

Our Mathematical Senses

The Geometry Vision

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Lecture-43

Video 9B: the criss cross construction

Okay, we're now going to go over a construction known as the crisscross construction, which is going to come in really handy for what we want to do next. So the next step is Pappus' theorem, and we're going to use the fundamental theorem of projective geometry, which we just proved, to prove Pappus' theorem. But before we do that, we're going to need a little construction, which will be key to the proof. So I want to examine that first and then get to the proof of Pappus' theorem. So the construction, the crisscross construction, is as follows. Let's take lines capital L and little l, which are coplanar.

So they're in a P2, not in a P3. They're coplanar, they share a plane. And let's look at another construction, sending capital A, capital B, and capital C to little a, little b, and little c. We've already seen a construction a few lectures back, which in fact worked even if these lines were not coplanar.

Now let's assume they're coplanar and see a more specialized construction known as the crisscross construction. So I want to consider the cross joints, ab intersect ab, capital A, little b intersect capital B, little a, and the cross joint, capital B, little c, intersect little b, capital C. So by cross joint, I just made up that word, but it just means these two intersections, these two points there. So this is one cross joint and this is the other one. And let's let mb be the line connecting these two points.

Now I want to consider a perspective now, f, little b from the line L to the line mb. And it's just perspective centered at little b from this line to this line. And what is this perspective going to do? Well it's going to take a, b, and c, remember it's centered here at b, so it's going to pull them into this line, intermediate line mb. And it's going to pull them into these three points. Now I want to consider a second perspective centered at capital B, fb from mb to L.

What is this perspective going to do to these same three points? Well it's going to push them out. It's going to push them out to little a, little b, and little c. So the projectivity γ_b , which I'm defining to be f , little b, followed by f , capital B, that's effect, that's, we've just done that. We've just seen that in action and it sent a, b, and c to little a, little b, and little c. So this is our construction.

This is another projectivity which takes these three arbitrary points to these three arbitrary points, but it did rely on these lines being coplanar. Now it's nice, it's a very elegant construction, it's kind of very slick. But there's something even slicker about it, which is that we actually, we centered it at a b, but there's a complete symmetry here between the points a, b, and c. There's no reason that we had to center our perspectives at capital B and little b. We could have just as well done the exact same construction, but centered them at capital C and little c, or centered them at capital A and little a.

So actually we could have chosen any of these. And so we could have centered our perspectives at a and little a. So for example, we would have in that case chosen all of the cross-joins involving a, so one here and one here. And then we would have formed our line ma, connecting them. And then we would have looked at the projectivity f little a, followed by f capital A.

So what would those do? Well f little a would pull these points in from capital L to ma. So it would pull a, b, and c to this point, this point, and this point. And then f capital A would push them down to little a, little b, and little c. So we would have gotten another projectivity mapping capital A, capital B, and capital C to little a, little b, and little c.