have so our next aim is naturally to compute the pressure because in most of the squeeze flows see the velocity whatever is applied so that play a role and corresponding to that what is the pressure and then corresponding to the pressure what is the force okay.

Because most of the times the physical quantity that is of interest is force which is required to estimate various physical mechanisms okay. So therefore, correspondingly what we do is we first integrate for the pressure and then go for the computation of the force okay.

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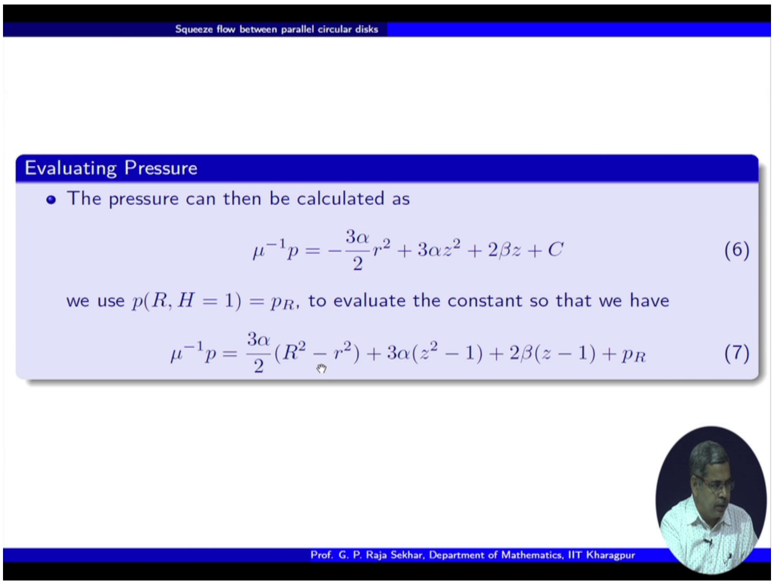


So this is again very straightforward okay. So this is again I can just make you understand.

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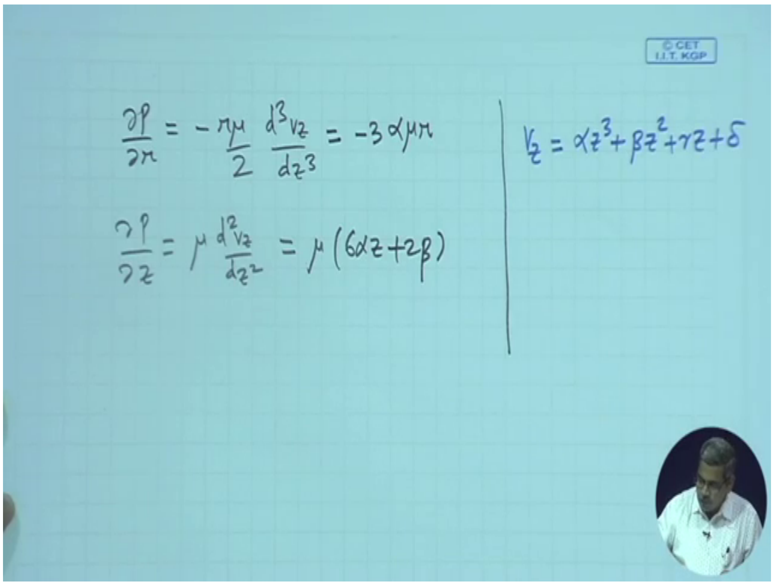


So we have the r momentum equation which is reduced already so that is okay. And we have vz structure here which is vz is written as Alpha z3 + Beta z power 2 + Gamma z + Delta okay. So therefore if we use three times okay so what we are going to get? This will be -3Alpha Mu r okay. So this one can just we have to compute third derivative substitute and we get this okay. Similarly we have okay.

So from here we can get Mu to be okay. So, this one can calculate very easily. So that is what we have done then we need to integrate okay. So what I have done is Mu has been brought okay. So we integrate this with respect to r so then you get a function of z and then you can use this to evaluate that. So once you do that exercise you will get this. So this is again very straightforward so we get the corresponding expression for pressure is this.

And this constant has to be evaluated okay. So typically the corresponding condition for the pressure is it is taken. At the centre typically the pressure equal 0 and at the periphery some pressure is given. So one can take some non-trivial PR at edge of the disk and then compute or one can take zero pressure okay. So now we for simple calculation purposes we take H to be 1 and then we are computing PR that is at the at the R edges okay.

