Design of Mechatronic Systems
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Lecture No 23
Mathematical Modelling: Overview and Context

Let's start with this new topic. Today, we will look at, and I will split this lecture into two kind of modules and we will start with the first module. We will look at this mathematical modeling part. We will give an overview and context and possibly see the model of the motor, how do you model the motor, that is what we will do. And then, we will move on in the next module to modeling the friction in the motor. So, we will talk about friction in lot more details, and see a lot of different aspects of it.

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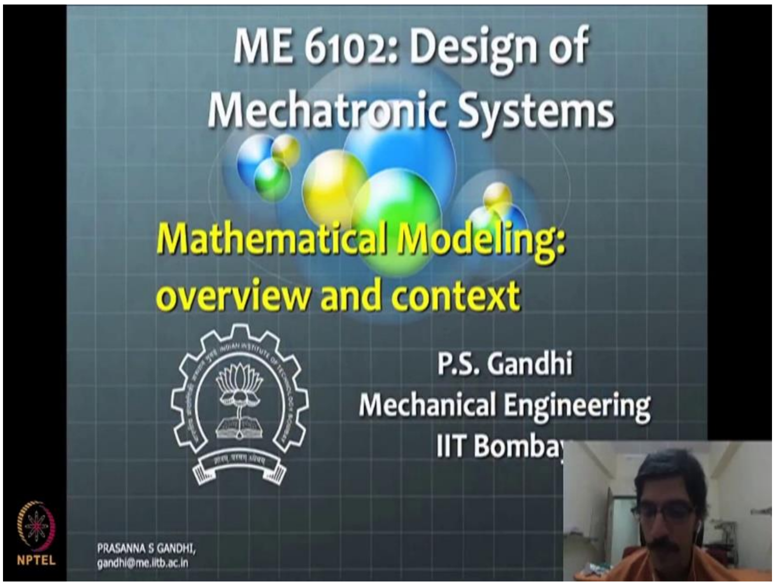

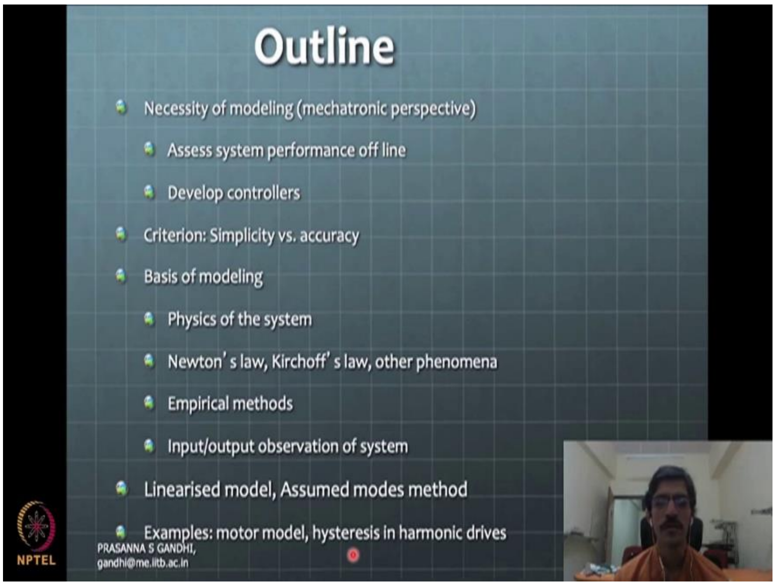

So, now to begin with, we will look at the slides. This is like a rough outline of the entire part. But in the modeling aspect, we are going to look at some of the parts here. So, we will start with these points first, and then, actually as we get this context, we will start about. So, why do we need to model?

So, if you think about the models, let us assess the system performance, and then we can develop some controllers. Why they have assess or how can you assess this performance with models because we can simulate them offline. And without doing much of experimentation, you will be able to get effect of one of the changes of the parameters or how the system behaves, how do you, get all the details about study about how the system behaves and what is its physics.

And then, further utility of it for, especially, for mechatronics purposes is for development of controllers. So, this is very, there are these two key aspects for development of the model or necessity for the development of the model. And the criteria for modeling will typically be simplicity versus accuracy.

So, we want very simple models, because you want to develop further controllers. So, your partial differential equation models are a FEM-based models which are huge matrices and many degrees of freedom that will be typically difficult to handle in control. So, you need to have a simple model, and it should have enough accuracy, reasonable accuracy to represent whatever physics that we are talking about.

And basis of modeling you all know. We have done a lot of these studies in prior classes, prior courses. You typically model the system based on the physics of the system. Typically, in mechatronic systems you will have Newton's law, Kirckoff's law or many other phenomena. In the fluid mechanics you may have some laws that may govern the flow behavior or you may have a temperature behavior thermal actuator you want to control temperature. So, those kinds of models may be there. So, basically, they are all based on the physics of the system.

