Introduction to Probability and Statistics
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Lecture – 12
Probability

In this lecture we introduce Probability and begin the discussion on probability. From now on till the end of this course, we would be discussing concepts in probability and then we will look at random variables and then we would also study some known distributions such as binomial and normal. So, let us begin the discussion on probability.

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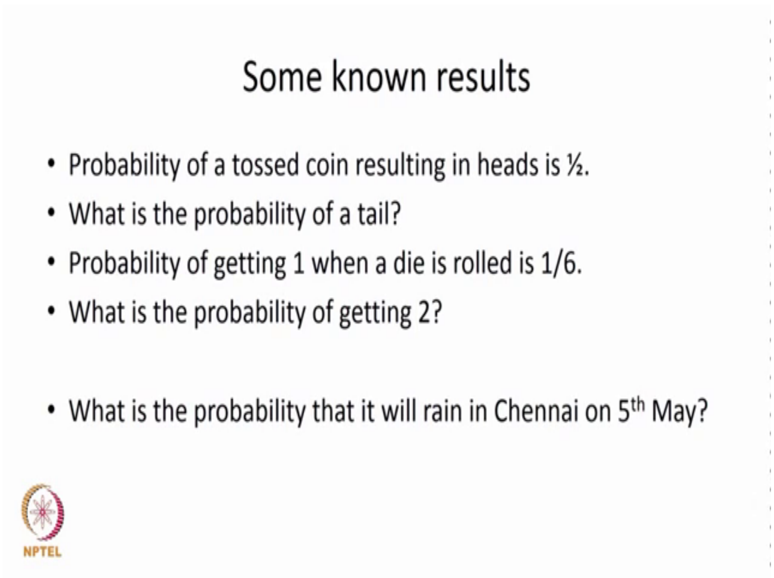

We will start with some simple well known results; probability of a tossed coin fair coin resulting in heads is half. So, this is something that all of us have learnt and all of us know that, when you toss a coin probability of getting a head is 0.5 and probability of getting a tail is also 0.5.

So, what is the probability of getting a tail? So, probability of getting a tail is 0.5. Now how does that happen? So, when we toss the coin we believe that there are only 2 outcomes; it is either a head or a tail. And then if we know that the probability of getting a head is half then the sum of the probabilities has to be 1.

And therefore, the probability of getting a tail is also half. The other way of looking at it is conduct a large number of experiments and generalize it. We will see about that as we move along. So, probability of getting a tail is half. Probability of getting 1 when a die is rolled is 1 by 6. So, a die has 6 faces, and we assume that there are dots representing numbers. So, if there is 1 dot it represents 1, 2 dots it represents 2 and so on.

Since there are 6 faces numbers one to 6 are there. One in each face and when it is rolled probability of getting a 1 is 1 by 6 because, any one of the 6 faces can show up, and with equal probability; so, it is 1 by 6. Now what is the probability of getting 2? It is the same it is 1 by 6, because any one of the 6 faces can show up. So, probability of getting a 2 is also 1 by 6. Now, these are some known results, these are things that we have learnt right from high school to where we are. Now, let us ask another question what is the probability that it will rain in Chennai on 5th may.

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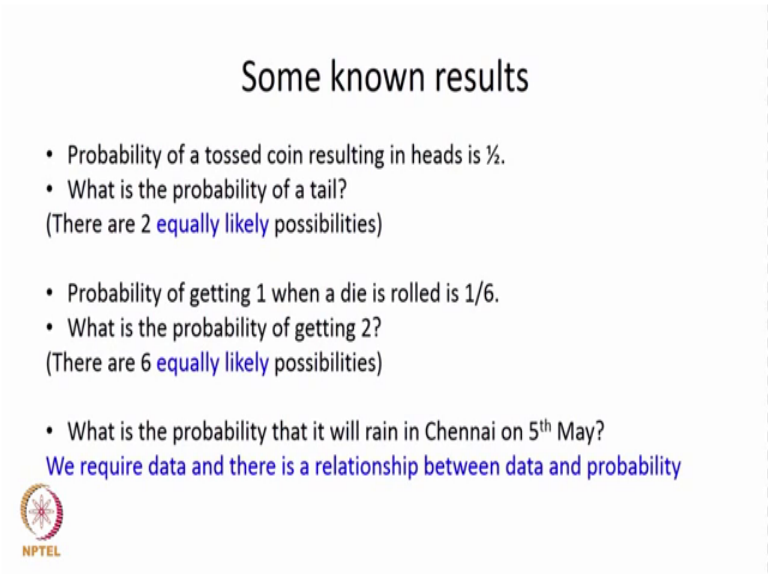

So, the answers to the first 2 things which I any way discussed just before is the head and tail there are 2 equally likely possibilities or outcomes, and therefore, each is half. When a die is rolled there are 6 equally likely outcome and possibilities.

