Introduction to Abstract and Linear Algebra
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Lecture – 02
Set Operations

Ok so we are talking about sets; we already defined the sets and we have discuss the we have talk about the subset. Now today we will discuss the now will we will define set operations before that we already define the subsets subset we have defined in the last class so; that means, A is a subset of B if and only if element x belongs to A imply x belongs to B ok.

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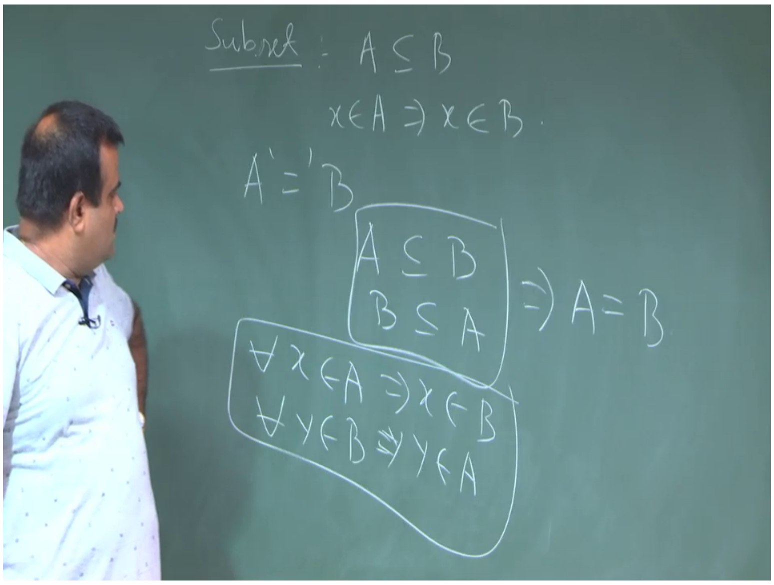

So now, we defined when you say two sets are equivalent A equal to B this is the equality; when you say two sets are equal. So we say two sets are equal if and only if A is a subset of B and both the condition and B is a subset of A if both satisfy then only say is A is equal to B this is the definition of equality of two sets.

So, what is the meaning of this? Meaning of this is so for all x belongs to A we have to show x belongs to B and for all a y belongs to B you have to show you have to show that y belongs to A. If this is satisfy then we say this two sets are equal A equal to B ok. So now, we defined some operation on the sets basically defined union and intersection so this are called set operations.

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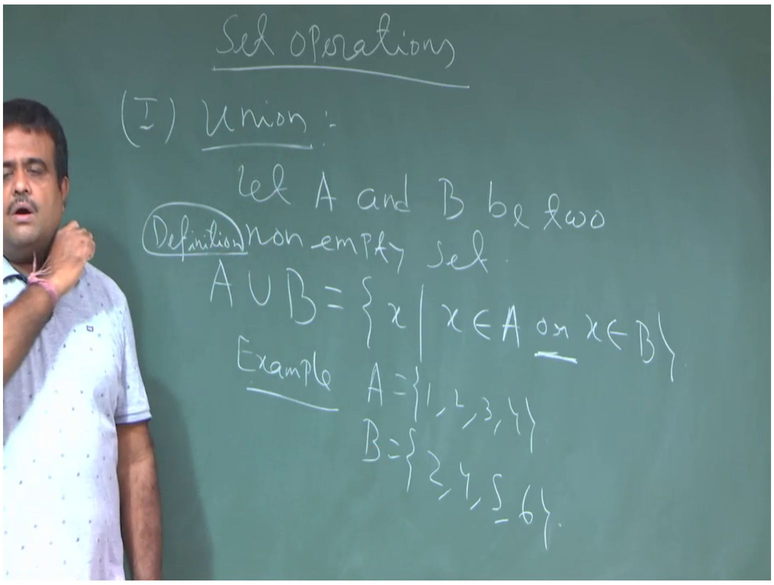

So, first one is say union; union operations. So, we have two sets A B; so what do you mean by A union B ok; so that you define. So let A and B be two non empty set then we define A union B is basically this is again a set; so when you are operating union on two set this will give us another set and what was the properties of these set; properties of these set is the element in this new set will be either will be either from A or from B. So it is basically this is the x such that x is from A or x is from B so this is the, or so that means, this means set consist of all the elements that elements coming from either from A or from B ok.