And then you may have some empirical methods of modeling. We will not get into whole lot of details about empirical methods of modeling, but there can be a lot of different models possible just based on the input/output data, you gave the input system behaves in this kind of a fashion and then you can represent that data by fitting some kind of a model.

It need not be just a mathematical curve fitting, but it can be actually a dynamic model which will feed the system. And then that is what imperial model, typically, we will need input/ output observation of the system. So, for example, if you experimentally get a Bode plot of the system, then one can predict a similar kind of a transfer function, which will have the same Bode plot as the system has, and then, that is when we are modeling the system.

More advanced versions in this form can be in the domain of your artificial intelligence or machine learning-based kind of neurological kind of a neurons-based model. So, there is a lot of possibilities in further in this message of modeling. Then for compliant systems you can or for more complicated systems you may look at like linearized versions of the models. We have a nonlinear total model, but then you can linearize that model and then use for control.

So, like that there are these Assumed modes method. There are multiple kind of these methods that will be there in the literature of modeling. It is vast kind of a domain, but we are going to look at part of it, which is most important from many motion-based mechatronic systems. We will not get into thermal systems too much or any fluid-based systems too much, but we will just focus on motion-based systems, mainly.

And we will see particularly like the motor model. If we will get a chance to kind of see this hysteresis modeling in harmonic drives systems based on some kind of empirical methods, rather than the pure physics of the system. So, this is like the typically the outline of the thing.

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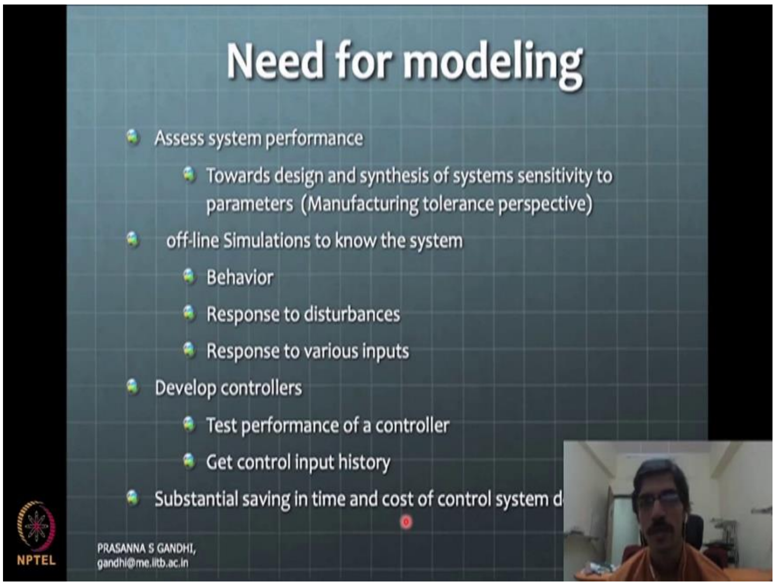

So, let's get in like talk a little bit more details about these points now. So, when we start say the need for modeling, you will have first assessment of the performance. What is the assessment can be for? You want to design some system parameters. I know, as a mechatronics engineer, we want to kind of see what is the kind of sizing of the system I need to do? What are the parameters I should use?

So, for that you need to study this model and like get these parameters and design, and then see that if you have some manufacturing tolerances, I do not have these dimensions coming up to the expectation, they are within some kind of a tolerances then how I can see that, okay, how this system will work or not work or what is the disable that will create in the system, how do I handle that? Those kind of sensitivity studies can be done by assessing this performance.

So, you can typically study an output variable, as a function of change in the parameter and see the sensitivity behavior there. So, if the, with the changing parameter if the output is changing a lot then that output is very sensitive to the changes in that parameter, and then, that particular parameter will need a better kind of a tolerance, strict tolerance controlled then maybe some other parameters in the system.

So, that is due to assess the performance of the system. And you have, I mean, you might have done like a lot of these kind of studies of modeling. May not be these sensitivity studies, but like modeling you might have done and seeing the changes in parameters. For example, if you take a pendulum kind of for example.

And I say, okay, if the change in the length is delta, then what will be its effect on the on some output parameter, which you want to control? Say for example, frequency of the pendulum.

So, you will be able to kind of say something about that. So, that kind of studies can be done with the model.

Then you can offline simulate the system, and then see its response or behavior to say various inputs or in the presence of noise, in the presence of disturbances, so all these kind of things can be done for the system. So, I mean a reasonable model will definitely help you to study the system in its totality to see, expect what you will expect in the experimental implementation of it.

And of course, last, a very important part is to develop controllers. So, test your performance in the controller, or tune the parameters of the controller, say PID controller to or gains of the PID controller to get desired output for the system. And other important thing is to get a control input history. You need to make sure that your control input is not getting saturated. We will talk about this point, multiple times because this is an important aspect that often gets neglected while studying like your basic automatic control kind of courses.

Where we are only focused on like the stability of the system, stability of the system. All the time we will talk about how to make the system stable. Are these gains step making the system stable, are these gains not making the system stable? Those kinds of things we talk about a whole lot in automatic control kind of a course, but nobody cares about, what is the input that I need to kind of maintain certain performance for the system.