So once we prescribe this pressure this C can be eliminated and we get the corresponding expression okay. So again this is very simple calculation, okay because one has to substitute this condition and eliminate C and then substitute it back. So then this can be nicely rearranged okay. So we have now got the pressure but however if you see we have the velocity expressions which contains these arbitrary constants and we have obtained expression for pressure in terms of these arbitrary constraints.

So still these arbitrary constants are to be determined okay. So these are these will be determined subject to the no-slip condition on the two disks. So that is the next task.

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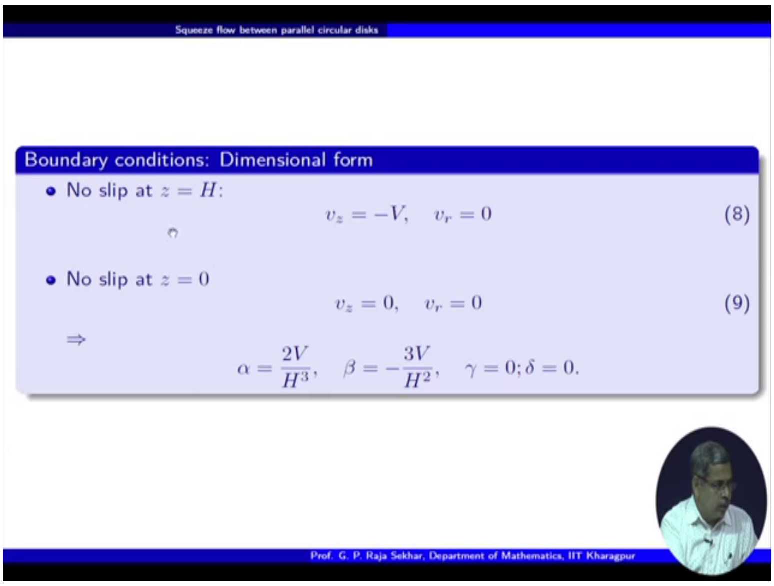


So then no-slip back to z = H. So you have vertical velocity is - V because we are pushing and then the radial velocity is 0. And at the bottom it is a stationary disc, therefore vz is 0 and vr is 0. So then one can use these to determine the arbitral constraints so again this is very trivial. So you should be able to solve and then match with these constants. So once we have these constants we have the complete velocity determined okay.

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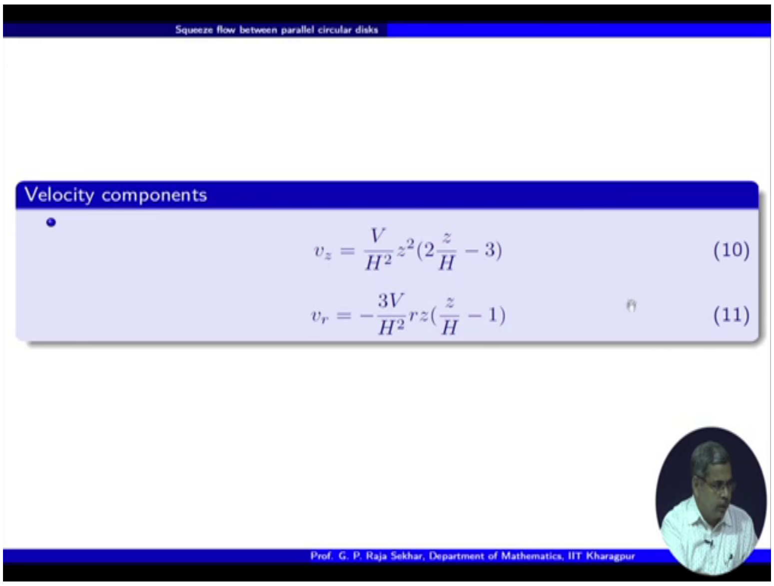


So we have the complete velocity determined right. So now as indicated so we have already obtained the velocity and pressure. So then the next task is to get the force. So now typically in a squeeze flows pressure play a vital role. So one can integrate the complete normal stress to get the force or get the component of the force that is due to the pressure alone okay. So to start with we would like to discuss the corresponding force due to pressure alone okay.