So, like if you have for example, this is the definition this is the definition of the union. Now if for example, if we have two set A is a 1 2 3 4 and if we have another set say 2 4 5 6 another set B ok. Now how we define union; union is basically another set C which is denoted by this symbol so then A union B is basically so 1 2 2 is in the both place, but we cannot write 2 twice because set is a distinct collection of object.

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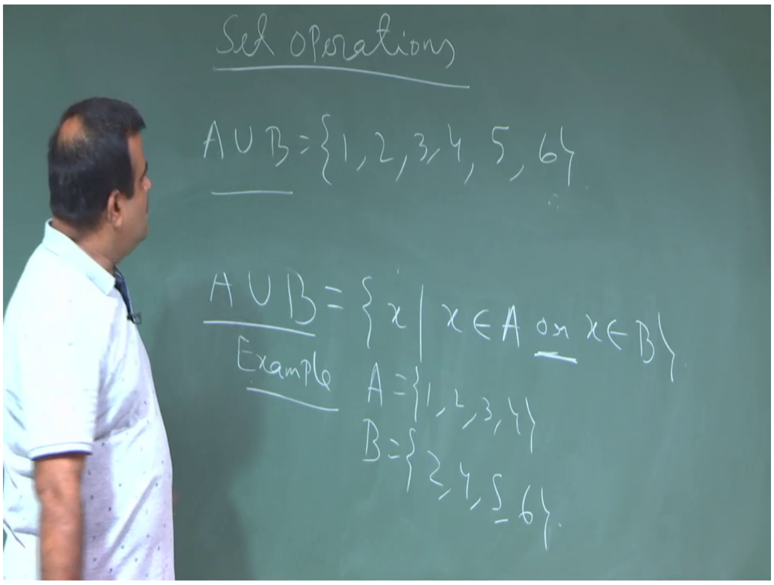

So, 1 2 3 again 4 become ones so these the distinct collection so this is the union. So union set is basically this is so the either A is belongs to x is belongs to A or x is belongs to B; so this is called union operations so now, we define the intersection operation.

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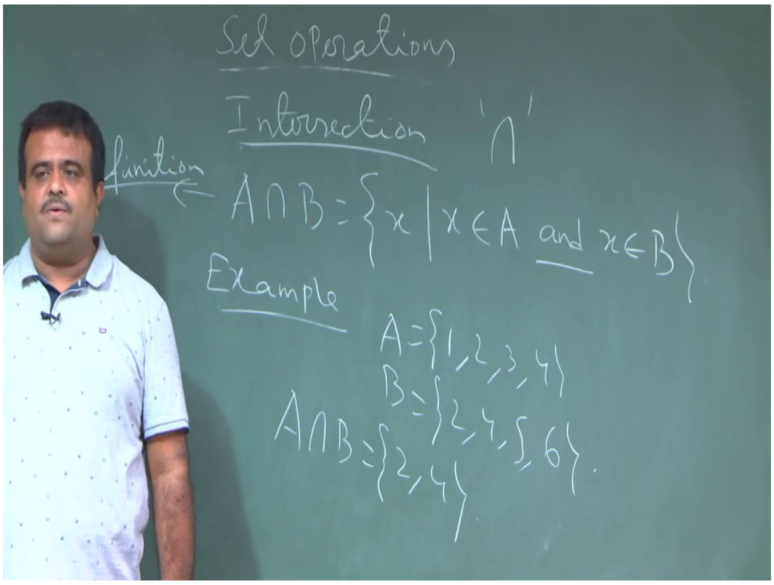

So, intersection which is denoting by this symbol so we have two again we have two non empty set A B; So A intersection B is a another set; which is having properties that this elements must be in both the set so this consist of the common element and so these elements has to be present in both the set then those collection of those elements is basically the intersection of this.

Now for example, this the definition; now example is this is the this is definition now the example is now if you have this set say 1 2 3 4 and if you have this set say 2 3 5 6 5 2 4 5 6 then what is the intersection is this 3 belongs to this intersection; intersection is again a another set, but that is it having the property that that element of this set must belongs to must be an element of both the set; is 3 belongs to these intersection no because 3 belongs to A, but 3 does not belongs to B; so this 3 is not having this property both it must belongs to both A and B, but 2 has this property 2 belongs to A and 2 belongs to B 2 has this property and 4 has this property, so this is the intersection set A intersection B.