That is very-very important aspect when you get into the practical world. Because always like your input is limited, you will have a saturation on your motor input or motor power that is there, and that needs to get factored into your simulation studies. And for last and foremost point you need to remember that you can save substantially in the time and cost if you do right now good amount of a nice representation of system, and do a lot of things on the soft than going in the experiments and doing some steps without any background in the real modeling.

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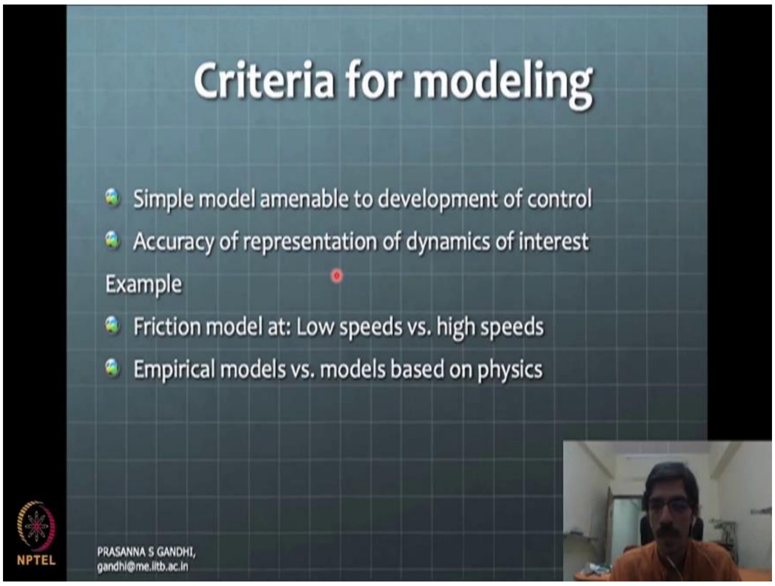

So, there are these criteria for modeling. This is another very important thing. Many times often mislead, people get misled into, oh, I want to capture this behavior accurately, this model must be capturing all the things whatever are there in the phenomena. So, that is not really of interest from control perspective. We want as simple model as possible, but at the same time capturing all the things that are of interest. Say, at different phenomena like if you observe different things may be of interest.

For example, if there is simple block moving on the, resting on the surface and is applied by the force and it is moving under the action of that force. What kind of model will come to your mind? If you say, reasonable accuracy "F=ma" that is a kind of a simple model that will be there. Now, when you say "F=ma", we are neglecting some parts of the dynamics of the interest. It is resting on a surface, so surface would offer some friction as a resistance, so that needs to kind of get factored in.

So, then for accuracy, we will need that friction to be accounted for. But now, one may say, okay, extending this well look there is air around the block and that air also is giving its resistance. When I kind of start pushing the block, the air is registering there. Now, if I start modeling that air resistance to the block, is it of what kind of a contribution to the entire model? That is one needs to think about that.

In some cases, this may be of a great importance. In some cases, maybe like a negligible important. Depending upon how fast you are trying to move or what is the shape of the block and things like that. So, for example, if I am attaching some load on the motor and that load is rotating. That versus, if that load happens to be a fan and I am driving a fan then that air flow, air resistance that comes into picture. So, now, one can think what is the way I would model such a kind of scenario, these different, different scenarios?

So, for example, if the block is, say, moving with a reasonably low speed I do not need to worry about air resistance at all. In other cases, where there is a fan which is attached to the motor then I would say, okay, if I start modeling this fluid structure interaction of the fan and then create some kind of a huge numerical kind of a model or some partial differential equation-based model is going to be hell for me to control this fan or this fan system in some way, if it is needed to be control.

But, then, that model will get simplified. So, overall, this air is giving some resistance to the to the motion. And that resistance if I am able to capture in reasonably well kind of a form, I do not need to worry about. So, one can use some kind of a simple linear damping kind of a model or some kind of a nonlinear damping model.

So, those are aspects of accuracy of representation of what is a dynamic of interest. So, this is what is an important point or important role for the designer to play to see, what is the important part in the model. And that often comes by just imagining or actually carrying out some small experiments. So, as we go along, we can model or we can identify the friction, and model the friction in our own our motor kind of the system.

Then there are these empirical models, which are based on this. So, when the Physics is so complicated that you cannot capture. As I told about a fan, which is moving and churning out the air, it cannot, if you go to its real dynamics it will have lot of these fluid structure interaction along with some kind of a Navier Stokes Equation, Partial Differential Equations which will model this physics.

I mean, these models will not help us for development of control so much. So, then in that case, we will consider some empirical models where I consider this overall resistance of the friction to be, the overall resistance of air in terms of this some air friction to be of some kind, some non-linear kind. And that is like becomes like an empirical model. And I will identify the parameters of the model, such that, what are the experimental data that I gather for various speeds it is fitting reasonably well with this empirical kind of a model.

