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Nonlinear Programming

Lec-11
Geometric Programming-I

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Hello friends, so welcome to lecture series on nonlinear programming. So we will start a new topic geometric programming. So we will see what geometric programming are and how can we solve some nonlinear problems based on this.

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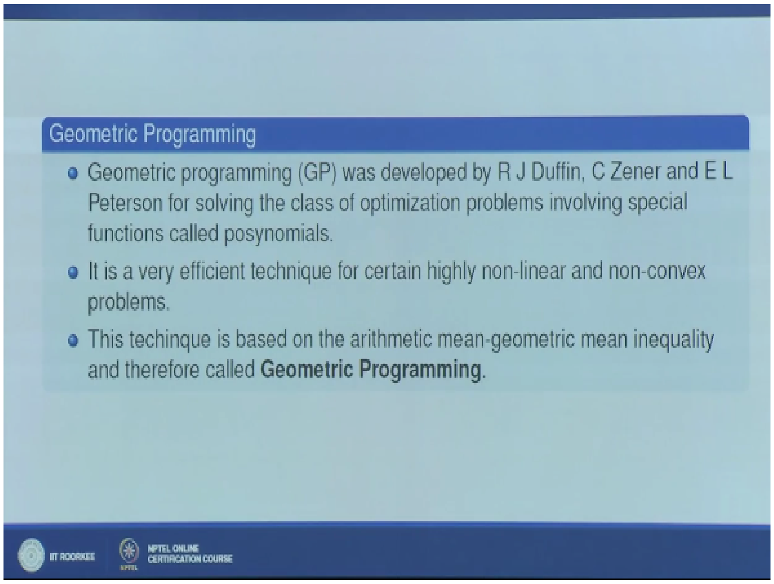

So geometric programming was developed by Duffin, Zener and Peterson for solving the class of optimization problems involving spatial function called posynomials. It is very efficient technique for certain highly non-linear and non-convex problems. If we have a non-linear problem which is a complicated one, or a non-convex problem geometric programming is efficient.

This technique is based on arithmetic and geometric mean inequality and therefore we call it geometric programming. We already know arithmetic mean and geometric mean inequality that is, arithmetic mean is always greater than equal to geometric mean, we will use this inequality while we develop the algorithm and therefore, this problems are sometimes called geometric programming problems. Now what is the theory, let us discuss the theory basically.

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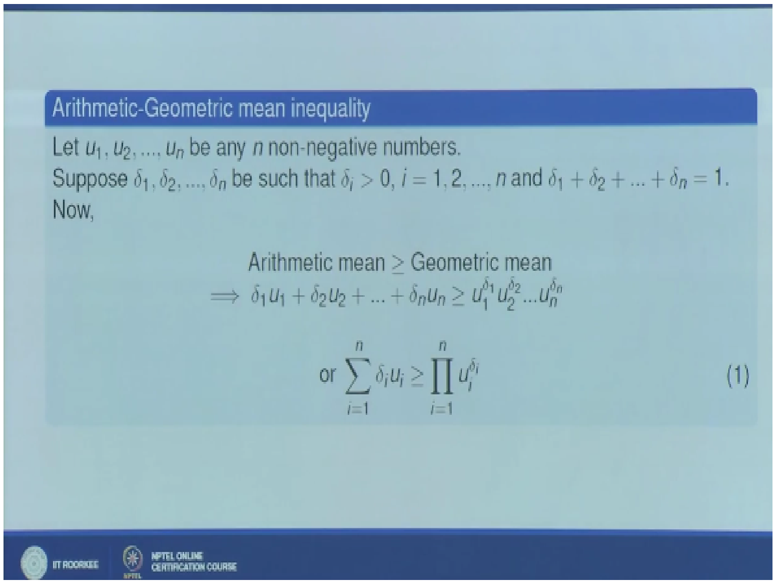

Now suppose u_1, u_2 up to un be any n non-negative numbers okay. So we have u_1, u_2, u_3 and so on up to u_n these are n non-negative numbers okay.

