**INDIAN INSTITUTE OF TECHNOLOGY ROORKEE**

**NPTEL**

**NPTEL ONLINE CERTIFICATION COURSE**

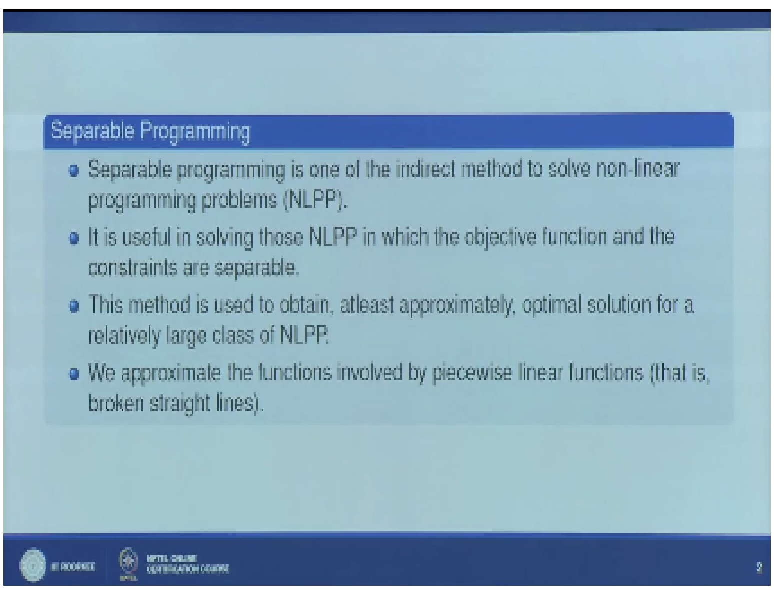
**Nonlinear Programming**

**Lec-9**
**Separable Programming-I**

**Dr. S. K. Gupta**
**Department of Mathematics**
**Indian Institute of Technology Roorkee**

Hello friends welcome lecture series on non linear programming, now next topic is survival programming so in this we will see what the specula programming is and how can we solve separable programming programmers, so what separable programmed coming is let us see so separable programmed is one of the intact method.

(Refer Slide Time: 00:40)

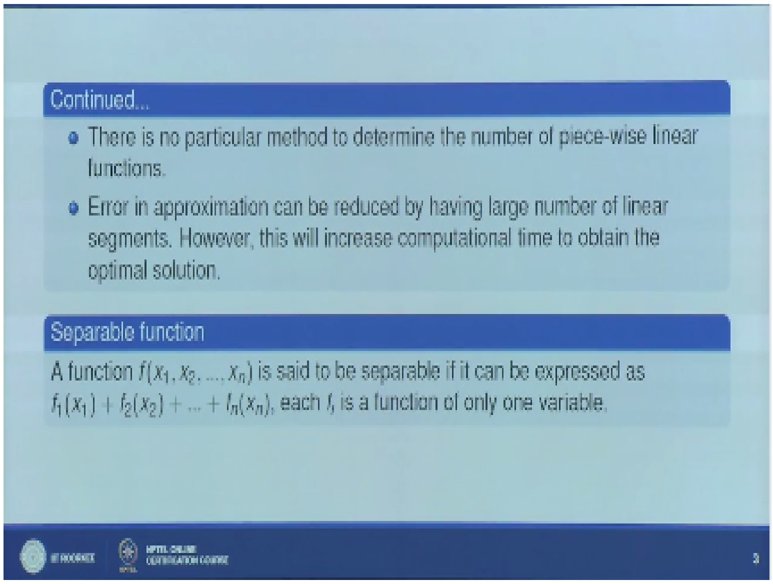



To solve nonlinear programming programme you see solving we do not have any unique method top solve a nonlinear programming problem okay, but if we have aseprable problems objective function as well as consaints of the seperable what do you mean by seperable in we will discuss in this lecture so we can solve it using the seperable programming okay.

