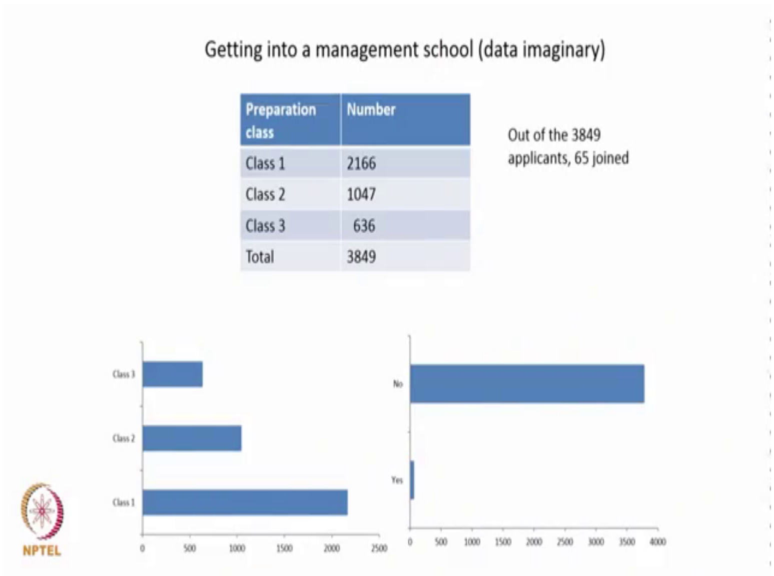
Introduction to Probability and Statistics
Prof. G. Srinivasan
Department of Management Studies
Indian Institute of Technology, Madras

Lecture – 15
Conditional Probability

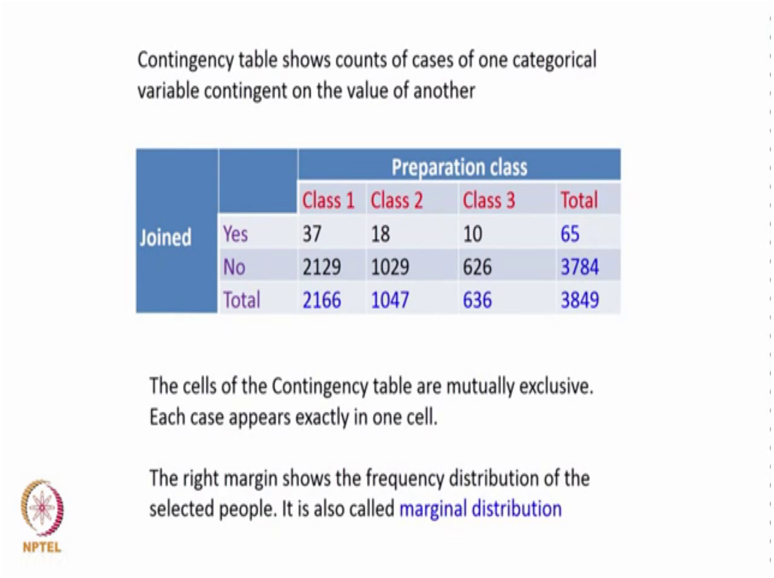
In this lecture, we discuss Conditional Probability. In an earlier lecture, when we were discussing statistics, we also looked at this in a certain way and when we looked at categorical variables, we discussed the case of admission to a program through different training classes and so on. So, we take the same example and then we try to convert the proportions into probabilities to try to understand these concepts in conditional probability.

(Refer Slide Time: 00:51)



So, we have seen this before we looked at this data of getting into a management school, imaginary data and let us assume that 3849 applicants were there and they belong to 3 preparation classes with the numbers given 2166, 1047, 636. And then we say that 65 people joined and this number is given here, those who joined and those who did not or could not join and then we also have this number of 2166, 1047 and 636.