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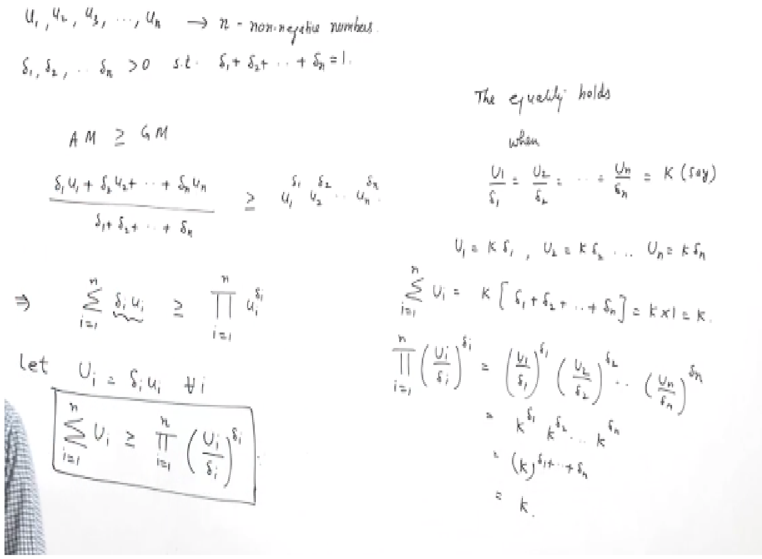

Now suppose we have ∂_1, ∂_2 up to ∂_n greater than 0 okay, such that sum of ∂ is 1 okay. We have ∂_1, ∂_2 up to ∂n which are strictly greater than 0 such that sum of ∂ is 1 as we have already discussed in the slide okay. So we know that the arithmetic mean is always greater than equal to GM okay. Now we can say these are the weights okay. So this into this that is $\partial_1 u_1 + \partial_2 u_2$ and so on ∂nun upon some of these things sum of $\partial_1 + \partial_2 +$ and so on up to ∂_n that is the arithmetic mean okay.

Arithmetic mean of these numbers is greater than equals to $u_1^{\partial 1}$, $u_2^{\partial 2}$ and so on $u_n^{\partial n}$ this is the geometric mean of these numbers okay. This you already know, this is by the inequality arithmetic mean greater than equal to geometric mean. Now sum of ∂ is 1, we are already assuming that sum of ∂ is 1, so we can say that sum of ∂_i u_i, i varying from 1 to n is greater than equal to product of i from 1 to n u_i ∂_i, so this can be easily obtained from this expression.

Now if you take say if you take $\partial i=1/n$ for all i, if we take suppose we take $\partial i=1/n$ so what is the sum of all ∂I $\delta 1 + \delta 2 + \delta 3 \ldots . \delta n$ is nothing but n/n which is 1, so this then this condition holds, okay. So for this δi we can apply we can use this inequality so we can obtain if we put if we substitute this δi in this expression so we will obtain Σ I from 1 to n, ui/n ≥ product of I from 1 to n, $ui^{1/n}$ so that is simple arithmetic geometric inequality.

You see what is this, this is simply u1+ u2 ……..un/n ≥ $(u1, u2 \ldots \ldots . un)^{1/n}$ so that is the sum of numbers arithmetic mean of this numbers that is sum of the number divided by number these are n in numbers that is $(n \geq u1, u2 \ldots un)^{1/n}$ which we call as geometric mean of n non negative numbers, okay. So these inequalities we can easily obtain from this inequality now let us assume that this term δi ui as some capital UI suppose, okay.

So let capital UI is something δi ui this we are assuming, okay. So what we will obtain? If we substitute this over here that is of course for all I so if you substitute this over here what we obtain, the summation I running from 1 to n capital UI ≥ product of I from 1 to n and what is the small ui from here, it is capital UI/ δi so we substitute it over here what we obtain capital (UI/ $\delta i)^{\delta i}$.

So this is an inequality which we obtain from the arithmetic geometric mean inequality, okay. So this inequality we will use while we solve some geometric programming problems, okay. Now first we will see what geometric programming problems are. So this we have already discussed and this also we have discussed yes the equality this equality will hold when all are equal. The equality holds when U1/ $\delta 1$ = U2/ $\delta 2$ = un/ δn.

