Modeling Transport Phenomena of Microparticles
Prof. G. P. Raja Sekhar
Department of Mathematics
Indian Institute of Technology - Kharagpur

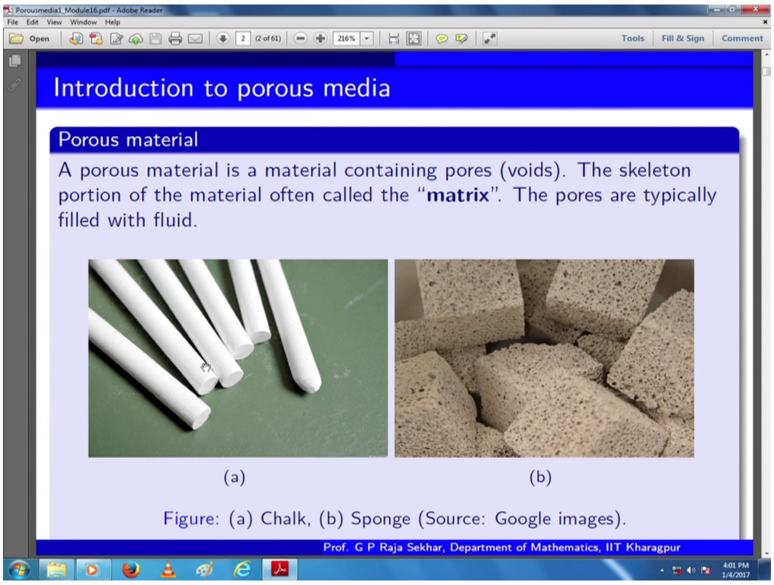
Lecture – 16
Introduction to Porous Media

Hello! So far we have discussed about Stokes flows and the flow past particles, Stokes flow past drops etc. But in many of the applications where various micro particles are involved so you come across either flow past micro porous particles or sometimes micro particles sedimenting inside some like arteries etc, okay. So these are some concepts which have applications of flow through porous media okay. So today we are going to discuss some introduction about flow through porous media.

So when you get a word like porous media, so intuitively one can think of a anything which has some pores okay. So pores in the sensor some gaps. So there is a material which is inter connected but there are some gaps. So then we call it porous okay. So sometimes some materials you can see immediately and conclude that this is porous but sometimes by looking at it you may think that this may be or may not be. But then when you put in some fluid then you will see whether it is really porous or not.

That means so in the latter case the gaps will be very compact and hence with a naked eye we are not able to observe okay. So let us see as an introduction to porous media, so what is the basic definition and various examples okay. And typically when flow through such a porous media takes place what kind of physical insights one has to understand before one solves the real problem okay.

(Refer Slide Time: 02:20)



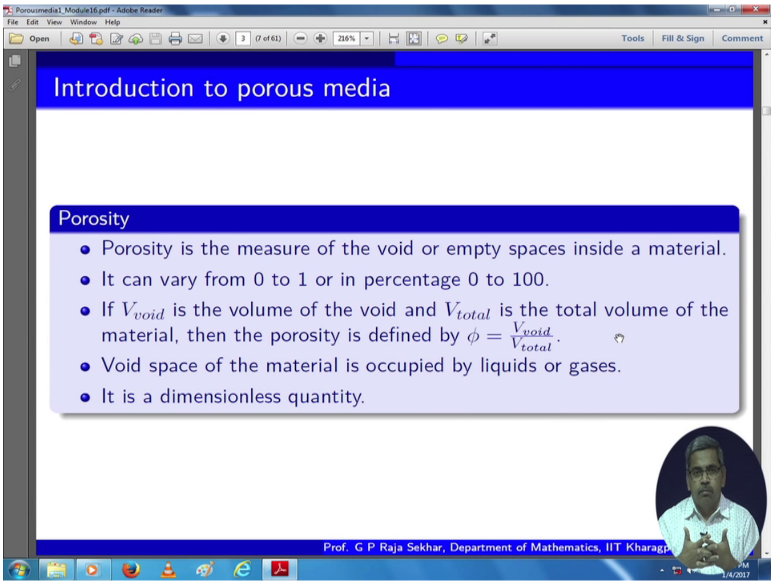
So as I indicated a typical example we see in classrooms, these are the chalk. So somebody can obviously conclude that this is a porous and somebody may say that I cannot see any pores and it is a solid. But then many times you see we put it into water and then try to write it on the board right. So when you put it on water so you can see some amount gets absorbed in this okay. So that means there are pores and this is a non deformable okay.

And on the other hand you have sponge so you can see the pores very well. What are the pores okay? Pores means look at the definition porous material is a material containing pores that is voids. So these are the voids you can see. Here also you have voids but only thing we are not able to see with our naked eye and the skeleton portion of the material often called the matrix. So this is the matrix okay. And these pores are typically filled with fluid.

So at this stage if you keep it in air so naturally air goes in. Then if you put water, naturally water goes in okay. So this is a non this is a rigid porous material whereas this is a deformable porous material okay. Because point if you apply some pressure then it gets a deformed. So then as we get some idea if you have dense particle packing so then if you put some liquid, so the liquid percolation inside will be little difficult but it will go with if you wait for a long time kind of.

Whereas here since you have large pores so liquid passes through very quickly. So that means the packing here is very dense whereas the packing here is less dense or sparse, if not very sparse but okay. So definitely there is some physical quantity that we can introduce characterizing the packing structure.

(Refer Slide Time: 04:46)



So that is nothing but porosity. So porosity is the measure of the void or empty spaces inside a material.It can vary from 0 to 1 or in percentage 0 to 100. So then how do you measure if V void is the volume of the void and V total is the total volume, porosity is the fraction okay. So this is a fraction void to the total right. So then a void space of the material is occupied by liquids or gases okay. So it is a dimensionless quantity right.

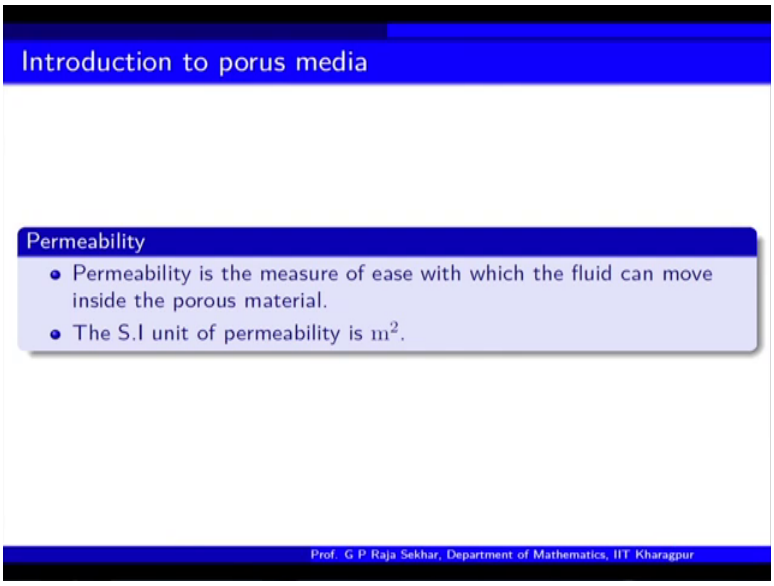
So that means if porosity is one then that means it is a completely void okay. So there is no obstruction kind of okay. Porosity is zero means there is no pore okay so these are the limiting cases. So I have given two examples one is a non-deformable other is deformable and let us see a small video.

