Introduction to Abstract and Linear Algebra

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Lecture – 10
Groupoid

Ok. So, we are talking about set along with a binary operator. So, we will just discuss some of the, we will we will talk about group. So, this is basically algebraic structure. So, before that let us talk about Groupoid.

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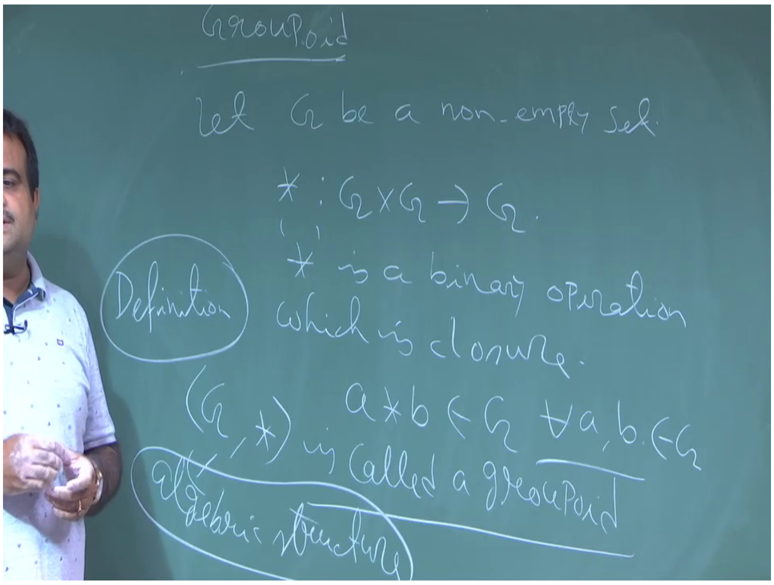

So, p o i d groupoid so, groupoid is basically we have a group we have a set G, let G be a non-empty set on which we have a operator star and if star is a closure operator so; that means, if it is binary operator which is closure.

So, this means star is a binary operator which is closure. So that means, what; that means, if you take any two element from G a star b will belongs to G this is true for all a b a b coming from G. So, this is the closure property of a star; then G along with this operator is called a groupoid. This is the algebraic structure, this is the algebra concept abstract concept, this is a algebraic structure. So, G along with this composition star binary composition star is called a groupoid ok.

So, this is the definition of groupoid; is the definition of groupoid we say G along with this binary operator which is closure is called groupoid ok. So, this is the algebraic algebraic structure, abstract concept this is an algebraic structure which is basically an abstract concept. So, we know the set now set along with this operator together we called is called as a groupoid.

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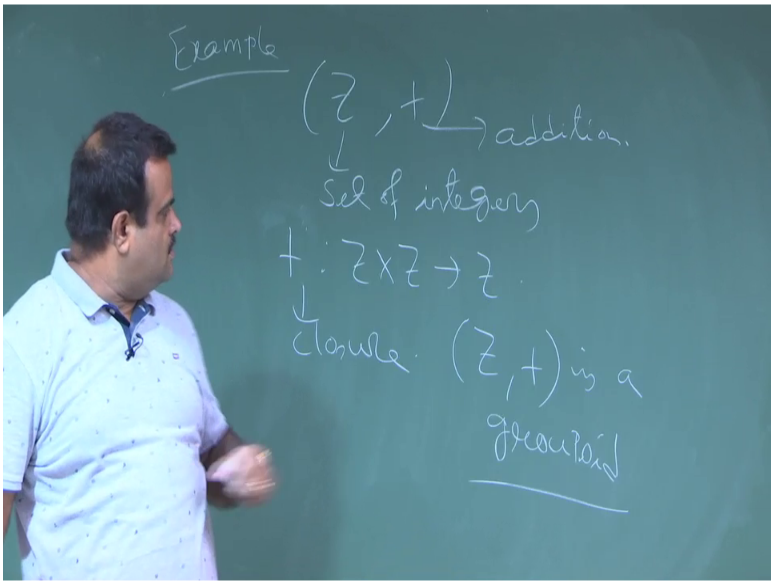

Now, let us take some example of groupoid say some of the example Z plus is a groupoid; Z Z is the set of integer and plus is the addition the real number addition. So that means, plus is a binary operator form Z cross Z to Z. So, like this is closure; that means, this is a groupoid this algebraic structure the set along with this operator is called a groupoid ok, set along with this operator we called a groupoid. So, this is an example.