So, empirical models understand that, they are not based on the physics, they are based on the input/output experimental data. And, typically, they cannot, they do not capture physics of a system.

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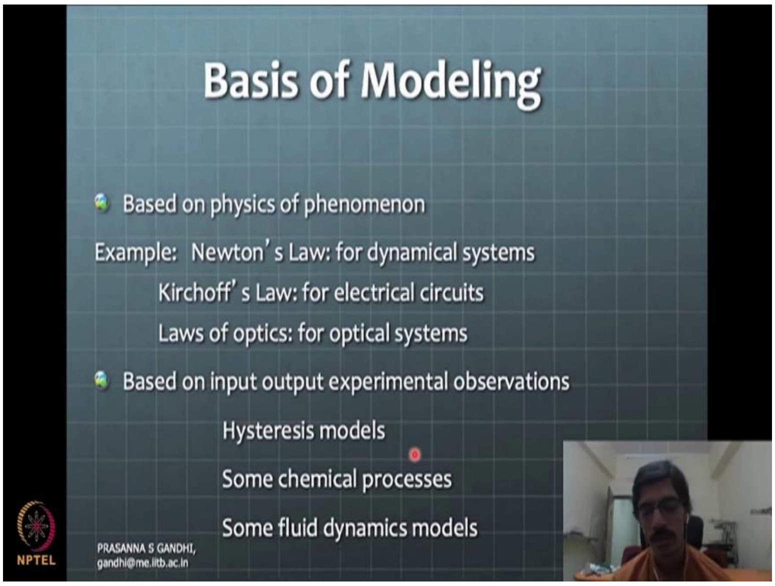

So, then, you have this basis of modeling that we talked about. There are typical laws. In addition, you may need to use. See, the laws of this electrical circuits Kircoff's law and the Newton's laws, they typically will be there in many, many mechatronic systems together. There will be some kind of a coupling between these two.

We will see, for example, in a motor model how this coupling happens. And in high-speed systems this coupling has a great effect or even in slow speed systems. But in high-speed systems there are some effects that are more specifically coming up from the electrical dynamics. In normal like a lower-speed things then, electrical dynamics will not come into picture because electrons will have flow velocities through the wire much, much higher order of magnitude than the mechanical motions in the system.

So, electrical dynamics will really not come into picture unless the speeds of motion of electrons or like some of the electronic elements that we are using. Say, your magnetic actuators or something has some dynamics, so that this electron dynamics matches with the frequencies of mechanical motion. So, mechanical motions need for that matter to be very, very fast to have electrical and mechanical forces, which are competing now with each other.

Otherwise, some simple electrical dynamics will not need to consider, but some kind of simple relationships will be sufficient. We will see this point in more detail when we see the model for the motor. Then you can use some additional like physics in the system and things like that.

So, for CD-ROM case, for example, you may need to kind of think about some small consideration about Gaussian beam propagation and laws of optics there. Like that, many other kinds of physics may come up. It depend upon what system you are handing here. Then based on input output observations it will be based on experimental observation. You carry out many different experiments and gather like the data of about these different complex numbers.

For example, Hysteresis or some chemical processes, fluid dynamic systems, they are kind of very complicated to a model from true physics of them. That is where you just give some input data and observe the output and then start looking at models, which can represent that data.

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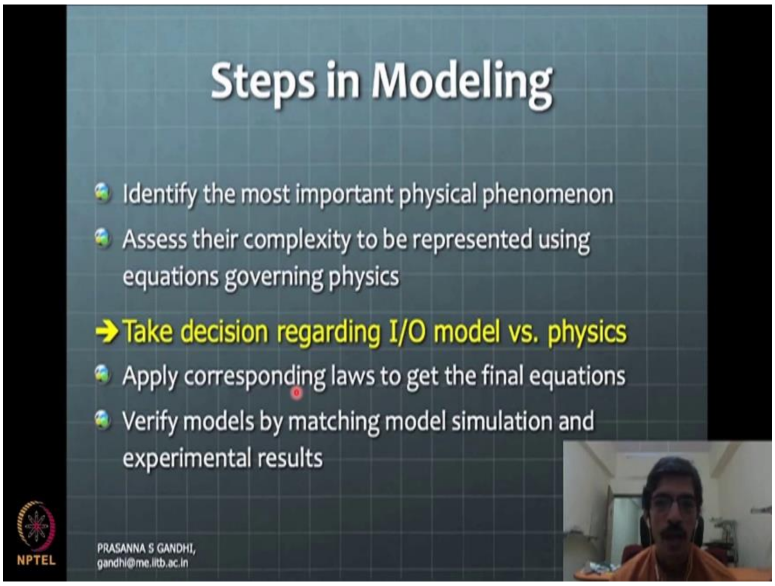

These are the typical steps in modeling. So, first, I were mentioning earlier also, identify the important physics of the phenomena and you may do a little bit in the alternative way this steps or if you are experienced you may kind of like directly jump on to knowing, okay, this will represent my physics of the model very well.

