Design of Mechatronic System
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Lecture: 24
Modelling Friction in a System

Now, let us begin with this next module which is on friction modeling. So, think of a friction what comes to your mind and let us see how this friction can be modeled. So, see in the day to day life we experienced this phenomena "friction", you have a block line on the surface you start trying to move it there is some friction resistance that will be great, you try to move it in opposite direction friction is turning into opposite direction that is a simple thing that everybody has experienced in their life.

So, what comes to your mind is simple friction model like that we will start kind of thinking about friction with some common sense understanding and like making it may be more deeper and deeper.

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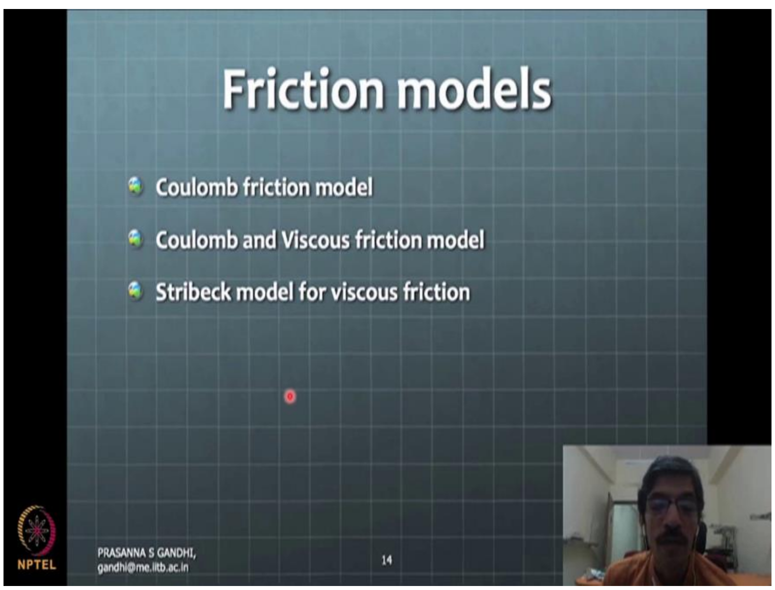

So, that is a model that you have been like familiar with probably in your "JEE" mechanics and other kind of courses is based on like say friction is equal to μN that is what you say. Now, can you represent really in mathematics restriction is equal to μN. How do you represent it in a mathematical form? Think about it write down the equation of friction model. So, while say you have a block that is resting on the surface and it is applied by some force F and it is opposed by say some friction force F_f.

Then how do you write dynamics of this system? So, now of course additional force F_f needs to be getting considered here. So, the F then will be equal to m times acceleration plus this additional force F_f. Now how do you express this F_f as a mathematical equation? That is a question so now we will see that different-different kinds of scenarios here.

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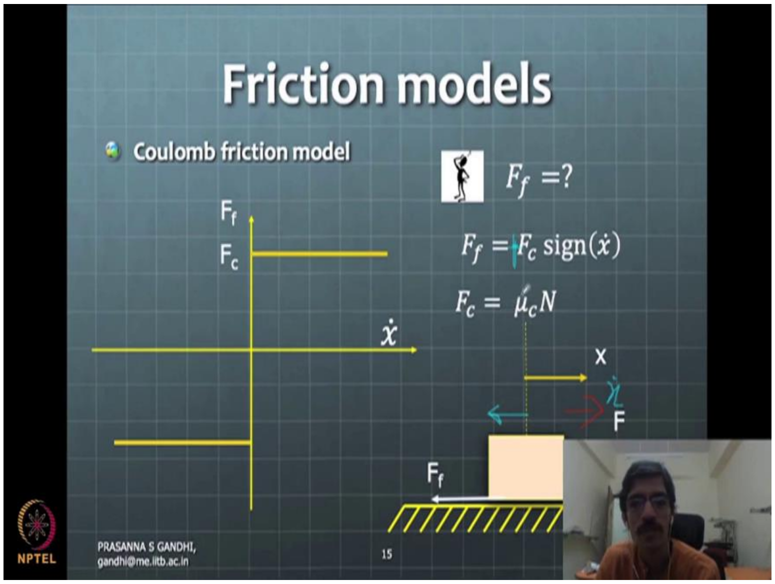

This is a scenario that we were just talking about, you have a force that is getting applied on the mass m as this F force in friction F_f is symbol and now we want to represent this force. So, we know that as I move the block in this direction. So, the block is if it is moved in one direction, then the friction forces in this direction, if the block gets moved in other direction then the friction forces in the opposite direction that is what we know.

