Our Mathematical Senses

The Geometry Vision

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Lecture-65

Conclusion to Geometry of Vision

How to OPEN the HHS deductions total templates We started the course with a simple question about two-dimensional images of a three-dimensional space, namely, what changes and what stays the same as we shift perspective. And now we have some answers. More specifically, the study of PGL3R gives an answer to the question, what changes under a projectivity? There are eight separate quantities that change. In particular, there are eight dimensions worth of projectivities, which our brains are constantly applying as we interpret visual data. On the other hand, our study of the cross ratio gives an answer to the question, what stays the same under a projectivity? If two images are related by a projectivity, then the cross ratio of any four collinear points will remain constant. As our journey comes to a close, we can also appreciate how projective geometry sits at a rich crossroads, where visual intuition meets foundational ideas in mathematics.

For example, perspective drawing suggested that we extend the Euclidean plane to include points at infinity. But in order to work analytically with this extended plane, it had to be reborn as RP2, the real projective plane, a space that is both intuitive but also really, really strange. The real projective plane is an example of a two-dimensional manifold. Other two-dimensional manifolds include the sphere, the torus, or just the regular Euclidean plane.

But unlike those examples, the real projective plane can't be fully embedded in three-dimensional space, making it one of the simplest manifolds that we can't fully visualize. However, pieces of the real projective plane can be visualized. One way is to construct a Möbius strip. If you haven't seen it before, it's just a strip of paper with a half-twist, whose ends are glued together. But the special thing about it is it has just one side, thanks to the half-twist.

Which is why cutting it down the middle doesn't actually split it into two pieces. If you haven't done this before, you should definitely give it a try. And for a little extra fun, you could try cutting it in half a second time. For right now, the most important property of the Möbius strip is that it has just one edge. Its boundary consists of a single circle.

Can you think of any other surfaces whose boundary consists of a single circle? How about this one? The circular disk. Now, believe it or not, if you were to sew the boundary circle of this disk to the boundary circle of this Möbius strip, you'd get the real projective plane. Unfortunately, you'd need a fourth dimension to complete your sewing, which we simply don't have at our disposal. Yet, we can train ourselves to imagine how that shape may behave. The real projective plane is just one of the mathematical objects we meet by taking a closer, more careful look at the geometry of our vision.

So as our journey comes to a close, do take a scroll down the companion page to see some of the other directions for further exploration, which we didn't have a chance to cover in this course. And let's not forget the many others that made this journey possible. It's goodbye for now, but I'm sure we'll meet again soon.