And then, the major decision to take is, whether, to go for empirical models, which are input/ output-based models or you want to model completely physics. This is an important decision to take. And then, once you take decision, you apply corresponding laws and get your models done. So, this is how we will proceed to do the modeling step.

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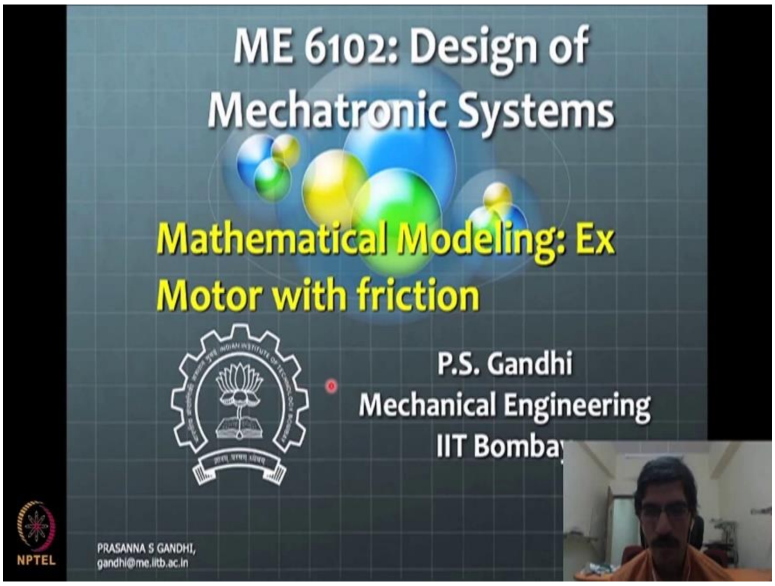

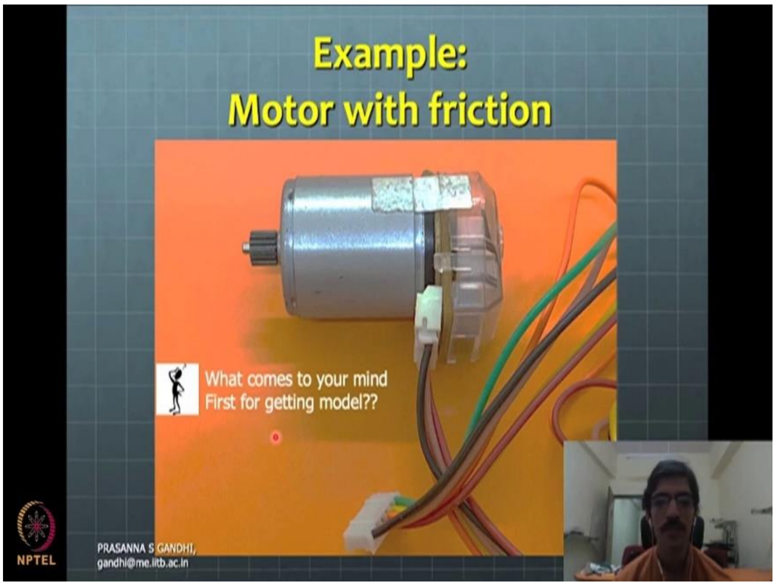

We will see a motor model, so that we apply now. These whatever things that we have gathered overall kind of a context of modeling we will apply it to this motor system now, and see. So, this is a motor. At the back what you see here is a encoder for the motor. And then there are these wirings for the motor whose terminals are given in that datasheet. And there is small little gear sitting on the shaft of the motor here. So, this motor we will try to kind of see whether we can model this motor or not.

So, we will work with some simple models of the motor to begin with. We may make it a little bit of a, maybe, we can work with the simple models only. I think, we will not get into more complicated models for motors in terms of any other kind of motors. We will stick to the DC circuit motor kind of a model, and later on we will see some other actuators as a solenoid or other kind of actuators.

Now, just pause here for a moment and think, how do you model this motor? What is a physics that is going on inside? What kind of ways that can be captured into some kind of a model? So, think in terms of the separation in mechanical and electrical domains. What is happening in mechanical domain, and what is happening in electrical domain? So, for example in electrical domain you are giving some supply to these lines to the motor, and motor is receiving some voltage signal, and it is exciting its coils, the coils when get excited with the magnetic they are suspended in typically in the permanent magnetic field.

And when the current is passed through the coil, which is suspended in a permanent magnet you know that by Lawrence Law you will get a force on that coil, and that force will cause the motor to rotate. That is what is going on, as a phenomena and as a physics in the system.

Now, we want to model this physics. So, what is that is going to happen? How do you kind of think about modeling this motor? So, let’s kind of get a mechanical domain separated with the electronics domain, and see through what are the laws which will work at mechanical. So, mechanical domain what happens here can you think about?