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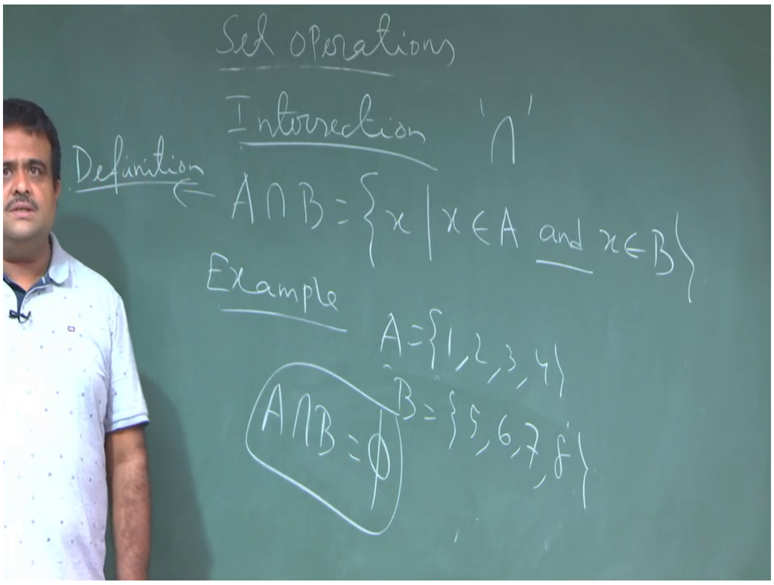

So, now you we have if you have say B is say like this so if A is this then say if you have B is say 5 6 7 8. Now what is the intersection of this is there any element which is common in both no so intersection is the null set. So A intersection B is null over here if they have they are having no common element in that set so now, we have we will be discuss some properties of this union and intersection ok.

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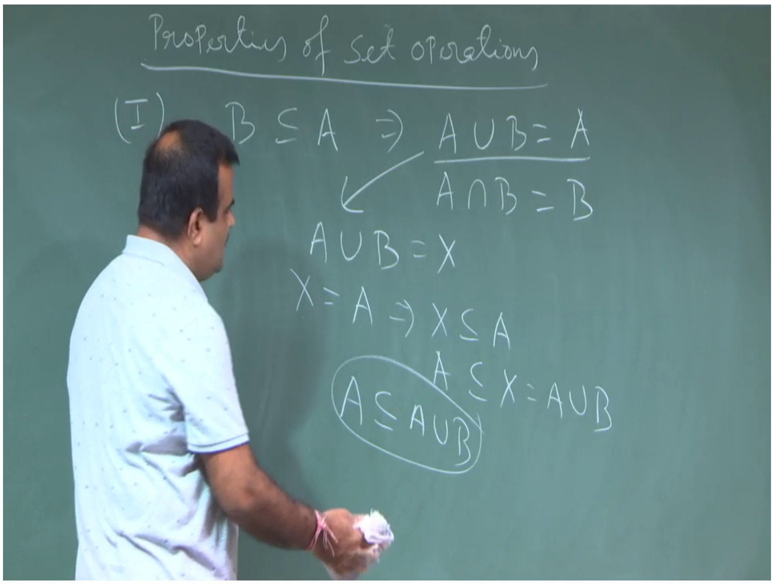

So, what are the properties say like first property is these are the properties of the set operations; basically you have learn two operations yeah union and intersections; so we will defined some of the will discuss some of the properties based on that the first property we discuss is suppose B is a subset of A then what is the union then A union B is basically B is a subset of A, so A union B is basically A and A intersection B is basically B ok.

So how to prove this; so to prove this so let us try to prove this one first so A union B is basically so this two are equal; equal means now this is say X this is say X so what you have to so you have to so X equal to A a set X equal to another set A; so to so this what you need to show you need to show the X is a subset of A and A is a subset of X. Now this part is of X because this X is nothing, but A union B so a every A is a subset of A union B because union means basically the elements of A along with the other elements of B which are not common in A; so this collection consists of A union B. So this part is obvious now we have to prove this part that X is a subset of this.