Now if I say the block moves in this direction, can I capture that intent of that by looking at x? Probably not because even if I am at some other different x value, I can be moving in the other direction I can be at a positive x but moving in the moving, towards a 0 value. So, x may not capture that intent but $\dot{x}$ may capture that intent? Because if $\dot{x}$ is positive that means I am moving in this direction.

My direction of velocity is this? If $\dot{x}$ is negative my direction of velocity is opposite. So, let us get pen here. So, if $\dot{x}$ is positive I am moving in this direction. So, $\dot{x}$ changes $\dot{x}$ this is a positive $\dot{x}$ here. This $\dot{x}$ is positive here and then we have this direction of $\dot{x}$ that will be negative. So, the friction will occur in the direction opposite to the direction of $\dot{x}$.

So, $\dot{x}$ if it is moving in this direction, the fictional being the opposite direction, if $\dot{x}$ moving in this direction, friction in the opposite that direction. So, that is how you need to capture the friction in the equation and the magnitude of friction say for simple when you have this μN kind of a way of representing the system. Typically, you say the friction force here is μ times N.

So, that is μ times N. only after the block starts moving, before the block starts moving friction is equal to this F force. So, we can see whether that can be also captured in the model. So, think about what is that way we can write now the equation for F_f here, the friction force here. So, one can use what kind of functions mathematical functions when can use to kind of get this direction, directionality dependence represented.

So, if I plot in with respect to $\dot{x}$ the friction force, then if the friction force is positive value here as $\dot{x}$ is positive and then friction force changes his direction and becomes negative as $\dot{x}$ becomes negative but for any other value of $\dot{x}$ fiction is still remaining the same that is like a coulomb fiction model. So, this μN. you say no matter what is the velocity of $\dot{x}$ you say your fiction is always μN?

That is a maximum limiting value of the friction that you consider we do not consider any dependence on $\dot{x}$ for the friction in our normal kind of a no coulomb friction kind of a model are normally like what we have studied in mechanics I mean the simple "JEE" kind of classes. So, write down this equation we develop this equation and develop this equation and then think see that. So, this F_f is F_c signum of $\dot{x}$.

So, F_f will be should be negative here you can put this sign negative here because it is in the opposite direction to force F. So, negative of this signum of $\dot{x}$, $\dot{x}$ in this direction, then friction is in the negative direction to $\dot{x}$. So, that may change this also in the opposite direction by the way. So, or else we can consider this as a F_f and in the equations you can use my negative F_f or whatever.

So, the intent is important here you can kind of correct little bit integrities of these equations, but when $\dot{x}$ is a positive direction, then like this friction should have a negative direction. So that is a way you can represent the friction, sorry, this may not be that because I am considering this so this is a positive only here because F_f itself is considered in this direction that is what is being considered here.

This is already taken care of here, so do not worry about the sign is really positive only yet because I am drawing this force in the negative direction here anyway. So, that sign is taken care of, because I am kind of representing is this force in the opposite direction. So when I am drawing the free body diagram that time also I need to have the friction also coming in the similar kind of direction then things will be fine. That is how things will go. And this F_c will be $\mu_c N$. So, this coulomb friction coefficient of friction will come into picture.

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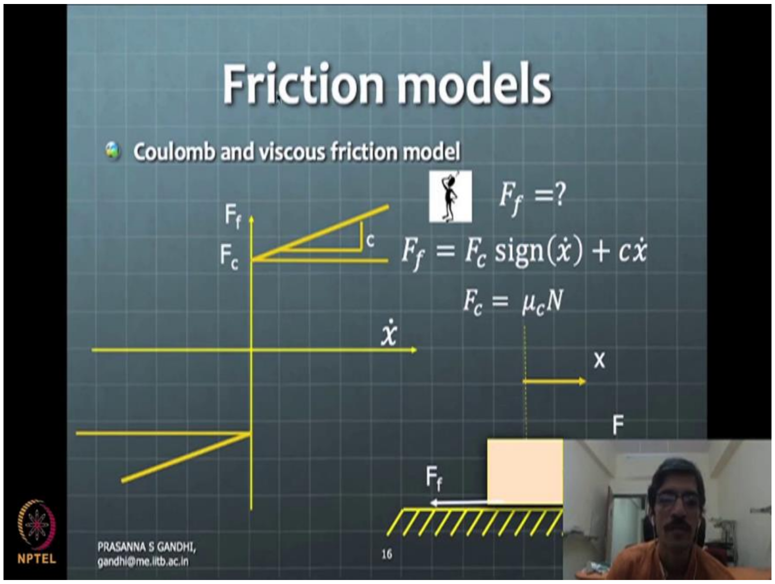

