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Nonlinear Programming

Lec-12
Geometric Programming-II

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Hello friends, so welcome to lecture series on nonlinear programming. In the last lecture we have seen what geometric programming problems are, we have seen that there are some complicated nonlinear problems which maybe non-convex and how to solve such type of problems using arithmetic geometric mean inequality. Now in that lecture we have seen only unconstraint posynomials optimization.

I mean if we have a problem of posynomials without any constraint then how can we use arithmetic geometric inequality to solve those type of problems. Now in this lecture we will see that if we have a constraint problem, constraint posynomial optimization with equality constraint, we are focusing only on equality constraint in this lecture. Then how can we solve such type of problems so let us see.

So what is the formulation we have a geometric programming problem with equality constraints, so consider the case of minimizing an objective function which is the sum of posynomials subject to the equality constraints okay.

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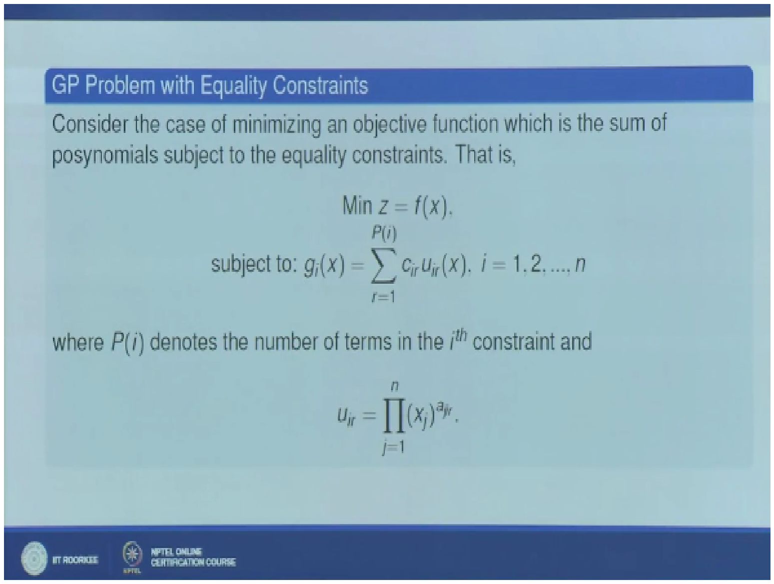

That is minimization of f f(x) subject to gi(x) = ∑r running from 1 to pi, cir and uir(x)I is running from 1 to n, where pi denotes the number of terms in the ith constraints okay, and uir is the same as the posynomial term okay. Now let us discuss the same thing again, now we already know that sum of Ui is where i is running from 1 to n is greater than equals to product i running from 1 to n, $Ui/ \partial i^{\partial i}$ this we have discussed in the last class, this we obtain from the arithmetic geometry mean inequality okay.

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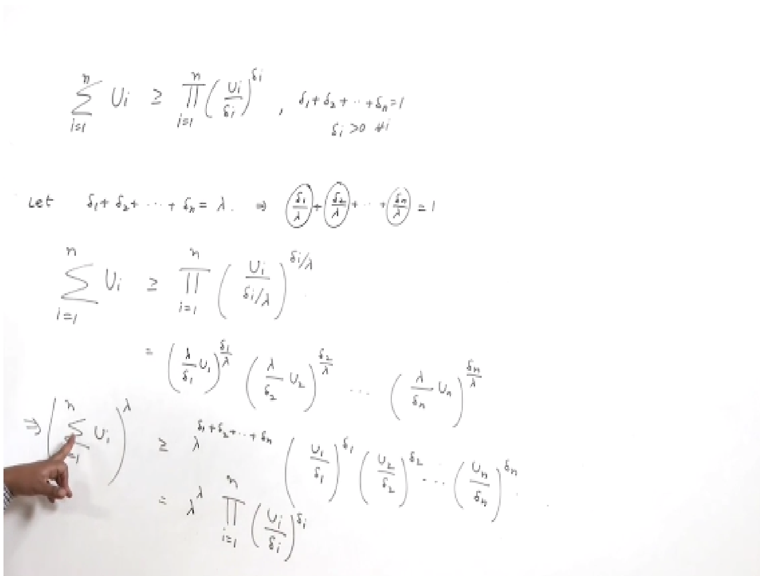

Where sum of ∂ is 1, and all ∂i are greater than 0. Now if sum of ∂ is not 1, suppose it is some λ. So how can we apply this inequality there okay. So suppose let us suppose sum of ∂i is some λ, if sum of ∂i is some λ so of course we cannot apply this inequality as much, because this is valid only when sum of ∂ is 1. So how can we apply this inequality, we divide each $\partial i/\lambda$ to make the sum equal to 1.