So, to prove this part what we need to show we need to take an element X from X and then we have to finally, show that we have to finally show X belongs to A.

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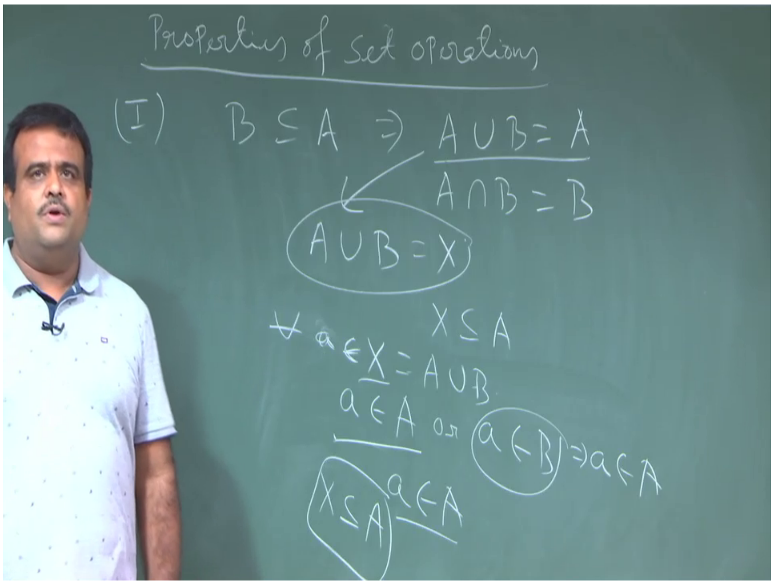

So now, what is X; X is A union so let us take small a now this is means a belongs to A or a belongs to B.

Now, suppose a belongs to A there are two cases either a belongs to A or a belongs to B. Now if a belongs to a then A a belongs A that is done. Now otherwise if a belongs to B now B is a subset of A; so that give us that gives us this implies a belongs to A because B is a subset of A. So in any case a is a, a belongs to A. So a belongs to X means a belongs to A so; that means, X is a subset of A; because this is an arbitrary element this is for all a belongs to X we taken arbitrary element from X and then we have we have prove that that a belongs to A; so X is a subset of A so; that means, and we have already a is A subset of X so that is why this is true so similarly this also we can solve ok. So this is one property.

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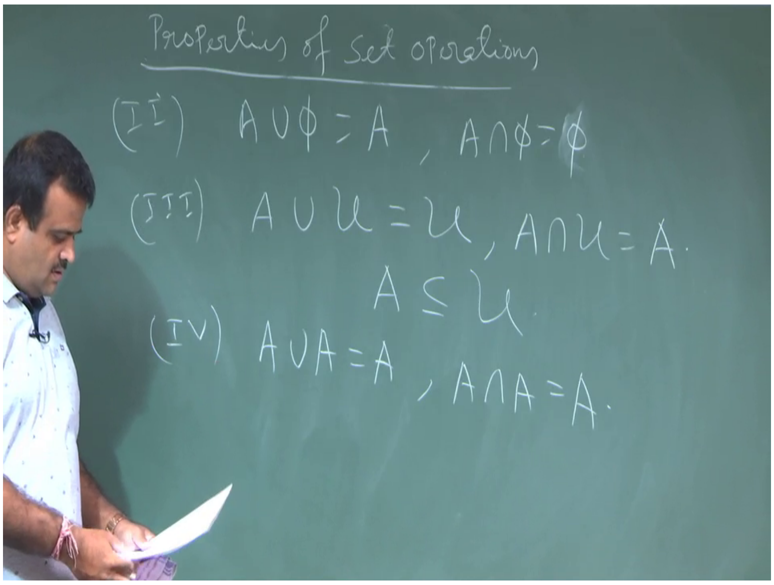

And another property is with the null set A union of null set equal to A, because null sets consist of no elements so if you take the union with any other set including null set then it will be that set.

