An Introduction to Riemann Surfaces and Algebraic Curves: Complex 1-dimensional Tori and Elliptic Curves

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Lecture – 14
The Construction of the Universal Covering Map

We are continuing with our discussion of trying to construct the universal covering for robotic space at least what we are interested in our universal covers for Riemann surfaces, but then this is completely of a topological nature. It applies to any topological space and of course, I miss again remind you that by any topological space we are of course, going to confine ourselves to topological spaces which are housed of arc wise connected locally arc wise connected and locally simply connected.

Let me recall quickly what I did in the previous lecture.

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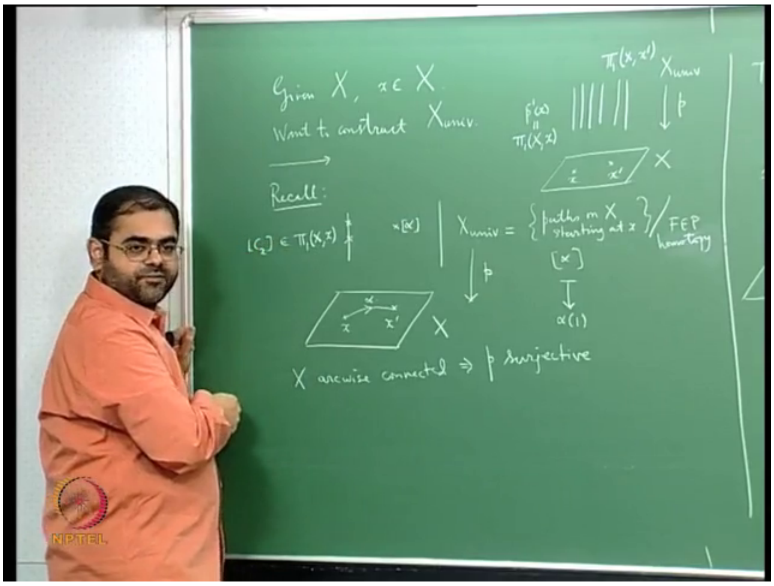

What had happened was given topological space X and a point of X and what we wanted what we wanted to construct to construct the universal covering space for X and you know we already we have already seen that I in the case of the universal covering the picture look like this. If P is the covering map from the universal covering space to x, then over each point small x in capital X what you get in the fiber P inverse of x is just the fundamental group first fundamental group of capital X based it small x and the and this happens for every point.

If X prime is another point then what you get above is if a copy of the fundamental group at that point X prime and in this way the universal covering. When I say equal to I mean bijective assets, there is a natural identification of P inverse x the fiber over the point X with the fundamental group based at X and, you get this picture of the universal covering being at least as I said just cotton by putting over each point X a copy of the fundamental group based at X. This was our starting point to construct this to space X of univ.

If you recall what we did was this is what we did, how did we define X of univ at least to begin with as a set. What we did was well we know what to put over x; over x you will have to put in you have to put pi inverse I mean you have to put the first fundamental group based at X and what do you put at a at a different point X prime above X prime what we are going to put is moment of equivalence classes of paths alpha which start from x and end at X prime and this, these are all paths and the homotopy the equivalence is with respect to fixed end point homotopy to and, in particular if the point X prime where X what we would do we would put the above would be homotopy equivalence fixed at end point homotopy equivalence of path starting at X and ending at X which is precisely the fundamental group at X.

When X prime is equal to X what you put above is exactly the fundamental group based at X and then I, this was this was how x of univ is defined because defined to be paths on capital x starting at small x modulo fixed end point homotopy.

When I say modulo fixed end point homotopy I mean take equivalence classes. This is the set of equivalence classes under fixed end point homotopy and what was the map from the from this set to X, this was a very natural map which would send a path alpha to the yeah it would send a point which is equivalence class alpha to the end point . In this case if alpha was a path ending at X prime, then this would go to X prime which is alpha of 1, this was the was the map from this set to X and since X is. So, you can see immediately that X arc wise connected would imply that this P is subjective and we also saw last time how to convert this set into a topological space.

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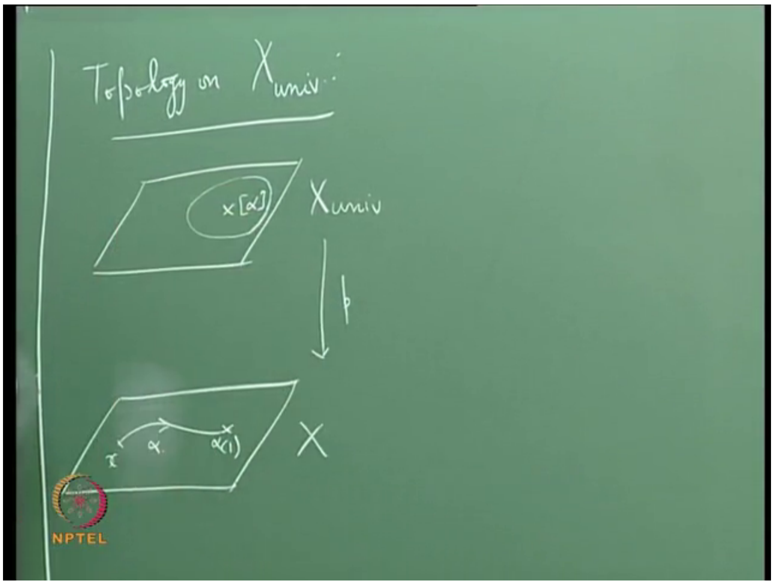

What we did was may be, I will just rub this and write it properly FEP homotopy. Top topology on this set, the topology on this set was also based on a very intuitive diagram. So, you see you have this set above which is X of u and unit and of course, that is map P and what you have below is your space X and well give me a point above which is a which is a homotopy equivalence class alpha.

Where alpha is therefore, going to be a path from X and it is going to end at alpha of 1. Now what do you what would you expect to have as an open neighborhood of alpha you know P finally, we have to show that P is a covering map in particular it is a local homeomorphism and you know it is an open map and therefore, you know if this if you if you are going to prescribe an open set above it is image has to be an open set below and therefore, it has to look like this and the therefore, what you do is you start with an open set U which is an open neighborhood of alpha of 1 and then what you do is you define a base.

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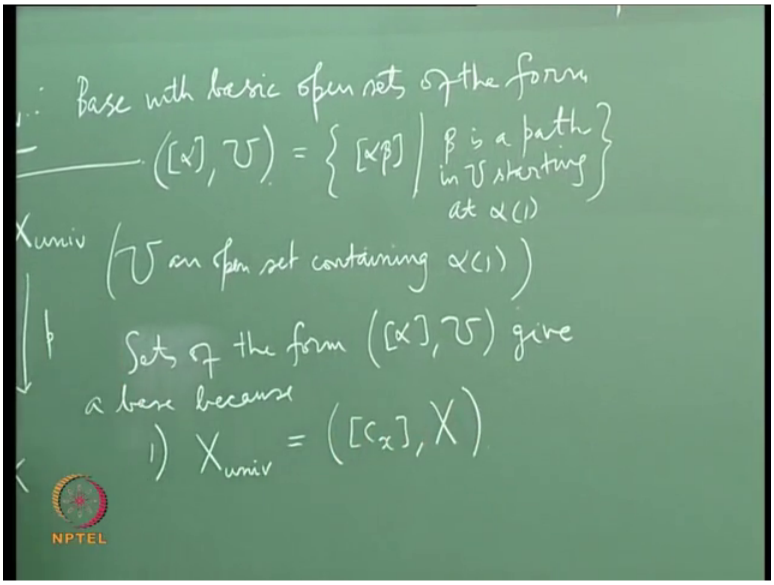

For the topology with basic open sets with basic open sets of the form we call it a notation was u comma alpha.