So what we have got to do basically we write $\partial 1/\lambda+ \partial 2/\lambda$ and so on $\partial n/\lambda=1$, so that means to apply this inequality for this type of problems we replace $\partial i/ \partial i/\lambda$. Because now the new ∂i is ∂/λ where sum is 1 okay, because sum of these, now this is one ∂, this is second ∂, this is third ∂ and another ∂ and sum of this is 1. And it is applicable only when the sum of ∂ is 1. So that means to apply this inequality for this problem we replace $\delta i/\lambda$, okay. So what we obtain it is summation I running from 1 to n Qi ≥ product I running from 1 to n, $(Ui / \delta i/\lambda)^{\delta i/\lambda}$. This inequality for which system this equality holds when sum of δ is λ, okay.

Now what is the right hand side let us see, you can simplify first it is $(\delta, \lambda 1,u1)^{\delta i/\lambda}$ the second term is $(\lambda, \delta 2,u2)^{\delta 2/\lambda}$ and another term is $(\delta, \lambda n\ un)^{\delta n/\lambda}$, okay. This is the right hands side now you first raised whole raise to the power λ both the side so this will implies summation I running from 1 to n, $(ui)^{\lambda}$ in the left hand side when you do this so it is and you collect the powers of λ, okay. It is λ raise to the power from here when you take this λ to the left hand side and collect the powers of λ from each term.

So it is $\lambda^{\delta 1}$, $\lambda^{\delta 2}$ so it will add up another remaining term will be $(u1/\lambda 1)^{\lambda 1}$ $(u2/\delta 2)^{\lambda 2}$ and so $(un/\lambda n)^{\lambda n}$ so this sum is again λ from this expression we are solving this expression fro this sum of δ is λ okay so this is equals to λ^{λ} and this is product I running from 1 to n, $(ui/\lambda i)^{\lambda i}$ so we have obtained another equality that is if u1 + u2 and so on up to un whole raise to the power λ is always ≥ λ^{λ} product of $(ui/\lambda i)^{\lambda i}$ where swum of δ is λ, okay.

So we will use this inequality for solving GP problem or geometric programming problem with equality type constraints. How we will do that let us see, let us discuss this by an example, okay.

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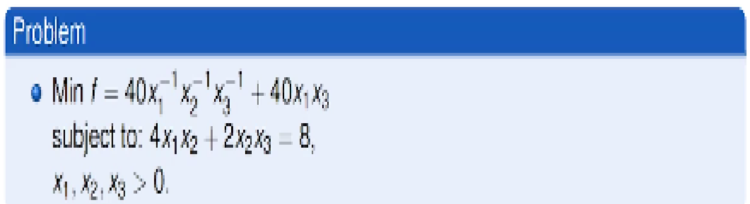

So to illustrate that how can we solve such type of problem using equality constraint let us discuss it by an example, suppose we have this example it is minimizing of f which is equal to 40 x1,x2 raised to power (-1), $x2^{-1}$, $x3^{-1}$ + 40 x1 x3 subject to you divide it by 8 this constraint you divide it by 8 so what we obtain it is ½ x1, x2 + 1/4 , x2 x3 = 1 and x1, x2,x3 are strictly greater than 0, okay.

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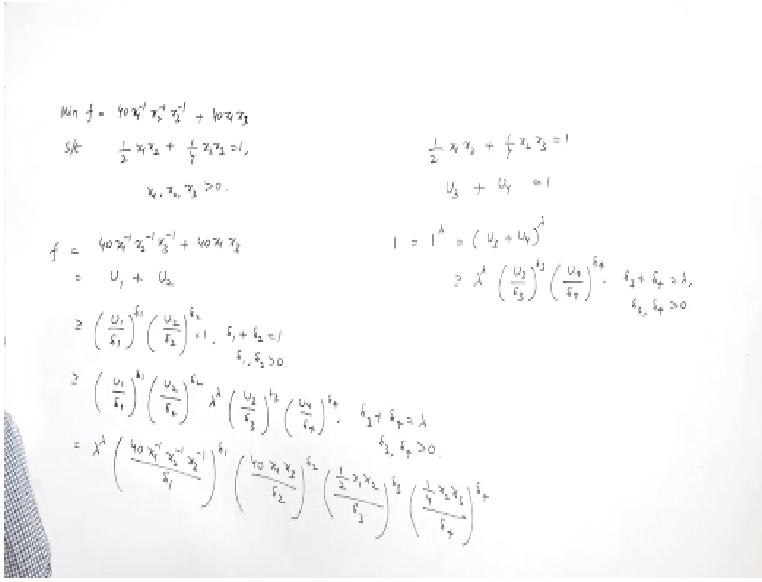