It is useful in solving those nonlinear programming in which the objective function and the constrains are seprable this method seprebable programming  is used to obtain atlest approxamately optimer solution for a reactively large classof the non linear programming problem in this we are actually not finding the exact solution but we are finding atleast the approxiamate otimer solution of a given nonlinear programming problem we approxiamate that function involved  by the peace wise linear function that is a broken estate lines what ever function we are given in the objective function as wel as in the consaint.

The approximate the function by number of linear function by the number of the broken a straight lines okay. now there is no particular method .

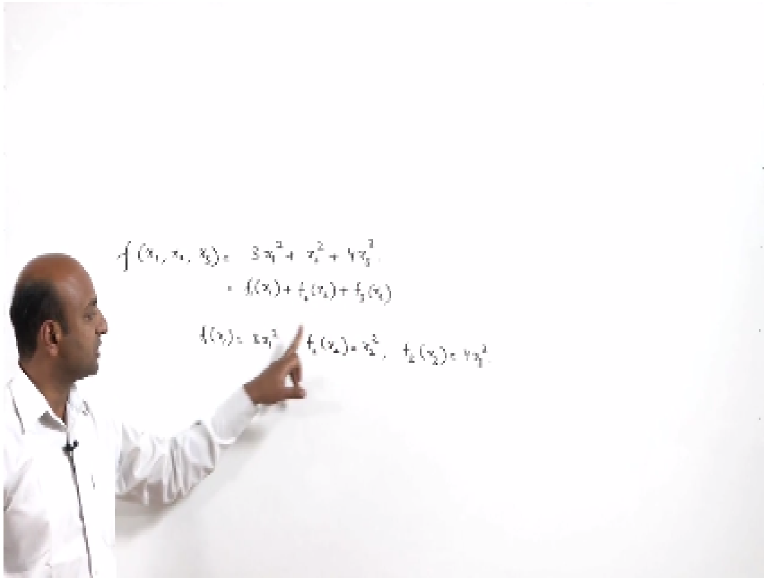
(Refer Slide Time: 02:10)




To determine the number of the peace wise in the linear function okay, we approximate the given function by a peace file linear functions but we do not have any method find the number of peace wise in the linear functions error in the approximatinon can be reduced by having the largr number of the linear segments however this will increase competational time to obtain the otimer solution offcourse if we have larger number linear function then the error will be lesser .

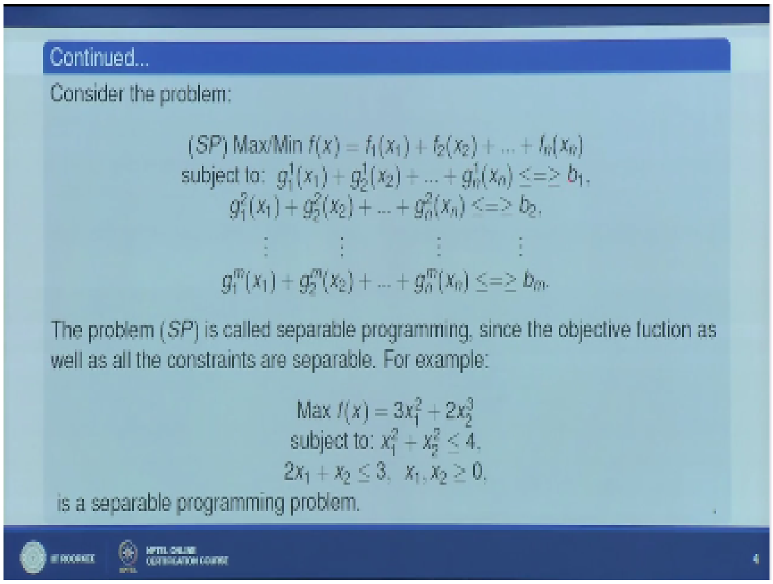
but however thiswill competational time to obtain the obtimer solution now let us start when the function issaid to be subtable now function involving and the variables X1 X2 X1 up to XN is seprate  to be a seprable if it can be expressed as that is each F5b is a function of only one variable suppose youhave the type of the function .