This is this U come alpha this U comma alpha actually is the set of all alpha beta where you know beta is a path in U starting at alpha of one and of course, U and open set containing alpha form. What are you going to what are what are the points in this neighborhood well as I, this is alpha 1 alpha of 1 this N point of alpha I have this open set you and then now I take another point I, I connect I take another path from alpha of one say beta from alpha of 1 to another point in U.

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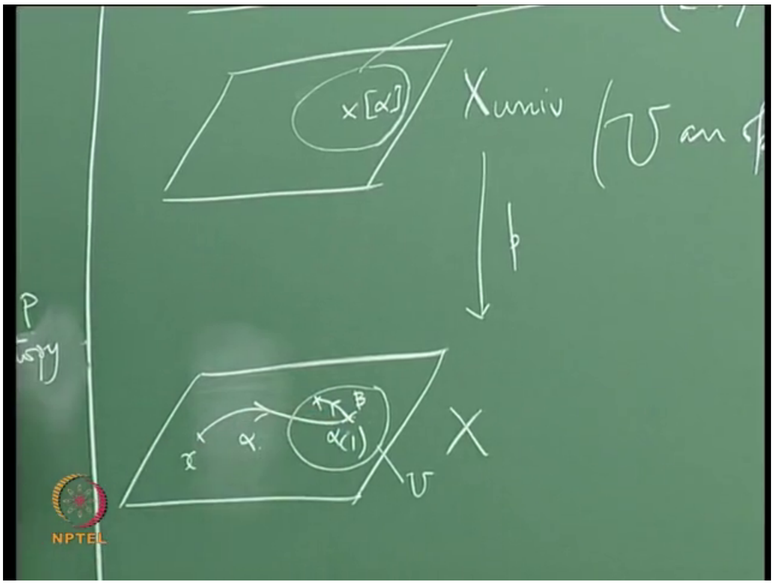

Then you see alpha followed by beta is certainly a path from X and that will give me a point here. That that will be this point alpha beta, nearby points here come from nearby points below and at least locally, but it is very clear that in this way nearby points go to nearby points and that is essentially saying that P is continuous.

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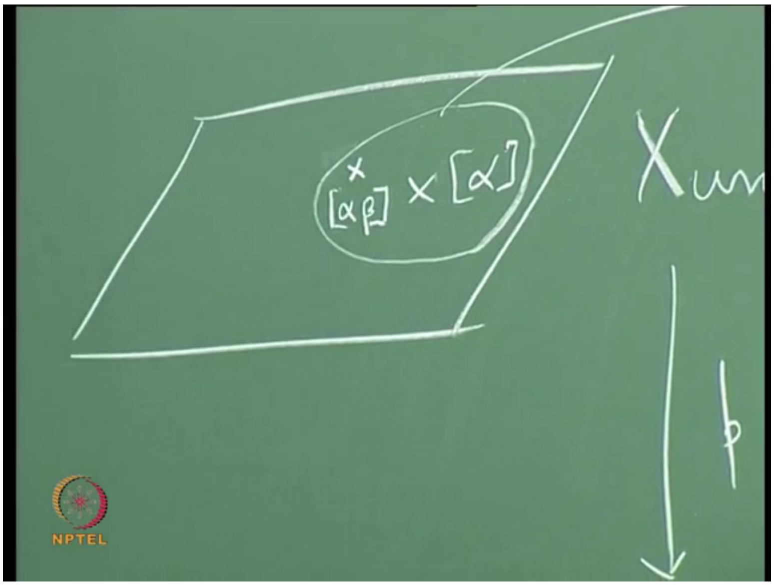

I verified in the last lecture that sets of this sets of this form a base for the topology; that means, you give the topology on this to be consisting of sets which I have gotten by taking finite intersections of sets of this form and then taking arbitrary unions.

Arbitrary unions are finite intersections of sets of this form and of course, any open set will then be can be expressed as a union of basic open sets.

Any point an open set can be surrounded by a basic open set and that is the reason why they are called basic sets of this form sets of the form alpha comma U give a base because number 1 the whole space the whole space X of univ is actually a set of this form namely I take the constant path at X and then I take X itself. Every point every point of this is just a path starting at X which is the end point of C of x mind you C of x is going to lie here. Here is X and in the fundamental group, there is a unit element that unit element is just C X the constant path at X and where will it go to it will go to the end point of this which is precisely X.

C X is here and you take C X it will go you take it ends it is ends point is X and then take the whole space X as an open set. You get this pair and this will include all points because X because X is arc wise connected then or even by definition.

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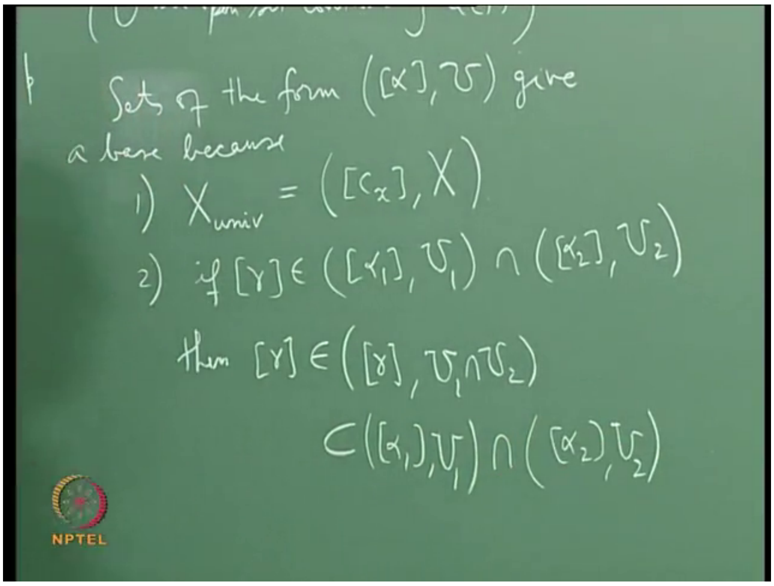

Then the second thing is you take what is the property of basic open sets you take any 2 basic open sets or for that matter any finite intersection of basic open sets if you given any point in the intersection you would find the another basic open set sitting inside that intersection. We verified this for 2 in the in the following way if gamma is a point lying in say alpha 1 comma U 1 intersection alpha 2 comma U 2 then gamma is contained in the set gamma comma U 1 intersection u 2 which is contained in this intersection which is further contained in alpha 1 comma U 1 intersection alpha 2 comma U 2.

Given a point in the intersection I am able to find another basic open set which is in the intersection which contains this point. Because of this condition you can see that all these open sets are just union are just given by unions of basic open sets and given any point in an open set you can always find the basic open set surrounding that point which is continued your; in your given open set. These basic opening sets are therefore, the building blocks for all open sets, well this makes this into a topological space and then this map becomes a continuous map, the map P becomes continuous and that is also very easy to see because I will have to just show that the inverse image of open sets are open.