(Video Starts: 05:53)

So we have taken a sponge and we are putting some dye okay. So you can see initially you have pores on the sponge but then if you put some liquid it may take some time slowly percolate because initially air and then you have some pour spacing. So depending on the competition between the pore space and the viscosity it gets. Then slowly you see you can so this is percolating okay. If you keep it with some weight it is a with some gravity it is going down. Then you apply some external force.

See we are deforming. So the tissue gets deformed and then more flow goes inside. So initial concentration was high and now once you apply some pressure it is fading out okay. So most of the biological materials are deformable porous and then various drug transport etc, happens by this kind of mechanism. Like so you have a lot of pumping systems and then using deformation via these pumping the various drug gets transported okay. So this is just to give you an idea about deformable porous media okay.**(Video Ends: 07:20)**

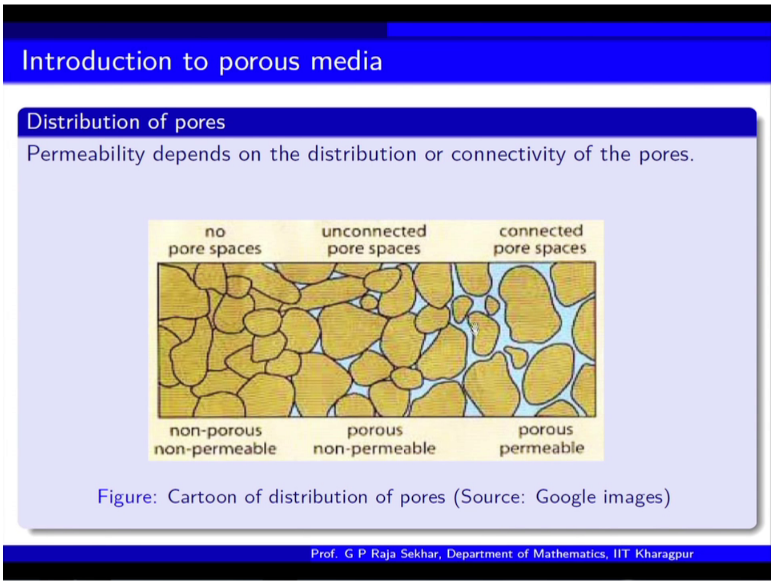
(Refer Slide Time: 07:22)



So now let us look at further physical insights. So once you have a porous material so then some material as I indicated flow is moving very quickly but in some cases it is not. So that means the ease with which the fluid can move inside the porous material that characterizes the porous media okay. So that is called a permeability. So permeability means the measure of the percolation okay measure of the percolation. And typically the S.I units of permeability is a meter square okay.

So in general the units are length square. So if it is a less permeable so naturally flow cannot pass through very easily. If it is more permeable flow can pass through very easily okay.

(Refer Slide Time: 08:20)

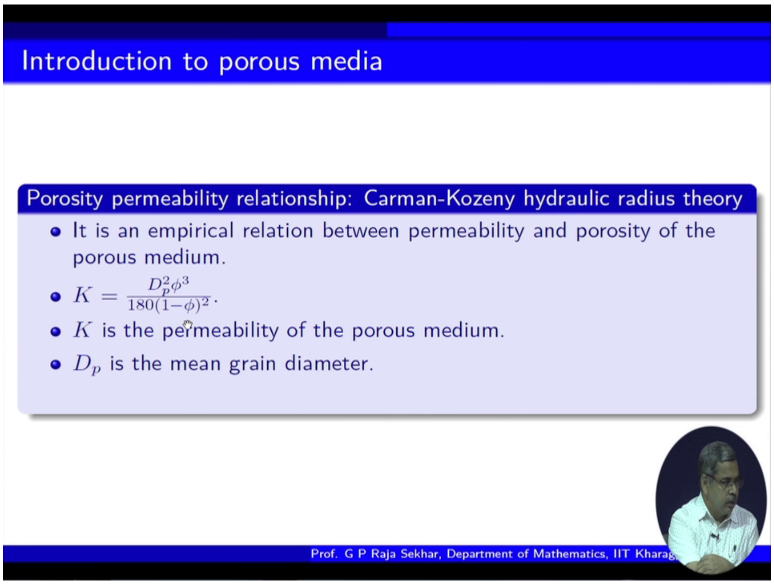


So again the pores distribution depends on the connectivity okay. So you can see so here there are no pore spaces okay. So you have some particles but they are connected no pore space. Here there are pore spaces but I mean if you look at any planner diagram so if there is some flow but it cannot go through because it is not connected okay.

These pores are not connected but whereas here pores are connected okay. So there is a general theory depending on the granule packing structure you get different permeability's that means there is a lattice arrangement and the corresponding relation one can obtain. But we are not going to those details. This will just give an idea about no pore, unconnected pore and connected pore okay.

So now what is the next question is? You have porosity you have permeability. So are they really connected okay. So that is the next question.

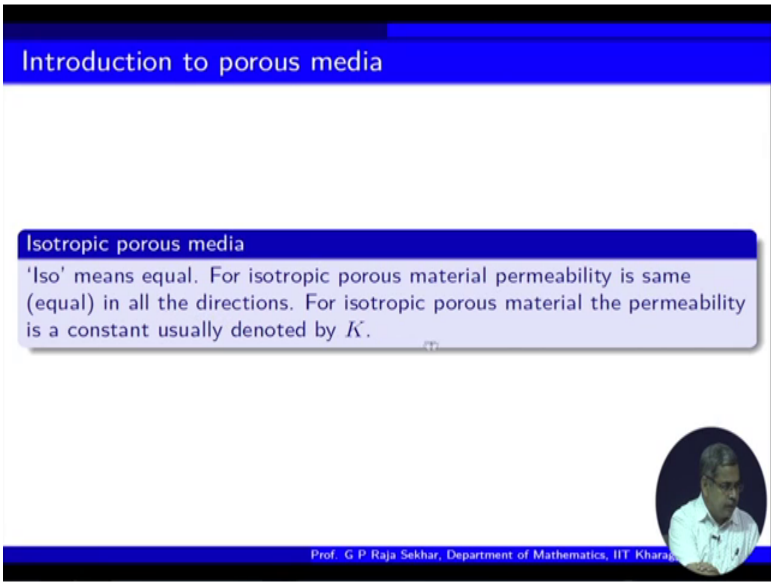
(Refer Slide Time: 09:42)



So to answer this Carmon-Kozeny they have given a formula so that is based on hydraulic radius theory. It is an empirical relation so where permeability is related to porosity in this order where Dp is the mean diameter of the grain okay. So is the mean diameter of the grains. Well this is not the very universal. So there are variety of ad hoc alterations of this and depending on the context.