These you can easily check you can take it say K, okay. If it is K then the left hand side will be nothing but what will be U1, U1 is K $\delta 1$, what is u2, u2 is k $\delta 2$ and similarly what will be unnecessary, it is k δn so what will be left hand side, left hand side is summation of UI, I running from 1 to n, so it is k times k will be common from all the UI's.

Then it will be nothing but $\delta 1 + \delta 2$ ….. δn and it is 1,so it is k x 1 which is k. Now if you see the right hand side of this expression then it is product of I from 1 to n, (ui/ $\delta i)^{\delta i}$ which is equal to u1/ $\delta 1$ which is k, u2/ $\delta 2$ which is again k that means it is nothing but (u1/ $\delta 1)^{\delta 1}$,(u2/ $\delta 2)^{\delta 2}$ and so on (un/ $\delta n)^{\delta n}$. So it is = u1 upon δ1 is k, k raise to part δ1 $k^{\delta 2}$ and so on kδn which is kδ1 + δ2 up to δn and sum of δ is 1 so it is k so this is also k and this is also k so equality holds when u1 upon δ1 is = u2 upon δ2 = u1 . δ this we can esily say now let us come to the problems geometric problems, now if we have this type of first we are discussing unconstraint problems without any constraint minimization of some function fx which is fx is ∑j running from 1 to n cj ujx.

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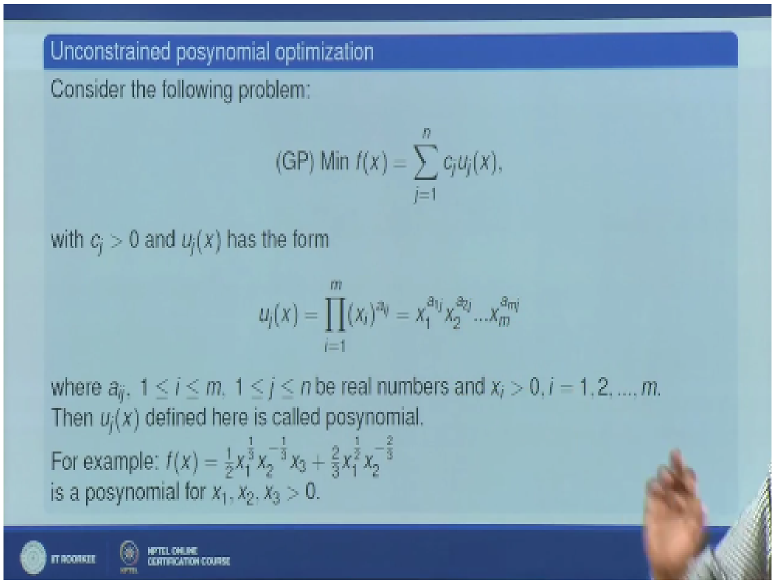

Okay cj is all coefficients are there assuming a restrict > 0 and uj has the form of product i from 1 to m xi^{aij} so if you simplify this then it is $x1^{a1j}$ x^{a2j} and so on x1 power these aij may be in fractions these are any real numbers may be fractions may be negative okay, and x size are we are assuming xi as significantly 0 i from 1 to m, now this type of expressions this type of expressions we call as posynomials okay this is what try from polynomials so we are some we are calling it as posynomials.

Because this may take fraction power also may be negative power okay so we are calling it as posynomials for example we have this type of expression function fx is imposed to 1 / 2 $x1^{1/3}$ $x2^{-1/3}$ it is a posynomial into x3 it is again a posynomial and some of two posynomials is again a posynomial for x1 x2 x3 strictly > 0 okay. So first type of problems where we have it is a non linear problem of course because non negative is there in the variables x1 x2 and x3 okay, so we have a non linearity involved in the problems so to solve such type of problems such type of non linear problems we use the concept of geometric programming problems okay.