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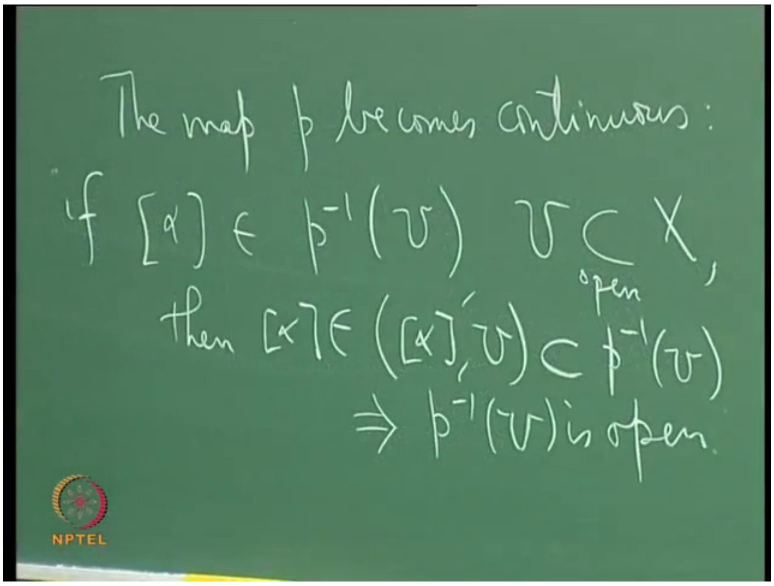

If you take if you take a point alpha in t inverse of U where U is open in X then you know this alpha is contained in this basic open set namely alpha comma U which is also in T inverse u. What you have what I have proved is, this is a diagram you suppose I have I have u and I take P inverse u and I take an alpha then alpha is certainly contained in this spare and that pair certainly under the map P goes into U; that means, this pair will be in P inverse u this open set defined by this data is going to be in P inverse u. What I have proved is that every point of P inverse u is surrounded by a basic open neighborhood a basic open set which is also contained in P inverse u and therefore, P inverse u is opened.

This implies this implies P inverse u is open, so that tells you that the map P is continuous then the third thing that I explained was that this topology, makes this with this topology the space the topological space that you get is housed off.

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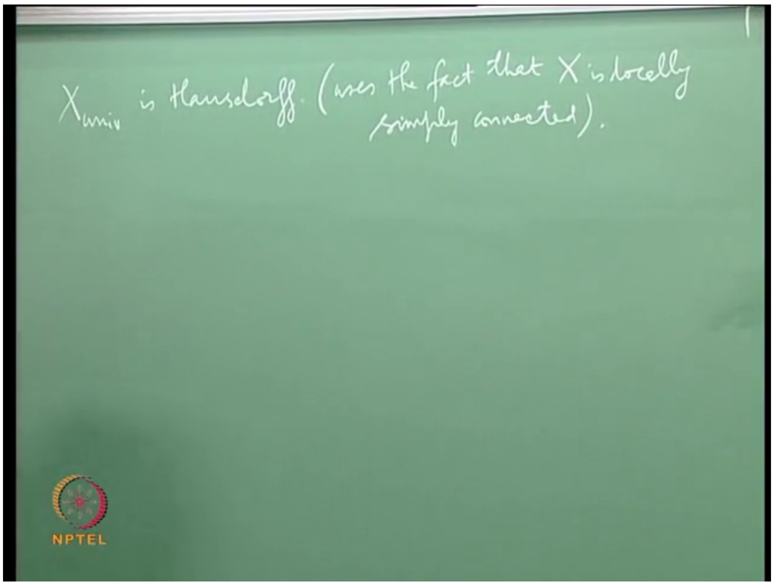

That is something that I again explained in the last lecture and you see the improving that this is housed off, we use the fact that capital X is locally simply connected. This uses this is the fact that a capital X is locally simply connected.

It uses this spectrum now we will have to do several other things what we need to prove. What is left what is it that is left out to be done I need to show that finally, I need to show that this is the universal covering I need to show this is universal covering. First of all I have to show I have to prove certain properties of this space this topological space what are those properties I need to show that the space is arc wise connected locally arc wise connected locally simply connected and then finally, I need to show it is also simply connected because I told you the universal covering space it is by definition a covering space for which the space above is simply connected. I need to prove that this is arc wise connected locally arc wise connected locally simply connected and simply connected I have to prove all that, then I will have to prove that this map is a covering map.

If I had; that means, what I have to prove that this is a local homeomorphism and in particular; that means, I have to it would follow that is also an open map. What I am going to do now is because X already has these local properties the local pub the properties of being locally connected locally simply connected the movement I prove this a local homeomorphism these properties will also pass on automatically to the universal coverings to this topological space. My aim would be first strategy would be to first somehow show that there is a local homeomorphism then after showing that this is local homeomorphism then I will show that it is also covering map; that means, there is an ad for every point below there is an admissible neighborhood such that the inverse image breaks up into a disjoint union of open neighborhoods each of which is homeomorphically mapped by P on to this neighborhood in X of that point in X.

The strategy now is to first show that the map P is an open map. That is what I am going to look at. You know I am trying to prove P is a local homeomorphism.

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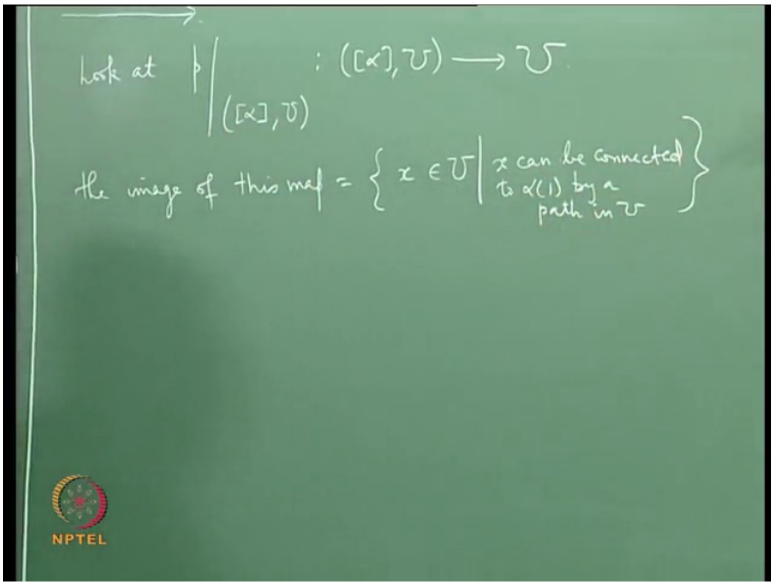

I am going to look at P restricted to this basic open neighborhood alpha comma U from you know alpha comma U to U I am going to look at this map of course, it is a continuous map it is restriction of a continuous map to an open set. It is a continuous map all right and now what is the image of this map what is the image of this map. You can see the image of this map of this map is the set of all X prime in U or let me say X belonging to u such that X can be connected to alpha of one by a path in U right.

What is in the image of this map they if there is a point? If there is a point if there is a point which is in the image of this map then it has to be ah. It has to come, if there is a point in the image of this map it has to come from a point like this and a point like this by definition involves path which existence of a path from alpha of 1 2, it is endpoint.