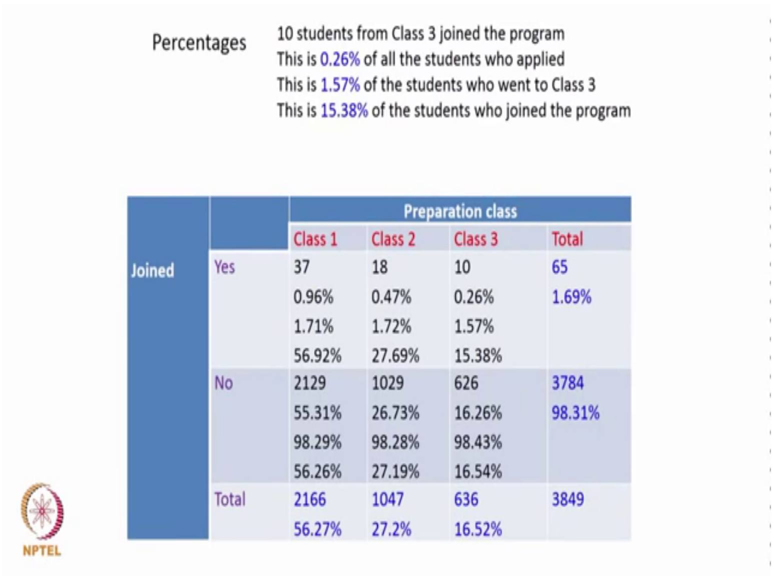
(Refer Slide Time: 01:30)



		Preparation class			
		Class 1	Class 2	Class 3	Total
Joined	Yes	37	18	10	65
	No	2129	1029	626	3784
	Total	2166	1047	636	3849

We also looked at this and the contingency table that we drew; showed the counts of cases of one categorical variable contingent on the value of an another. So, the 65 is divided to 37, 18 and 10 and the 2166 now gets distributed to 2129 and 37. So, out of 2166 who went to class 1; 2129 did not or could not join the program, while 37 dead and so on. So, we have already seen that the cells of the contingency table are mutually exclusive and each case appears exactly in one cell, we also looked at the frequency distribution of the selected people; that is called the marginal distribution we have seen this.

(Refer Slide Time: 02:14)



		Preparation class			
		Class 1	Class 2	Class 3	Total
Joined	Yes	37 0.96% 1.71% 56.92%	18 0.47% 1.72% 27.69%	10 0.26% 1.57% 15.38%	65 1.69%
	No	2129 55.31% 98.29% 56.26%	1029 26.73% 98.28% 27.19%	626 16.26% 98.43% 16.54%	3784 98.31%
	Total	2166 56.27%	1047 27.2%	636 16.52%	3849

The next thing we did when we looked at contingency tables was we then started computing these percentages. Now, what are these percentages and how are they computed. We now look at this the category under yes and then you looked at the category under class 3. So, then 10 students from class 3 joined the program, 10 out of 65 joined the program, but this 10 students out of 3849 who actually applied is 0.26 percent of the people who applied and joined went to class 3 for earlier preparation.

Now, this is 1.57 students who went to class 3. So, class 3 had a total of 636 people who went and 10 of them joined; so, 1.57 percent. So, 1.57 percent of those who went to class 3 as a preparation class joined the program and 15.38 percent of the students who joined the program. So, out of the 65 who joined, 10 went to class 3. So, 15.38 percent of the people who joined the program went to class 3. So, there we saw the proportions, now we are going to generalize these proportions as probabilities.

