Introduction to Abstract and Linear Algebra
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Lecture - 03
Set Operations (Contd.)

So, we are talking abouts the properties related to setting union intersection and compliment. So, this two properties this is called De Morgan's law, we will prove these two properties. So, first let us prove the first one.

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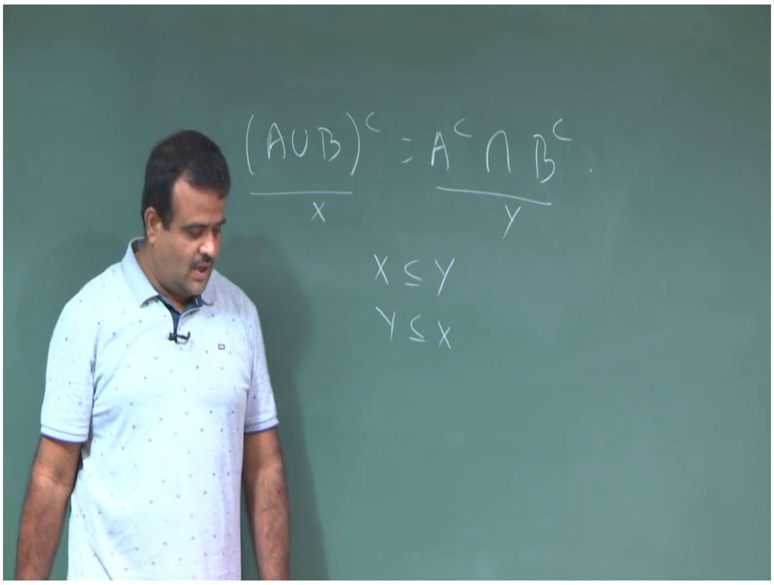

So, the this is so, now, this is say X set and this is Y set now, we have to show X equal to Y so, for that we know the approach.

So, first of all we have to show X is a subset of Y and then we have to show Y is a subset of X and if both the both we can prove then we can say X equal to Y

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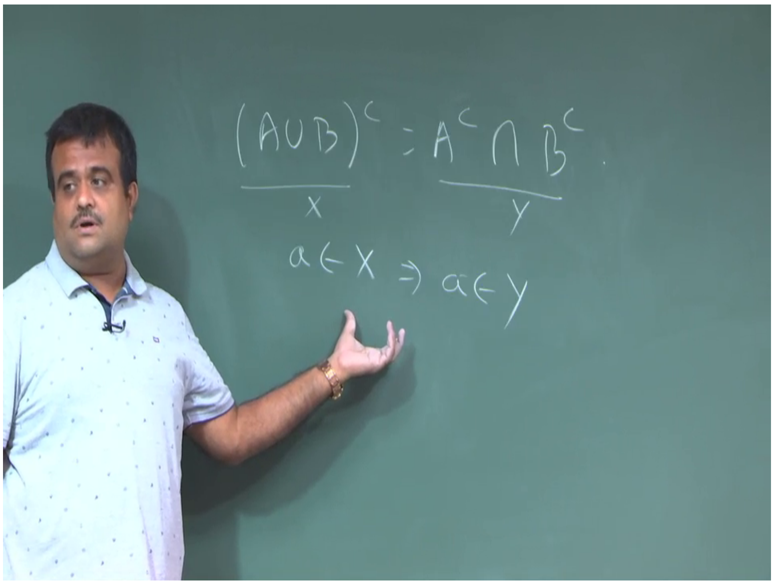

Now, to show this X is a subset of Y we have to take an element a from X and then at the end we have to show dot dot dot we have to show this a belongs to so, we have to show this imply a belongs to Y. This way to show then only we can say X is a subset of Y.

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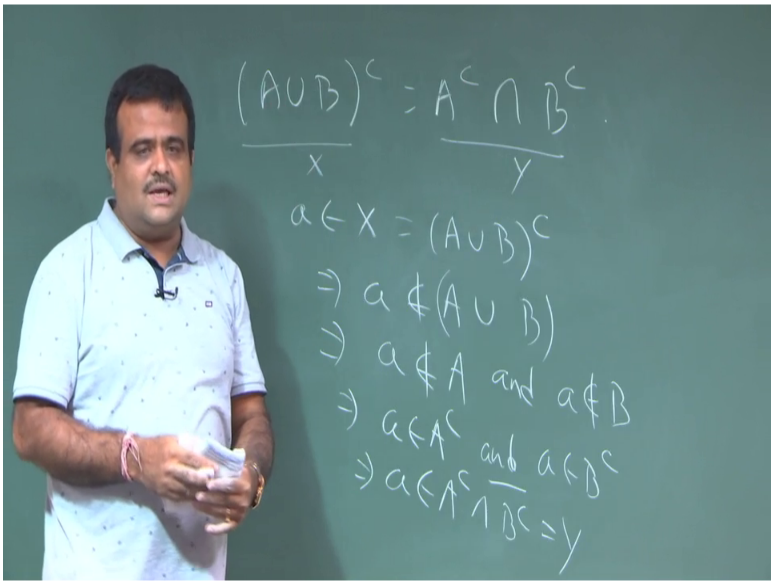

So, how to show this so, X is basically A complement. Now, if X is in the complement so that means, a does not belongs to this, if a belongs to the complement so; that means, a does not belongs to the, that set, that is the definition of the complement.