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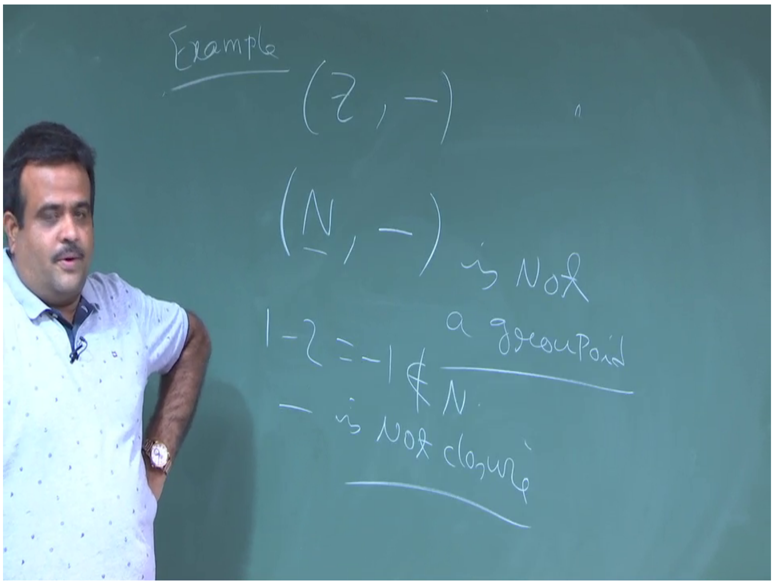

Even this minus subtraction this is also a groupoid, Z this is also a groupoid, but if we take the natural number set N and then the subtraction real number subtraction. This is not a groupoid because this is not a closure under this is not a groupoid because if we take 2 natural 2 number this is basically minus 1 which is not belongs to N.

So, this is not closure, not closure it is not satisfying the closure property ok.

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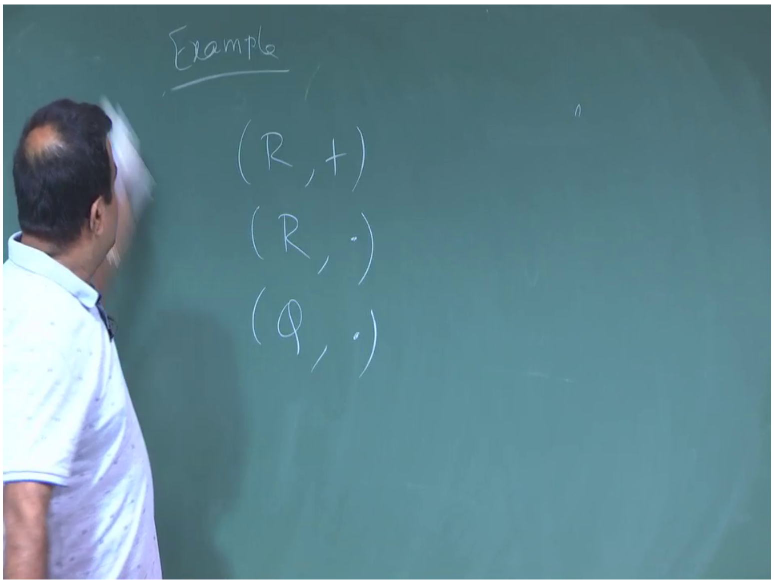

So, there are many other example of groupoid like R, R plus, R dot multiplication, we have Q dot ok. So, these are the all groupoid so; that means, if you take this is having the closure property.