Therefore, the probability is 1 by 6. Now if we ask the next question what is the probability that it will rain in Chennai on 5th May. We required data, otherwise it will be an opinion and we do not want to make a decision based on opinion. So, the next best thing we could do is ask for data. Now go back in the past and then find out on how many years on 5th May it has actually rained in Chennai. So, we require data and therefore, we understand that whatever result or answer that we give to this question will depend on the data that we have. And therefore, there is a relationship between data and probability.

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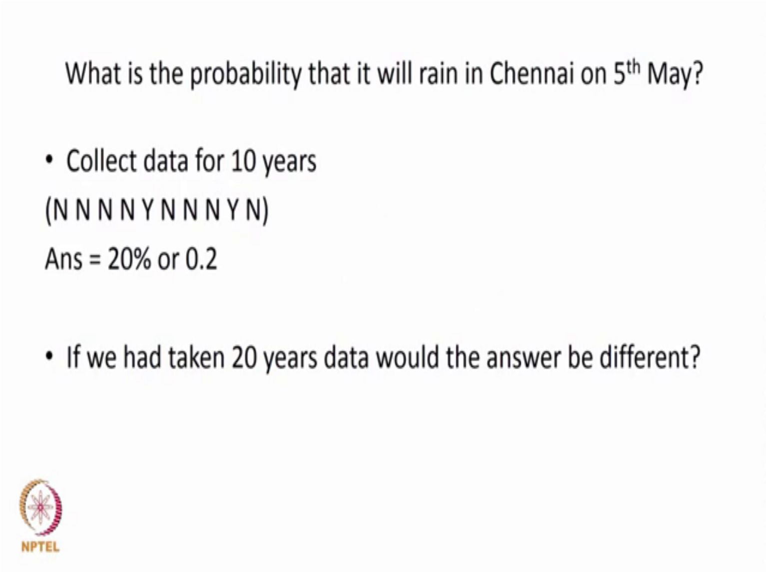

Now, what is the probability that it will rain in Chennai on 5th may. Now let us assume that we have collected data for 10 years. And I am not going to say that these are actual data, and let us assume that the data that we have with us right now which may not entirely represent the correct data, would be say n is no and say y is yes. So, if we are given these 10 pieces of data on yes and no, then we realize the 2 out of 10 there was a yes, and then we say the answer is 20 percent probability or probability of 0.2.

Now, that leads us to the next question now instead of 10 years data, if we had taken 20 years data would the answer be different; or to put it simply if we had taken 20 years data, would we have a situation where there are 4 yes out of 20, because we had 2 yes in 10. So, we need to answer that question.

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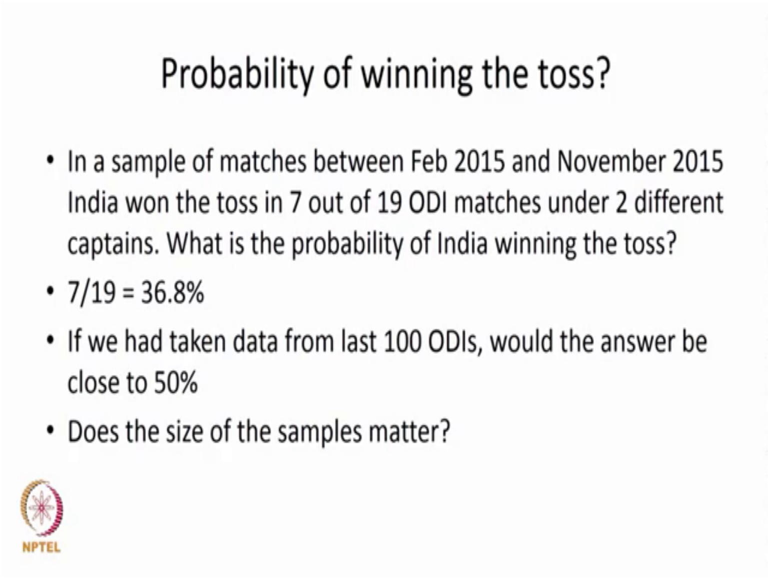

And we will do that as we move along. Now, let us look at another question, though we said the probability of getting the head is half, let us still continue the discussion on this. Probability of winning the toss, in a sample of matches say between February 2015 and November 2015; let us say the Indian captain won the toss, in 7 out of 19 matches under 2 different captains.