So, if A does not belongs to this set means this is union. So, a does not belongs to A and A does not belongs to B.

So, this implies if A does not belongs to that union; that means, A does not belongs to one of them, otherwise A belongs to the union. So, A does not belongs to this and a does not belongs to B.

Otherwise if a belongs to a capital A then a must belongs to A union B which is not correct or if a belongs to B then a belongs to A union B which is not correct so; that means, this is true. Now, this means what this means a belongs to A complement and a belongs to B complement.

So, now this is the intersection. So, this means a belongs to A complement intersection B complement which is the Y.

So that means, any element of this set is an element of this set so; that means, this is a subset of this so, for one way we have prove that. Now, the other way around, we have to take an element from this set and we have to show that is that is also an element of this set. Then we can say this is subset of this so, this both will tell us right these two set are equal.

So, let us try that. So, we take an element from this set.

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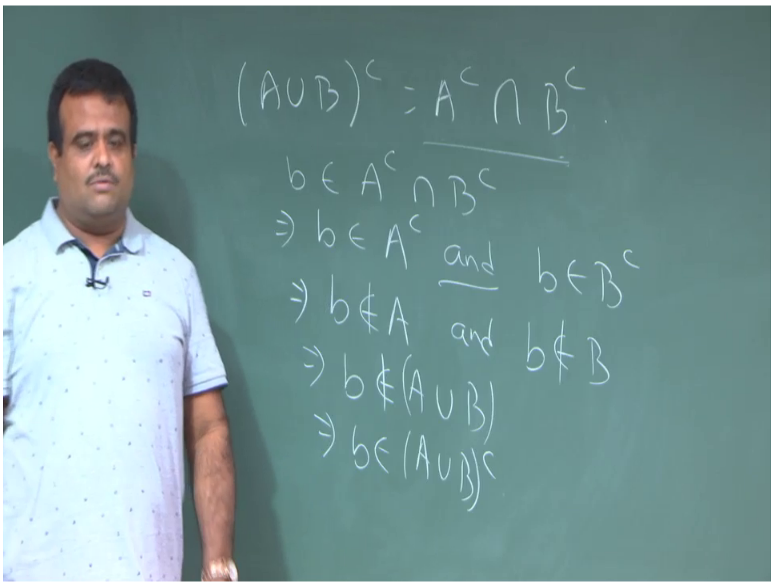

So, we take b belongs to A complement union B complement. So, now and now we have to show this belongs to this. So, let us try that so, this belongs to the intersection means this belongs to A complement and b belongs to B complement. And now its belongs to complement means it does not belongs to A and b does not belongs to B ok.

Now, if it does not belongs to both A and B then it does not belongs to the com union because union is the or collection of element which are either in A or in B. So, this implies b does not belongs to A union B set. So, this implies b belongs to their complement that is it. So, b belongs to this set so; that means, this is a subset of this and earlier we have prove this is a subset of this hence, this two sets are equal.

So, this is the first law and the second law is second law is A intersection B complement is equal to A complement union of B complement ok.

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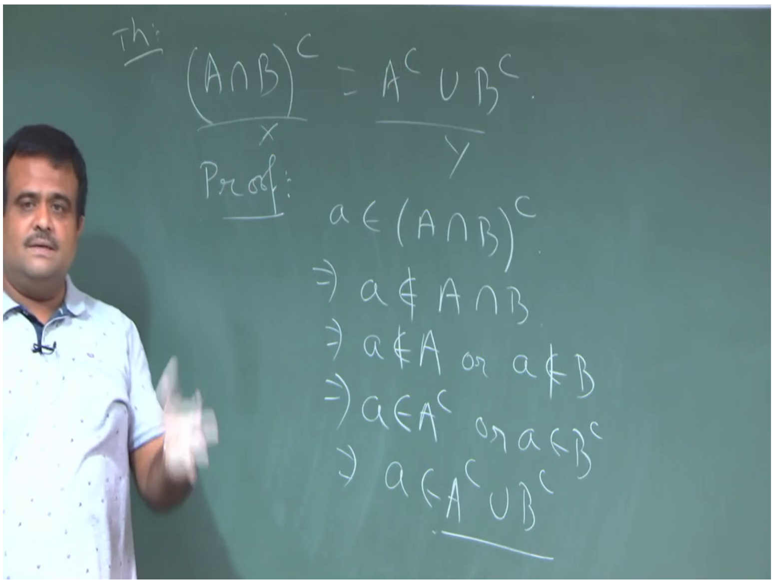