So first let us focus on the objective is f = 40 $x1^{-1}$ $x2^{-1}$ + 40 x1x3 now it is request to u1 + u2 okay, let u suppose a first term is u1 the second term is u2 so it is greater than equal to by the same inequality $(u1/\delta1)^{\delta1}$ $(u2/\delta2)^{\delta2}$ were $\delta1 + \delta2$ were $\delta1 + \delta2$ is 1 and $\delta1$, $\delta2$ is strictly greater than 0. This is by the objective function, now come to the constraint what is the constraint we are having it is ½ x1 x2 + ¼ x2 x3 = 1.

So it is u3 suppose it is u3 suppose it is u4= 1 were u3 is the first over the constraint and u4 is the second term over the constraint okay. Now to deal with this equation we discuss it like this one is always equals to 1 raise to the power λ for any λ okay so this one can be replaced by u3 + u4 is / λ because u3 + u4 is 1 so this one can be replaced by u3 + u4 and we just now discuss in the slide also that some of uy is whole raise to power λ is always greater than = λ^{λ} .

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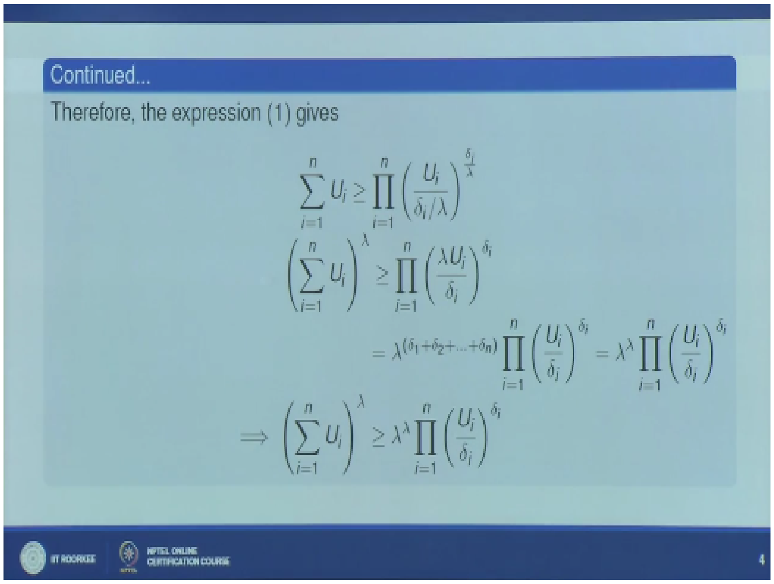

In two product of ui / δi $^{\delta i}$v where sum of δi is λ so this is u3 + u4$^{\lambda}$ so it will > = λ$^{\lambda}$ into u3/ δ3 $^{\delta 3}$ u4 /δ4$^{\delta 4}$ where δ3 + δ4 is λ and δ3 δ4 is r strictly written as 0, this is by in equality okay.

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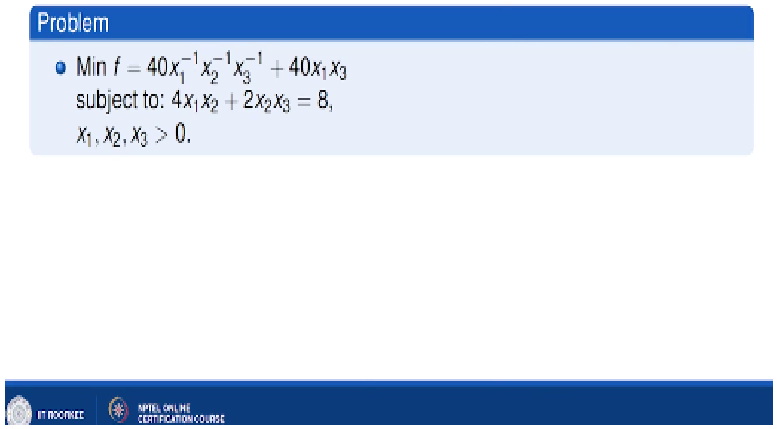