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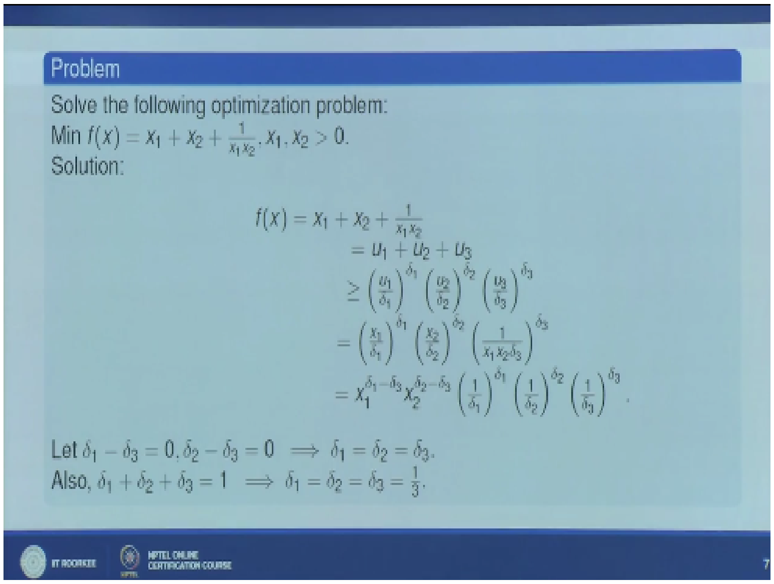

Now let us understand how to solve the problems of such type by the example okay, so let us discuss an example those things will be clear let us discuss simple problem first okay materials minimization.

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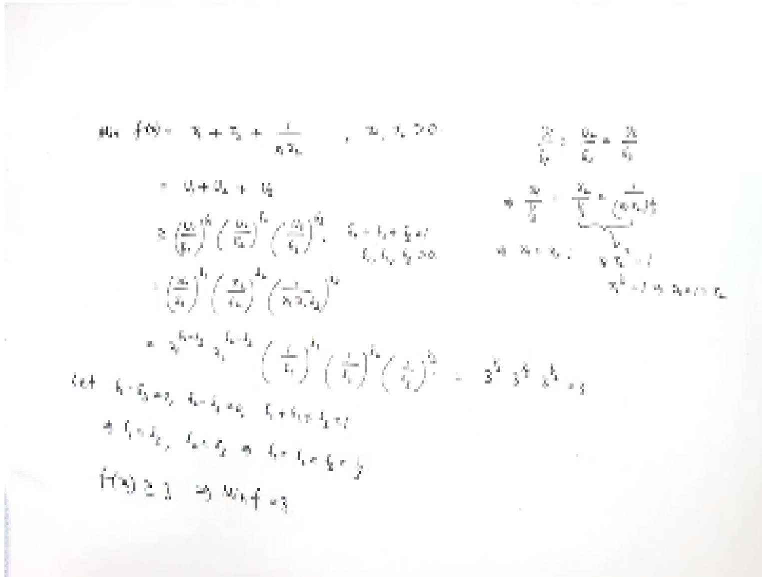

Of fx = x1 + x2 + 1 x1 x2 where x1 x2 is strictly written 0 now how to solve this problem of geometric programming approach now let us called this as u1 + u2 + u3 where u1 is the first term x1 u2 is second term x2 and u3 is a third term which is upon x1 x2 okay our aim is to find out the optimal solution of this problem okay, now we have just add it that sum of uy is > = we have already started sum of i from 1 to end ui is always > = product i from 1 to n $(u_i)^{\delta i}$ $u_i/\delta_i)$ $^{\delta i}$ where sum of δ_i is 1 i varying from 1 to n and all δ_i are strictly get to the 0 this in equality we have seen, so we will try to apply this inequality in this expression so u_1 $+u_2+u_3$ that is the sum of u_is $\geq (u_1/\ \delta_1)^{\delta 1}\ (u_2/\ \delta_2)^{\delta 2}\ (u_3/\ \delta_3)^{\delta 3}$ such that $\delta_1+\ \delta_2+\ \delta_3=1$ and δ_1, δ_2, δ_3 are strictly greater than 0 by this condition okay.