Now, what is a probability of India winning the toss when in a cricket match? From the data that we have we would say since India won the toss, remember winning the toss is slightly different is little more than tossing the coin, because one person tosses the coin the other person calls out whether it is head or tail. And if whatever the person has called out is correct, the person wins the toss. Otherwise the person tossing the coin wins the toss. And also remember that in these 19 matches, it is not necessary that the Indian captain had tossed in all 19 times or had called in all 19 times.

In spite of all these discussions, from the information that we have we say 7 out of 19 matches the Indian captain had won the toss and therefore, one would say that the probability of India winning the toss is 7 by 19 which is 36.80 percent. Now, let us ask another question, if we had taken data for 100 matches would the answer be close to 50 percent, because based on the discussion that we had on the heads and tails, we can extrapolate it to understand that in general probability of winning the toss is close to 50 percent.

So, if we had taken more data would the answer be close to 50 percent, and the next question is does the size of the sample actually matter.

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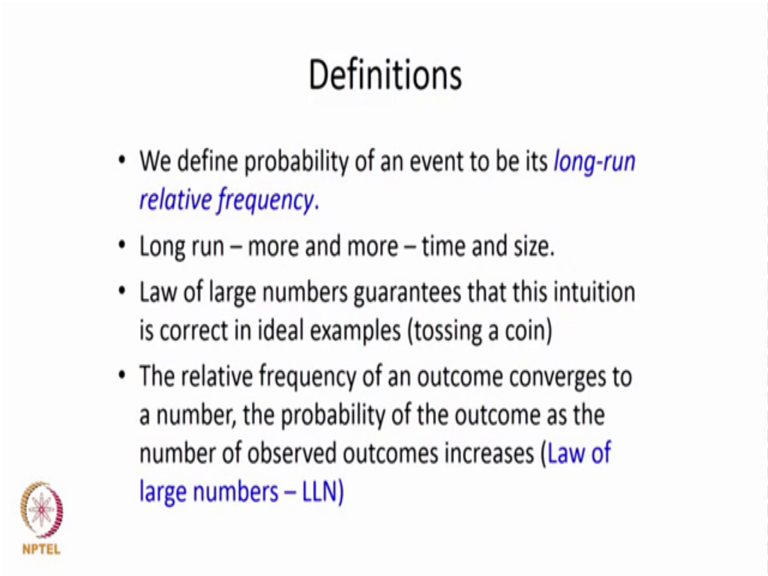

Now, we will look at some definitions and slowly try to answer some of these questions that we post. So, what is probability? Probability of an event is defined as it is long run relative frequency. So, the frequency is obviously, an occurrence so, it is long run relative frequency relative to something.

So, long run implies more and more, it also means time, it also could mean size. There is a very important concept called the law of large numbers. Now, the law of large numbers guarantees that this intuition is correct in ideal example such as tossing a coin. So, as we keep tossing a coin and keep doing these experiments more and more large number of times; we will observe that 50 percent of the times that is heads and 50 percent of the times it is tails.

So, relative frequency of an outcome converges to a number; which is the probability of the outcome as the number of observed outcomes increases is called the law of large numbers. Many times for example, if you if you take this 7 out of 19, and if 7 out of 19 is correct data, for example, we just went ahead and looked at 19 matches and it so happened that the Indian captain won the toss in 7 out of 19, and we realized that the probability of winning the toss based just on this is 36.8 percent and not 50 percent.

Whereas if we had taken 100 matches and looked at it would be much closer to 50 percent than 36.8. So, the law of large numbers is extremely important to compute and understand probability. Many times we in our discussions in our computations, we use proportions as probabilities. For example, if I had said the from this situation if I had said the Indian captain won the toss in 7 out of 19 matches.

So, the proportion of India captain winning the toss is 36.8 it is fine in most of our discussions we would even say the probability is 0.368 or 38 percent. So, proportions become probabilities under the assumption of the law of large numbers. So, whenever we substitute a proportion to a probability, we have to assume that the number of trials, the number of times we have done it is sufficiently large to make that generalization.