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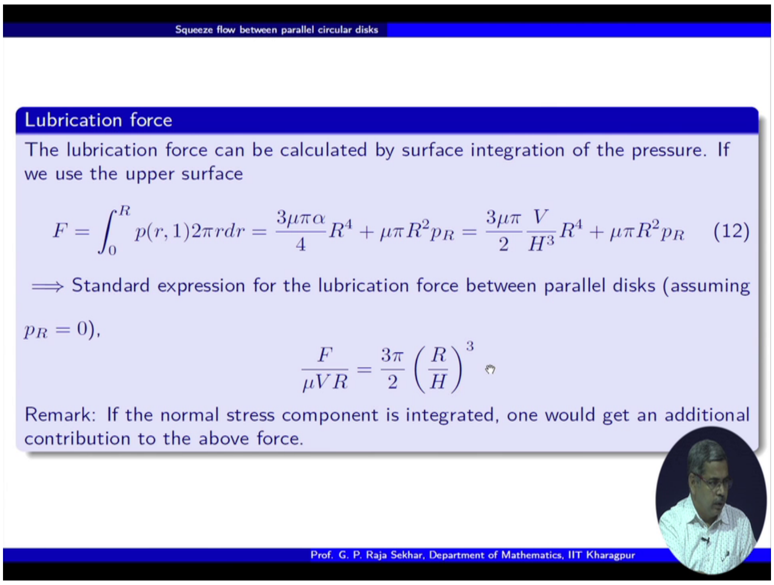


So this is called lubrication force and this can be calculated. So we are computing between 0 to R and at H equals to 1 considering H equals to 1. So we are integrating the pressure the corresponding areal element, then we have substituted p(r,1) okay that is computed. So then one can see so there is a contribution due to the pressure constant that is provided at the edge and then there is a component due to the pressure okay, hydrodynamic pressure.

So correspondingly we have the value of Alpha. So once that is substituted we get these two components okay. So this one if you take pr is 0 this component is not there. So typically this is termed as the lubrication force okay. So standard expression for the lubrication force between parallel discs is if you assume pr is 0, we get the force equals to this. But then if we non-dimensionalize, so then we get the corresponding force as 3Pi/2(R/H) power 3 okay.

So this is a competition between R and H so here we are assuming R is large, so therefore, this is a substantial force as well. So as I already indicated, so if the normal stress component is integrated, one would get an additional component here. So when I say what is the corresponding normal stress component?

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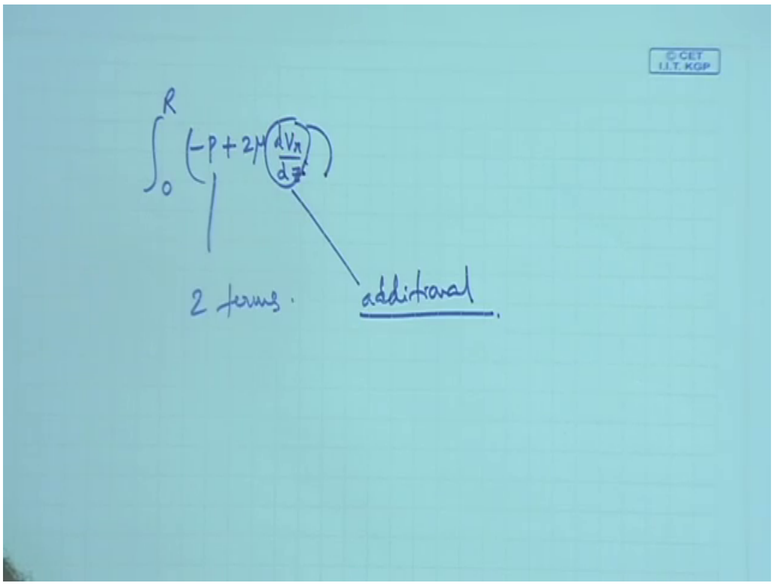


What I am trying to say is, so if you integrate p + 2Mu dvr/dz okay, so this is a normal stress component. So if you integrate so from here we got two terms. So from here we get an additional term okay. We get an additional term so this is normal stress component should be think the dvr/dz or so this one has to check okay. So correspondingly, we get the additional component okay.

So in this case we are not considering this we have only integrated the pressure. So correspondingly we are getting two components okay. So this is one and this is the other one okay. So with this we have got the velocity components, pressure and the force. So the next interesting is to compute the stream lines.