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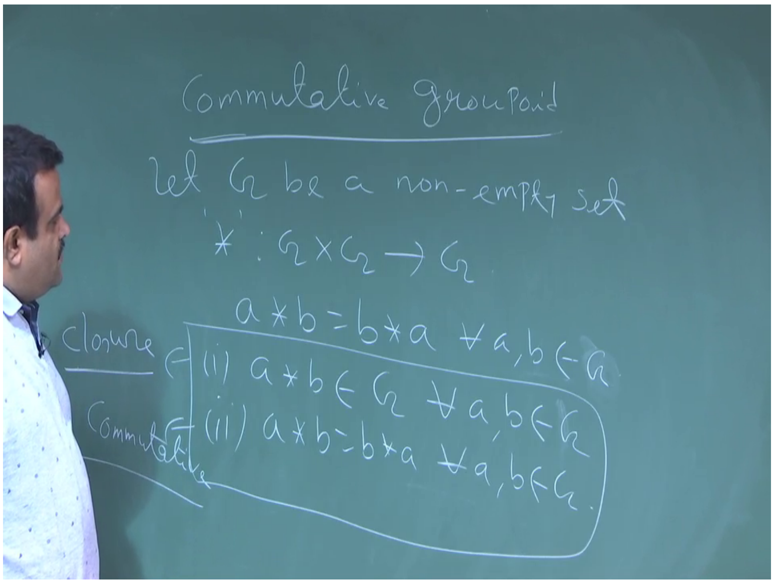

Now, we defined commutative groupoid commutative groupoid. So, so we have a let G be a set a non-empty set and then we have a operator star on G which is basically a having the closure property then it is a groupoid. So, when it is called commutative groupoid if the or star is commutative then it is called a commutative groupoid.

So, this means what this means this was the two properties need to be satisfied for commutative; first one is closure that is for groupoid sorry G. So, this is G and this is the closure property. So, this is the closure property and second one is commutative property. So, a star b is equal to b star a this is the commutative property. So, if these two property satisfy then it is called commutative groupoid. So, this is the definition of commutative groupoid ok.

Now, we defined associativity of the groupoid not associativity first we define the identity element existence of identity element in a groupoid ok.

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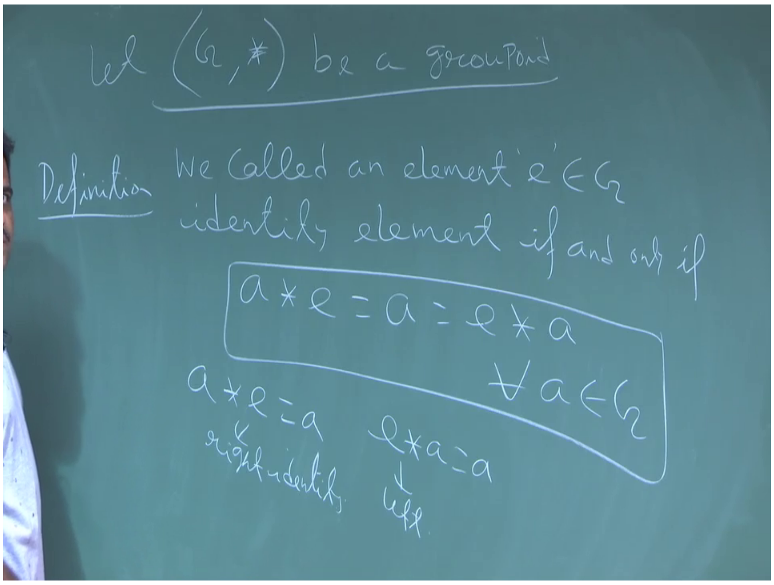

So, suppose you have a groupoid.

So, suppose you have a groupoid. Let G be a groupoid. Now, we called an element e called an element from G obviously, identity element if and only if this property satisfied; a star e is equal to a is equal to e star a and if it is true for all a ok.

If we have we take e we operate with any other element any element from G it will give us the same element and this is both the way. Now, if it is this way only if it is a star e is a then it is called the right identity element. And if it is e star a is equal to a for all a then it is called left identity.

But existence of right and existence of identity means it is from the both the both the way, it is from the right side as well as it is from the left side; then it is called identity element. If it is right identity element now as well as the left identity element then it is called an identity element of G.

And later on we will see if such element exists then it is unique we will prove that theorem. So, this is the definition of the identity element this is the definition of identity element. Now, we will define the inverse existence of existence of inverse of an element ok.

So, let G be a groupoid means we have only one property closure property; we take a set we take a binary operator this and which is closure then this algebraic structure said along with this operator is called groupoid. So, we take a groupoid and suppose there exist identity element ok.