(Refer Slide Time: 03:19)



Suppose you have F X 1X2 X3 are the three variable function and suppose it is like thisand the F 1is the $3X1^2$ $+X2^2$ +4 $X3^3$ okay thisfunction iti sis the subtiltle it is because we can right this function as F1 X1 +F2  X2 +F3 X3 we are the FEMALE_1 X1 is some thing but the $3X1^2$ and the 2X2 is nothiong but the $2\ ^2$ and the f3 x3 is nothing but 4 $x3\ ^3$ okay sense each functionisafunction of only one variable it is the X1 X2 X3only so we said that it is function of the F is seprebable okay now in this consider non linear type of the problem and the in the special case of the nonlinear programmnig oroblem ofcourse so what we have the objective function invoved in the problem .

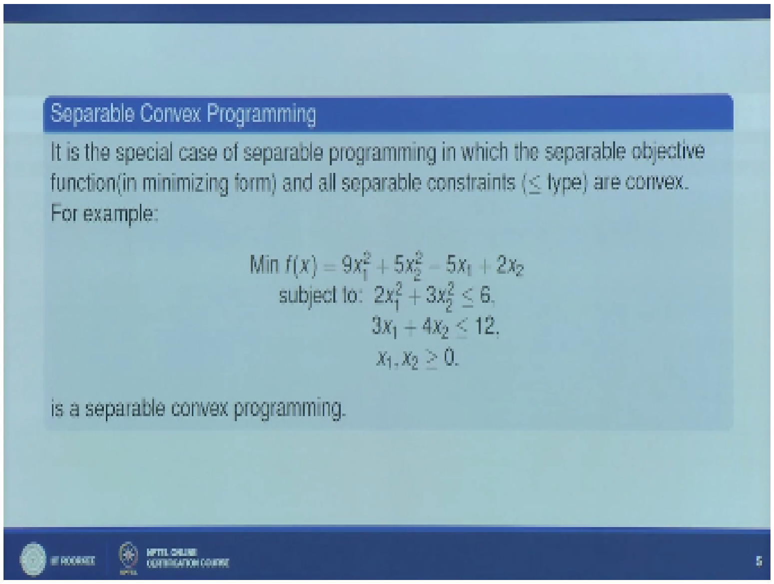
(Refer Slide Time: 04:31)




And the problem issepreble iokay and all the construction is the seprable the first cconbstrait is also the sepreble second constraint in the constaits and it is seperable in the such problem were the objective functionis as well as the constraitsas the upreble are called superable programme problems.

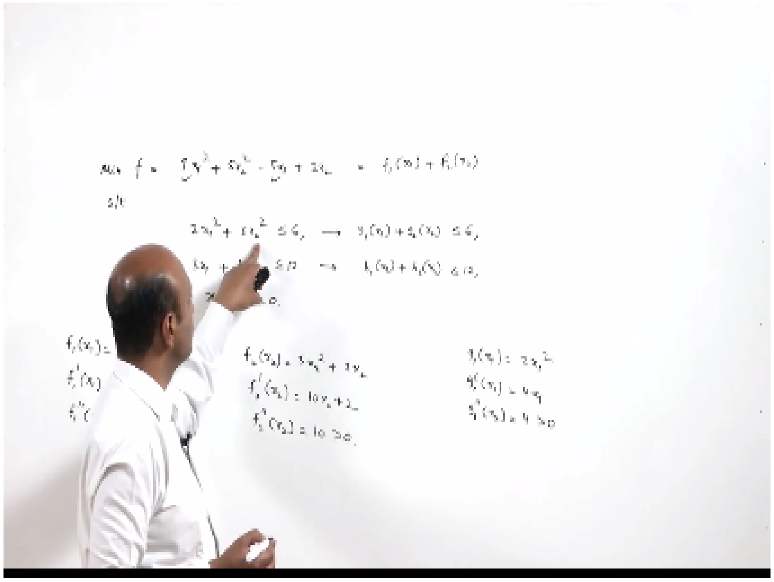
Now the first example we have thisproblem we have the maximze and the X=to and there is the 31X$^2$ and the 2X2$^3$ subject to the first one of the firstt constait as the 1$^2$ 2X$^2$ and that is = to 4 the second constait is 2 x 1 +2x 2 and the 3 and x1 x2 negative so here youcan easily see thsat the objective function is seprable because it is we can write this easily request to F 1 X1 =F 2 X2 we are theF 1 X1 is the 3X1 $^2$ and the F 2 vx2 is 2 x 2 $^{3.}$
Now the first constait is also seprable by because it is written as and the F 1 X1 + G 2 X 2 lesson = to 4 herethe G 1 X 1 is and the X 1 $^2$ and G 2 X 2 is X 2 $^2$ the next constait is also seprable and the H 1 X1 H 2 X 2 lesson = to 3 were the H 1 X 1 is2 X 1 and the 2 X 2 is andthese constaints are obsiouly seprable so we can say that this problem is a seprable programmnig problem okay ,nowwhat is the seperable convex programmnig .