(Refer Slide Time: 03:46)

		Preparation class			
		Class1	Class2	class3	Total
Joined	Yes	0.01	0.005	0.003	0.02
	No	0.55	0.267	0.163	0.98
	Total	0.56	0.272	0.166	1

- Proportions become probabilities.
- The probability that a person applied and has gone to class1 is selected is 0.01

- Each of the outcomes describes two attributes. Joined/Not and the preparation class. It is the joint probability of joining and preparation
- P (Yes and class1) = 0.01
- Joint probability is a probability of intersection

So, then we start writing this; out of the people who applied 0.02 totally joined, 0.98 did not or could not join and then we represent this 0.2 as proportions of the other numbers. So, out of those who applied 0.56 came from class, 1.272 from plus 2 and 0.166 from class 3 and that gets distributed in this manner. So, proportions become probabilities; the probability that a person who is applied and gone to class 1 and got selected or joined is 0.01 which is given here. So, each of these outcomes describes two attributes joined and not join and the preparation class. So, it is the joint probability of joining the program and the preparation class. So, probability of yes and class 1 is 0.01 that is people who join the program who went to class 1 is 0.01.

(Refer Slide Time: 04:54)

A marginal probability is the probability of observing an outcome with a single attribute, regardless of its other attributes

		Preparation class			
		class1	class2	class3	Total
Joined	Yes	0.01	0.005	0.003	0.02
	No	0.55	0.267	0.163	0.98
	Total	0.56	0.272	0.166	1

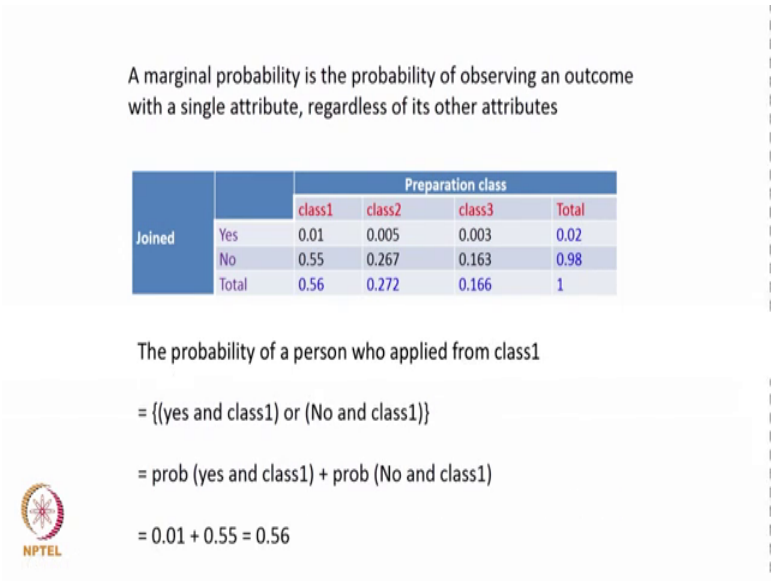
The probability of a person who applied and joined the program (next year) is 0.01 + 0.005 + 0.003 = 0.02

= {(yes and class1) or (YES and class2) or (YES and class3)}

= prob (yes and class1) + prob (YES and class2) + prob (YES and class3)

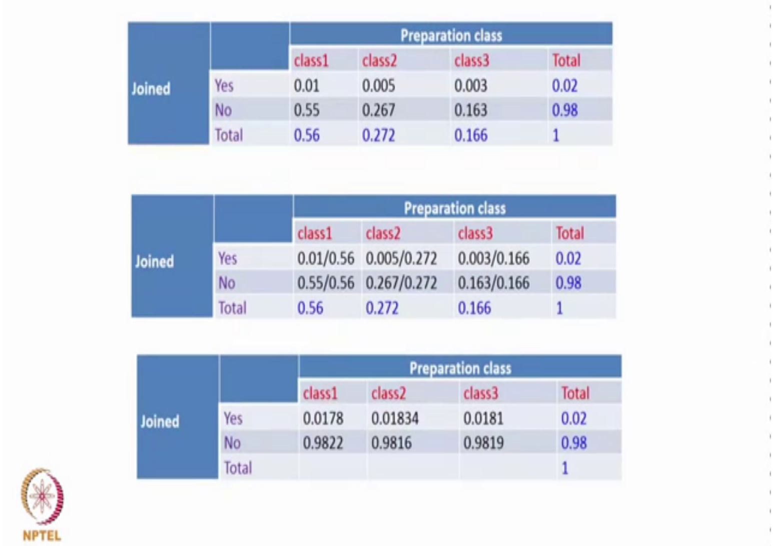
So, joint probability is the probability of intersection, marginal probability is the probability of observing an outcome with a single attribute regardless of other attributes. So, probability of a person joining the program is 0.01 plus 0.005 plus 0.003 its actually approximates to 0.02 because these are all got from the numbers like 65, 3900 and so on. So, this is probability of joining and going to class 1 or probability of joining and going to class 2 and probability or probability of joining and going to class 3 which is sum of probability of joining and class 1 plus probability of joining and class 2 plus probability of joining and class 3 which is this 0.02.