So what we obtained from here by the constraint we obtained this in equality now this we can use it here you see this we can always write this into 1 okay and this one is 1 > = this in equality so this is > = u1 / δ1 $^{\delta 1}$ u2 / δδ^{2} into 1 and 1 is > = λλ u3 / λ3 $^{\lambda 3}$ and u4 / δ$4^{\delta 4}$ where λ3 + λ4 is λ λ3 λ4 are strictly > 0 okay so it is equals to λ^{λ} this can be put it outside the brackets okay what us u1, u1 first sum of the objective function that is 40 s1 $^{-1}$ x2^{x2-1} x2 $^{-1}$ of δ1, what us u2, u2 is 40 x1 x3 / δ2 $^{\delta 2}$ what is u3, u3 is a third term.

First we constraint x1 x2 / δ $^{\delta 3}$ and u4 is 1/ 4 x2 x3 / δ4 $^{\delta 4}$ okay now again you will collect the powers of the variables here we are having 3 variables x1 x2 x2 you collect the power of x1 x2 and x3 put it equal to 0 and also δ1 + δ2 = 1 to the adjacent condition and try to find out the maximum value of the right hand side, so what are the equations we will obtained from this we can obtained.

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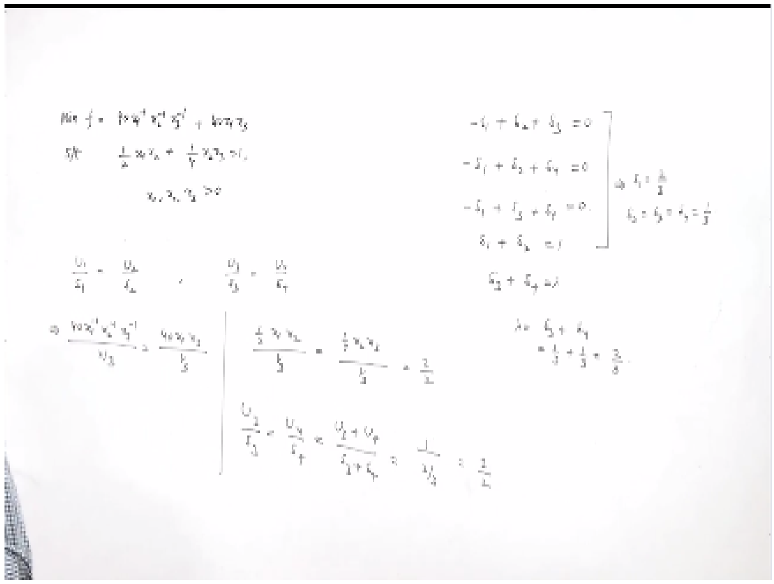

If you contain the powers of x1 it is – δ1 – δ from 1 here + δ2 + δ3 = 0 δ1c+ δ2 + δ3 = 0 for second term that is a power of x2 it is – δ1 no x2 is here so no power of δ2 for x2 and it is δ3 + δ4 that is – δ1 + δ2 + δ4 = 0 okay and the power of x3, x3 is – δ1 + δ2 – δ1 + δ2 / x2 +δ2 + δ4 power of x1 is – δ1c+ δ2 + δ3 = 0 – δ1 + δ3 + δ4 = 0 power of x2 is – 1 and + δ3 okay +δ3+δ4=0 and δ1+δ2=1 and δ3+δ4 is λ.

So here we are having four equation four unknowns okay, we can solve it and find the values of δ1, δ2 and δ3 and δ4 of course okay. So how to now when we solve this equation what are values δ1, δ2, δ3, δ4 we can obtain so you can solve it I know the values of δ1 δ2 δ3 for this system δ1 can be obtain as 2/3 and δ1= δ2= δ3 are 1/3 you can solve it and find these values of δ1, δ2, δ3 and δ4 okay, you get simply verify that these are satisfying all the constraints all the equations okay.

So what will be λ, λ is nothing but δ3+ δ4 which is okay, and δ2+ δ3 and δ4 you can easily verify okay. Now δ3+ δ4 is 1/3+1/3 that is 2/3, so in this way we can find out the values of δ and λ. Now how we find out the value of the unknowns that x1,x2 and x3 okay, so for that we will again apply the equality condition from the equality condition we obtain the equality hold everywhere for δ1=u2/ δ2 for the first one.