You will see that because of this current that is flowing in the wires the force is applied on the wires, so there is some kind of a torque that is generated inside. So, now, there is a torque on the mass which is now rotating. So, it is similar kind of a situation, as if the block is lying on the surface and applied by the force. It is the same situation where it is only in the variable of theta or motion of the motor.

So, the torque is applied on a system of some mass, and there is some bearings are there to this mass. So, like that we start, and like model the mechanical dynamics. So, see what are the resistances that are provided. The generation of the torque is like same motor is generating torque which is τ_m, and like that, you can start yourself first to write some equations.

So, do that step. You write your own equations in mechanical domain. Think about electronics domain also in a similar kind of a fashion, and then, we see okay in electrical domain what all equations or what all the elements in the electronic circuit we need to consider.

So, you are applying voltage, so voltage source is there. Then there will be some resistance that will be applied by the wires. Then since the wires are in coil form there will be some inductance that will be there. So, like that we can think about these different-different elements that will be there as a part of the electrical circuit and start modeling this motor.

And then, once you are finished your own kind of a things, then you can proceed from this. So, pause at this moment I would say, and then, once you have done something on the paper, then you can proceed to the next part of the lecture.

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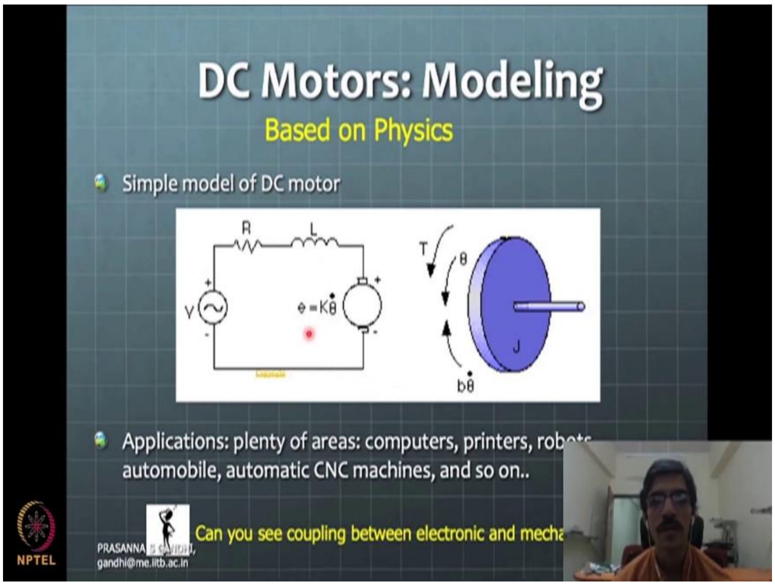

So, here, you have a representation in the form of a electrical circuit, and in the form of a mechanical free body diagram. That is what we do for any mechanical system. So, you can see that, here the torque that is applied here will have, will be through the motor wires, windings that is like motor torque, τ_m there will be a torque generated by the motor, and then there will be motion in the same direction, and then you have some resisting torque, which is given by the viscous kind of a friction.

And the important aspect here is in the electrical domain. So, important aspect is like where is a coupling, what is a coupling between the electrical and electronics or electrical domain and mechanical domain. Say, for example, if you see this torque that is getting generated, as we have seen by Lorentz law the current is related to this torque. So, this torque will be proportional to the current that is flowing into the motor, motor windings.

And, similarly, if you see in the electrical domain, you will have the resistance, inductance and then there will be this back EMF that is generated. Do you know what is this? This is because suppose you do not apply voltage, and you rotate the motor shaft. When you are not applying any voltage, and rotating the motor shaft, if you see the terminus of your motor wires, they will show some voltage.

So, that is like a electrical conductor placed in the magnetic field, and it is moving, it will generate current. That we term as a back EMF, and more faster you move the motor more will be this this current that is generated or this back EMF that is generated. So, this is back EMF is proportional to the $\dot{\theta}$ to this of velocity of motor shaft rotation.

So, this is, these are the two areas of the coupling. Coupling in the, coupling from mechanical to electrical is through these back EMF and coupling from electrical to the mechanical is through the torque that is getting applied, which is proportional to τ_i, a current torque which is proportional to the current that is flowing in the model, proportional with the current I.

So, now, how do you write the equations? So, if you check here whatever your equations you have written, have you missed anything? Probably you might have missed this back EMF part. So, you add that back EMF part and see what is your enhanced model, and then you move forward. Then you will see that you will get some equations of this kind.