So, now again how to prove this; this is a property of the law. Now, how to prove this. Now, again same way this is say X set and this is say Y set, we have to show X equal to two sets are equal. So, in other way we have to show X is a subset of Y and Y is a subset of X then we have done

So, for that what we need to take we need to take an element from this a then we have to show this is an element of this set. So, this means what a must not belongs to A intersection B, if it is belongs to the complement then a must not belong to A intersection

B. So, A intersection B, a must not belongs to means so, when a either it not in a or not in b. So that means, a does not belongs to A or a does not belongs to B, if it is or not and because this is this does not belongs to their intersection; that means, it is not a common point of A intersection B AB.

So, if it is not a common point means either a not belongs to A small a or small a not belongs to B so; that means, a belongs to A complement or a belongs to B complement. So, this implies a belongs to A complement union B complement.

So, this part is done here we have taken an element from this set, we have prove that that is an element of this set. So, this is a subset of this and similarly we can take an element from this set and then we can show this is also a element from this way that let us try that.

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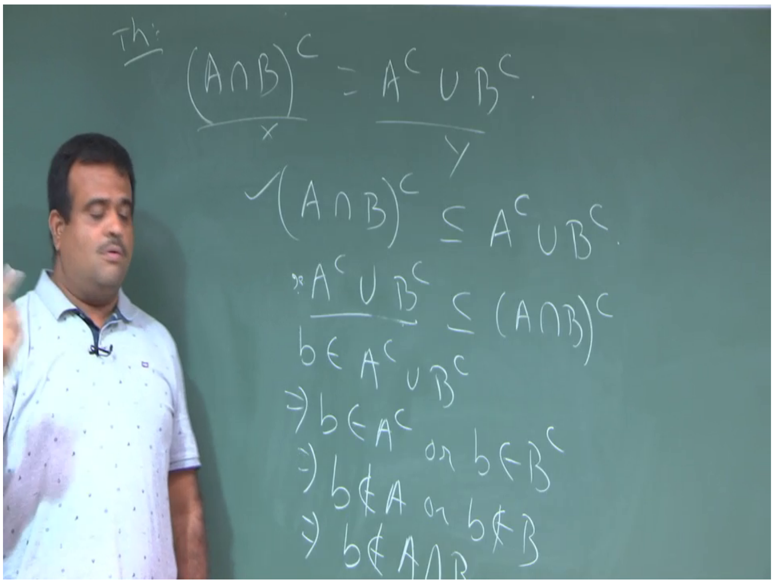

So, this part is showing that so, whatever we have seen this is showing that this is a subset of this, the first part is done.

Now, what we have to show we have to show that this is a subset of this is a subset of this set. Now, for that what this need to show this is done and this is yet to show.

So, now for that what we need to take we need to take an element from this set. So, let us take an b belongs to A complement intersection B complement sorry, this will be union this set this will be union, this union of this.

Now, if this is belongs to this so that; that means, b belongs to either A complement or b belongs to B complement. Now, if b belongs to A complement this implies b does not belongs to A or b does not belongs to B. Now, if b b either not in A or not in B.

So, this implies this implies b does not belongs to their intersection; for intersection b has to be in both the place.

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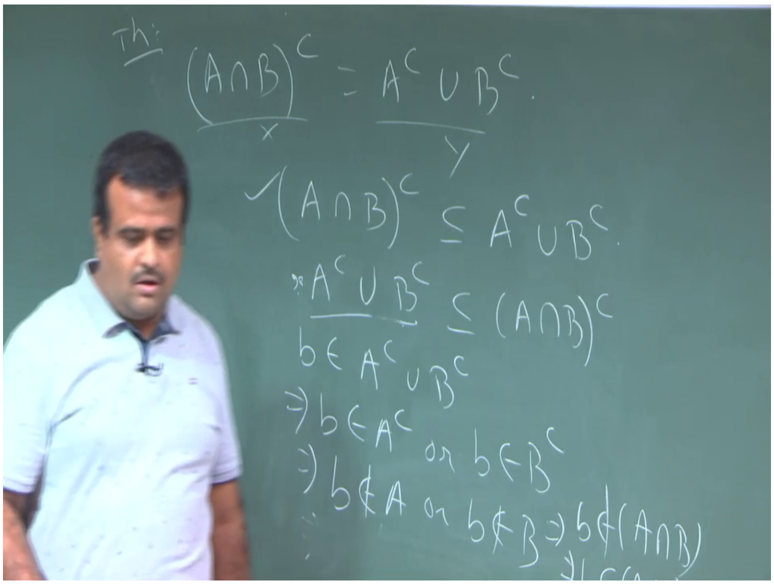

So, this implies b does not belongs to A intersection B does not belongs to. So, this implies b belongs to A intersection B complement so; that means, if we take an element of this set we just prove that is an element of this set. So, that will give us the, that is the proof ok. So, this is called De Morgan’s law.