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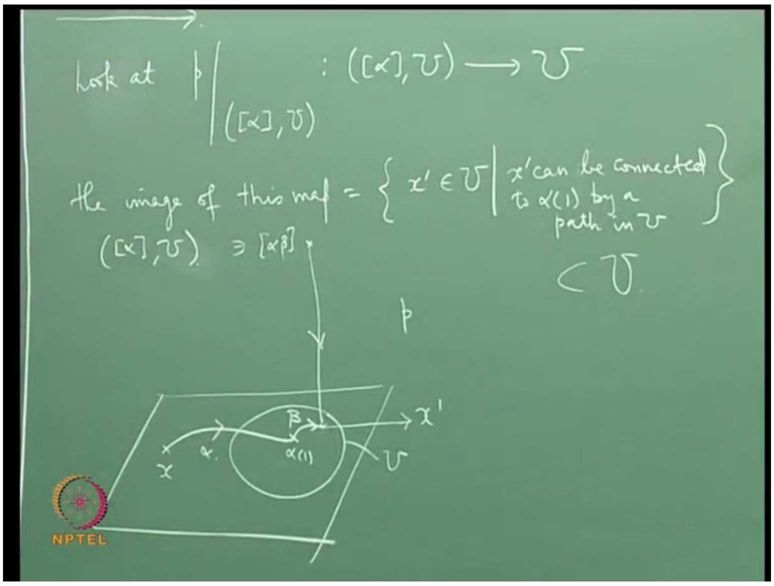

Here is my X oh I think my X is already fixed I should not use this X again fortunately. Let me use X prime please change C 2 X prime. X is confusing because X has already been fixed the point X is fixed. The image of this map is those entire X prime which can be connected to alpha of 1 by a path in U that is my claim all right. This is this is quite clear because you see. Here is my alpha and this is alpha of 1 and this is my U and supposes there is a point here let me call this point as X prime.

Suppose or this is a point X prime and suppose is this in the image of this map P from a point above the point above the point above has to come from here because I am only looking at the restriction of this map from here and I am looking at it is image then, but what is the point here a point here is the form alpha followed by beta it is alpha yeah. Here alpha followed by a beta all right. What it means is that is some alpha followed by beta here that will go that that is the point here in a in X of univ is. In fact, in this you know in this in this basic open neighborhood namely alpha comma U and you see that is going to this point here under P.

Which by definition means that beta if you look at the definition of this basic open neighborhood beta should be a path in U starting at alpha of 1 and the projection map takes this alpha beta to the endpoint of beta? What is going to happen is that this alpha beta. Beta is going to be a path from alpha of 1 to the N point of beta which is X prime; that means, X prime can be connect connected to alpha of 1 by a path in U conversely if x prime is connect can be connected to alpha of 1 by a path beta in U then X prime is exactly the image of alpha beta. The image of this map is this which is a subset of U this is subset of U.

Now, the beautiful thing is that this subset of U turns out to be an open set because X is locally arc wise connected this set. You know I will I will give it a name I will just I will just call U sub X U sub alpha one and I will put A R C.

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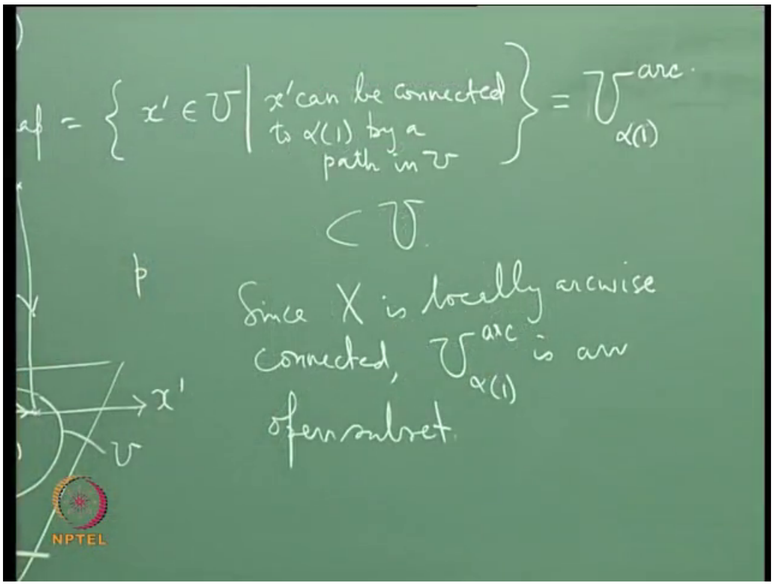

This notation means look at all those points in U which can be connected by an arc inside U to alpha of 1 which is a point of view. This is of course, a subset of U now what is my claim the claim is if. Since X is locally arc wise connected U the; this set U arc sub also 1 is an open subset is an open sub set. Here again you see where this local arc wise connectedness of X is being used and why is this true. It is quite obvious, but let me write it, you see I will have to tell you.

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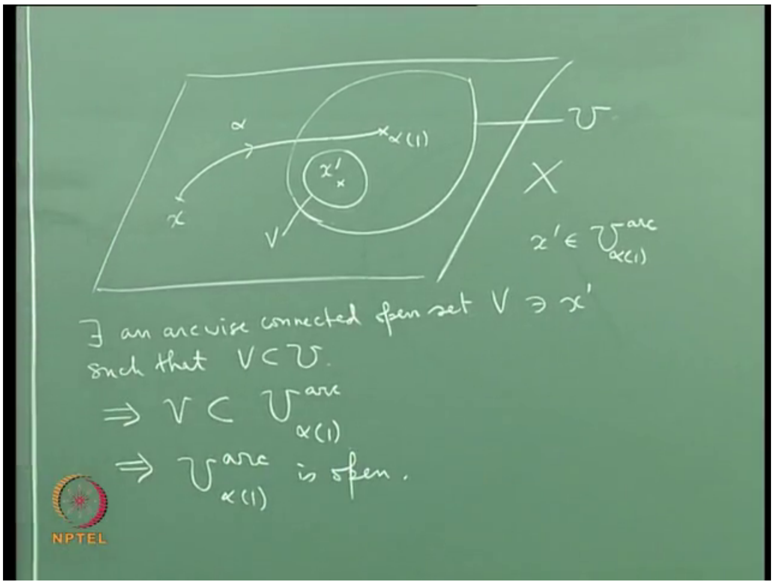

Here is my situation, I am on, I am on X while here is my, here is my small x which is fixed then there is some alpha this is alpha of 1 and then I have this. This is my U and I have taken a point of U of this subset of U. This is a point let me call it as X prime. Let me have some more space let me write it here X prime and X prime is a point in U which can be correct which in which can be connected to a alpha of 1 by an arc now what is the condition that X is local arc wise connected the condition is given any point and given any open sets and containing that point you can find a smaller if you can find a smaller open set if not that open set itself a smaller open set which is arc wise connected.

Now, therefore, here is my point I apply that condition to this point. Here is my point x prime and then there is open set U. There is going to be a smaller open set which contains X prime and which is arc wise connected. What I will get is I will get I will get a I will get a V there exists an open set and arc wise connected open set V containing X prime such that V is also V is inside U this is locally arc wise connected how give me a point and give me a neighborhood no matter; however, small I can find a smaller neighborhood surrounding that point in your given neighborhood which is arc wise connected therefore, now you see this V is arc wise connected. Every point in V can be connected to X prime, but X prime is in this.

X and X prime can be connected by an arc to alpha 1 therefore, every point in v can also be connected by an arc in alpha 2 alpha of 1 by an arc in U. So, what this means is it is obvious that V is contained in this subset. What we have proved you take any point of that set I can find an open sub I can fineness and an open set which contains that point and which slights inside this set so; that means, the set itself is an open set. This implies that this set is open. What the upshot of this is that P restricted to this takes this basic open set to this subset and that subset is an open set. It is very clear that P maps this basic open set every basic open set to an open set.