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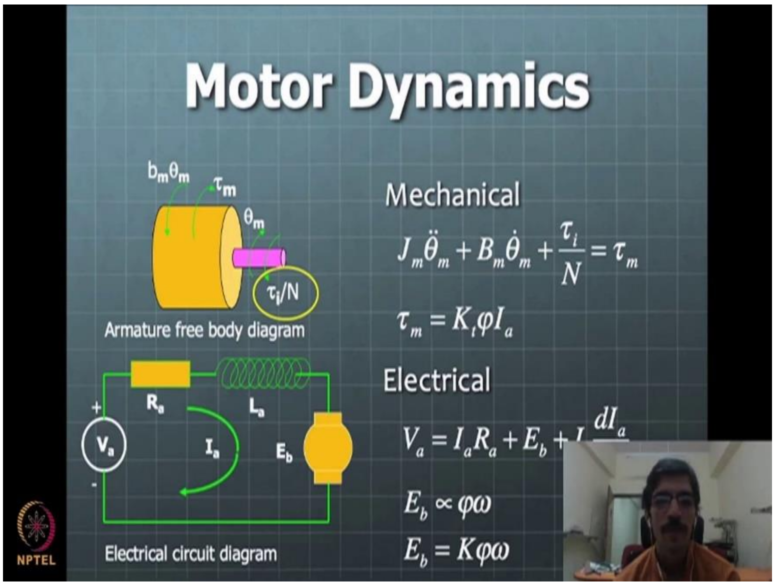

So, now, here this so mechanical domain we apply like J $\ddot{\theta}$. So, J is the inertia of the mass moment of inertia of the motor, whatever is rotating in the motor shaft, along with the motor shaft. $\ddot{\theta}$ will be acceleration, angular acceleration, then B_m θ_m double dot is resisting torque because of some kind of a bearing friction, viscous friction in a bearing.

And then, if you are applying some additional load on the motor then this τ_i and there is some gearbox also. The load is not directly applied on the motor shaft, but it is applied through a gearbox, which is having n as a gear ratio, then you will get this τ_i/N term here, which is equal to τ_m. And this τ_m is like the torque which is generated by the motor.

So, you need to be careful about the tau torque, which is applied externally on the motor, and motor is like working against that torque versus a torque that is generated by the motor. So, this is a torque that is generated by the motor typically is proportional to the current that is going into the armature windings. And this is like a constant factor.

So, this φ is a flux of magnetic field in the permanent magnet motor that flux will be constant in the separately excited motor, there is no permanent magnet there, there is a magnetic coil through which like the motor permanent. Instead of creating permanent magnet you create a electromagnetic magnet there.

So, that flux will be bearing, it may change. So, we are not talking about a separately excited motors, we mainly talk about a permanent magnet kind of motors. So, this both the terms can be single constant. And in the electrical domain, if you see now, the voltage that is applied to the motor is consumed in like the resistive losses I R and then back EMF and then in the inductance of the motor, inductance of the motor is if it is L, (L (dI)/ dt) would be the part of the.

It is not resistance, it is some kind of fluctuation of the voltage across the inductor. And according to that your complete balance of the voltage will take place. And then, this is a back EMF if that is in this system. And as we know, the back EMF is proportional to the speed ω. So, we can write it as a, again, this flux may be showing as a separate component or may not show as a separate component depending upon permanent magnet motor or a separately excited motor. So, this is a basic kind of a motor dynamics model.

Now, one can simplify this further by considering like this coupling of the current, eliminate the current and you will get like some more simplification for the model. So, you can work about that and see, okay, how you can get a unified model of these, both these things considered together, are all these relationships considered together and you get like, the entire equation of the motion of the motor that may have this (L (dI)/ dt) term coming.

See, typically, like L inductance of these motors is very-very low. So, many times for all practical purposes this inductance is neglected. So, these electrical dynamics, as I was saying before that it is very fast dynamics here. So, that dynamics is a seldom kind of affects the mechanical coupling effects. It does not matter really.

So, there are two things. Although, this current may be changing so fast, if the low inductance of the motor itself, like the motor winding inductance is very-very low value, as compared to these back EMF or these voltages that are given here other quantities of the given. So, this term gets neglected in the actual representation practically.

So, with this term, there are not there. You can kind of try to see, what is a motor-model equations look like, and then, you can kind of take some example and simulate also. They have some kind of a, you go to this Maxon Motor website and take some motor, and put some parameters and check out. Those kind of exercise will be nice to do to see, okay, well how this motor will have different effects coming up in the modeling.

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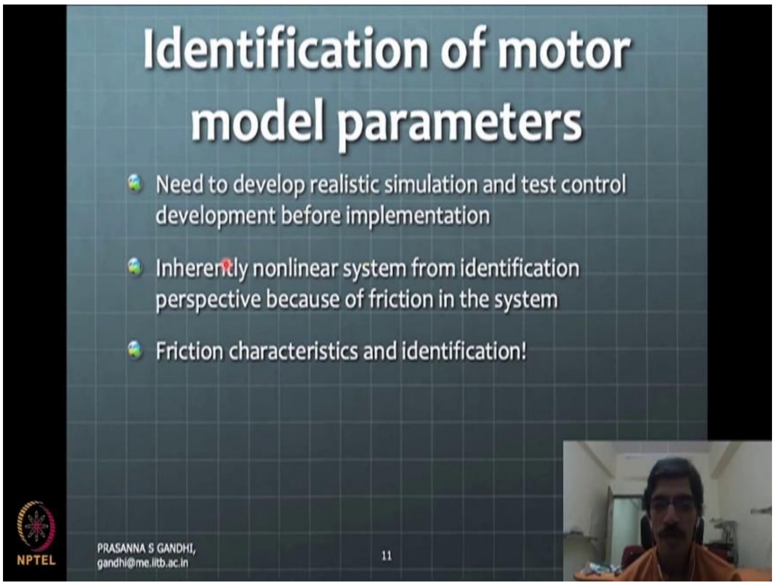