(Refer Slide Time: 05:44)



So, probability of a person applied from class 1 is this 0.56. So, of all the people who attended the interview or who applied so, that is probability of people who joined and went to class 1 and probability of people who did not or could not join and went to class 1. Therefore, it is probability of join and class 1 plus probability of not joined and class 1; 0.01 plus 0.55 which is equal to 0.56.

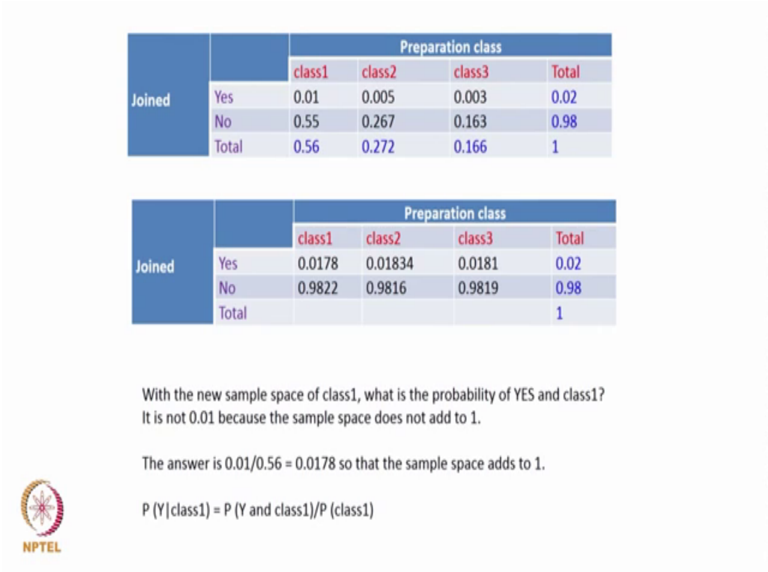
(Refer Slide Time: 06:13)



So, we show all these computations this way. So, this is 0.01; now if you say what percentage of people who of the people who came from this went there. So, it is 0.01 by

0.56 which is 0.0178 and so on. So, we get the last table where we do the divisions and say that 0.0178 comes from 0.01, 0.56. So, what is the probability that if the person going to class 1 joins the program would be 0.01 by 0.56 and so on.

(Refer Slide Time: 06:51)



