Algebra - I
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ALGEBRA I

1. Lecture 78: Structure of finitely generated modules over a PID

Let R be a principal ideal domain and suppose M is the finitely generated R-module . So, we can assume that M is generated by some finite set of elements $x_1, \ldots, x_m$ for some positive integer m. Define an R-module homomorphism $\phi : R^m \to M$ by $\phi((a_1, \ldots, a_m)) = \sum_{i=1}^{m} a_i x_i$. $\ker(\phi)$ is a sub module of M and (since R is a principal ideal domain; which also a noetherian ring) therefore finitely generated. So, let us say it is generated by $v_1, \ldots, v_n$. Form the matrix with the columns are $v_1, \ldots, v_n$. So $M \equiv R^m / \ker(\phi)$.

Theorem 1.1 (Structure theorem for finitely generated modules)**.** *Every finitely generated R-module M may be expressed as*

$$M \equiv R^s \oplus R/(d_1) \oplus \ldots \oplus R/(d_r),$$

where $d_1, \ldots, d_r$ can be chosen in such a way that $(d_1) \supseteq \ldots \supseteq (d_r)$, and this decomposition is unique.