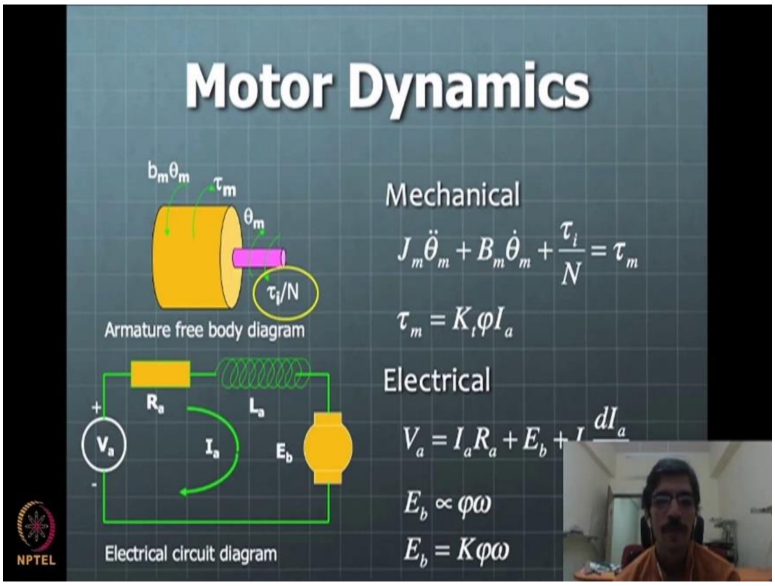

Now, you need to, so now, based on this model itself you can get know the realistic simulation for the motor done. So, we are, we can do that without adding any load okay so we do not have this term τ_l/N we are not adding any load on the motor, that gear is just sitting on our motor.

But if you apply some, you put some linkage there or some kind of mechanisms there, then that is additional load that is coming up. But currently like, right now, we will not be considering that any external load to the motor, and then we can start thinking, okay, how we can identify the parameters of motor.

So, we will just think about this. Given this now, like the motor model, what are the ways in which one can kind of see and identify. What is say, for example, the inertia of the motor? So, consider these equations together and think about these this. This is very important and nice thinking that you can have to say, okay, I want to say, get inertia of the motor, what I need to measure.

What you have to, what sensors you have on your motor to measure is a encoder. So encoded data if I measure, can I get these parameters. That is a kind of thinking one can go ahead and do.

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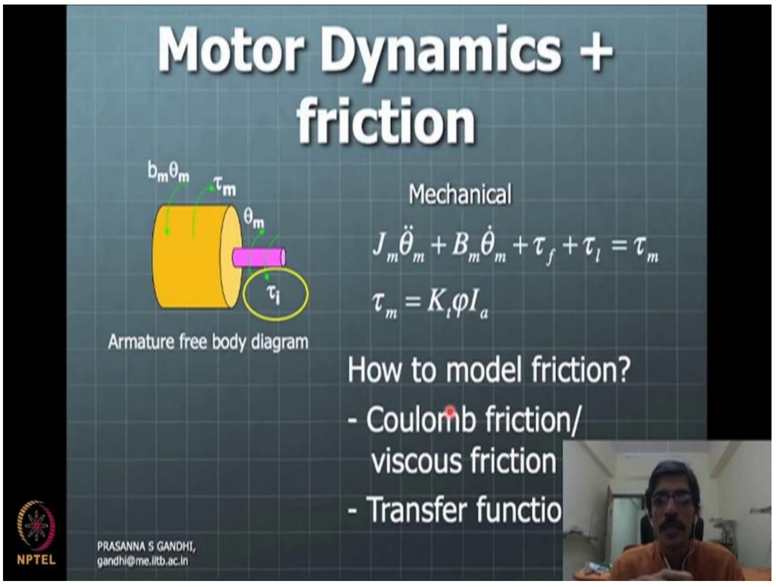

Then, typically, there will be a friction in the motor. So, right now, whatever we have modelled, we have not considered friction in the motor. But when we are doing this parameter identification, and there will be a part, friction will be part of that identification process also, so that will be coming up here. So, you need to be aware about that.

And, then now, we will have a question. How do you model friction? So, we will discuss now in the next module in a great detail like what is a friction and how do you kind of model friction, and represent friction in the motor model. And then we will be able to see, a little more about this friction knowledge of having this good knowledge of the friction and the model of the friction is very, very important aspect for mechanics engineer to consider or not consider friction or take some decisions based on the situation at hand, in terms of friction. So, we will do that in the next module. So, I will stop here for this module.