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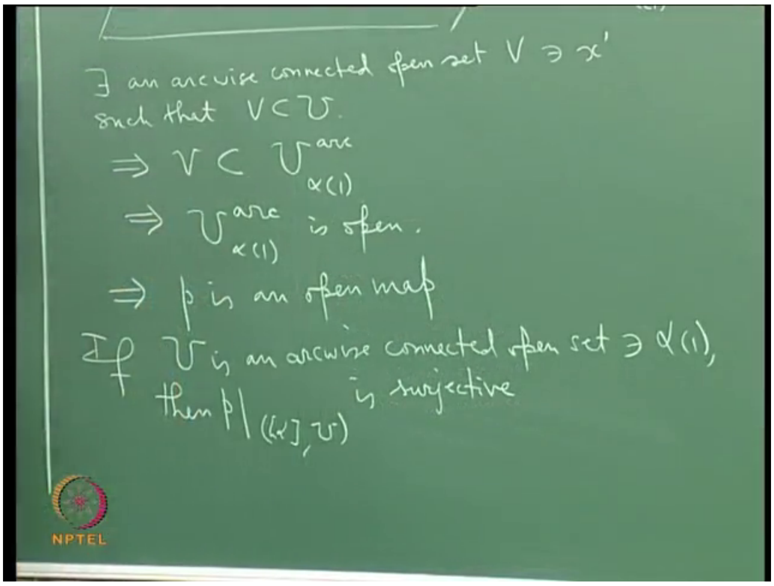

P is an open map this implies p is an open map this implies P is an open map and you know and it also implies 1 more thing it tells you that if my U to begin with was already suppose this U was already an arc wise connected subset then P restricted to this will be subjective if u is an arc wise connected open set containing alpha of 1 then P restricted to this basic open set alpha comma U is subjective because, since in this case U will be the same is the set of points in U which can be connected by an arc or a path from alpha of 1.

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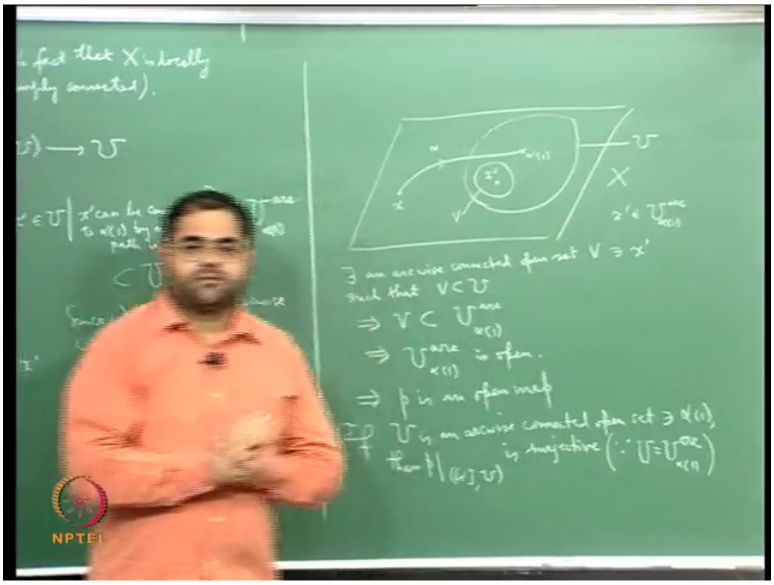

You know these are all the steps in trying to get hold of neighborhood of alpha which is homeomorphic to a neighborhood below.

The first thing that I get is you take any basic open set the image is open and the second thing is; if in particular my basic open set the second member was arc wise connected then this is subjective. If u is arc is you can see that if U is arc wise connected then these 2 are the same every point in U can be connected by an arc to alpha of 1 right. U get subjectivity all right now we will have to go on to injectivity. You know I have I have this map. I have this map is already continuous it is open if I choose U to be arc wise connected is already subjective if I just proved it is injected to become a local homeomorphism because it is an open map the U s map the moment I prove it is injective I have a set theoretic inverse and because it is open the set theoretic inverse will become continuous.

I have to only put more conditions on U to make it injective and then I will get a locally homeomorphism and what is that condition on U the condition on U is you choose it further to be simply connected.

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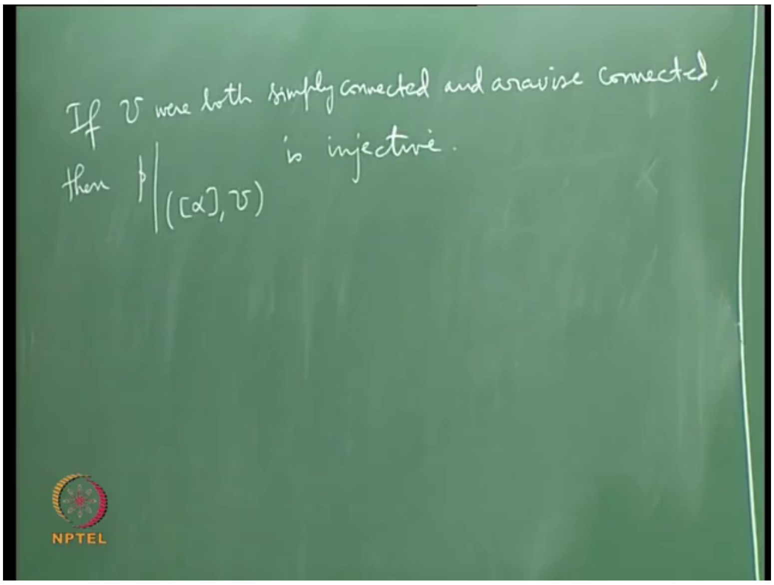

Let me write the down, let me rub this, what we do is if U were both simply connected and arc wise connected then P restricted to this alpha comma U is injective, let us prove this the situation is as follows, here let me draw on the diagram.

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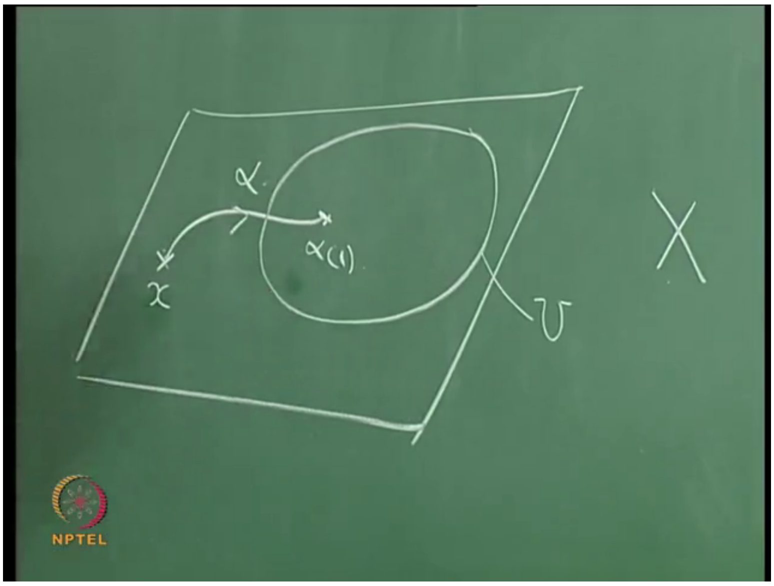

This is my X and, here is my small x. Here is alpha is the path here and the endpoint is alpha of 1 and then that then I have a neighborhood U of alpha of 1 which is both simply connected and arc wise connected I can get such a neighborhood that is because capital X is locally arc wise connected and locally simply connected because capital X is locally arc wise connected and locally simply connected I can get a neighborhood of any point of X which is both arc wise connected and simply connected and the claim is for such neighborhood U for such an open set u this pair will be homeomorphic to U under the map P and why is that true.