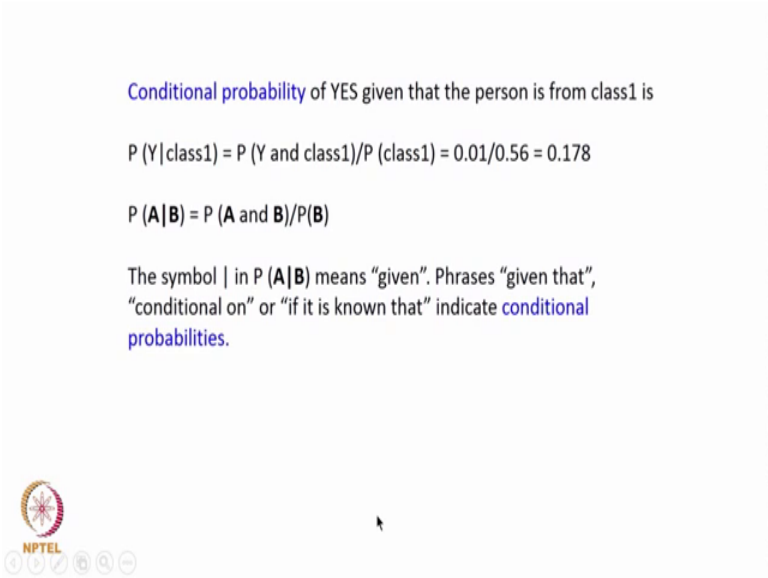
		Preparation class			
		class1	class2	class3	Total
Joined	Yes	0.01	0.005	0.003	0.02
	No	0.55	0.267	0.163	0.98
	Total	0.56	0.272	0.166	1

		Preparation class			
		class1	class2	class3	Total
Joined	Yes	0.0178	0.01834	0.0181	0.02
	No	0.9822	0.9816	0.9819	0.98
	Total				1

Now with a new sample space of class 1 what is the probability of yes and class 1, it is not 0.01 because it does not add up to one the answer is 0.01 by 0.56 which is 0.0178 so that the sample space adds up to 1 here. So, probability of joining given class 1 is probability of joining and went to class 1 divided by the probability of going to class 1.

So, probability of joining and class 1 is given by this 0.0178 which is probability of joining and going to class 1 here is 0.01 divided by 0.56. So, this is probability of joining given that they went to class 1 is probability of joining and went to class 1 divided by the probability of class 1.

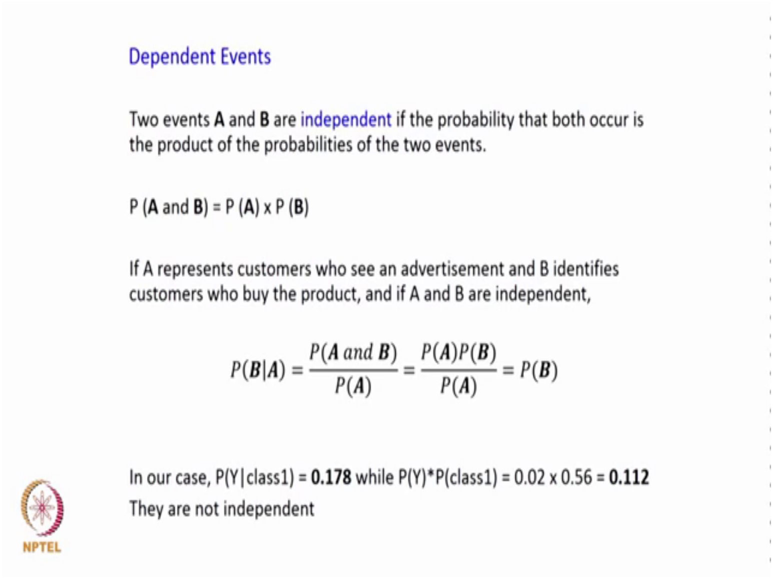
(Refer Slide Time: 07:44)



So, conditional probability of yes that is joining given the person is from class 1 is probability of joining given the person is from class 1 is equal to probability of joining and the person going to class 1 divided by the probability of person from class 1 which is 0.01 by 0.56 which is 0.178.

So, probability of A given B is equal to probability of A and B divided by probability of B this is the conditional probability equation probability of A given B this line has to be read as given B this vertical line. So, probability of A given B is equal to probability of A and B divided by probability of B the symbol verticals line symbol in a given B means given phrases given that conditional on it known that they all indicate conditional probabilities.