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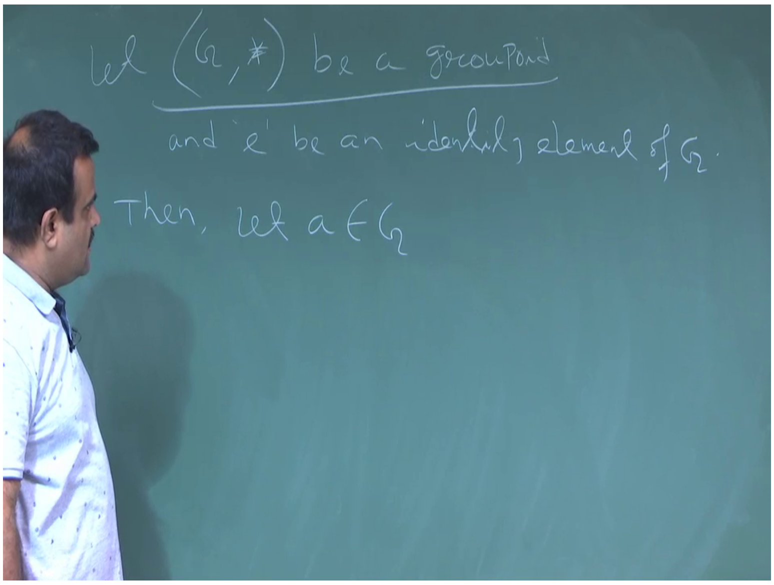

So, this groupoid and e be an identity element of G. So, identity element exists.

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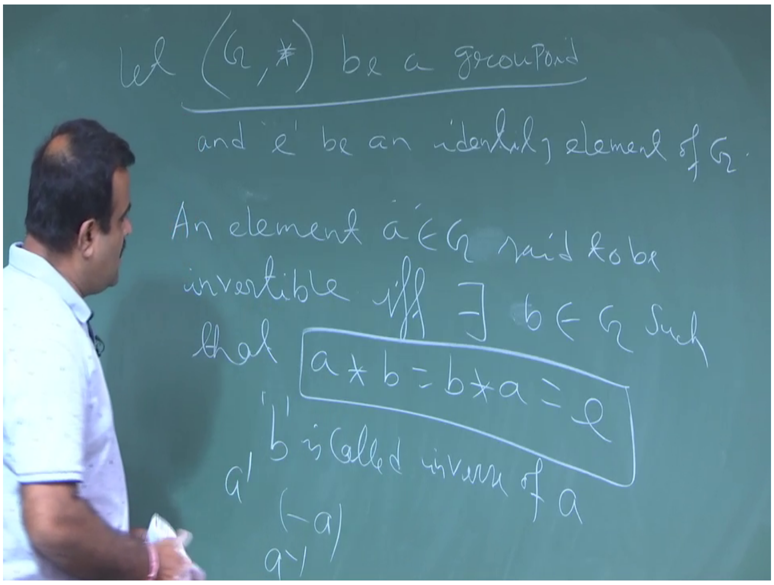

Then what let a be an, a be an element in G is a a be an element is G is said to be. Suppose then an element a belongs to G said to be said to be invertible if and only if there exists an element b from G such that a star b is equal to b star a is equal to e identity element.

So, if a is called invertible if such a b exist, if there exists a b if and only if there exists a b such that this then this b is called inverse of a, inverse of a. Sometimes it is denoted by a star if it is if the operator is additive sense then it is a minus, if the operator is in multiplicative sign this then this is a to the power minus 1. So, these are all notation depending on the type of operation we are dealing with ok; so, so, this is that this is called inverse ok.

Now, we defined the semi-group what do you mean by the algebraic structure called semi-group.