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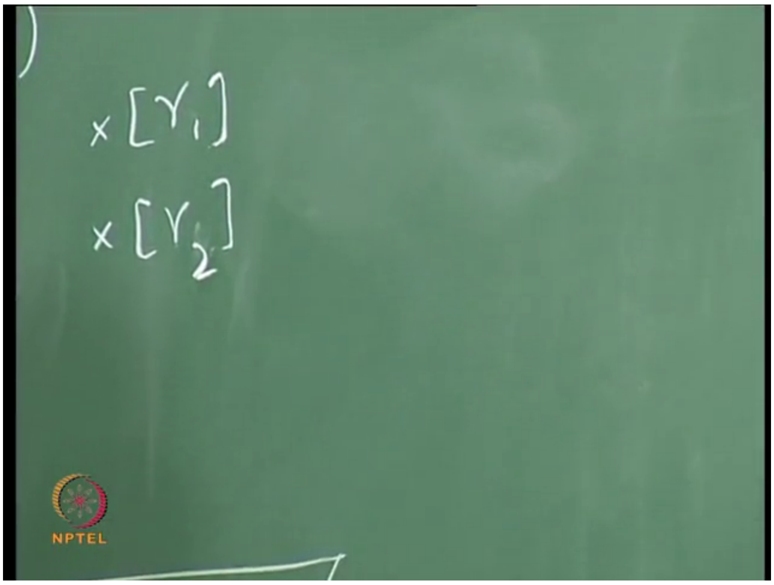

See suppose you take 2 points. Gamma 1 and gamma 2 suppose there are 2 points and these are both in they are both in.

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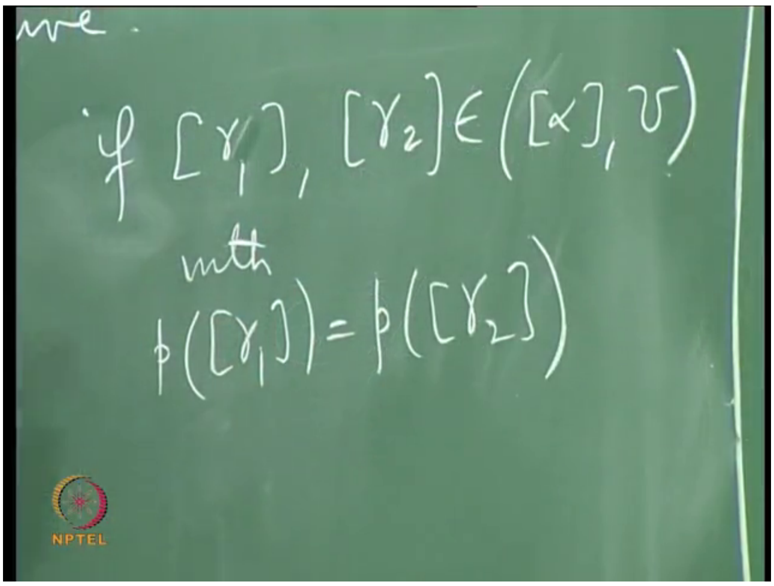

Gamma 1 comma gamma 2 in this neighborhood above basic open set above with both of them goes into the same point below, with T of gamma 1 is equal to P of gamma 2 then what does it mean it means that both of these points will go to this point and what is this point this point will be just the endpoints of gamma 1 it will also be the end point of gamma 2, that is what this condition means.

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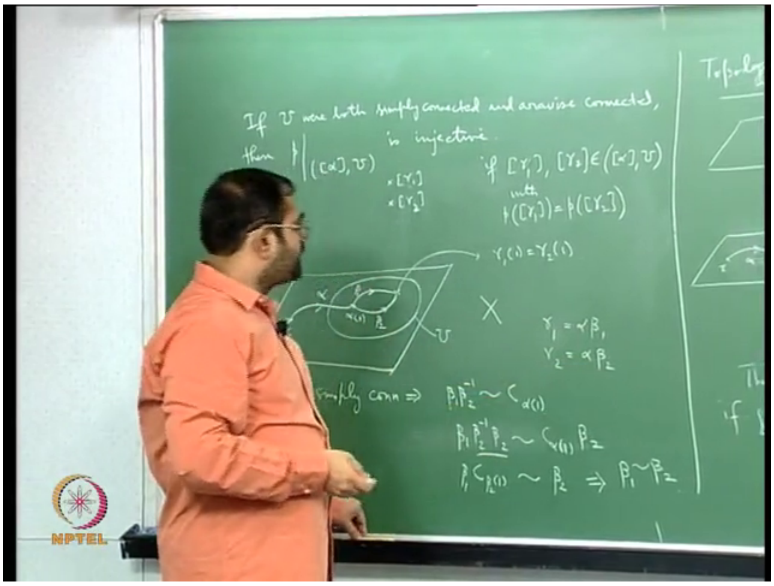

Now, what is the meaning of saying that gamma 1 gamma 2 are in this see the it means by this definition it means that gamma 1 is actually alpha followed by certain beta 1 and gamma 2 is again this alpha followed by certain beta 2 where beta 1 and beta 2 are paths from alpha of 1 to this common endpoint. It will look like this. Here is beta 1 and here is beta 2, alpha followed by beta 1 is the path whose homotopy class is gamma 1, an alpha followed by beta 2 is your gamma 2 and you see the point is U simply connected.

Therefore any closed loop in YU can be shrunk to a point. This beta 1 followed by beta 2 inverse it is a closed loop in U and it is ending at alpha of 1. I can shrink it to a point. It is homotopic to the constant path at alpha 1. U simply connected implies that beta 1 followed by beta 2 inverse is homotopic fixed endpoint homotopic to the constant path at alpha 1 and of course, you know this is the this is the same as saying that beta 1 is fixed endpoint homotopic to beta 2 because I will have to concatenate it both sides with beta 2 and I will get this. I hope this is clear or if not I will write 1 more step.

It if this means that beta 1 beta 2 inverse beta 2 is homotopic to C alpha 1 beta 2 and beta 2 inverse beta 2 inverses followed by beta 2 is the constant path at this point. This will be beta one followed by the constant path at beta 2 of 1 which is also the same as beta 1 of 1 that will be homotopic to the constant both at alpha one followed by beta followed by beta 2 is just homotopic to beta 2 and beta 1 followed by this constant path will be just homotopic to beta 1.

Essentially what I will get is beta 1 is homotopic to beta 2 and if beta 1 is homotopic to beta 2 then alpha beta 1 is also homotopic to alpha beta 2.What it will tell you is therefore, that alpha beta 1 is homotopic to alpha beta 2 which means that these 2 hometopic classes are the same. These 2 points have to be the same and that will give you injectivity.

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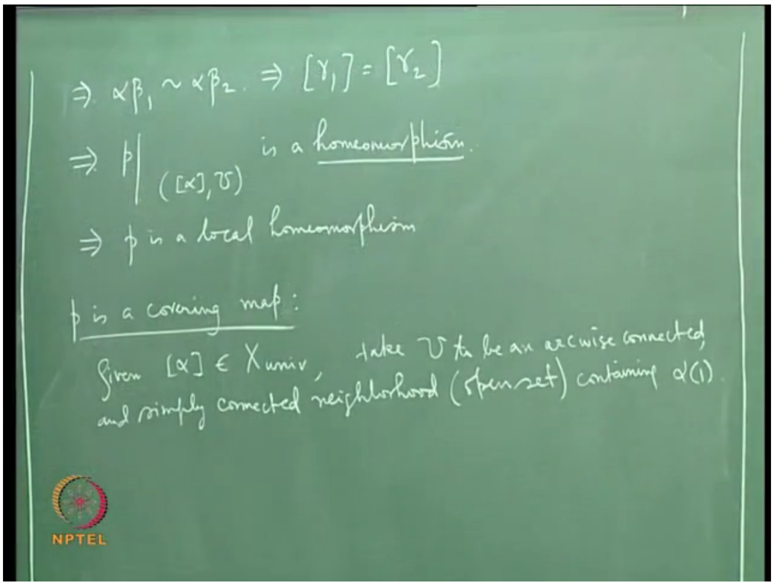

Let me write the term implies that alpha beta 1 is homotopic to alpha beta 2 and then that tells you that gamma 1 these 2 are the same point above and, we have got injectivity .What has happened is that if u were both simply connected and arc wise connected neighborhood of this point alpha of one then this map is bijective continuous and open.