Suppose sometimes somebody is considering elongated fibres okay, so in various randomized or perpendicular parallel fibre. So bundles in that sense. So the corresponding Carman Kozeny may vary and things like that okay. So but this is just a generic indicator to connect permeability and porosity okay.

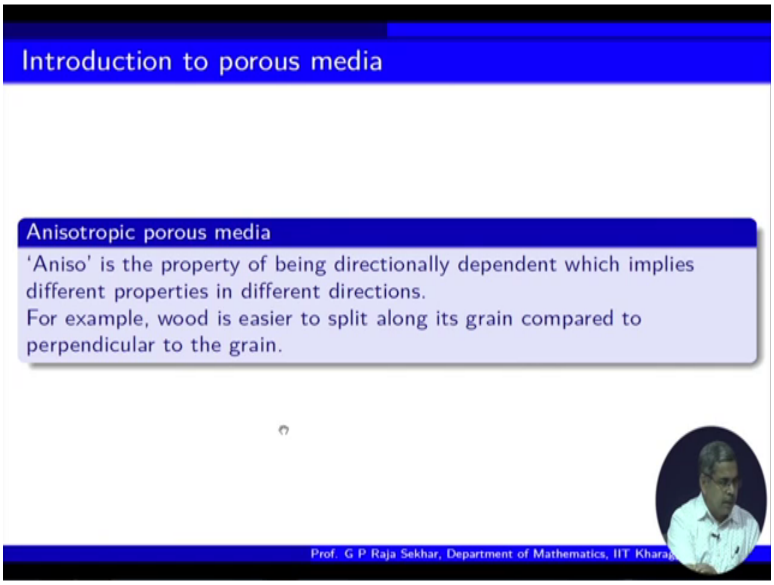
(Refer Slide Time: 10:43)



So the natural question is how the direction dependency play a role in the permeability. Suppose your fibres are very nicely organized and then they are homogeneous and there is no change with the direction etc, so then we say the permeability is simply isotropic. That means so equal in all directions. So that is what the first characterization Iso means equal. So for isotropic porous material permeability same in all directions okay.

And then K which we denote it by K it is a simply a constant.

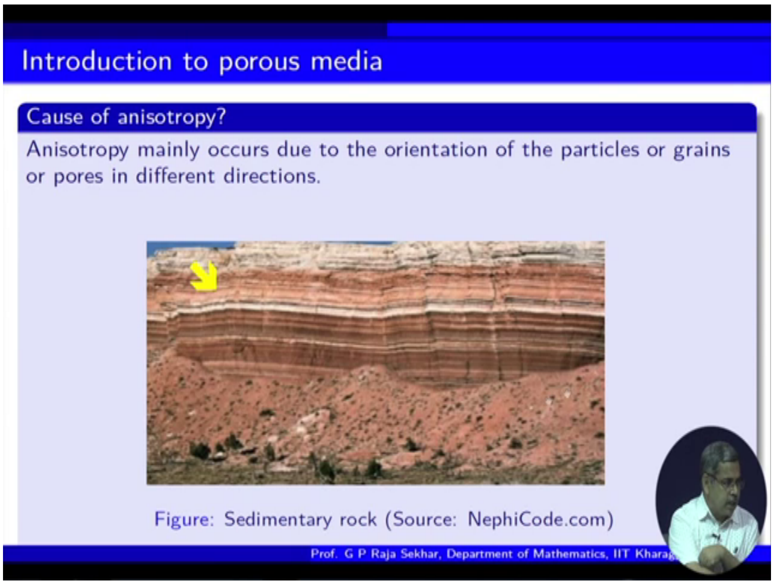
(Refer Slide Time: 11:32)



Whereas in some cases the bundles are arranged very in a particular direction. That means they are nicely arranged with within a particular pattern. So in that direction the permeability is more whereas across the permeability is less. So kind of so that means the permeability that is the ease of percolation is depending on the direction okay.

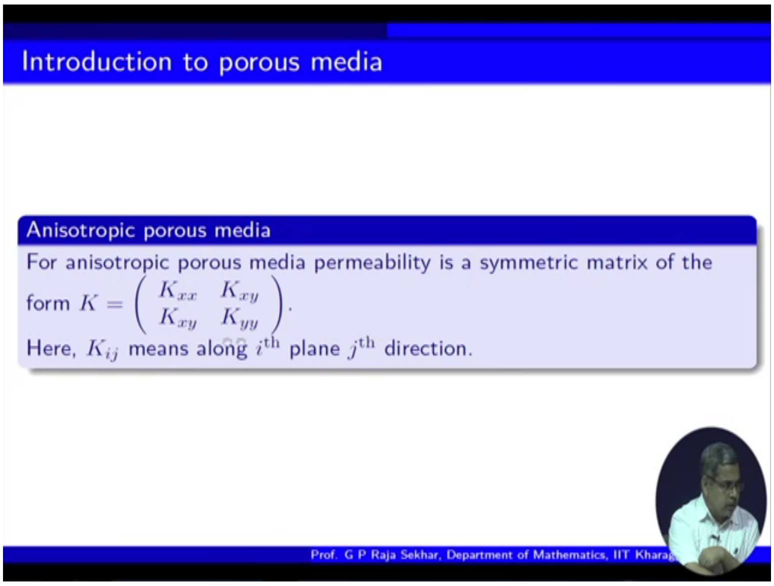
So in that case definitely we expect that the permeability is directionally dependent and correspondingly we call it an isotropic okay. So in this case for example if you take a typical wood we see now wood, due to the layers of the tree we get wood structure in some eccentric circles. So the layers and layers so that means horizontally percolation is expected difficult but vertically it may be easier. So that means it is a direction dependent.

(Refer Slide Time: 12:33)



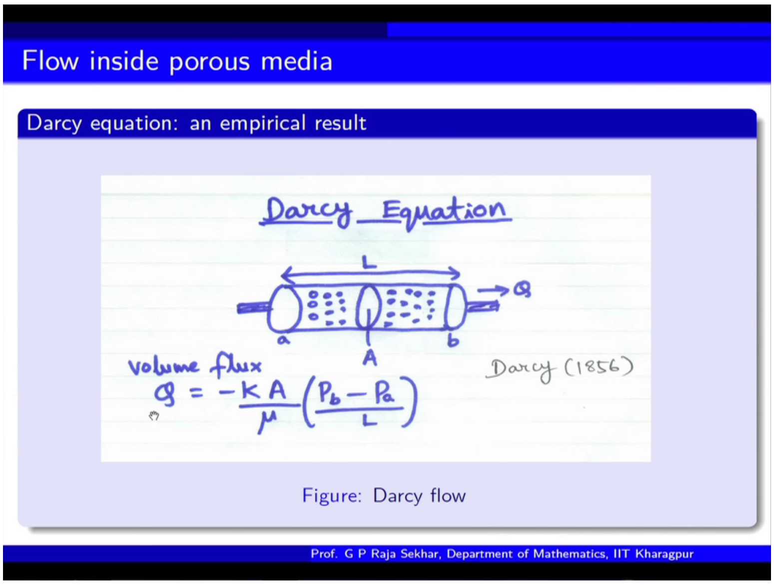
So this is one example. Rock you can see here anisotropy is occurring due to the orientation of the grains. So you can see so one would expect percolation this direction is a slightly difficult compared to percolation in this direction okay. So this is one case where the anisotropic permeability is seen. So there are a variety of four structures okay. So if such an isotropic structure is there of the soil or rock or any porous media then what will be the structure of the permeability? So that is the next question right.