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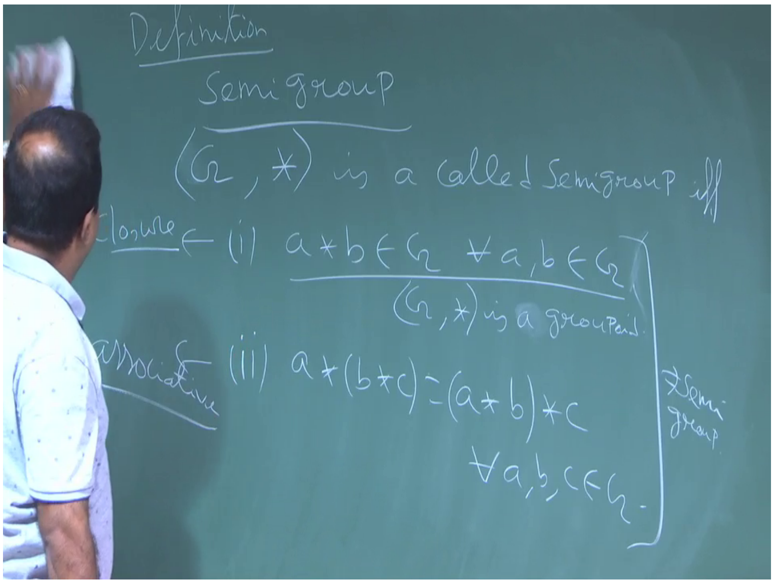

So, this is the definition semi-group; we will slowly move to the define the group ok. Semi-group means so, first of all we have this G the set along with a binary operation star will be called semi group is called semi group if and only if star is closure. So that means, this is a groupoid first of all this has to be a groupoid and then it is associativity associative.

So, closure property so; that means, a star b belongs to G for all a b; this is the closure property and this will this is basically telling us this is the property for groupoid. So, this is; that means, this must be a groupoid this must be a groupoid and along with the associativity property.

So, a star b star c is equal to a star b star c this is true for all a, b, c ok. So, if this is true for all a b c this is the associativity property associative property. So, if both the properties so, this will this property will give you all the groupoid and the both the property satisfied then this is called semi group ok; then this is called semi-group.

So now, we define the monoid.

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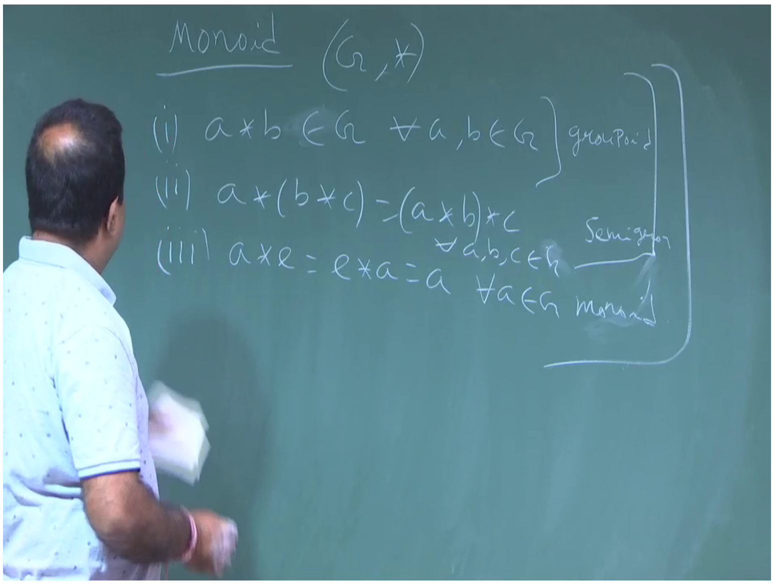

So, we define the monoid ok. So, monoid means it is first of all it has to be groupoid. So, monoid will call this is this is a monoid if and only if first of all this is a groupoid. So, that is a star b is equal to a star b belongs to G for all a b. So, closure property and then the associativity. So, it is a semi group also so, it is a semi-group also and along with the existence of identity.

So, if it if it has a identity element then it is called monoid for all this is also for all a, b, c belongs to G sorry belongs to G. So, this is groupoid up to these it is sorry up to this it is semi-group and up to these it is monoid ok.

So, now we define now we will move to the definition of the group. So, before that let us just define what is called quasi group.