It is a homeomorphism .This implies that P restricted to alpha comma U is a homeomorphism. What this now tells you is that it tells you that P is a local homeomorphism from this from this you can immediately reduce that the topological space X univ has the properties of being locally arc wise connected and locally simply connected that you will get immediately all right. What would be left out is to show that it is arc wise connected and then I will have to show that P is a covering map I will have to show it is a covering map I. For we know that P is only a surjective local homeomorphism.

I told you the difference between this and a covering map is that it should have the unique path lifting property, but we can directly see that P is covering map. We can see we can see that next P is a covering map. How does 1 prove that a given alpha in this in X of univ take u to be a an arc wise connected arc wise connected and simply connected neighborhood that is open set containing alpha of 1. We take the same kind of neighborhood that we wanted to give we wanted us to give a set above which is a homeomorphic to it. You take that kind of neighborhood the claim is you take just inverse image of that neighborhood then the inverse image is going down going to break down into pieces disjoint union of pieces each of which is homeomorphically mapped by P on to this neighborhood.

Therefore the alpha of 1 you see the admissible neighborhood for alpha of 1 is simply a an arc wise connected simply connected neighborhood.

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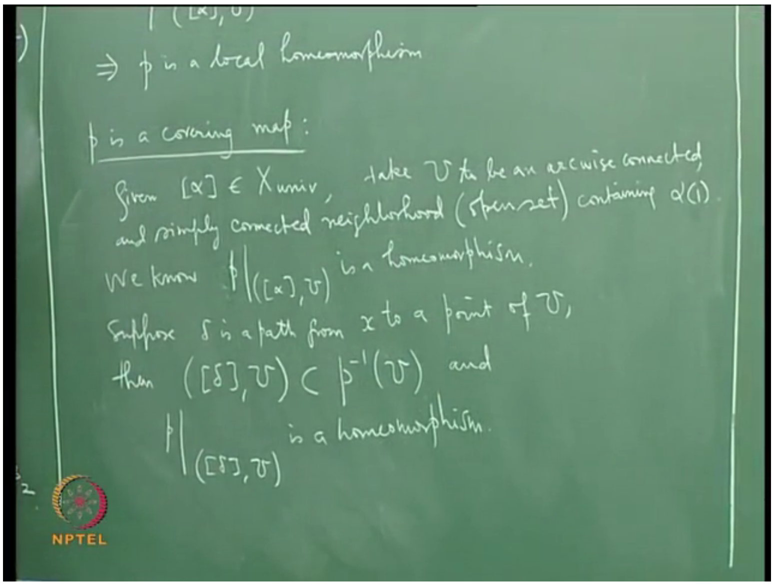

We know alpha P restricted to alpha comma U is a homomorphism, we know this already. Suppose delta is a path from X to a point of U then you know at this open set delta comma U this open set delta comma U is going to be contained in p inverse u and P restricted to this delta comma U is homeomorphism the only thing that it uses is that U is both simply connected and arc wise connected all right.

It is clear that all sets of this form are going to be in the inverse image and I claim that the inverse image is exactly the union of sets of this form and I claim also that if the first members are different then all these sets are disjoint.

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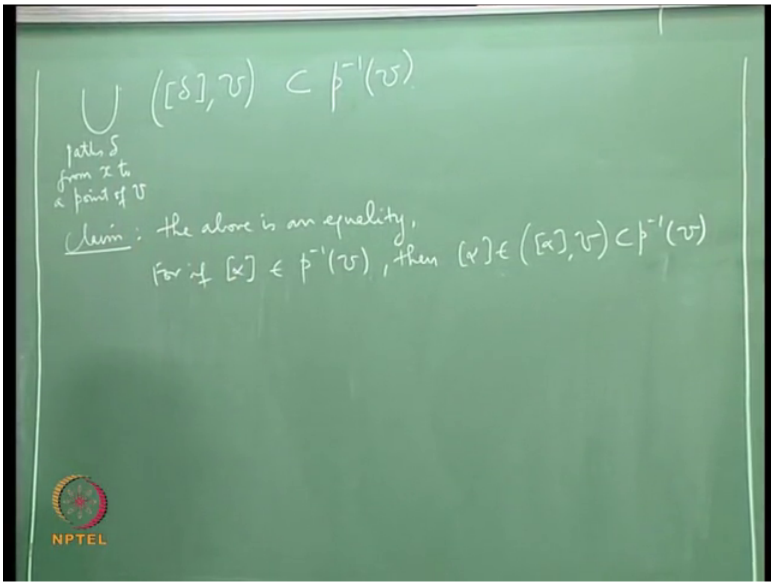

The union over paths delta from X to a point of U of basic open neighborhoods of from delta comma U this is contained in P inverse u and the claim is that this is an equality the claim is the above is an equality that is because of the following reason you take an element of P inverse u then this element of P inverse u suppose I call it as alpha then the endpoint of alpha is going to be a point of u all right and by what I said this basic open neighborhood alpha comma U is going to be inside P inverse u.

For if alpha, you know I should not have I should not a fixed an alpha here I am not fixing an alpha here I am just saying that I mean this alpha could vary, what is really being fixed is this open secure these alpha could actually vary all right the only condition is that H the endpoint of alpha lies inside U, long as endpoint of alpha lies inside U then this P restricted to this basic open set is a local is a homeomorphism, for if, if alpha is in P inverse u then you know alpha is certainly a point of this basic open set alpha comma U which is also in P inverse u. Therefore, this union is P inverse u then I will have to tell you that these are all disjoint.

If I take 2 of these things if they intersect I should say they are the same. If the first member a different then I should say that they are disjoint.

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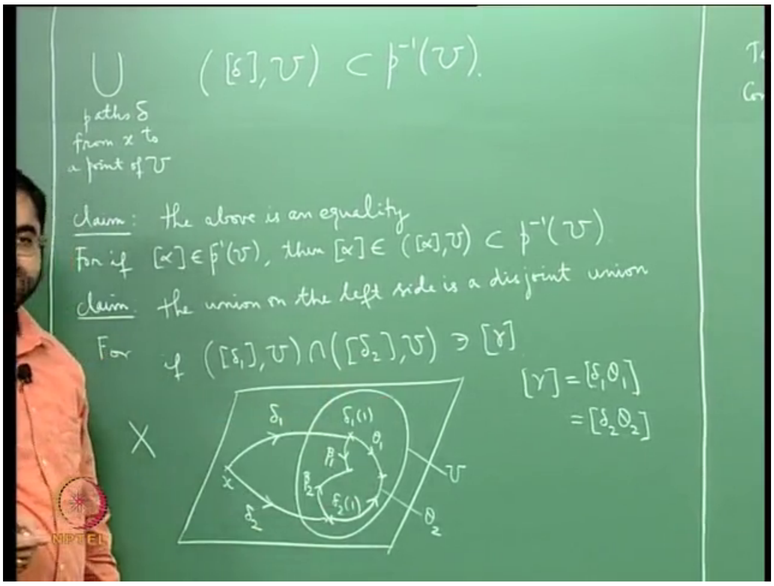

Clean the union on the left side is a disjoint union if we consider is a is a disjoint union let me answer anything else, if for if let us say some let us say some delta 1 comma U intersects delta 2 comma U suppose the intersect all right.