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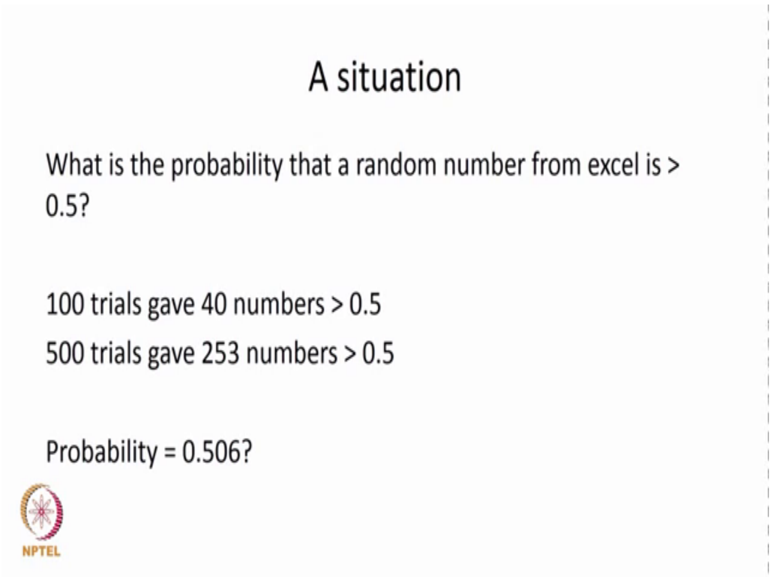

Now, let us look at continue this discussion a little more. So, what is the probability that a random number generated from say an excel sheet or from a calculator is greater than 0.5. So, we did a small experiment and said we did 100 trials and it gave 40 numbers to be greater than 0.5. But when we did 500 trials, it gave 253 numbers to be greater than 0.5.

So now, do we answer the first question saying: what is the probability that the random number is 0.5. So, actually what is the probability of getting a number greater than 0.5? Is it 0.5 or is it 0.506 and so on? But then we realized if we did 1000 trials 10,000 trials, they realize that 50 percent of the times the value is more than 0.5.

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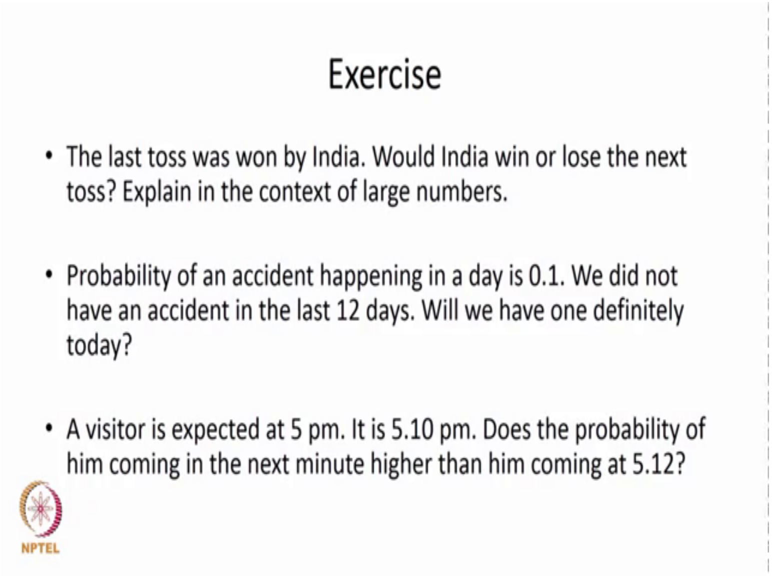

The simple exercises, the last toss was won by India by the Indian captain would the Indian captain win or lose the next toss explained in the context of large numbers. There are times we answer this question by saying oh last toss was won by the Indian captain. And therefore, it is quite likely that the person may not win the toss now so that the average becomes 0.5.

Does not happen all the time; this is a separate event or a separate thing, and the probability of winning the toss or losing the toss does not change, because the previous toss was won or lost. So, we have to understand the idea of large numbers, it is not that we have making a decision based on 2 numbers. But we have to understand the probability based on repeating the experiment a large number of times. Probability of an accident happening in a day is 0.1.

We did not have an accident in the last 12 days will we have won definitely today. If the question is will we have won definitely today, the answer is no, and we cannot say that that we will have won definitely today. The only thing we can say is, yes there is a 10 percent chance that there will be an accident today. It does not matter whether one accident happened yesterday or 2 accidents happened yesterday, it has nothing to do with it.