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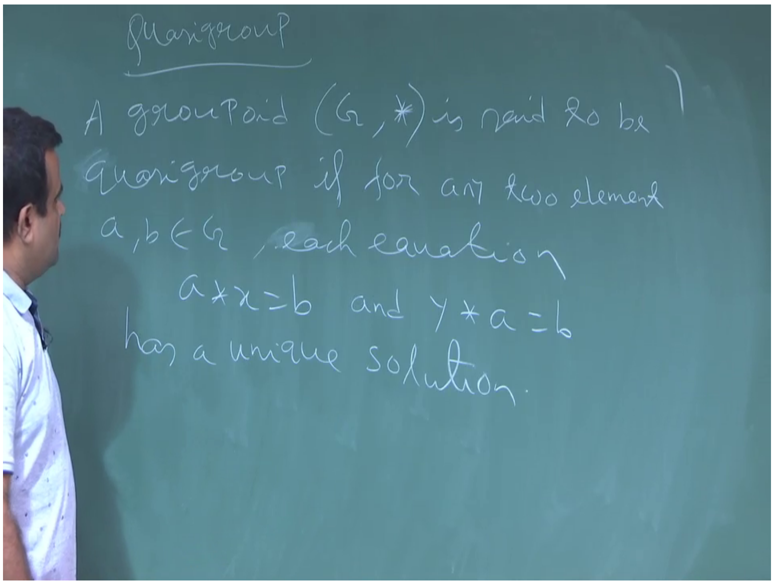

So, quasi group this is the definition of quasi group ok. So, a groupiod G is said to be it is the definition is said to be quasi group, if for any two element a b if for any two elements a b belongs to G, each equation equations like a star x equal to b, and x and y star a is equal to b has a unique solution.

So that means, they are the has a unique solution then it is called a quasi group ok. Now, if it is say if we take this to be e b to b e then this will be happen to be the inverse. So, from here we can fill of concept of existence of inverse.

Now, we finally, move to the defining the group of group.

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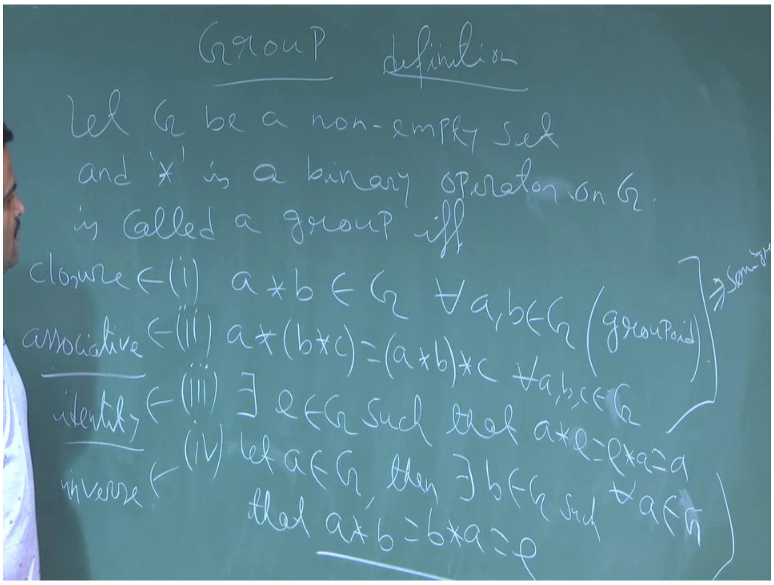

So, now, we going to define the group this is definition ok. So, let G be a non-empty set and this is an this is a binary operator; binary operator on G need not be closure.

So, it is a function from G star G 2 if it is closure to G otherwise it is to some other set is called a group if the following condition satisfied; if and only if first condition is closure a star b belongs to G. This is the closure property; that means, it must be a groupoid.

So, this must be a groupoid a star b is equal to a so, this must be closure. Then it must be associativity a star b star c is equal to a star b star c this is true for all a b c. So, this two will give us this is the associativity property associative and the next one is existence of identity.

So, they are must exist an element e from G such that such that a star e is equal to e star a is equal to a and this is true for all a so, this is G ok. So, this is the existence of identity so, this is the identity element.

And last one is the existence of inverse. So, for every element has inverse so, given a let yeah for let a belongs to G then there exist a b such that such that a star b is equal to b star a is equal to e.

So, this is the existence of inverse. So, this is the inverse. So, every element has the inverse, every element has a inverse. So, this given a element a we must have a element b such that a star a star b is equal to identity which is same as b star a. So, this is telling us every element of G has a inverse.

So, if this for pub so, up to this it is a semi-group; semi-group means closure and associativity. Now, if we have existence of identity then it is called monoid and then if we have existence of inverse then a along with that if we have also existence of inverse; before every element we have inverse then it is called it is called a group ok.