And A intersection null set is sorry empty because there is no common element because null sets consist of no element; it is just a symbol it is a convention to have such a set because when we have two set which having no common element then there intersection is containing no element; how to represent that, to represent that we have to bring the notion of the null set, so null set is the set with no element ok. So the next property is with the with the universal set A union universal set is basically universal set A intersection universal set is A because every subset is a every set is a subset of the universal set; so it is coming from there ok.

Now, next property is A union A is A is very easy to convince A intersection A is A quite obvious because just we are taking this; so now, the next property is the commutative property.

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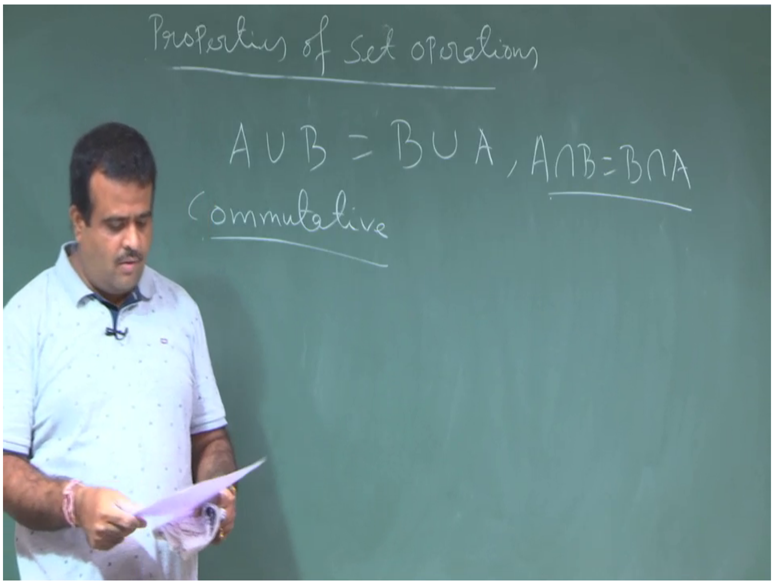

Like A union B is same as B union A this is called commutative property. Commutative it does not matter which I mean we take A first or B first is only matters is the elements of that set; so A union B same as B union A.

Similarly, A intersection B is same as B intersection A. So the ordering really I mean ordering is not so important so this is a commutative sense and we have another property which is the associativity property; this property is called associativity.

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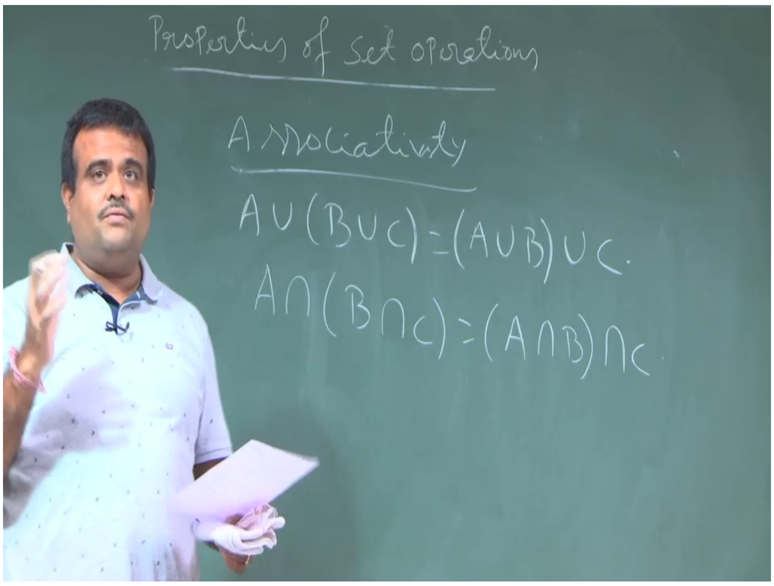

So if you take three sets A B C three non empty set even this is true for empty set also; if we take 3 arbitrary set A B C so then A union B union C is basically A union B union C and this is true for intersection also A sorry A intersection B intersection C ok; so this is the associativity property.

Now, we have if we want to combine this union and intersection then you have a another property which we will prove that so the combination so suppose we have A union B intersection C so this is basically A union B then intersection A union C this is a property.