Now, we will defined two more operation on this set operation.

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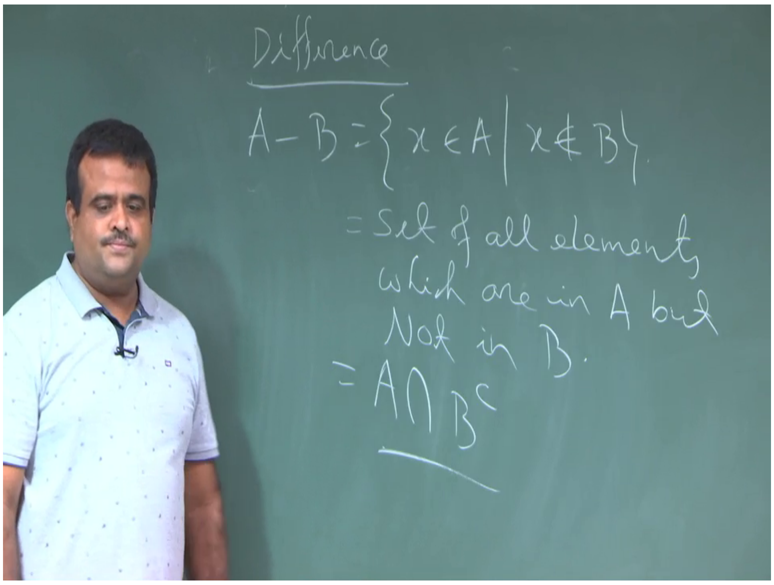

The first one is the difference; difference operation on of A and B and it is denoted by A minus B. If you take two set A B, A minus B is basically set of all element which are in A, but not in B.

So, this is the element in A, but it must not be in B ok. So, this is the set of all element all elements which are which are in A, but not in B. So, this is called this is the A union B sorry A difference B so, this is basically we can say this is basically A intersection with the B complement.

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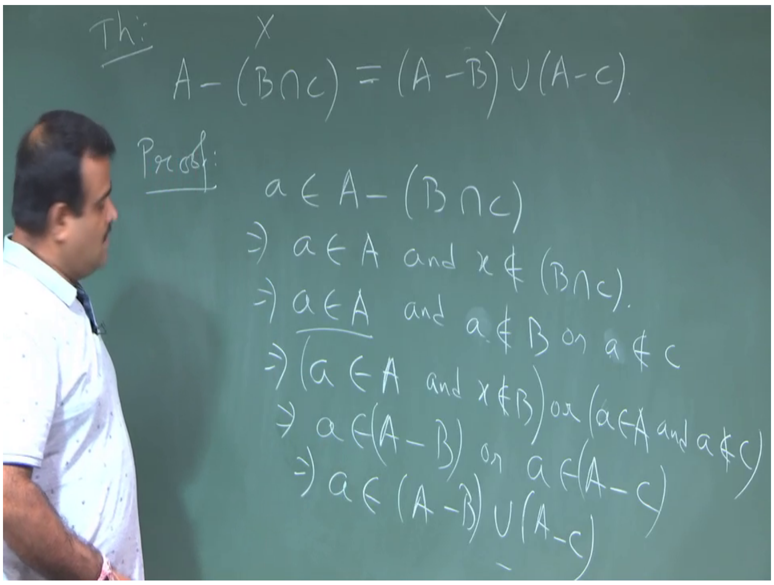

Now, we can have some properties on this say; the first property is A minus B intersection C is basically A minus B union A minus C ok. If you take three sets A B C, then this is a this is a result so, how to prove this; let us try to prove this.

So, we have two set X and Y, we need to set similar way that X equal to Y so, we have to show. So, we take an element from this set X so, this means what this means a belongs to A, but a does not belongs to this ok; a belongs to A, but this.

Now, this imply what this imply a belongs to A, but and a does not belongs to their intersection means a does not belongs to B or either one of this, a does not belongs to C.

So that means, a does not belongs to A and either one of this is true. So, we take a does not belong a belong this is always true now, with this we can take this a belongs to capital A and a belongs does not belongs to B.

This is one case or a belongs to this is always true, a belongs to A and sorry this is a and a does not belongs to C.

Now, this is giving us a belongs to A minus B or this is giving us a belongs to A minus C. Now, this two will give us a belongs to A minus B intersection A minus C sorry union A minus C.