Then I need to say that delta 1 and delta 2 are 1 and the same and that should be obvious if you remember the ideas following the proof of house toughness. What does it mean that these 2 intersect so; that means, there is a gamma there is a gamma in the intersection all right. Let us draw a diagram it means that. I have something like this. We have gamma a point in the intersection of these 2 open seats basic open sets and what does it mean it means that gamma is because it is in here it is equal to delta 1 followed by a theta 1 delta 1 followed by a beta 1 let it be as this and then it is also equal to delta 2 followed by theta 2 because it is in here.

Let me let me draw this and of course, these 2 being equal tells you that the end point of theta 1 should be equal to the end point of theta 2 and of course, the starting point of theta 1 is the endpoint of delta 1 the starting point of theta 2 is the endpoint of delta 2. Situation is the diagram looks like this, this is this is theta 1 and then there is 1 there is 1 more here and, this is theta 2, this is theta 2 and of course, theta 1 is a path in u and theta 2 is also a path in u by the very definition of these basic of neighborhoods and now what

I will do is that I will I will have I just want to say that this is a disjoint union. I will have to say that if 2 members intersect then they are completely equal. I will just show that this is contained in that and that is contained in this.

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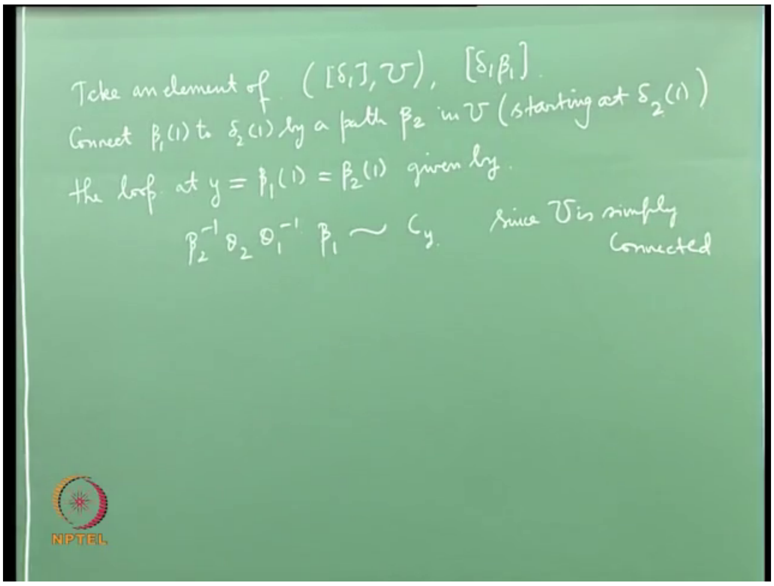

What I will do let me start with take an element of taken element of the first member delta 1 comma U the first basic open set it will be of the form delta 1 beta 1 . It is going to look like this it is going to be delta 1 followed by some path beta 1 which is a again a path inside U and I will I am I am going to show that this homotopic class fixed endpoint homotopic class delta 1 beta 1 is also in this basic open neighborhood. For that what I do is I make use of the fact that U being arc wise connected I can find a path from delta 2 of 1 to the endpoint of beta 1 and let me call there had beta 2.

Connect beta 1 of 1 to delta 2 of 1 by a path beta 2 in U, So, of course, I should say that is starting point is del delta 2 of 1 starting at delta 2 of 1, connected by this path and then now notice that you see let us calculate let us look at this there is a closed loop here there is a closed loop here and that closed loop is going to be homotopic to a constant path that is because the neighborhood U is arc wise connected and simply connected. If I call this points let me call this might as small y let me call this point as small y then I have the following thing. I take I take beta 1 followed by beta 2 inverse then theta 2 and then theta 1 inverse. If I do that then I will get, then I have to start at this point, but let me start at this point. If I start at this point I should take beta 2 inverse then theta 2 theta 1 inverse beta and beta 1.

Let me look at that the loop. Let me write that down at y is just beta 1 of beta 1 of one and it is also equal to beta 2 of 1 given by this is my loop beta 2 inverse and then go by delta by theta 2 and then go by theta 1 inverse and then go by beta 1 is homotopic to the constant path that y since u is simply connected now from this I am I am just going to try to calculate what beta 1 is homotopic 2. You know I operate on the left by beta 2.

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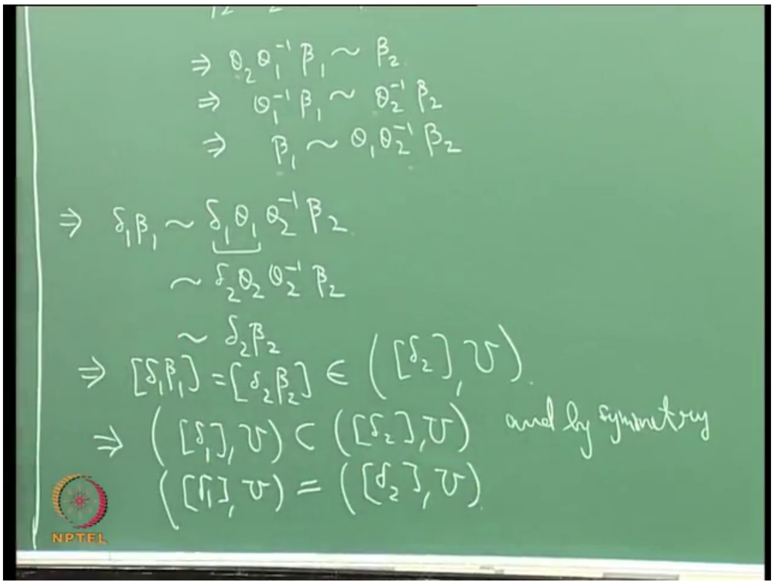

I will get theta 2 theta 1 inverse beta 1 is homotopic to beta 2 then I operate by theta 2 inverse I will get theta 1 inverse beta 1 is homotopic to theta 2 inverse beta 2 and again I operate by theta 1 on the left. I will get beta 1 is going to give me theta 1 theta 2 inverse beta 2.

Beta 1 is homotopic this now. Now, let me look at delta 1 beta 1, delta 1 beta 1 will be homotopic 2 delta 1 followed by theta 1 theta 2 inverse beta 2, but you see delta 1 theta 1 is homotopic delta 2 theta 2. You know I can write this as homotopic to delta 2 theta 2 theta 2 inverse beta 2 and that will be homotopic because theta 2 theta 2 inverse will be homotopic of to constant path I will get delta to beta 2 and delta 2 beta 2 is now. What this tells you is that which in terms of fixed endpoint homotopy will mean you will mean the delta and beta 1 will be equal to delta 2 beta 2, but the latter name is delta 2 beta 2 is here.

This is an element of this basic open able delta 2 comma U. The modern the story is you start with an element in delta 1 comma U you get you see that it is also in delta 2 comma U. This is a subset of that and therefore, by symmetry that is also a subset of this. They are equal. This will imply that the 2 are equal. Delta 1 comma U, contained in delta 2 comma U and by symmetry 1 will get well delta 1 comma U will be equal to delta 2 comma U, this completes the proof that this is the disjoint union and it is equal to P inverse u all right and therefore, what we have proved is that if you start with A an arc wise connected simply connected open set U, then that open set itself serves as an admissible neighborhood for every point that it contains and therefore, P becomes A covering map. That that gives you the proof of the fact that P is a covering map all right. I will stop here.