(Refer Slide Time: 13:17)



So in two dimensions we are showing it. So in this case if the porous media is anisotropic we expect that the permeability will be a matrix rather than a constant. So these are the principle directions. Then you have the off diagonal directions so Kij means along ith plane in jth direction okay. So this is a for anisotropic and typically it is a symmetry okay.

(Refer Slide Time: 13:49)



Suppose one can consider what will be the corresponding dependency on the volume flow okay of the permeability. So Darcy is the one, Darcy is an engineer who was so working with waterworks and then by part of his job once he tried to measure this empirical relation. So if you see so this is a pipe and filled with some particles okay. Instead of clear flow, so then suppose at A some pressure is maintained at B some pressure is maintained and length of the pipe is L.

So typically as a layman if you consider a pipe okay of this length then what happens? If you have certain length and then you maintain pressure and then you want to measure velocity. So keep the length fixed and then you change the pressure okay. So then naturally if you are increasing the pressure reference velocity will be increased. On the other hand keep the pressure fixed and then change a length.

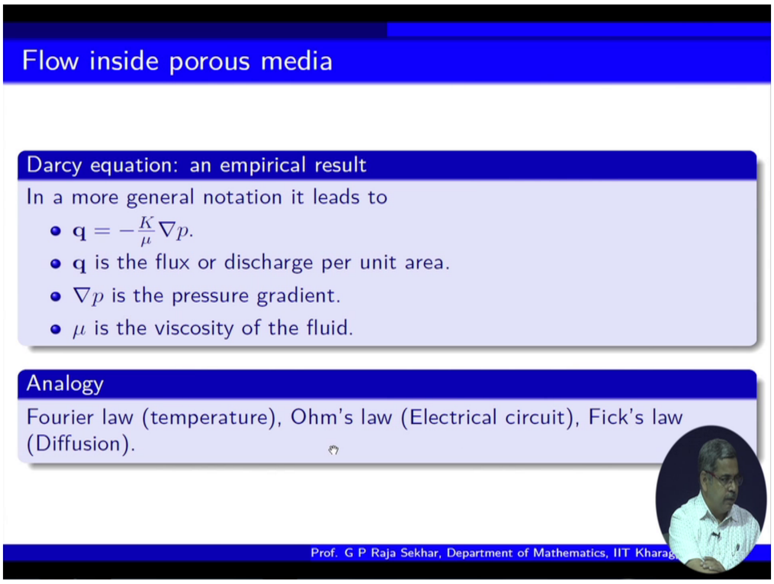
So you are increasing the length then what happens velocity will be reduced if you are decreasing the length. Then the velocity will be higher. So this is a general intuition right. So now what we are trying to do? Put more particles into it so then what happens. Naturally for this pressure difference whatever maintaining you expect certain velocity to that you are putting some resistance that is the particles.

So you expect that naturally the velocity will be further reduced. Then the question comes how much it is reduced? It depends on the particle spacing. That is the permeability okay. So if it is more permeable than the velocity will be more if it is a less permeable velocity will be less.

So that is what Darcy observed so the total volume flux is, see this is the already I mentioned. It is proportional to the pressure difference inversely proportional to the length that I already mentioned then one parameter that is depending on the volume flux will depend on is viscosity. So if it is a high viscous flow when we are keeping other things fixed naturally the velocity will be less if it is a less viscous then velocity will be more. Therefore it is a in the denominator.

Then if it is less permeable volume flux is less if it is more permeable and area, naturally if you are having larger pipe with everything fixed so then you get more area so this is the empirical relation.

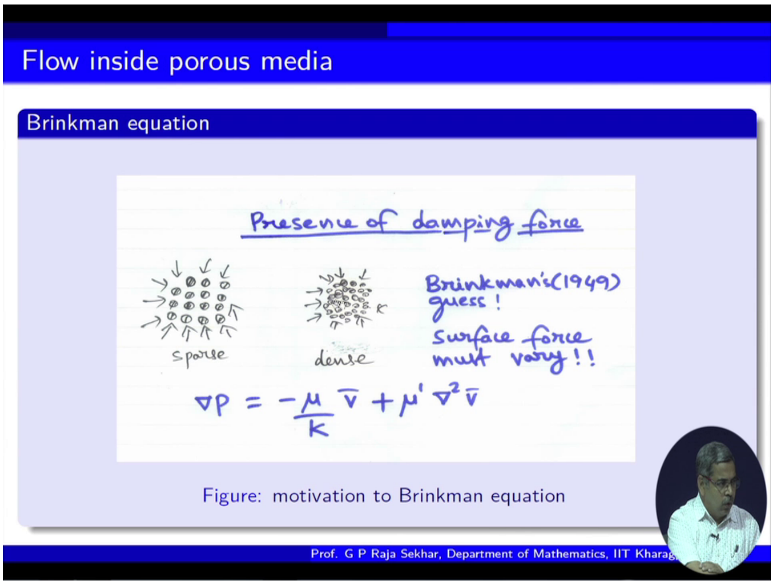
(Refer Slide Time: 16:44)



So this can be generalized and that change the complete understanding of a flow through porous media. So volume flux is minus K by Mu Grad p where this is the pressure or viscosity permeability and volume flux okay. So this is an empirical result but mathematically people have shown it via various techniques.

And one can quickly look at this and they can correlate with Fourier law of temperature or Ohm's law or Ficks law of diffusion okay. So that is flux is proportional to some gradient of a scalar. So that is each of this law is general indicating this okay.

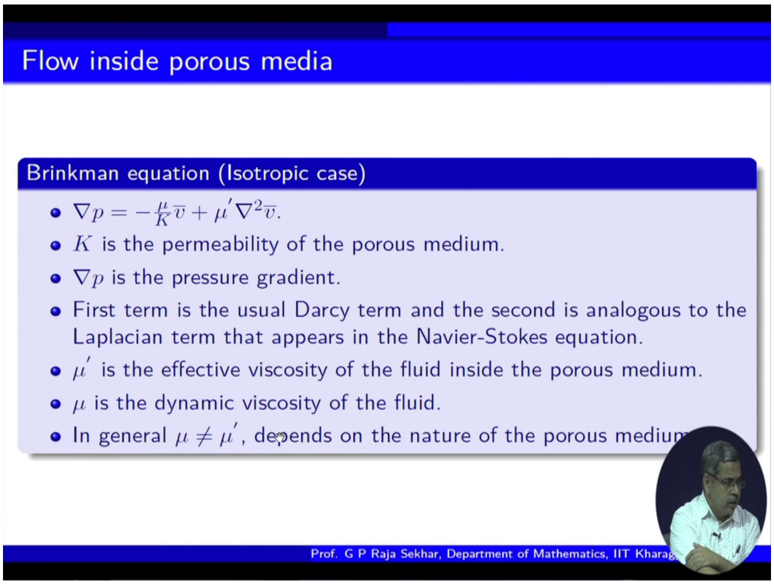
(Refer Slide Time: 17:35)



So now okay we have particles but then we are saying if the particles are sparse and dense what is the difference okay. So we expect corresponding damping should change right. So this was thought of by Brinkman. What he thought so correspondingly the surface force must very. Surface force means forces acting on the surface of these particles. So that should also contribute so correspondingly this is the typical Darcy law that we have seen just now.