(Refer Slide Time: 06:40)




So if it is aspecial case of the appreable programming in which the seprable objective which is the minimizing form and all seprate constaint of the lesson= to type or convex so if the involt objective function which are the seprabletype and all constain lesson equal to type are all the convexthen we say that it is conve seprable programming problem it is minmizing as.

(Refer Slide Time: 07:15)



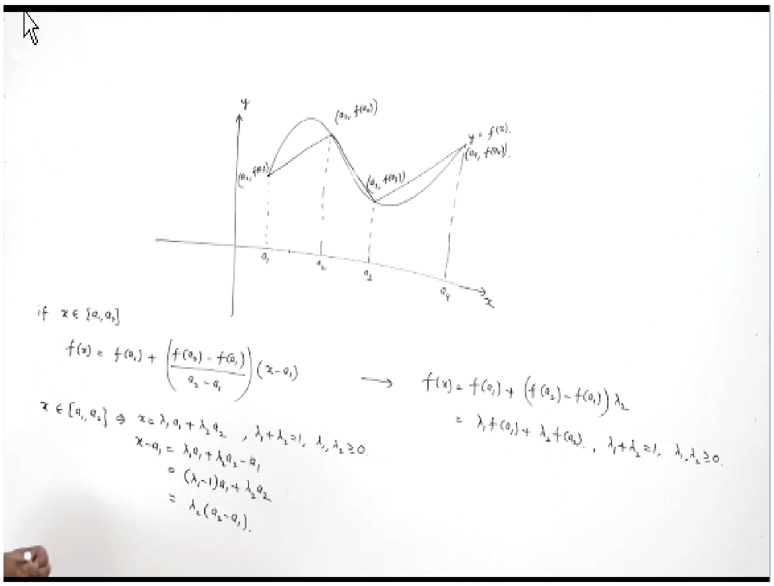
And the 9 X$^{2}$ +5 X 2$^{2}$-5 X1 =2 X 2 and the subjected to the condition are the 2 X 1v and the 3 X 1 +4 X 2 and X 1 X if you take thisproblem so you can explain F 1 as and F 2 X 2 youcan explAIN THIS AS g 1 x 1 +gG 2 X 2 and the lesson =to 6 you can explain thisconstrait as and H 1 S 1 +H 2 S2 that is =to 12 okay now what is F 1 F 1 is the function of the X 1 and it will be (9 X 1 5 X 1 these are the function of the X 1 ,so this will come in the X 1 okay and what will be F 1 2 and the F 2 X2 5 X2$^{2}$ and +2 h 2 now if you take the first derivative of this F 1 and the 18 X 1 -5 if the second derivative of this is18which is positive .

So this is convex thisfunction  is convexbecause it is the second derivative is thtechnicall is the0 and the greater than the 0 ,nowif it isif you take first derivative of this andthe 10 X2 +2 andthe second derivative of this is 10 which is greater than the 0 again this convex okay the F 1 F 2 both are the convexandthe sum of the 2 convex also thwe convection this is convex .are yopu canm take the matrix of the F andit can out to be positive andthe nevt in the positive then we can see the function is ands thw convexfunction andthe nowif you take G 1as1so the G 1as the G 1 okay.