So, that this is the set so; that means, if we take an element from this set we can easily show that this is an element of this set. So that means, this is a subset of this.

Now, in other way around we can show if we take an element of this set that will be an element of this set. So, that that is telling us this is a this is a subset of this. So, if you combined these two we can get this result ok.

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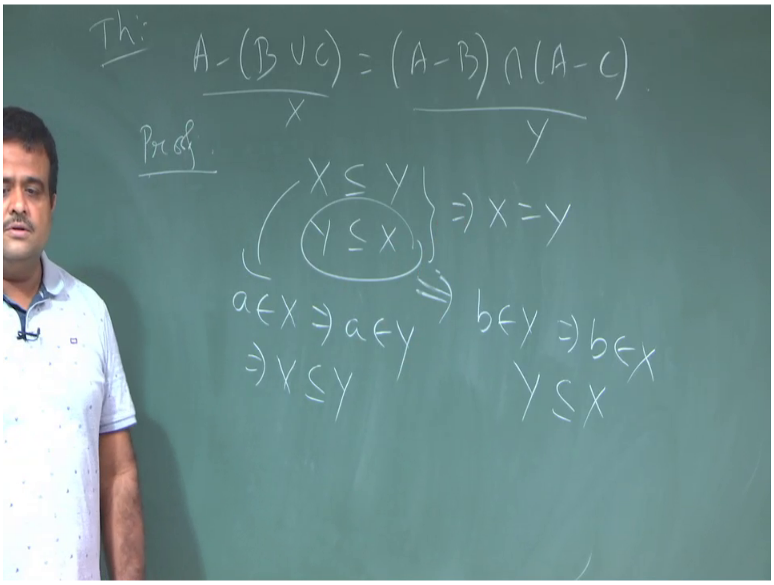

Now, the second property is so, second property for this is A minus B intersection C is basically A minus B intersection of A minus C. So, earlier one here this was union intersection here we have union and here we have intersection.

So, how to prove this again by similar way we take an element form here and that we have to show that is an element of this set. So that means, this is the subset of this and again we take an element from here and we have to show this is a so, let us try that. So, this is say X set, this is say Y set. So, basically we have to show X is a subset of Y and Y is a subset of X and this both will give a X equal to Y.

Now, to show this one what we need to show we need to take an element from X and we need to prove that this is a element of Y. Then this will give a X is a subset of Y and to show this what we need to show we need to take an element from Y and we need to show that that is an element of X.

So that means, Y is a subset of X and combined these two will give a X equal to Y. So, let us try that ok.

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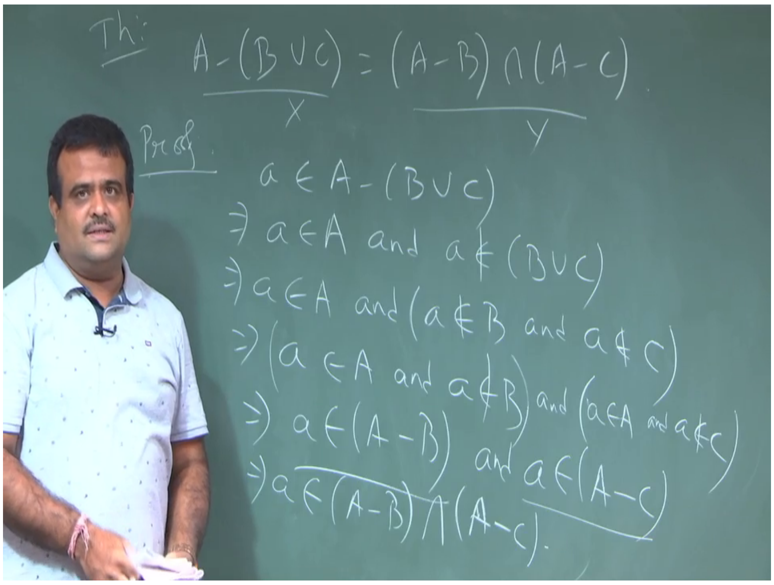

So, let us take an element form this set so, this set is A minus B union C. So, now, we have to show that this is an element of this set also so, let us try that. So, this means a belongs to A and a does not belongs to B union C.

So, this implies a belongs to A that is always true and a does not belongs to B union C means, a is not in B or not in B; because other way is if a is either of this B or C then a belongs to B or C.

So, this means a does not belongs to B or and sorry not or so, a does not belongs to B union C means, he does not belongs to any one of this otherwise a, a belongs to B, B union C.

So, this and a does not belongs to C you understand, otherwise if a belongs to small a belongs to B or C any one of this then small a belongs to B union C, but that is not a does not belongs to this means this is true. Now, these two we can combined and we can have a belongs to A and a does not belongs to B and a belongs to A and a does not belongs to C.