So then Brinkman's proposal is there should be a viscous term and this is the viscous force which will contribute as a as a body force as a surface force. Because viscosity acts it is more prevalent when it is a coming in fluid and coming in contact with the surface of these particles. So this is the proposal by Brinkman and how Brinkman has proposed this. He considered Stokes flow past collection of particles and then averaging the corresponding volume flux obtained this okay.

(Refer Slide Time: 19:03)



So in isotropic case permeability is a constant so we have this okay. One should make a note of this Mu prime that is given so what we are saying is you have a fluid of some viscosity and that viscosity with that viscosity it is entering a porous medium. So then we expect fluid to have similar porous media in within the porous media similar viscosity. But typically speaking why is it a point measurement.

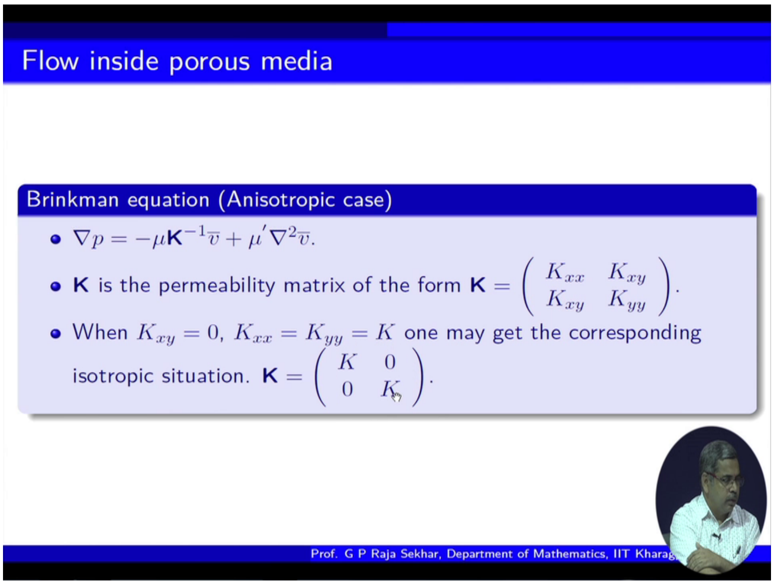
So that is the question because what we are saying in porous media you have a solid space or solid particles and then you have spacing okay. Then we are saying one can measure a velocity V bar given at some position but what is the guarantee that that position is really occupied by the void. So that is the question right. So which means when we define velocity or pressure in porous media it is not really at a particular location; it is an averaged quantity okay.

So these are averaged quantities so and hence in a generic sense so we are using this notation. So now when we have such averaged sense do we still expect the viscosity of the fluid to how same as whatever it has outside the porous media. So the observation is a not and the corresponding viscosity is called effective viscosity. So that is what we have indicated Mu prime.

So typically this effective viscosity is supposed to be different from this viscosity okay. So that is the thing and there are some correlations between this okay but most of the studies they assume that these two are equal okay. But however if somebody would like to really capture the complete phenomena so they can consider both different and then use various correlations while considering a particular analysis okay.

So this is Brinkman isotropic as I indicated in general these are not equal okay.

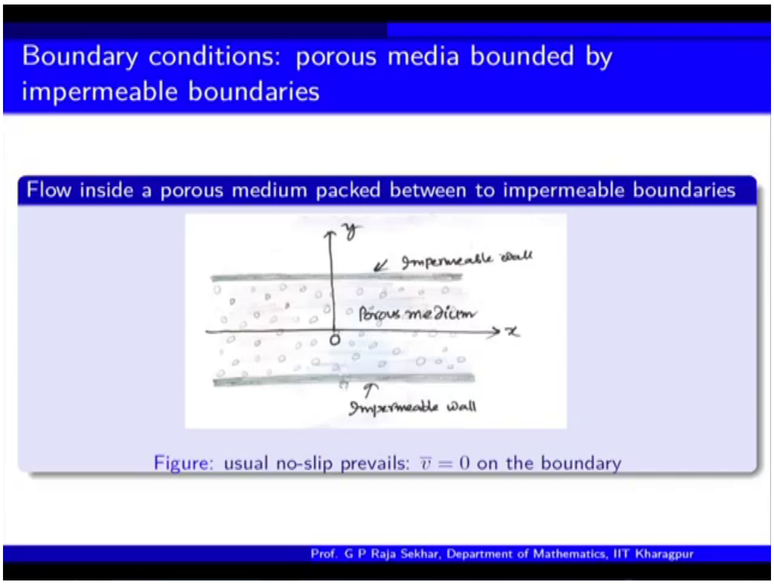
(Refer Slide Time: 21:34)



So now let us say for anisotropic case the permeability will be a matrix. So here we are putting 2D if you can extend to 3D. So it will be the corresponding tensor will come and then so you have the corresponding Brinkman equation. Suppose you have Kxy is 0 these two and each is equals to constant so then the matrix reduce to this. Which is nothing but the isotropic situation okay.

So, far we have discussed how the corresponding governing equations when you have a fluid flow inside. So these are empirical and then using various concepts like for example mathematical homogenization theory etc, mathematically these equations have been derived okay from Navier-Stokes equations okay. So that is a different story anyway.

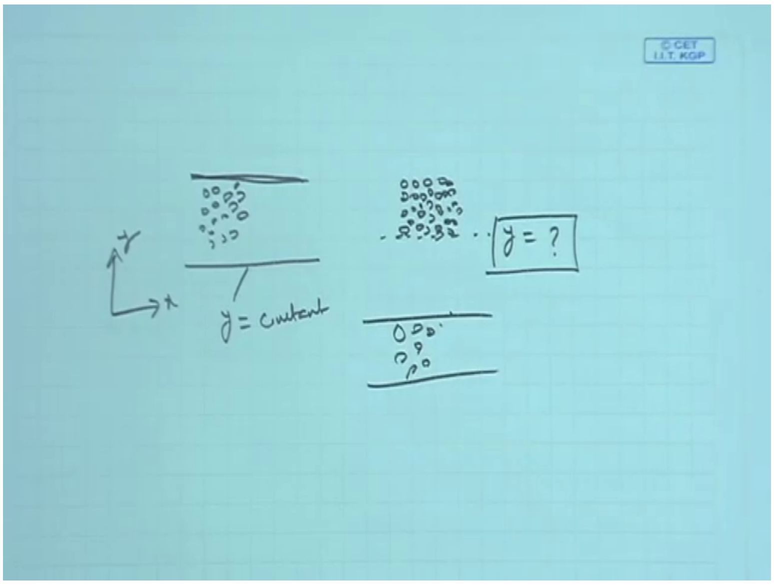
(Refer Slide Time: 22:36)



So now let us see what happens to boundary conditions when you have a porous media. So we are considering flow packed between impermeable boundary. So you have two impermeable boundary, you have a porous media and it is filled with the fluid. So then if you would like to solve a fluid mechanical problem what are the boundary conditions one would okay. So the question is since these are impermeable so usual no-slip prevails okay.