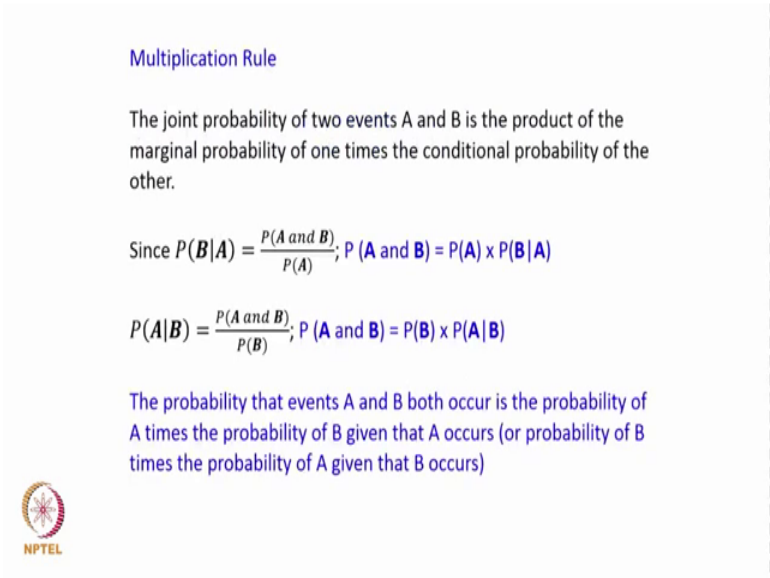
(Refer Slide Time: 08:45)



Now, dependent events two events A and B are independent if the probability that both occur is the product of the probabilities of the two events. So, we know that P of A and B is equal to P of A into P of B for independent events. For example, if A represents customers who see an advertisement and B identifies customers who buy the product probability of buying the product given seeing the advertisement this P A and B divided by P of A. But if A and B are independent then P of A and B is P of A into P of B which is therefore, P B given A is P of B it does not depend on A.

So, in our case, probability of joining given that they went to class 1 is 0.178 that we calculated while probability of joining multiplied by probability of going to class 1 is 0.02 into 0.56 which is 0.112 and therefore, we can say that these two are not independent and there is a dependency which is that. So, there is another way of checking whether events are dependent or independent using conditional probabilities.

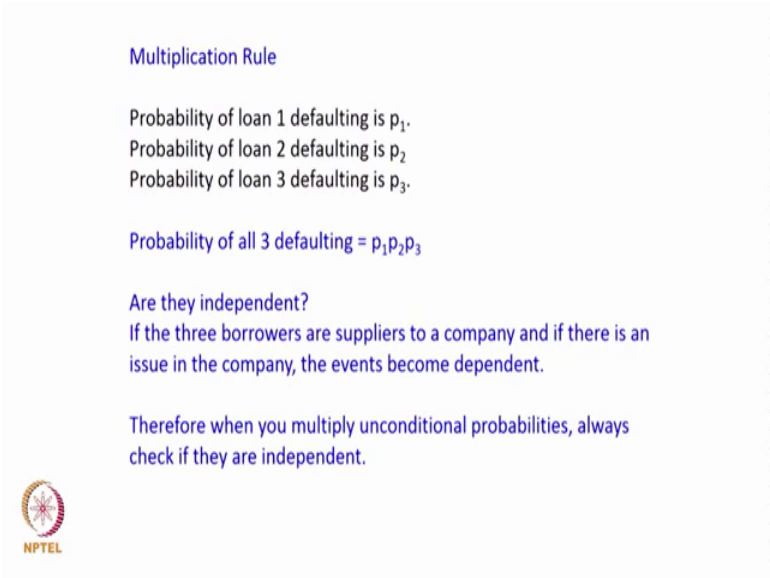
(Refer Slide Time: 10:03)



Now, let us look at this multiplication rule joint probability of two events; A and B is the product of the marginal probability of 1 times the conditional probability of another. So, since probability of B given A is probability of A and B divided by probability of a probability of A and B is equal to probability of a multiplied by probability of B given A is called the multiplication rule. So, joint probability of two events A and B P of A and B is the product of the marginal probability of one which is P of A multiplied by the conditional probability of the other P of B given A.