So, this is the property of this is the definition of a group. So, every group has this many properties; this basically four properties this and this then this if this four properties satisfied then I called this G along with this is this algebraic structure is called a group, called a group ok; if this four properties satisfied.

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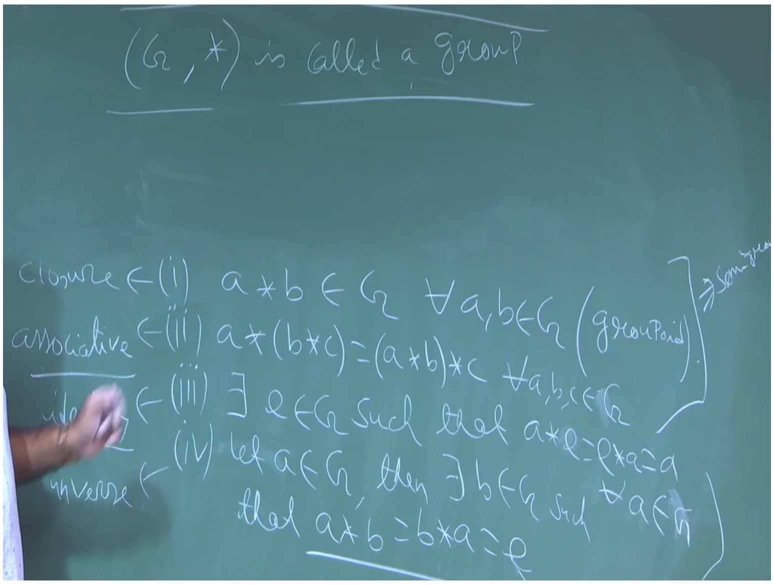

Now, when we called this is a commutative group. So, for group we just need this four property closure, associativity, a closure; that means, if we take any two element if we operate this by operation it must belongs to G. Associativity a star b star c is b star a star b star c associativity. Existence of identity we must have an element which is called identity element; such that if we operate with any other element with the identity element it will give us the that that element a.

And existence of inverse for every element in a there must exist the corresponding b, such that a star b is equal to b star a is equal to the identity element; so, these four properties are the proper properties of a group.

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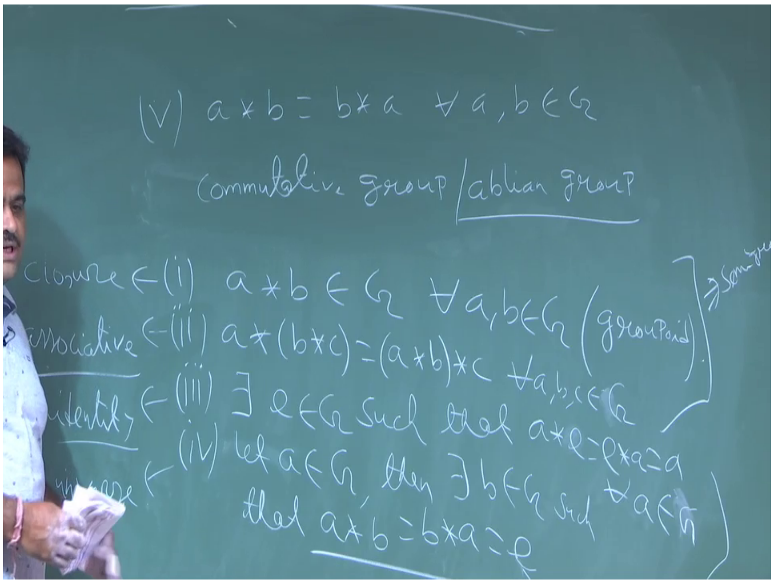

Now, when we called a group to be Abelian group; if along with this if we have a another property fifth property which is commutative property; If we have a if if the star is commutative then along with this four property this will called a commutative group or Abelian group, Abelian group.

So that means, for Abelian group first of all it has to be a group by these four properties and then this star must be that operator must be a Abelian operator or the commutative operator. Then it will be a it will be called a commutative group or Abelian group. So, this is the definition of a group.

Now, in the next class we will discuss some properties of a group and we discuss some of the example of the group.

Thank you.