Now this is very interesting topic and definitely not resolved.

(Refer Slide Time: 23:24)

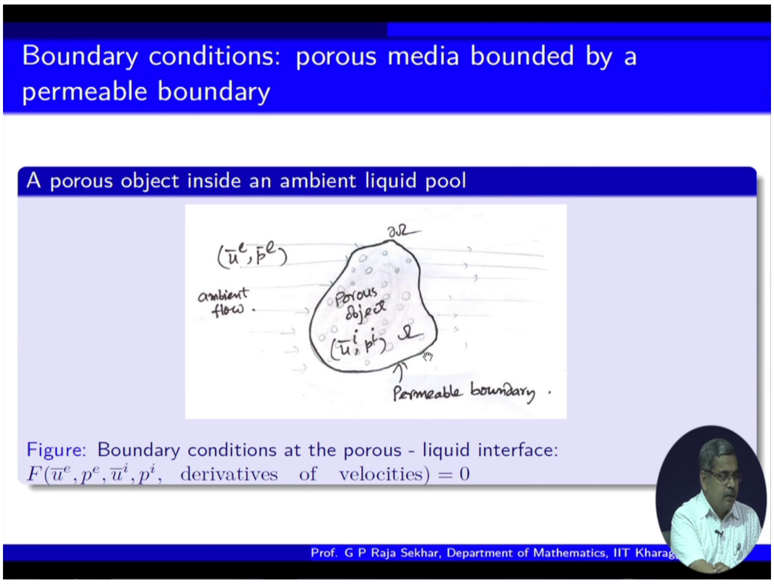


So it is like see when you say you have two plates you kept plates and then you have filled with particles. So in this case we know that this is some suppose it is x and y you know that this is y equal to constant. On the other hand suppose no plates you have a stack of say pebbles okay, you arrange like this then you want to define the boundary.

So is it y equal to constant? So that is the question because you cannot expect that the packing at the boundary will be very smooth and in line. So that you get y equal to constant okay. So it depends on the packing so therefore this is a debatable issue okay. So any case the same is also true when you have a fluid porous which we will discuss in detail later.

So for the time being when we are assuming that it is two plates and then you have some porous and these are impermeable. So within this context we have no slip okay.

(Refer Slide Time: 24:41)



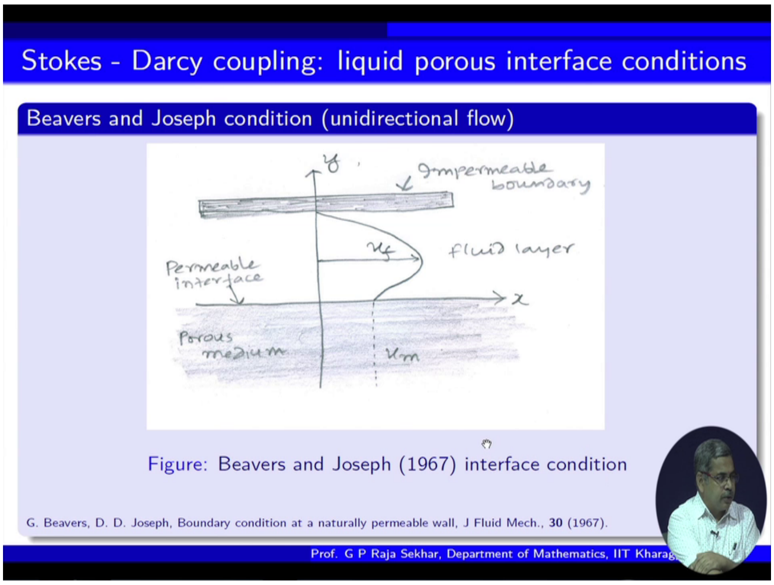
The other case is let us say you have a porous object in a fluid pool okay. So then what happens? You have external velocity, pressure in some ambient flow. Then within the porous object the flow it comes and percolates within the porous object. So you have internal velocity and pressure. And this boundary is permeable. So then what are the boundary conditions if somebody has to solve the problem?

Here you have let us say Navier-Stokes equations; here you have a porous object correspondingly you have Darcy or Brinkman equation. So then if you set up a boundary value problem what are the corresponding interface conditions? That is a natural question. So let me tell you one would intuitively expect the flow quantities outside and flow quantities inside should be interacting in some sense at the boundary.

Therefore, one natural assumption one can think of is a functional relation between the velocity, pressure of exterior, velocity pressure interior and I put here derivatives of velocities but not pressures. Well this is not based on any intuition. Basic rule is typically when you have viscous forces interacting along a boundary so you have stresses come in play okay.

So we expect that some combination of stresses also have exchange. Because see at the interface what happens? So the momentum coming from the free flow is expected to be percolated into the porous media. So there will be a momentum exchange happening between the phases. So therefore, what best can be expected is a functional combination of velocity pressure. And when I say derivative of velocities we would like to hint it as stresses. So this is the general structure okay.

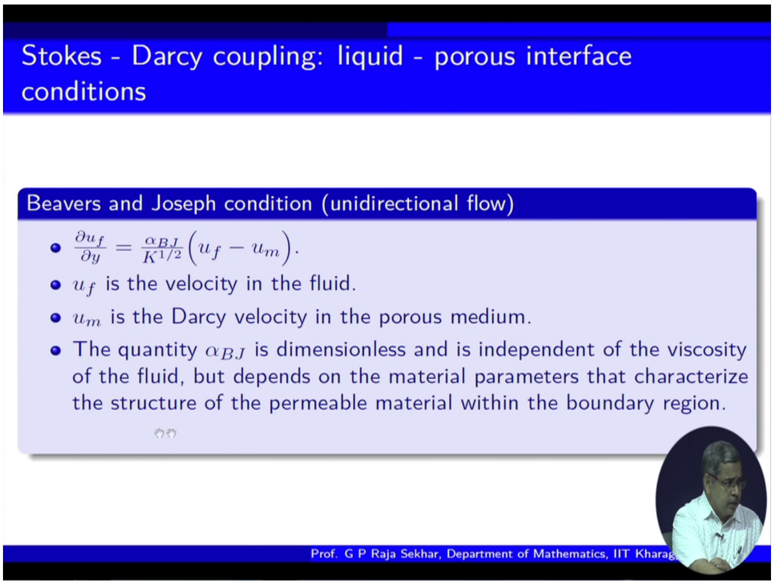
(Refer Slide Time: 27:05)



So this is still not resolved okay. But Beaver Joseph is the first one. For a unidirectional flow, suppose you take a fluid flow and then you have a porous bed okay, so then let us say this is impermeable so no slip prevails. Then this is free fluid flow, so therefore, free flow. So velocity profile can be expected like this. But then since this is permeable so velocity is not coming and becoming zero here but due to some flow seepage into the porous media, so you have such profile is expected okay.