And the second derivative of this is4 and it is positive again it is convexand similarly G 2X 2 is3 X 2$^{2}$ and take the derivative and the second derivative of this is simply 6 with the again positive and G 2isalso convex okay so this consent is a convex consent and it is the convexfunction and similarly it is the linear function and these two are also linear so it is also convexso you say that that problem is seprable aswell as convex so it is caled as seprable convexprogrammnig programme okay so if the nonlinear programme if it is the convex programming problem as well as the satisfied as the seprable ability of the function so we can easily say that they are seprable convex programming programme.

Now come to our seprebale rogramme let usto understand seprable programme and function of the seprable programing okay now suppose we have the function we have the cdommon variable here as given in the diagram here okay .

(Refer Slide Time: 11:06)




You for this curve you make this linear approximation of thisfunction okay now for this curve you make thislinear approxiamtion suppose up to here and then up to here how many grid pointsare here we have the four grid points assuppose it is the linear process ,so this is A 1 and this is A 2 and this A 3 and thisis A 4 and this is the rise to the effects basically what we havedone we have given function of the variable to the effects we are trying to find out the linear approximationof the now this function .

So we have is splitted the curves in to the 5 8 number of in to the finite numkber of broken lines this are the linear function okay this curve is the procressand this straight line and this curve is approximately by thisd straight line and thiscurve is approximately by this straight line okay Offcourse if we increase the number of we can make the two straight line of the straight line also here if we approximate this given cut okay.

If weincrease more straight lines so a complexity to find the optimer solution increase okay here for the simplicity I am taking only thyree linear approximation of this curve now this A 1 A 2 A

3 A 4 these are called grid points .now how to finear linear approximation of the F let us see now thispoint nothing but it is A 1 F 1 and F of A 1 okay,thispoint is A 2 and F of A 2 thispoint isA 3 F A 3 this point is A 4 F A 4 okay nowif X belong to A 1 to A 2 this is the x okay andthisisF X or Y okay nowthw X belongs to X in thisinterval given to the A2 ,so how to find Y in this interval thisis like equation of straight line of passing through two points passing through F A 1 and A 2 F A 2 F A 2 ,so what will be the value of the function that we can see it the F X =to F A1 +FA 2 –F A 1 .

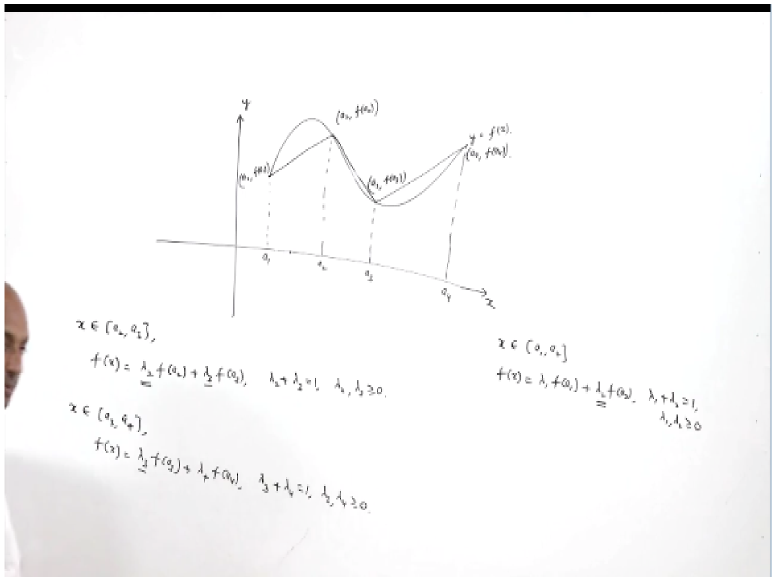
So what is the value of the function that will see that f(x) = f(a1) + (f(a2) – f(a1)/a2 – a1) (x-a1). That will be the may equation of straight line passing though two points. Afa1 and a2fa2 okay? So in this way we can find the value of f at the point in between this line segment okay? Now this x in is in between a1 and a2 okay.