Now what is u_1, u_1 is nothing but x1 so $(x_1/\delta_1)^{\delta 1}$, $(x_2/\delta_2)^{\delta 2}$ and it is $(1/x_1x_2\ \delta_3)^{\delta 3}$ so it is equal to $x1^{\delta 1-\delta 3}$,$x2^{\delta 2-\delta 3}$ and next remaining terms are $(1/\delta 1)^{\delta 1}$ $(1/\delta 2)^{\delta 2}$ $(1/\delta 3)^{\delta 3}$ okay, now we want to minimize fx, fx is nothing but sum of u1+u2+u3 okay, we want to minimize it. In order to minimize this we are getting an inequality fx is greater than equal to something, so if we are getting an maximum value of this expression so that will give a lower bond of fx okay.

You see fx is greater than equals to this expression okay, so to find out the minimum value of this f we have to find out the maximum value of this expression. Now in order to find out the maximum value of this we first make it free from the variables because the value of the variables are not known we first make it, we first choose δ is such that it will become free from the variables, okay.

If variables are involved we are unable to maximize this expression that is why we first make it free from the variables, now how to make free from the variables you can choose δ_1-δ_3=0, δ_2-δ_3=0 so that it will free from the variables okay, so let δ_1-δ_3=0 and δ_2-δ_3=0 and we know that δ_1+δ_2+δ_3 is 1 so here we obtain δ_1=δ_3 and δ_2=δ_3 this means δ_1=δ_2=δ_3 and sum is one so this means is equal to 1/3.

Basically we have three unknowns with three equations so we are getting a unique solution of the right hand side also which is 6 okay, so what we obtain basically now this is this values we obtain now the maximum value of the right hand side is fixed because δ is fix δ_1 δ_2 δ_3 all are 1/3 when you substitute it over here where getting a fix value of the right hand side, that means what is the fix value a fix value will be you see x_1 x_2 power to 0 is one actually x_2 power 0 is one 1/ δ_1, δ_1 is 1/3 that is 3 1/3 x $_3$1/3 x $_3$1/3 which is 3 that is fx ≥ 3.

That means the lower bound or the minimum value of fx 3 now the question is at which point what is the optimal solution what is the optimal values of s1 s_2 at which value of f_x is 3, you can simply said at minimum of fx_3 now the question is how we can find out the values of x_1 x_2 such that minimum value of fs_3 so for that we already know this equality will hold when all are equal. So the quality will hold when μ_1 / δ_1 = u/ δ_2 = u_2 / δ_3 this we will use to find out because this is an equality where we other minimum value.

So this equality will be obtain this is an equality this equality we are using here this equality we will obtain when all are equal when these all are equal, so this will use to find out the value of x_1 and x_2, so what us u_1, u_1 is x_1 and δ_1 is 1 /3 what is u_2, u_2 is x_2 again this is 1/3 and u_3 is 1/ x_1x_2 and it is 1/3. So this implies x1 = x_2 and when you take these two expression form these two expressions what we obtain we obtain x_1 x x_2^2 = 1 that is x_1 q = 1 implies x_1 = 1 which is = x_2.

So that means x_1x_2 = 1 and these are the optimal solution of the problem okay. So when x_1 and x_2 both are one then 1 + 1+ 1 is 3 which the minimum value of f, okay so in this way if we have some problems if we have problem involving the negative or the fraction of variables we can solve a simple using arithmetic geometric mean in equality. So let us discuses one more problem based on this, this is a simple illustration of same thing.

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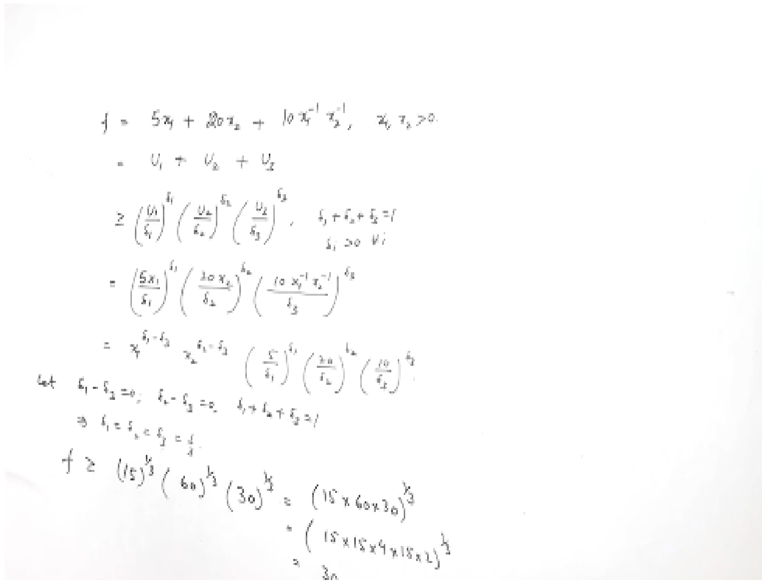