So, this two will give us this will give us does not belongs to sorry this will give us a belongs to A minus B first one and a belongs to A minus C second one. So, a belongs to this means this imply a belongs to then intersection A minus B intersection of A minus C so, that this is the set. So, if we take an element from this that will be an element of that set.

Now, we have to show the other way around we will take an element from this we have to show this is an element of this; let us just try that quickly.

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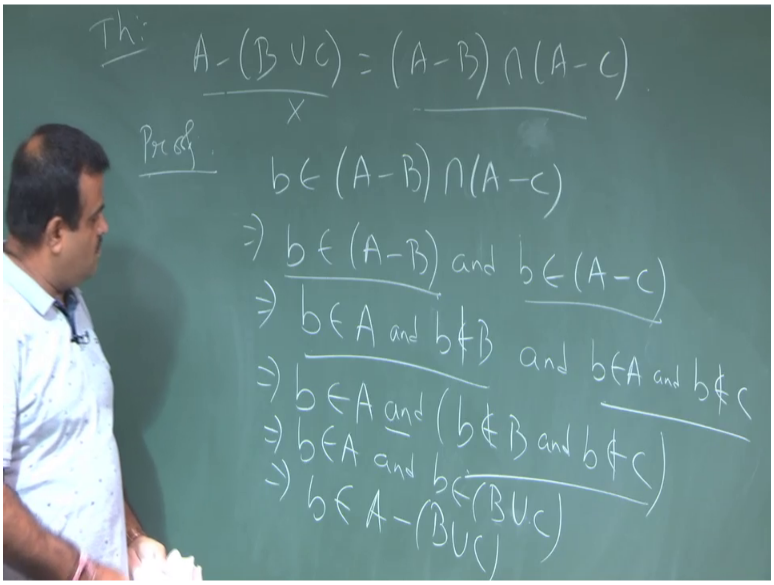

So, let us take an element from this set ok. Now, we need to show this is an element of this set so, how to proceed. So, yeah so, let us try that so, this means a b belongs to so, b is in this means b belongs to A intersection B and b belongs to this.

So, this implies b belongs to so, the first one means b belongs to A and b does not belongs to B, this is the first part. And this is and similarly here also b belongs to A and b does not belongs to C this is the second part.

When the both the cases b belongs to A no doubt so, b belongs to A. And so, if you combined this ands it will give us and B b b sorry b does not this is b does not belong. So, b does not belongs to B and b does not belongs to C. So, b does not belongs to B and b does not belongs to C.

So, if you combined these two this will give us b does not belongs to B union C. So, this imply and b does not belongs to B sorry difference C sorry B union C. So, this implies b does b belongs to A minus B union C this is the set.

So, if we take an element from this set we just prove that is also an element of this set. So, this is the way we can prove that ok.

So, let us have a another symmetric difference notation of two set A B.

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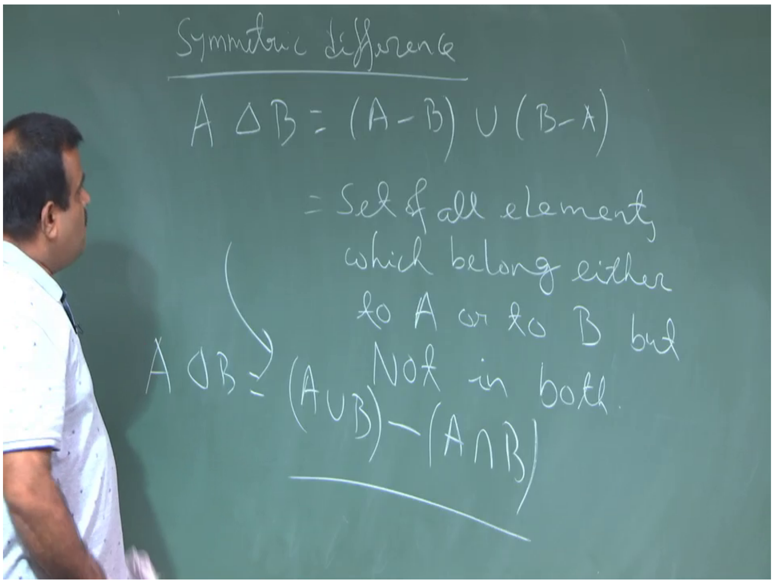

So, that is basically denoted by this symbol symmetric difference this is a set notation set operation between two set so, this is denoted by this. This is basically the element which are either belongs to A or belongs to B, but not the both so, this is basically the A minus B union of B minus A.

So, so this is basically this is basically set of all elements, elements which belongs to belongs either to A or to B, but not in both ok. So, this is also equal to A intersection B minus of A this so, this is basically A set difference of B ok. So, this is the way we defined this.