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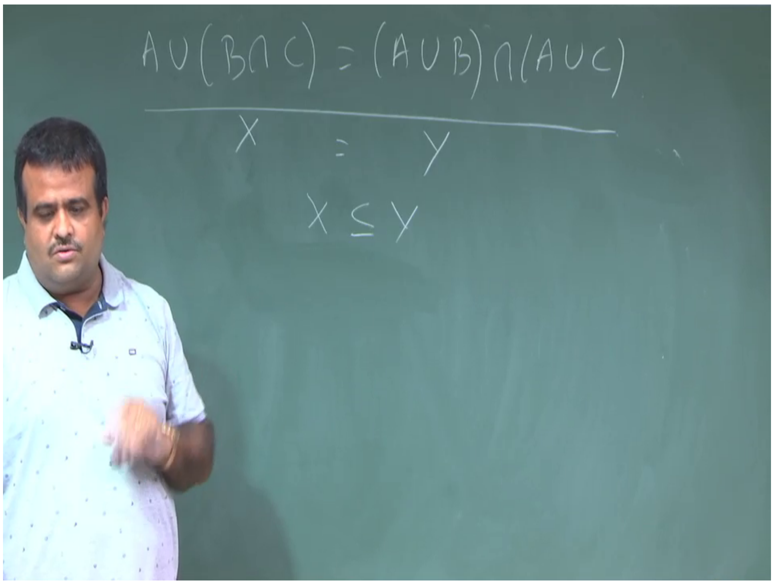

So now, how to prove this property we are going to prove this property yeah so A union B intersection C here we have union here we have intersection then it will be A union B intersection A union C.

Now, now we have two sets to prove equal so this is say X set this is Y set you have to prove X equal to Y so how to prove that; we know when the two sets are equal if X is the subset of Y first of all you have to see X is the subset of Y or not and if it is done then you have to check Y is a subset of X or not. So let us try the first one; here going to see whether X is a subset of Y or not, so for that what you need to take we need to take an element of X arbitrary element and you have to show that that element belongs to Y so if this is true for all element then we are true, so let us try that.

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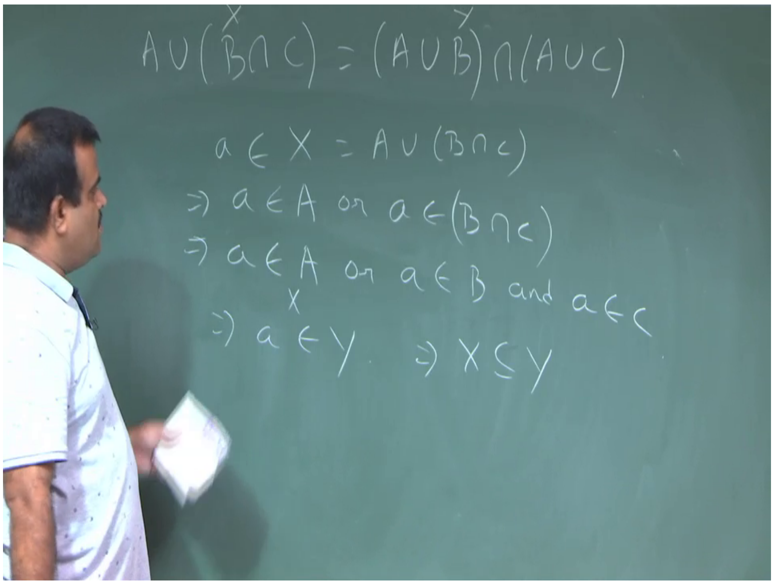

So, this is our X this is our Y this whole set so let us take an element from X so a belongs to X; so this whole set is Y not only this so a belongs to X means X is nothing, but A union B intersection C so; that means, a belongs to A or a belongs to B intersection C. Now this means what, this means a belongs to A or so a belongs to B intersection C means a has to belongs to B and a belongs to C ok.

Now, if so this is basically A belongs to these or this so suppose a belongs to there are two cases now; if a belongs to A first case if a belongs to A then a must belongs to both of this a must belongs to A union B and A union C; so a must belongs to this both intersection second case, suppose a belongs to this two suppose this is not happening this two is happening a belongs to B and if a belongs to B then a has to belongs to C in that case A belongs to this one and A belongs to this one.