Let us discuss about more problem based on this the problem is minimizing of f which is equals to 5 x_1 + 20 x_2 + 10 s_1 s_2 $^{-1}$ x_2 $^{-1}$ and $x_1x_2 > 0$, now it is same type of problem let us see how to solve it again it is say it is equals to $u_1 + u_2 + u_3$ okay we have to find this is f and this is equal to 0 we have to find the minimum value of f okay. Now what is u_1, u_1 is the first term $5x_1$ u_2 is the second term $20x_2$ and u3 a third term 10 $x_1x_2^{-1}$ x_1x_2 $^{-1}$ now we will apply this, the same in equality that is this is greater than or equal to u_1 / $\delta_1^{\delta 1}$ u_2 / δ_2 $^{\delta 2}$ u_3/ δ_3 $^{\delta 3}$, such that $\delta_1 + \delta_2 + \delta_3$ and all Δ is greater than 0 okay now this is equal to what is u1 u1 is $5x_1$ /Δ_1^2 what is u_2 $20x_2$/ $\Delta 2^2$ what is u_3 it is $10x_1$ x_2-1 / Δ_3^3 it is further equal to x_1 x_2^2 you collect that power of x_1 x_2 so x_1 $\Delta 1$ and it is $-\Delta 3$ x_2^2- Δ_3 and the remaining terms are $\Delta 1$ 20/Δ_2^2 and 10/Δ_3^3.

Now again in order to find out the minimum value of this f we have to find out the maximum value of this expression and for that we have to first make it free from the variables and to make it free from variables we have to choose those Δ_i such that powers will become 0 powers of variable will become 0 will suppose Δ_1-Δ_3=0 and Δ_2-Δ_3=0 and also some of Δ is 1 so from here we obtained $\Delta_1=\Delta_2=\Delta_3=1/3$ we have a unique solution.

Because we are having three equations with three unknowns so we are having a unique solution so we affix value of right hand side what are the values in right hand side that we can obtain that is f will be equal to this x_1^2 x_2^2 is 0,1 five upon $\Delta 1$ is 1/3 that is 15 that is 1/3* 20*1/3 and 10 1/3 that is 30 1/3 so this is nothing but this is 15*60*30 1/3 so that we compute it is 15*15*4*15*2 that is 15*2 that is 30.

So this is the minimum value of this f, f is greater to 30 that means minimum value of f is 30 now for which again for finding out the values of variables at which x1 x2 this is 30 for that we will apply that equality holds and u1 upon $\Delta 1$=u2 upon $\Delta 2$= u3 upon $\Delta 3$ so what are the values for x1 x2 x3 and u1 upon $\Delta 1$=u2 upon $\Delta 2$= u3 upon $\Delta 3$ so this implies what is u1, u1 is 5x1 =20x2=10x1 1/3 from here what we obtain from here we obtain x1 has 4x2 and if you take these two so we obtained 2x2 = 1/x1 x2 or 2x1x2 ^{2} = 1 when you substitute x1 = 4x2 substitute then it is 8x2 q = 1. So we obtain 8x2^{3}= 1 so this implies x = ½ and when you substitute ½ in this suspension, so we obtain x1 as 4 x ½ which is 2 so the optimal solution of this problem is x1 = 2 x2 = ½ and the mini value of f is 30, so in this we can solve such type of problems oaky.