Now, we will this is the another set operation. Now, we will defined the notation of Venn diagram and then we will see using the Venn diagram most of the operation are very easily to prove.

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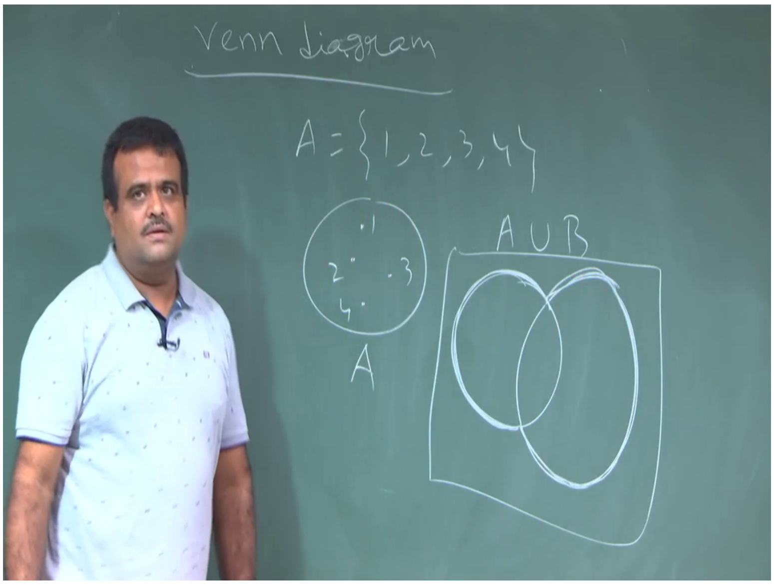

Venn diagram so, we can represent a set in a diagram way. So, suppose A is a set say 1, 2, 3, 4 these are the element so, this is the pictorial way. So, this is 1, this is 2, this is 3, this is 4, this is called Venn diagram. So, this is a set we are seeing this in a pictorial way.

Now, if we have this notation then how to defined A union B. So, we have a set A we have another set B now, A union is B is so, this is the universe. So, A union B is this set the this set this is basically A union B ok. Now, what is the A intersection B.

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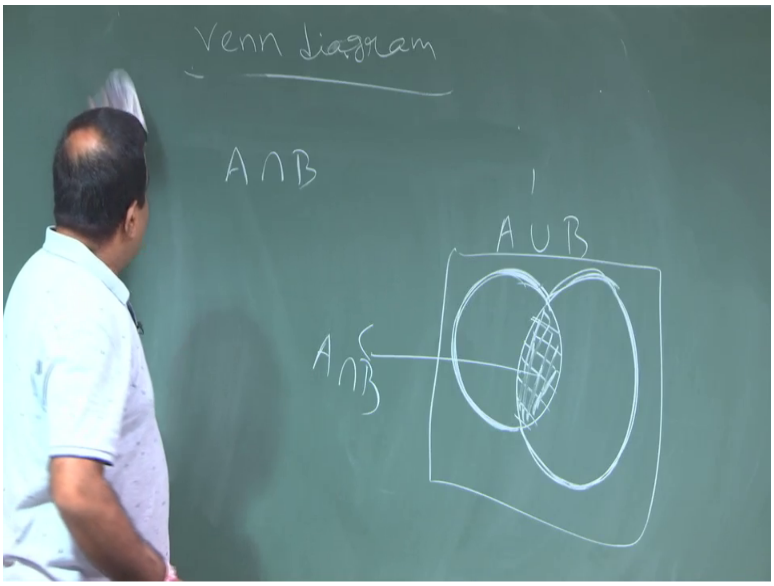

So, if we have this intersection is basically that this set common point. So, this dark one this is basically A intersection B and this whole set is this set is basically A union B ok.

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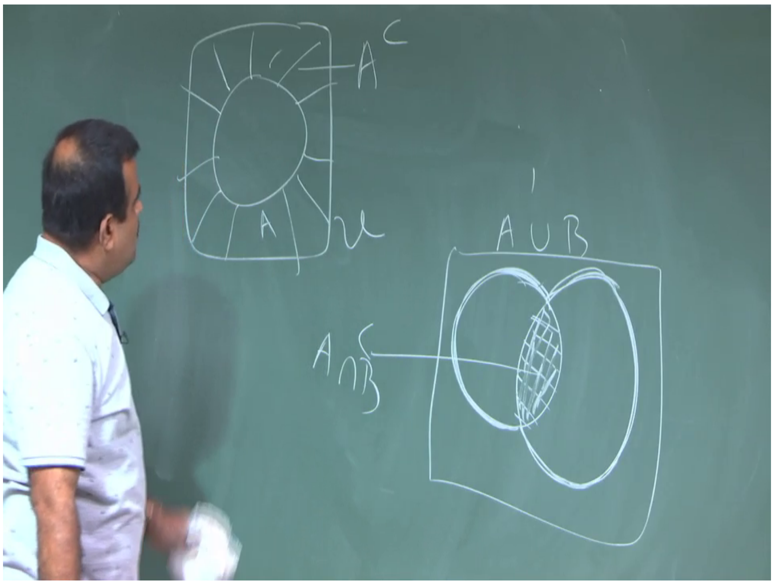

