Our Mathematical Senses

The Geometry Vision

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Lecture - 16

Video 4A: Incidence relations

I want to take a closer look at incidence relations. So our goal this week is to build a geometry that's invariant under changes in perspective. Now what is a geometry? What does it mean to build a geometry? Well geometry can include many different properties of interest. It can include study of length, angle, area, curvature, congruence, parallelism, similarity. So these are all words and properties that you may have studied under the name geometry. But we've already seen that when we start shifting perspective all of these like length, angle, area, and all the rest go haywire.

So somehow our geometry has to be based on something more fundamental. But we've also seen that lines are taken to lines, points are taken to points, and intersections are taken to intersections. So we have something to work with, namely the incidence relations. So we introduced those in the intro video, but maybe those could be our basic building blocks of this geometry.

Now incidence is just a technical way of saying that two objects meet somewhere. In other words, they share one or more points. For example, in 3D, a point is incident to a line or to a plane if it's contained in that line or that plane. Two lines are incident if they intersect or if they're the same line, in which case they share infinitely many points. And a line and a plane are going to be incident if the line is contained within the plane, they'll also share infinitely many points then, or if the line passes through the plane, if it pierces the plane, in which case they'll share a single point.

So since incidence relations are preserved under perspective shifts, we could try building a geometry of incidence based entirely on incidence relations. So we'll see what we mean by that as we build it. But first let's take a moment to review some basic statements of incidence in the Euclidean plane R2. So here's a statement of incidence.

Any two distinct points are incident with exactly one common line.

Any two distinct points in R2 are incident with exactly one common line. They determine a unique line. And that's true in R2. Any two points determine a unique line. So this is a true statement of incidence involving points, lines, and incidence relations.

Here's another statement of incidence. Any two distinct lines are incident with exactly one common point. So we're in R2, the Euclidean plane. Any two lines in R2 are incident with exactly one common point. They share exactly one common point.

They intersect in one common point. So this is a statement of incidence in R2. Any two lines share a common point. Unfortunately, this statement of incidence is false. This one is true, but this one is false.

So, let's see. The exactly one is the problem because we know that parallel lines exist in R2. There are distinct lines which don't share a common point. So really to make this a true statement of incidence, we better change it to saying at most one common point, which also reveals something about points and lines. They behave a little differently from one another in the geometry of R2.