So what will be the exchange here happening? So Beaver Joseph has proposed the following.

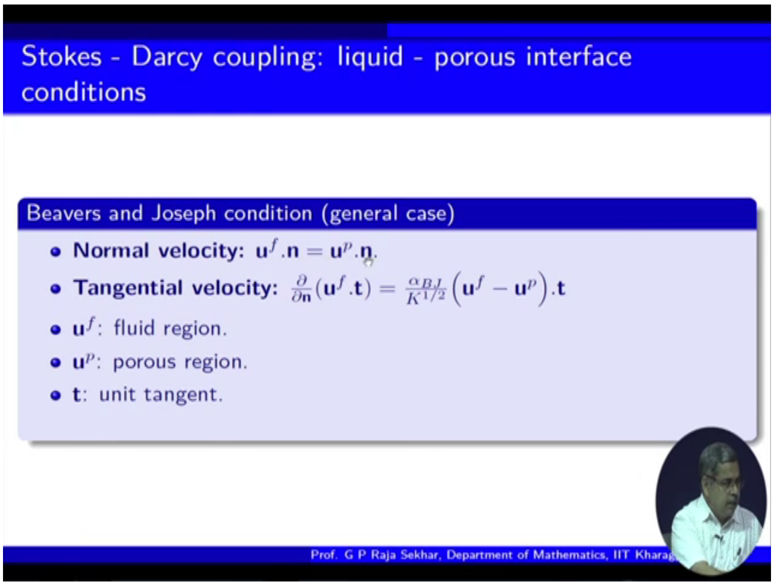
(Refer Slide Time: 27:57)



So the fluid velocity with the corresponding normal derivative is proportional to the relative velocity. So this is fluid velocity and this is velocity in the porous media and the proportionalities such that there will be a constant okay. Alpha and then here it is a root K okay. So this is permeability K is the permeability and root K and Alpha is slip coefficient.

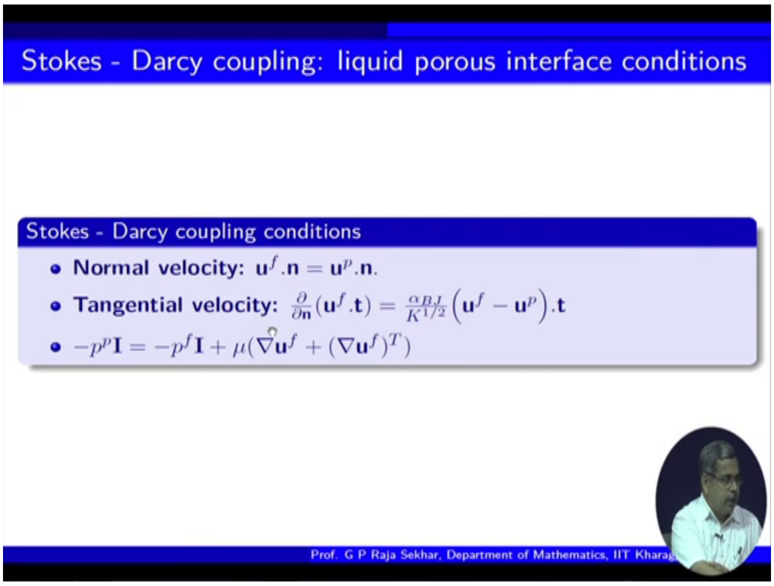
So this is expected to in some sense behave depending on the structure of the porous media. So that is what I indicated. So this is the dimensionless and is independent of the viscosity of the fluid but depends on the material parameters. For example, you have a fluid layer on a wood then fluid layer on a sand bed. So then this slip coefficient is expected to be different. So again this is empirical. Beaver Joseph has given this relation okay.

(Refer Slide Time: 29:06)



So then when we come to general case the same can be extended Beaver Joseph. What we want to say is normal velocity is expected to be continuous whereas in case of the tangential velocity you expect such slip okay. So this is treated as a slip condition okay. So this is the slip condition of Beaver Joseph for a general case. So now you take Stokes Darcy so this whole thing is at a Stokes Darcy coupling okay.

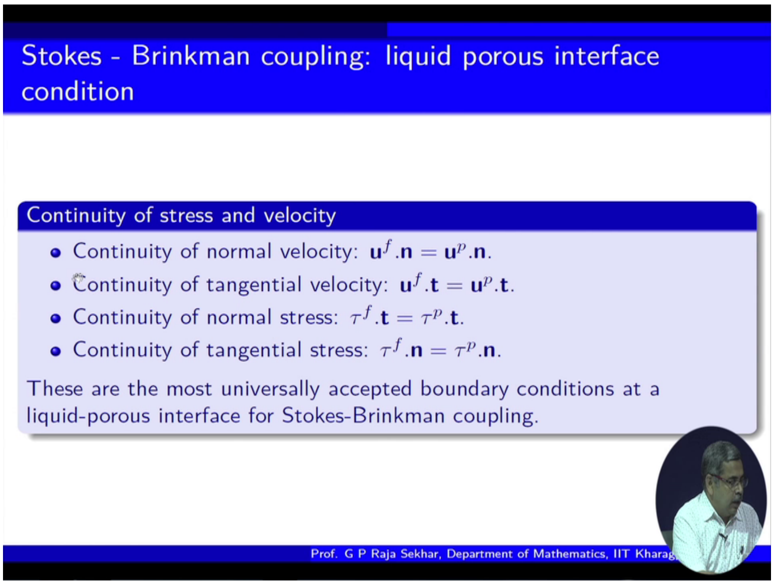
(Refer Slide Time: 29:39)



Suppose you take a Stokes Darcy system what are the full set of boundary conditions. Normal velocity is continuous, tangential velocity as indicated by Beaver Joseph and then these are not sufficient because you have a Stokes equation which is second order and then Darcy equation first order. So you require one more boundary condition so that is expected to be normal stress.

But Darcy is a is a potential flow because U is minus Grad P. So there are no stresses in there okay. So corresponding normal stress means just a pressure and that must be equal to the corresponding normal fluid stress okay. So that is what is happening so this is the Darcy pressure and this is the fluid pressure in the free flow region. So these are the set of typically accepted boundary conditions at a Stokes Darcy okay.