Now, using Venn diagram it is easy to see the complement suppose we have a set A and this is the universe; this is set A. Now, A complement is basically set of all elements belongs to here these are the A complement. So, this is the universe this is A so, A complement is not in A ok.

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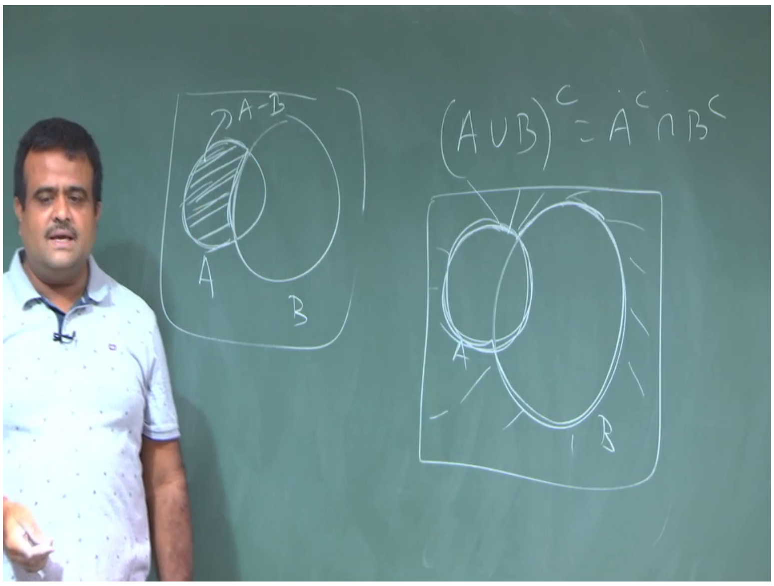

So, now this is the complement now, what is the difference A minus B. So, we have a set this is the, this is A, this is B. So, A minus B means this one. So, is it is in A, but not in B. So, this set is basically A minus B and similarly this set is B minus A.

So, this is the way we represent this represent is in its called Venn diagram representation. So, this is the, this is one of the pictorial way we can represent a set ok.

So, this way it will help us to see the proof like if we want to see whether what is the A union B complement ok. Suppose this is a set A, this is a set B, this is universe so this is A, this is B. Now, what is A union B? A union B is this one, this is A union B set ok; all the elements over here.

Now, its complement means this one now that is nothing, but A complement intersection B complement because A complement is what is A complement, A complement is this one after outside A, B complement is this one. So, their intersection means basically this one.

So, by seeing this Venn diagram we can easily those proof will be little more easier.

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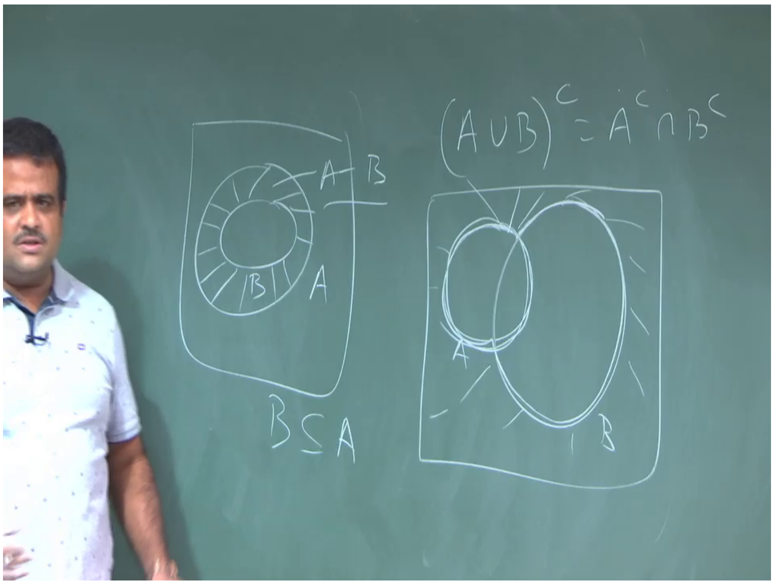

Now, subset means if we have two set A B, this is say A set, this is say B set. Now, B is a subset of A; if every element of B is an element of A and then A is a superset of B and these are the these are the element in these are the element in A minus B. So, this is the way we view this set in a Venn diagram.

Thank you.