And u3/ δ3=u4/ δ4 for the constraint also the equality will hold when this condition holds we can easily verify. Now from here we can obtain u1 is 40 $x1^{-1}$ $x2^{-1}$, $x3^{-1}$/δ1 is 2/3 u2 is 40 x1x3/ δ2 which is 1/3 and from u3, u3 is ½ x1x2/ δ3, δ3 is 1/3 which is equal to 1/4 x2x3/ δ4 is 1/3 also you can easily see that if you u3+ δ3=u4/ δ4 so this also equal to u3+u4/ δ3+ δ4 okay, because these are the ratios and u3+u4 is 1 so it is 1 and δ3+ δ4 is λ and λ is 2/3 so it is 2/3 so it is 3/2.

So we can say write it is equal to 3/2 so using these equations you can easily find the values of x1, x2 and x3 that will be the optimal solution of this problem, okay. So in this way we can find out the optimal solution of a sustain problem with equality constrict, okay. now let us discuss degree of difficulty.

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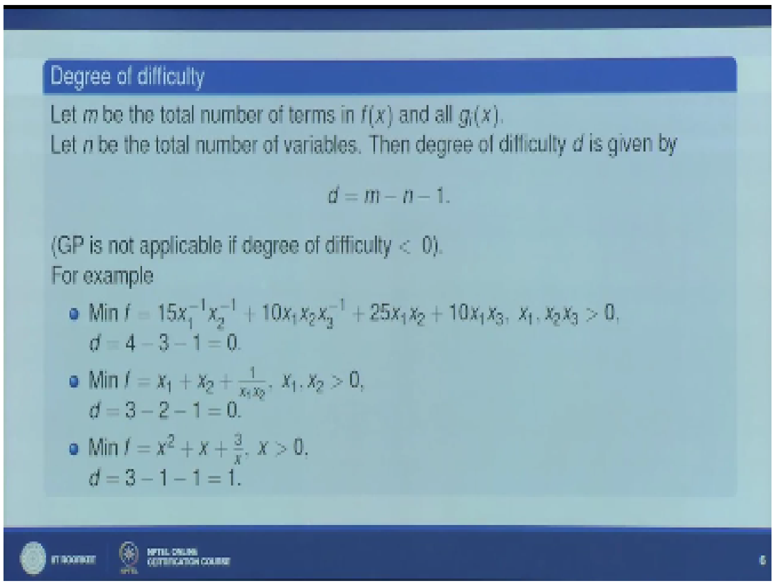

Now let m be the total number of terms in fx, fx the objective function and all gix so included term of fx and included in term in all g_ix we are calling it m okay, and n be the total number of variables then the degree of difficulty d is given by m-n-1. For example, for this problem for this problem how many terms are there 1,2,3,4 including all terms in the gix here only one constraint so 1,2,3,4 so m is 4 how many variables we are having 3 so answer is 3.

So degree of difficulty will be 4-3-1 that is 0 so degree of difficulty is 0 for this problem that is why we are obtaining a unique solution for δ_is okay, if degree of difficulty is less than 0 so gp is not applicable for those types of problems okay. Now if you consider these problems suppose the first problem now in the first problem how many terms we are having 1,2,3,4, 4 terms.

So M is four how many variables how many unknowns we are having 3 x1x2x3 three unknowns. So M is 4N is 3 so degree of difficulty will be 4 – 3- 1 by this expression that is c2 okay again in this we are having three terms 1, 2, 3 how many unknowns 2 unknowns 3 – 2- 1 that is degree of difficulty 0.

Was this problem degree of difficulty is because we are having only three terms with one unknown so 3-1-1 that is one so degree of difficulty for this is one. Now it is easy it is I mean easy if a degree of difficulty is 0 because in that case we are having a unique solution if exist okay so the right hand side of the, an equality will be having a fixed value. Now if that degree of difficulty is more than or equal to one so how can we solve this type of problems. So for illustration let us discuss this problem.