So x ∈ this means x is a convex in a combination of these two points, because x is in between a1 and a2. So f will be something λ1 a1 +λ2 a2 so here λ1+ λ2 =1 and λ1, λ2 are non negative okay? Because x is in between a1 and a2 okay? Now what is x-a1 from here this implies λ1a2 + λ2a2 – a1, so this will give (λ1-1)a1+ λ2a2 and λ1-1 from here is -λ2, so this will give λ2(a2-a1) okay? Now when you substitute this over here.

Why x-a1 is this quantity when you substituted over here in this equation a2=a1 will cancel from the denominator λ2 will come here. So what you obtain you obtain f(x)= f(a1)+)(f(a2)-f(a1)) λ2. Because this is a cancelling both the sides okay? From the denominator this will cancel so this will give—now this is 1-λ2, 1-λ2 is λ1. So it is λ1 f(a1)+λ2f(a2). So were λ1+ λ2=1 λ1,λ2 are non negative, so that means if x is an convex in a combination of a1 and a2.

The similarly f(x) will be the convex in a combination of f(a1) and f(a2). If we are talking in between a1 and a2, now similarly if we talk x in between a2 and a3 then f(x) will be similarly f(λ2) f(a2)+ λ3 f(a3) where λ2 + λ3 =1 and λ2, λ3 are non negative okay.

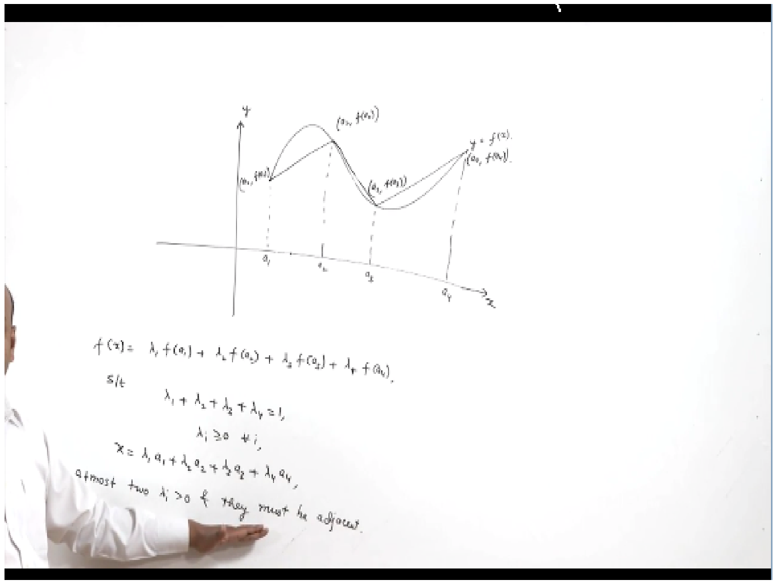
(Refer Slide Time: 17:21)




On the same lines we can obtain if x line between a2 and a3 so corresponding f(x) which is in the straight line will be the convex in a combination of f(a2)and f(a3). Now here it to be noted that λ2 is same, what you see when x is in between a1 and a2? So f(x) be obtain as λ1f(a1)+ λ2f(a2). Where λ1 + λ2 =1 and λ1, λ2 are non negative. So whatever λ2 is here, the same λ2 is here. This is the important condition here okay.

Now similarly if f is in between a3 and a4 so what will be f(x)? if it is in between a3 and a4 so f(x) will be nothing but now it is in λ2 f(a3) + λ4 f(a4), such that λ3 + λ4 =1 and λ3, λ4 >= 0. And λ3 is here whatever here same here okay? So what is with the linear approximation of these steps? Now if you combine all the three terms.