So, in the both the cases A belongs to intersection of this two so this implies a belongs to Y. So this implies X is a subset of Y ok. Now the other way round we have to prove Y is a subset of X. So we take an element from Y so these this type of argument you have to do when we try to prove this type of property; so we have to take we have to show now Y is a subset of X.

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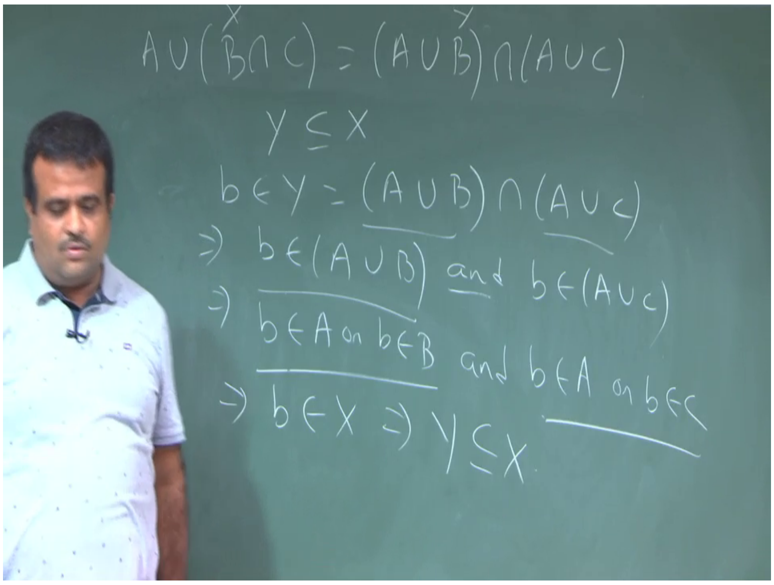

So for that we need to take an element say b b from Y so Y is basically A intersection A union B intersection A union C. So this implies this implies so Y belongs to he has to belongs to both of these A intersection B and b belongs to A intersection C.

Now, this implies first one implies b belongs to either A or b belongs to capital B and this is together and b belongs to A or B belongs to C ok. So now, there are few cases like b belongs to A and here so there are four options b belongs to A and b belongs to C there.

So if B belongs to A then B belongs to this union otherwise if b does not belongs to A so then b must belongs to capital B and if b does not belongs to A then b has to b must belongs to capital C. So in the both the cases so if these b belongs to X so; that means, this implies Y is a subset of X ok. So these imply Y is a subset of X; so hence we prove these two sets are equal so this is one of the properties, so you prove this type of properties by using this argument ok.

So, another properties is based on the complement, so you first defined the complement of a set complement operation complementation.

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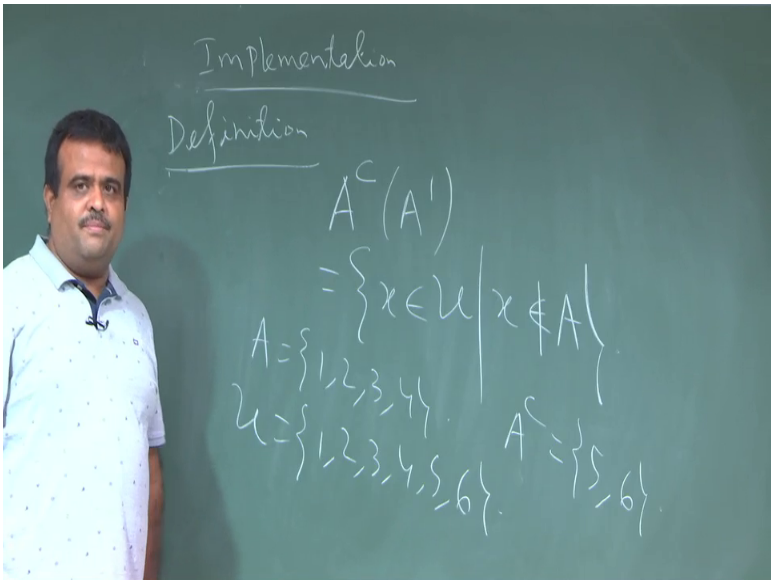

So you define this is the definition so let A is a subset then what when the complement of A we denote complement of A by this or sometimes it is denoted by this symbol. So this is basically set of all element from the universe where X does not belongs to A, so this is basically the element which is not belongs to A, which are coming from the universe.