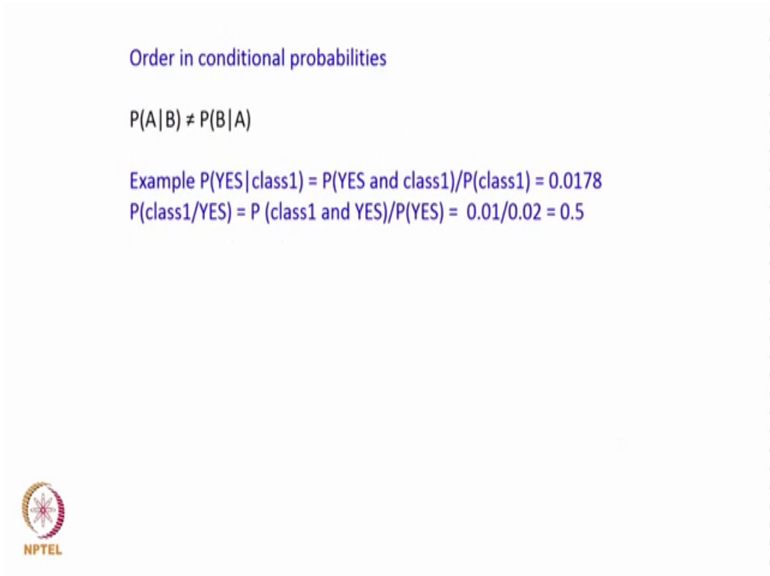
Now, P of A given B is P of A and B divided by P of B therefore, P of A and B is equal to P of B into P of A given B remember, now P of A and B is equal to P of A multiplied by P given A, it is also equal to P of B multiplied by P of A given B. So, the probability that events A and B both occur which is A and B is the probability of A times the probability of B given A or probability of B times probability of a given B occur. So, both are valid now let us look at another.

(Refer Slide Time: 11:28)



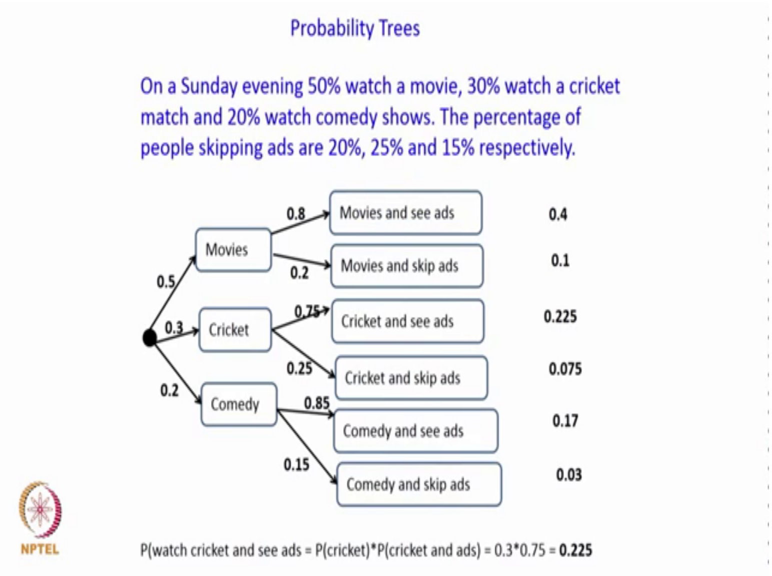
Question probability of loan one defaulting is p 1 probability of loan to defaulting is p 2 and probability of loan 3 defaulting is P 3. So, probability of all defaulting is p 1, p 2, p 3 are they independent if they are borrowers if the 3 borrowers are suppliers to the same company and because of some issue it is defaulting, then there is a dependency. Therefore, when we multiply unconditional probabilities check always whether they are independent. So, only when they are independent, we can do this multiplication rule.