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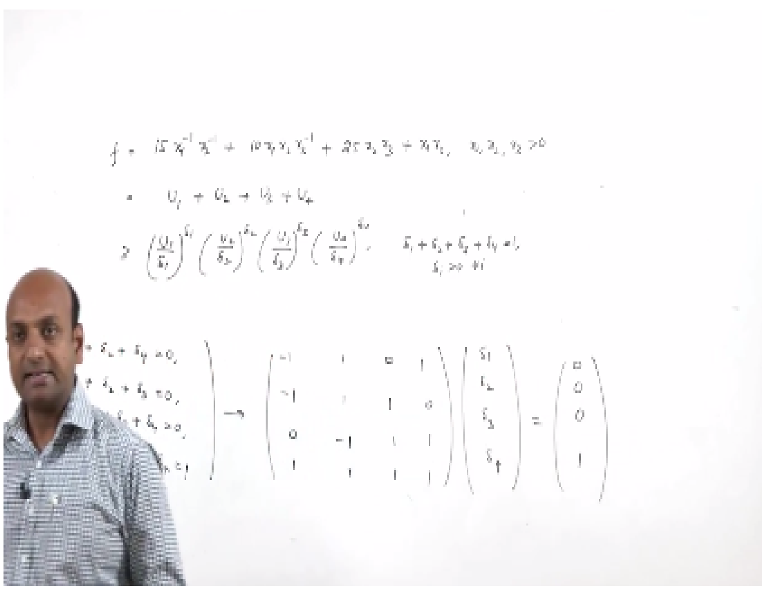

Now let us discuss one more problem based on the same lines we write the conditions directly so how to write let us see, now what is f here, f is 15 x^{-1} + 10 x1 x2 x^{-1} + 25 x2 x3 and + x1 x3 and x1, x2 , x3 becomes 0 okay, now let us try to solve this problem let us try to find out the mini value of this f and what is the optimal solution let us try to find it. So again we will write it how much term 1234 so it is u1+ u2+u3+u4, where u1 indicate the 1st term, u2 indicate the 2nd term, u3 indicate the 3rd term, and u4 indicate the 4th term.

Again we will use the equality that in equality arithmetic geometry equality it is > Δ1 u2 $Δ2^2$ u3Δ3 u4 Δ4, such that Δ1 + Δ2+ Δ3+ Δ4 is 1 and Δ i> 0 okay. Now what is u1, u1 is 15 x^{-1} u2 is 10 x1x2x3 x^{-1} similarly u3 at 3rd and u4 at the 4th term okay. Now how many variables are involved here, here we are having 3 variables x1x2 x3, okay now we will 1st make it free from variables, now let us collect the powers of x1 x2 x3 directly we can see, that the x^{-1} which we obtained from this expression will be substituted over here is - Δ1.

When you substitute u2 over here the power of will be Δ2 here there is no x1 so no power, so what is the power of x1 we are obtaining, it is - Δ1 + Δ2+ Δ3 we can substitute it =0, because we want to make 3 from the variables. So - Δ1 + Δ2+ Δ4 okay, so last term we obtain Δ4. Now 2nd equation is x2 from this expression is -Δ1, so it is - Δ1 + when you substitute this expression over here, so the power of x2 will be Δ2 and + Δ3 = 0.

There is no term of x2 over here so no Δ4, now from the 3rd variable x3 okay , 3rd variable x3 it is 0 no power and it is - Δ2 + Δ3+ Δ4 = 0and also sum of Δ is 1 as we already know. So here also we are obtaining 4 equations 4 unknowns that means unique solution okay. That means the fixed value is in right hand side. No w1st we will find the value of Δi how we will find the value of Δi we form the corresponding matrix. Matrix will -1,1,01, it is -1,1,1,0 it is 0, -1, 1,1 it is 1111 and the variables are Δ1 Δ2Δ3Δ4 = 0001.

So it is the system of linear equation you can apply any technique solve to find the values of Δ1 Δ2,Δ3,Δ4 oaky you can make it upper travel matrix anything to find out the values of these unknowns. Once you find out the value of ΔI then using the equality constraints that u1/Δ1, u2/Δ2, u3/Δ3, u4/Δ4 you will find out the values x1, x2, x3 okay which will give the optimal solution of the problem. So in this way we have seen that if we have some typical non linear problems which is other ways difficult to solve but by using arithmetic it can be easily solved. So thank you very much.

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