Now, for example, suppose our A is a 1 2 3 4 and our universe is a 1 2 3 4 5 6; suppose this is our universe then what is A complement A complement is nothing, but set of all element coming from the universe which will not in A so; that means 5 and 6 ok. So these are the element not in A so now, will have some this is the definition of the complement now we will have some properties of this complement.

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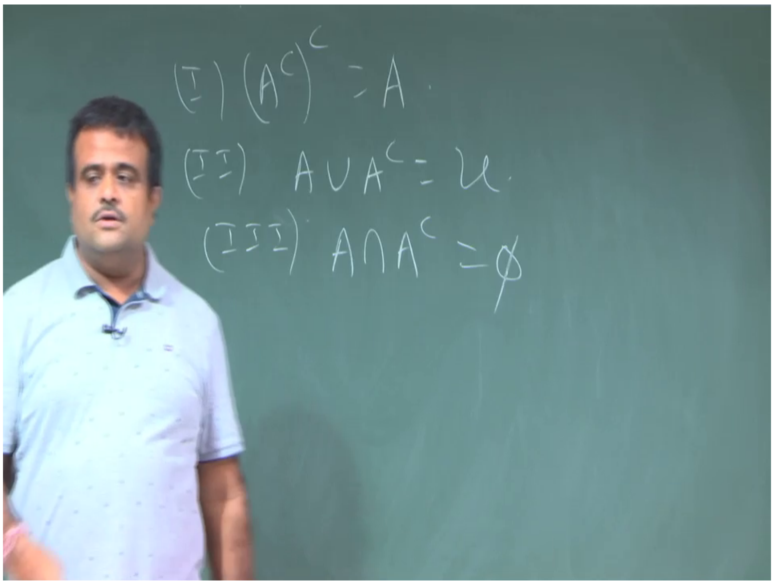

So first property is A complement, complement it is basically A this is quite easy to convince or easy to prove. Now second property is A complement if you take the universe A with A complement this is the universe and this is quite obvious property is complement consist of the element which are not in A; so there intersection must not have there are no common element so the intersection will be null.

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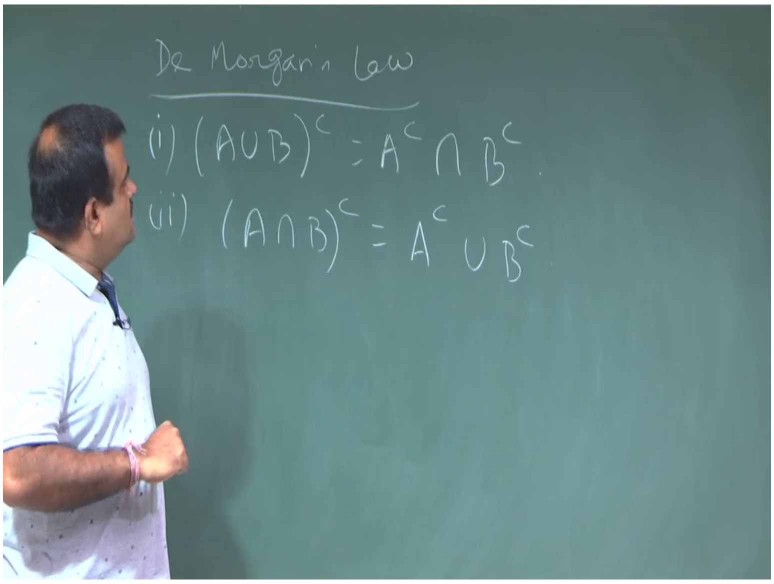

Now another property which is called De Morgan's law which is basically De Morgan's law which is basically A intersection B complement is equal to A complement this is union and this is intersection A complement and B complement this is first De Morgan's law and second law is with the intersection this is intersection so it will be union over here.

So, A intersection B complement will be is equal to will be is equal to A, A complement union of B complement so this is the this two laws are called De Morgan's law will prove this two properties in the next class so before and also will define the what is called Venn diagram method. So we can represent the graph in a pictorial way the diagram way and then we will then it will help us to prove all those property very easily; so we will discuss those in the next class.

Thank you.