(Refer Slide Time: 30:44)

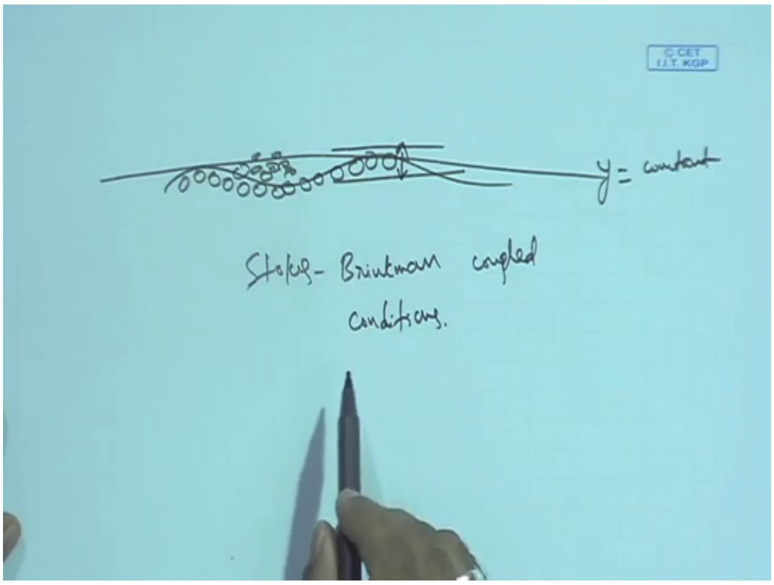


Now when you replace Stokes Darcy by Stokes Brinkman. What do you mean by Stokes Brinkman? What I am trying to say is you have such configuration so this is porous and this is free flow okay. So suppose you have Brinkman equation in a domain, porous domain and then here Stokes what happens at the interface? So that is the question we are asking okay.

So for this configuration what we are trying to discuss these are the corresponding interface conditions one would expect. So you have continuity of normal velocity then tangential velocities not slipping as Beaver Joseph expected. Because of this Beaver Joseph slip is typically due to the Darcy equation. But since so we have both higher order equations, that is, Stokes second order and Brinkman second-order, so we have the corresponding continuity of the velocities and the continuity of normal stress and tangential stress have been okay.

So this is a typical of the Stokes Brinkman coupling. But the question to be asked is are these are universally accepted or the exact interface conditions at a porous liquid interface is resolves. That is a question to be asked. Unfortunately till today that is not resolved because the reason that I have mentioned.

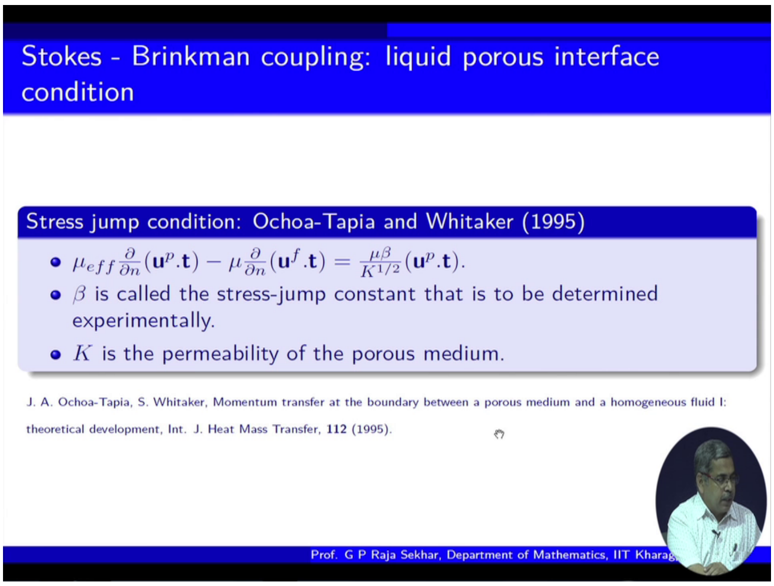
(Refer Slide Time: 32:36)



So there is no crisp boundary when you have a porous bed. The particles are irregularly okay. So you whatever you are defining y equal to constant so that is not. So you have particles which are okay irregularly. So therefore, depending on the particle spacing so that means one is expected to take a, some kind of a boundary layer. Then you take it into account of this boundary layer and then try to pose valid boundary conditions okay.

So this is Stokes Brinkman coupled conditions. So this is very interesting and very challenging issues. So this is not resolved yet so there is a lot of research happening. However Ochoa-Tapia, recently proposed that stress there will be a jump in the stress.

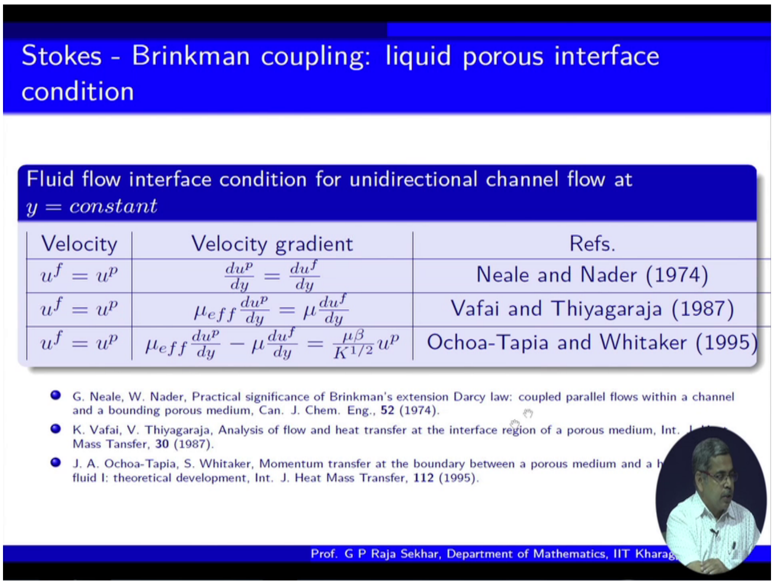
(Refer Slide Time: 33:44)



So you can see this is effect of viscosity and then this is the tangential velocity with a normal derivative. So the difference between the corresponding quantities is again this is viscosity and root of the permeability and proportional to the tangential velocity in the porous media. And this is called stress jump coefficient okay. So this stress jump coefficient is a non-dimensional scalar quantity which according to Ochoa-Tapia should depend on the again like Beaver Joseph's condition this depends on the structure of the porous media okay.

So material properties. So one can refer for more details okay. But the issue is not settled because how one can estimate this? So Ochoa-Tapia has considered a volume averaging approach and given some estimates for Beta. However various people use some others use these set of boundary conditions, some others replace continuity of tangential stress with this okay.

(Refer Slide Time: 35:06)



So to summarize for a unidirectional flow, these are the popularly accepted conditions. So this is a velocity continuity and this is velocity gradient okay. So then this is a corresponding condition given by Vafai and Thiyagaraja is effective viscosity multiplied by the shear rate of porous velocity okay. So corresponding to the fluid and then this is the Ochoa-Tapia. So this is a recent so a lot of work using this is happening.

But as I indicated the exact set of interface conditions at a porous liquid interface are not yet resolved. But whatever the interface conditions we have discussed are generally universally accepted and whenever we solve problems we adopt one of them depending on the algebra and depending on the physical problem okay. So in coming lectures we discuss some problems. Thank you!