(Refer Slide Time: 12:08)



Now order in conditional probabilities its very very important to know this P of A given B is not equal to P of B given A. So, example probability of joining given class 1 is probability of joining and from class 1 divided by class 1 which was 0.0178 probability of class 1 given joining is probability of class 1 and joining divided by probability of joining which happened to be 0.5. So, P of A given B is not equal to P of B given A.

(Refer Slide Time: 12:42)

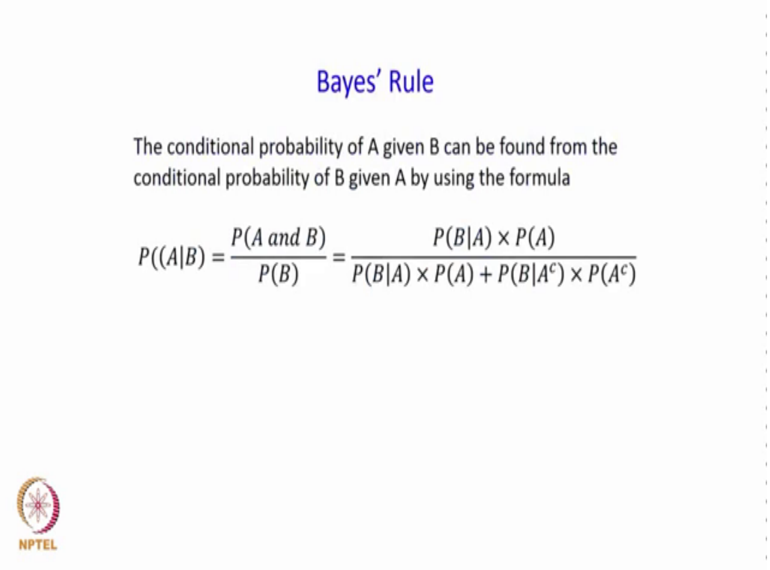


We now look at probability trees and try to solve some problems using probability trees. In fact, indirectly we have seen this in one of the examples where we looked at the batsman having to score 2 runs to win a match with 1 ball remaining. And we looked at a tree like solution where if the person went for the clean hit there is a probability and then when the person does not there is another probability and then there is an outcome and so on.

The example where there is a 30 percent if they goes for a clean hit and 50 percent, if tie and another 50 percent of winning. Now we look at another example on a Sunday evening 50 percent of the people watch movies, 30 percent watch a cricket match and 20 percent watch comedy shows. The percentage of people skipping adds are 20 percent, 25 percent and 15 percent respectively. Now what happens? Now we have this probability tree. So, 0.5 movies, 0.3 cricket, 0.2 comedy; now within this percentage of people, skipping ads are 20 percent if they watch a movie.

So, people who watch a movie and see ads is 0.8 people who watch a movie and skip adds 0.2. Similarly, 0.75, 0.25 and 0.85, 0.15; so, we now realize that people who watch movies and see ads are 0.5 into 0.8 which is 0.4; the other one is 0.5 into 0.2 which is 0.1 and so on. Therefore, people who watch cricket and see ads is probability of people watching cricket multiplied by people watching cricket and seeing ads. So, this is 0.3 into 0.75 which is 0.225 and so on.

(Refer Slide Time: 14:35)



We also have this very very important result called the Bayes' rule conditional probability of A given B can be found from the conditional probability of B given A by using this formula. So, probability of A given B is probability of A and B divided by P of B. So, this is B given A into P of A divided by probability of B given A into P of A plus probability of B given A complement into probability of A complement. So, now, B is divided into B is expanded into B given a into P of A plus B given A complement into P of A complement P of A and B is expanded by the original multiplication rule formula; so, B of A into P of A.

So, we could use the Bayes' rule and try to solve some problems. So, with this, we come to the end on this lecture and we will look at discussion questions on this lecture and continue our discussion on probability in the next lecture.