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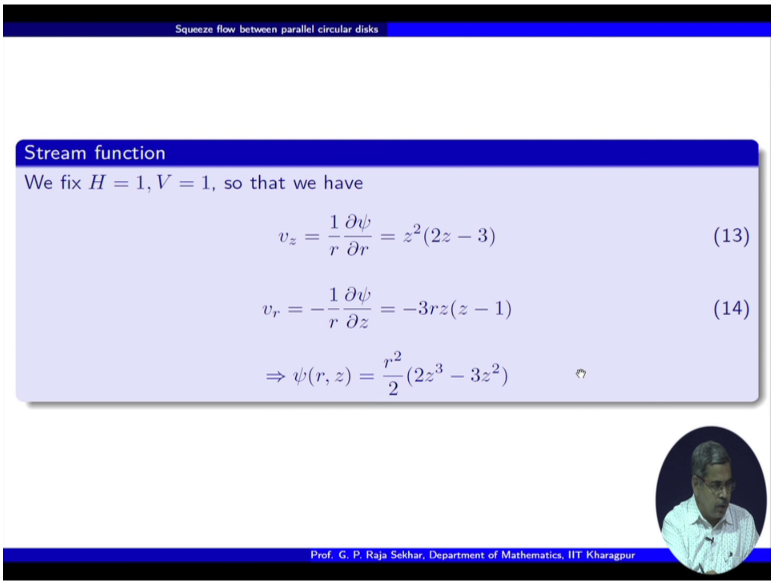


So we fix H = 1 and V = 1 just for the calculation purposes and then try to get the corresponding stream function okay. So vz is given by this and vr is given by this where Psi is a steam function and when we take H =1 and V = 1, we get to the simplified expressions for the velocity components. So then one can integrate this to get the stream function okay. So the calculation is very straightforward so not explaining the integration.

So you can try yourself and see that the corresponding stream function is this okay. So as you can see so this is streamlines are with respect to the radial distance, they are quadratic okay. So naturally one would be interested in seeing the corresponding contours okay. So how the corresponding contours are behaving okay? So one can use some mathematica or Mapple.

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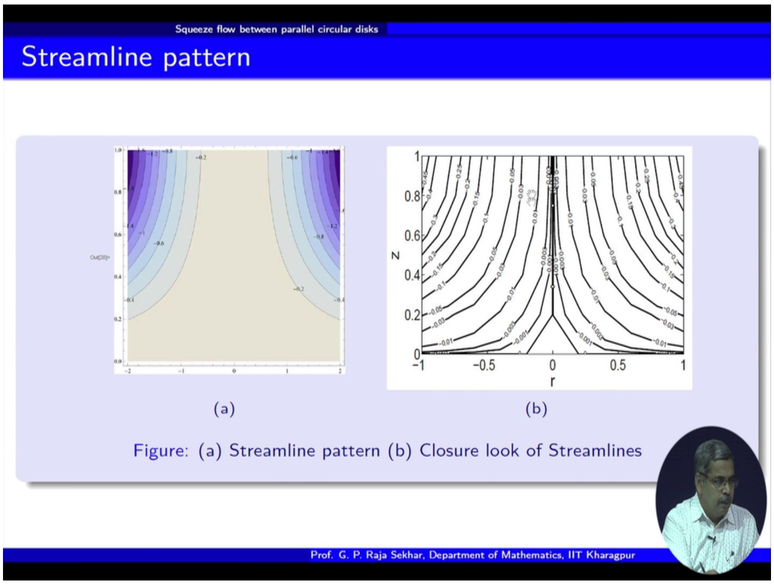


Figure: (a) Streamline pattern (b) Closure look of Streamlines

So this is a done in mathematica with a simple command of contour plot with the values shown. So these are the Psi = -0.2, Psi = -0.6, Psi = -0.8, so on so forth. But we would like to see what really happens at a closer look. So these are using Maple so these are drawn with you can see much closer denser plot. So you can see the radial flow going up and then towards the centre it is a more straight okay.

So the flow gets radially distribute so you are squeezing it and then the flow is getting distributed okay. So radially but the radial velocity is a very much substantial not only in radial direction also along the axial direction. So there is a lot of deviation okay. So this is a very important application in various lubrication equipments and I hope or you get an idea of a squeeze flow.

And then one can study advanced configurations like squeeze law of two immiscible flows or like you have a soil above and then porous soil and then you are filtering something through a porous pack. So this is another kind of configuration. So these are very much useful in various micro-fluidic environments. Thank you!