(Refer Slide Time: 18:53)




So the linear approximation of this f will be $\lambda_1 f(a_1) + \lambda_2 f(a_2) + \lambda_3 f(a_3) + \lambda_4 f(a_4)$ subject to $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1$. And all $\lambda_1$ is are non negative. And what will be x? $x = \lambda_1 a_1 + \lambda_2 a_2 + \lambda_3 a_3 + \lambda_4 a_4$, and now the important condition is at most 2 $\lambda_i > 0$ and they must be adjust. Why that is the same. You see either we have to minimize all we have to maximize the given objective function okay, Okay.

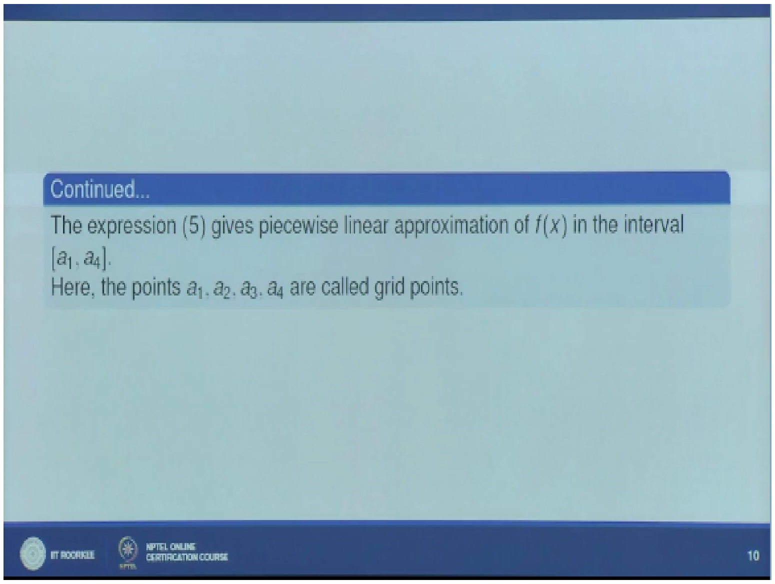
Now for this particular function if you say for this for the particular function. It will again suppose we are taking a minimization problem okay? It will again minimization on one of the line segment okay? We have—we have this approximate this even knows by a finite number of pieces wise linear functions okay. If it is a minimization problem so it will again minimum at one of the linear function okay.

Suppose here it is minimum suppose for this function here it is minimum. So it will again minimum if it is again in any minimum over here so this linear straight line will approximate this optimal solution okay? Not the entire straight lines, only that the straight line is important. Which where obtain the minima okay. Now if this is a straight line which is giving an approximate optimal solution for this particular problem then only $\lambda_3$ and $\lambda_4$ are important, I mean greater than 0.

Thought the conjugative and all other the 0, that is why at most two $\lambda_i$ are must greater than 0 they must be adjacent okay. Either these two or these two if it is optimal over here only $\lambda_3$ and $\lambda_4$ are important. They are adjacent $\lambda_3$ and $\lambda_4$ and must be positive, and all other must be 0 in that case okay? So whenever we have a separate programming problem and we are taking respect to the reverse x1.

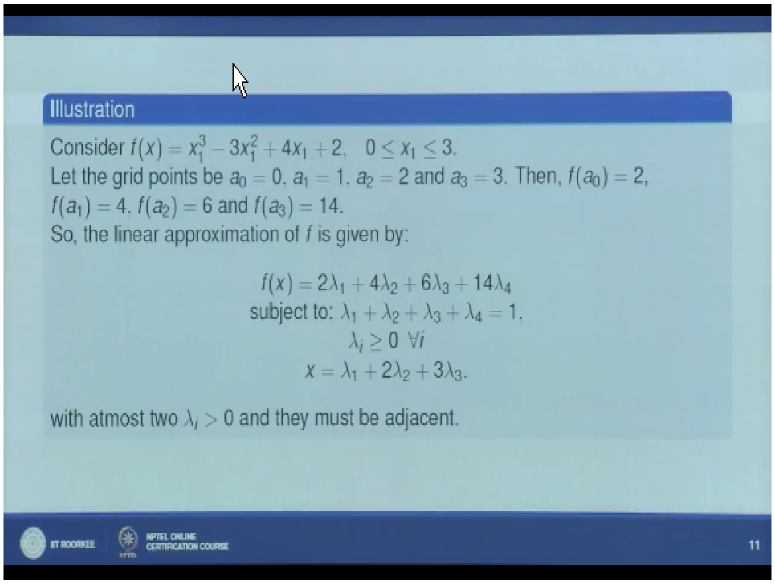
Among the reveal function we are approximating functions by number of linear functions okay Piece wise linear functions. So in that with respect to that variable, only two λ must be greater 0 and they must be adjacent okay. So in this way we can find out linear approximation of a given function f. So all these description over here in the slides okay.