So, one more time we have to understand the large law of large numbers, and then give answers to these questions. A visitor is expected at 5 pm, it is right now 5 10 pm, does the probability of him coming in the next minute higher than him coming at 5 12, once again we have to look at this from the low context of large numbers, and say no, no, no, the probability of the person coming is still the same. So, these 3 different situations actually helped us understand the role of large numbers, while many times we convert proportions to probabilities based on limited sample or small number of trials. And we always have to keep in mind that the probabilities numbers are computed with a large number of trials or experiments.

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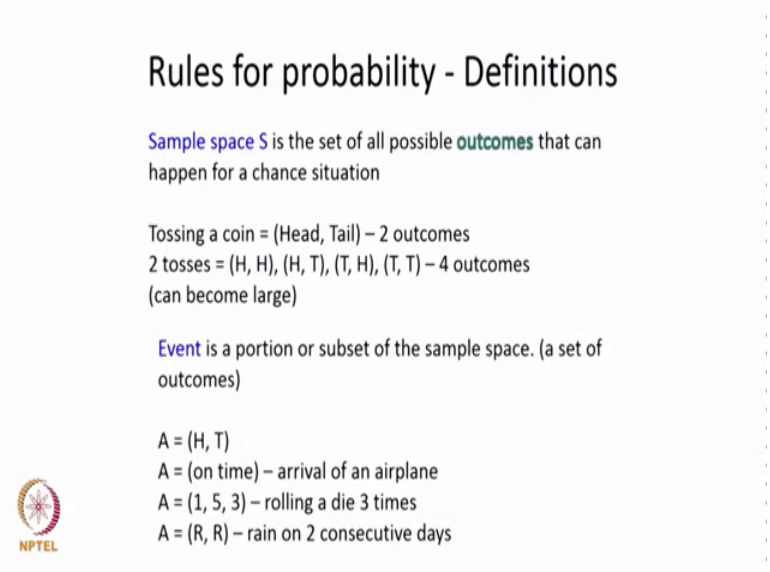

Now, we start introducing some rules and some notation and some description now what is probability, some definitions which would help us. So, the first definition is called a sample space, a sample space is a set of all possible outcomes that can happen for a situation. Sample space is the set of all possible outcomes that can happen for a given situation or a chance situation.

So, tossing a coin can have 2 outcomes, head and a tail so, 2 outcomes. So, if we do toss the coin 2 times, then it is 4 outcomes, head head, head tail, tail head and tail tail. And the number of outcomes can become very large. By definition an event is a portion or a subset of the sample space and it is a set of outcomes.

So, there can be an event, where I toss the coin 2 times, and I have head and tail. There can be an event call on time arrival of an airplane. So, there can be an event which has numbers 1 5 and 3 which were the outcomes when a die was rolled 3 times. In the second example, we could have 2 outcomes, which could be the plane arrives in time or the plane arrived late. So, arrival on time is one of the outcome.

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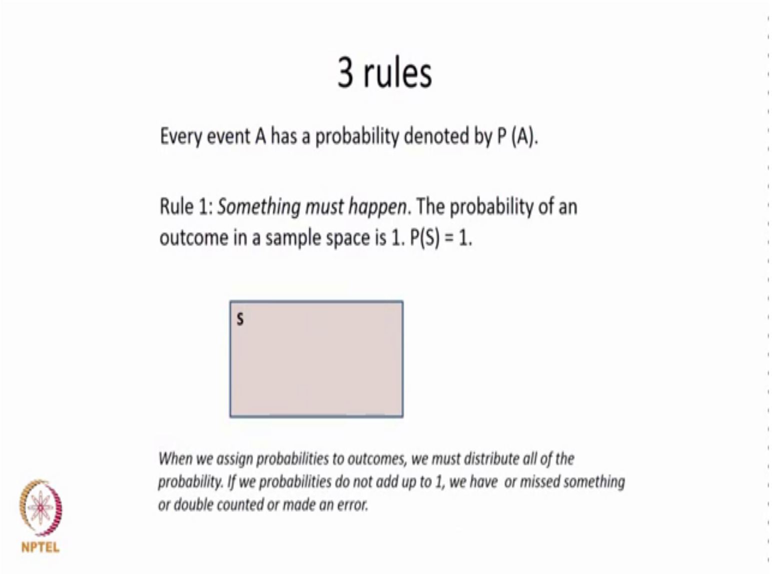

Now, R R would be rained on 2 consecutive days and so on. There are 3 important rules in probability. Now every event has a probability denoted by P A. So, the first rule is called something must happen. So, probability of an outcome in a sample space is one. When we assign probabilities to outcomes we must distribute all of the probability. So, the probabilities do not add up to 1, then it means we have missed something or we have double counted, or we have made an error.