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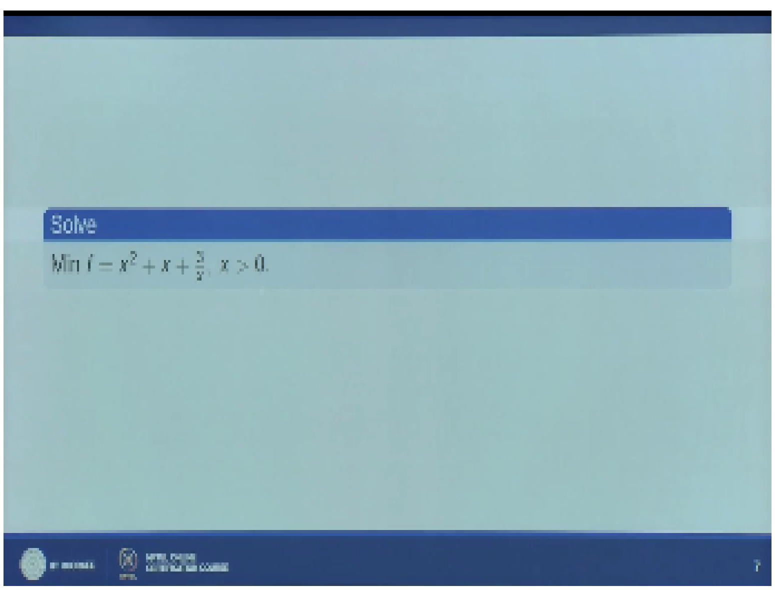

We already know a degree of difficulty is this is one because it is having one unknown at three terms so 3-1-1 that is one, so for this problem it is again unconstraint problem this is simply illustration that u1 + u2 + u3 is ≥ u1 / $\delta 1^{\delta 1}$ u2 / $\delta 2^{\delta 2}$ u3 / $\delta 3^{\delta 3,}$ we are sum out δ is one and δ I are written 0 for all I. now when you connect the power of x here only one variable is there, so how many equation we obtain only one.

And the second constraint is this and how many unknown's 3 unknown's δ1 δ2 δ3 okay so will be the equation it is where is substitute it here it is x^2 / $δ1^{δ1}$ it is x/ $δ2^{δ2}$ it is 3/ $δ3^{δ3}$, so when you connect the power of x put it equal to 0 we obtain 2 δ1 + δ2 – δ3 = 0. So here we are having only two equations with three unknowns so it will be having infinitely minor solutions, so how can we solve such problems.

Now what is our right hand side our right hand side it when we make it free from x it is 1/ $δ1^{δ1}$ 1/ $δ2^{δ2}$ 3/ $δ3^{δ3}$ we want to find out the maximum value of this expression maximum value will give the mini value of F okay. Now what we will do we will try to express all the variables in terms of one variable say it δ3 okay we will try to express that we can easily do you see δ1 + δ2+ δ3 is one and δ2 δ1 + δ2 – δ3 = 0 okay.
So you can subtract these two equations so it is – δ1 + 2 δ3 = 1 so δ 1 will be nothing but 2 δ3-1 so that is in terms of δ3 so you can substitute do this δ3 over here or in the any one of the equation when we substituted over here suppose so δ1 is 2 δ3 – 1 + δ2 δ3 = 1, so δ 2 we obtain as 2 – 3 δ3 okay it is δ2 and it is one 2 – δ3 okay, so we have express all the variables in terms of one variable that is δ3.

Now you can put it over here now you focus only on this term okay what is this term say it is g δ, g δ is a right hand side 1/ δ1, δ1 is 2 δ3 – $1^{2\ δ3-1}$ okay. So in this way we have expressed in right hand side only in terms of one variable now what we have to do we have to find out the maximum value of this expression which is only one variable how we can do that you just take the logarithm for both the side you just take the log for both the sides find out the first derivative respective $Δ_3$ put it in equal to 0 that will give the maximum value of g Δ for which you can find out the value for $Δ_3$ for which gΔ is maximum once you find the value of $Δ_3$ for which gΔ is maximum you can substitute $Δ_3$ over here .

And you can find out the value of $Δ_1$ and$Δ_2$ and once you found out the value of Δ1 and Δ2 then you can easily say that this equality will hold when are all equal that is $u_1/Δ_1$ u_2 upon $Δ_2$ and u_3 upon $Δ_3$ and that using that you can easily solve you can easily find out the value of x for which affix is minimum okay.

So in order to find out the maximum value of this you simply take log book both the sides differentiate respective Δ^3 put it in equal to 0 in the first derivative and you can find out the value of Δ_3 for which gΔ is maximum you can substitute that Δ_3 over here to find out the value of Δ_1 and Δ_2.

And that will give the values of $\Delta_1\Delta_2$and Δ_3 to find out the maximum value of minimum value of f you use the equality condition okay so in this way we can solve the problems with constraint normal optimal with equality constraint or if we have a problem having difficulty more than 0 so thank you very much.

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