(Refer Slide Time: 22:38)




So the expression (5) gives piecewise this is expression (5). So this gives piecewise linear approximation of f(x) in the interval a1 to a4. Here the points a1, a2, a3, a4 are called grid points okay.

(Refer Slide Time: 22:38)




Suppose so this functions okay now this function is also of only of one variable, so $f(x) = x_1^3 -3x_1^2 +4x_1 +2$ and subject 2 conditions are 0<= x1<=3. Now suppose this function is a function in only one variable and we want to find out linear approximation of this function how to find. So the bound of $x_1$ should must be be greater than 0 or less than equal to 3. So first we find out the grid points.

Finding find points is in our hand oaky? So we take grid points like this. This is o and it is 3, you see $x_1$ is between o and 3, it is 0 and it is 3 okay? So we can divide grid points as 1 and 2, it is one grid point 0, 1, 2, 3. These are the grid points; we can take grid pointers half also 3/2 also but it will make the calculation difficult okay for convenience we take grid points as 0, 1, 2, 3 okay we can increase the grid points also.

No problem okay? So for convince we are taking only these grid points 0, 1, 2, 3 okay? Now first grid point $a_0$ is 0 $a_1$ is 1 $a_2$ 2 and $a_3$ is 3. So what is $f(a_0)$ then $f(a_0)$ is substitute over here. So it will be 2. What is $f(a_1)$ is here you can simply see the 6+1=7 ,7-3=4 and what is $f(a_2)$? $f(a_2)$ substitute a $a_2$ $x_1$ =2. So when you substitute $x_1$ = 2 it is 8 8 + 2=10 10+8 ia 18 18-12 is 6 and $f(a_3)$ is 14. So these are the values you obtain okay?

So what will a linear approximation of this f(x) nothing but it is suppose it is $\lambda_0$ over here λ1 here λ2 here λ3 here so it is $2\lambda_0$ +4λ1+6λ2+14λ3 it is λ0f(a0) +λ1f(a1)+ λ2f(a2)+ λ3f(a3) so 2(f(a0) λ0 4λ1 6λ2 14λ3, subject to λ0+λ1+λ2+λ3 must be one. All λi is must be >= 0. And at most 2λi=>=0 and they must be adjacent. Okay. And what will be x the x1 the optimal solution the action will be 0λ0 +1λ1+2λ2+3λ3. So that will be the optimal solution. 0λ0 +1λ1+2λ2+3λ3. So this is how we can find out linear approximation of a given function okay so thank you.

**For Further Details Contact**
**Coordinator, Educational Technology Cell**
**Indian Institute of Technology Roorkee**
**Roorkee – 247667**
**E Mail: etcell.iitrke@gmail.com. etcell@iitr.ernet.in**
**Website: www.iitr.ac.in/centers/ETC, www.nptel.ac.in**

**Production Team**
**Sarath Koovery**
**Arun. S**
**Pankaj Saini**
**Neetesh Kumar**
**Jitender Kumar**
**Nibedita Bisoyi**

**Camera**
**Jithin. K**
**Graphics**
**Binoy. V. P**
**Online & Video Editing**
**Mohan Raj**