Example, the first one is the easiest to understand head and tail, 2 outcomes each has a probability of half so, it adds up to one. 2 tosses we know quickly that head head, head tail, tail head, tail tail. There are 4 outcomes; all of them are equally likely. So, each one of them has a probability of 1 by 4 and the probabilities add up to 1 which is what is told as the first rule.

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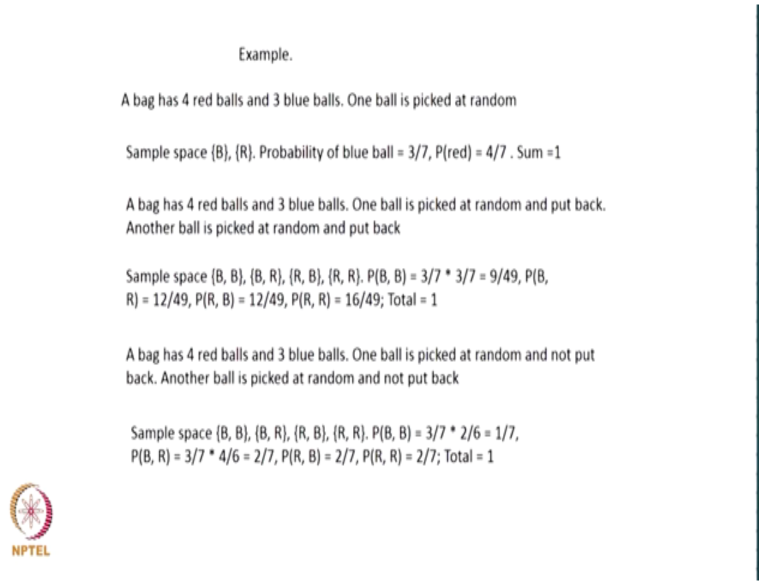

Let us look at this example a bag has 4 red balls and 3 blue balls and one ball is picked at random. So now, what happens? Since one ball is picked at random, the sample space has 2 events, and the ball that is picked is either a blue ball or a red ball. Since there are 3 blue balls out of a total of 7 probability of picking a blue ball is 3 by 7, probability of picking a red ball is 4 by 7, and the sum is equal to 1.

Now, look at the second situation, the bag has 4 red balls and 3 blue balls as before. One ball is picked at random and put back. Another ball is picked at random and again put back. So now, since we have picked 2 balls, the sample space can be blue and blue, blue and red, red and blue and red and red. Now probability of picking 2 blue balls is 3 by 7 into 3 by 7, because there are 3 blue balls out of 7, so 3 by 7.

And since the ball has been put back the probability stays at 3 by 7, so 9 by 49. Probability of blue and red is 12 by 49, first picking red and then picking blue is 12 by 49. And red and red is 16 by 49, 4 by 7 into 4 by 7, 16 by 49 if we add all these events, we get 9 plus 12 21, plus 12 33 plus 16 49 by 49 which is equal to 1.

Now, look at a third scenario, again the bag has 4 red balls and 3 blue balls. One ball is picked at random, it is not put back. Another ball is picked at random and again it is not put back. Again the sample space can be blue and blue, blue and red, red and blue and red and red. The probability of blue and blue is 3 by 7 into 2 by 6, the 2 by 6 comes, because we pick the first ball, and we assume that this ball is blue with a probability of 3 by 7. So, if this ball is a blue ball and it is not replaced or put back, then there are 2 remaining blue balls out of 6 balls that are inside and therefore, the next blue ball has a probability of picking equal to 2 by 6. Therefore, this is 3 by 7 into 2 by 6, which is 6 by 42, which is 1 by 7.

Now, blue and red will be 3 by 7 into 4 by 6, because the first ball is blue 3 by 7. It is not put back. So, there are 6 balls remaining, out of which 4 balls are red, so 4 by 6. So, again 12 by 42 which is 2 by 7. Red and blue is also 2 by 7 by the same reasoning. And red and red is also 2 by 7. And therefore, the total is 1 by 7 plus 2 by 7 plus 2 by 7 plus 2 by 7, which is equal to 1. So, we now realize that when we write the sample space as a set of all possible outcomes, and if we compute probabilities for these they add up to 1. Remaining important concepts in probability we will look at in the next lecture.