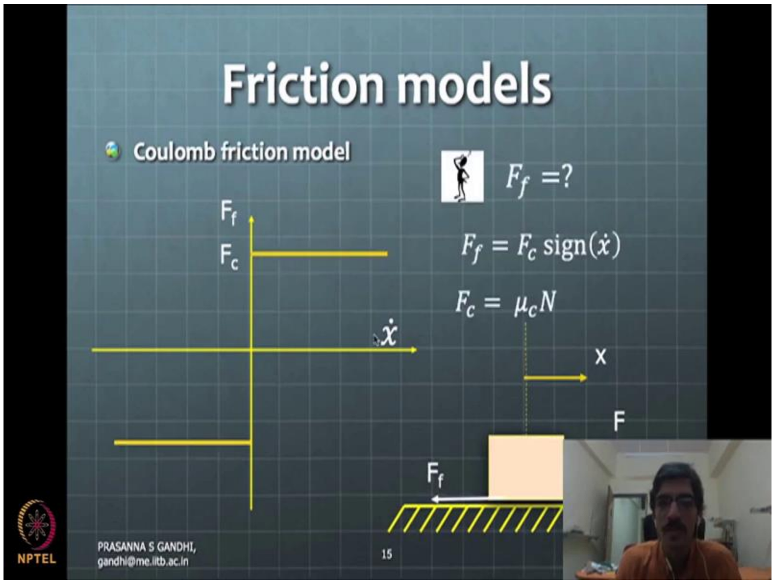

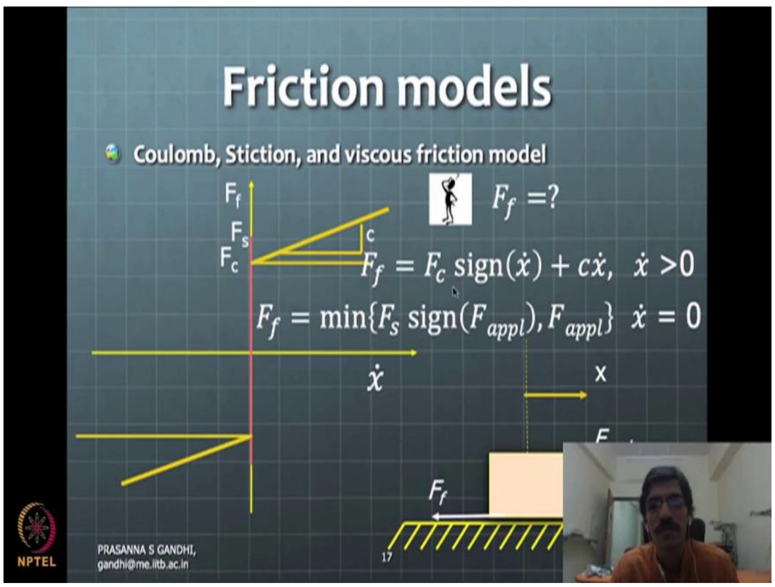

Now, we want to kind of little bit complicated matter. So, now we know already that you know this ẋ is also going to affect your friction force, because we know that viscous component we have talked about in the motor model, that bearing has some viscous friction in it. So, typically, people normally consider only the specific viscous part and like they do not consider coulomb part because viscous part, use nice ẋ relationship.

Without this coulomb component, if ẋ that relationship is linear in nature. It does not have this non-linearity. So, you see this like a sudden jump from here to here for the friction as ẋ changes is some kind of a non-linearity in the system. Or if you see in terms of this signum function, signum function itself is a nonlinear function that is why like this becomes a nonlinear equations. So, your system becomes nonlinear with these kind of functions.

So, again one can think about what is this friction force? You pause here again think about and write the equation considering now this viscous part also and you will come back to this. So, this will find now that this friction force is equal to now this $\{F_c \text{ sign}(\dot{x}) + c\ \dot{x}\}$. So, this is a additional component corresponding to the velocity or viscous friction that will get added. So, this is how you will represent this coulomb plus viscous friction kind of a model. Is that sufficient? What do you think?

Think about this part, is this sufficient yet? You see, you remember we have done this small; I asked you to kind of take out your mobile and keep it on the surface and apply a force to control. So, when you apply a force does it starts start moving immediately or you feel some stickiness, especially if you have cover on your mobile you will feel that stickiness with that rubbery kind of material. So, that stickiness comes as an additional kind of a friction part in the model. How do you consider that it is called stiction, that is there only to begin with?

Once emotion starts, then you do not feel that much resistance, that is called a stiction or a stickiness in the system. Stiction is a term that is used for that. So, with the stiction elevation how this picture for the model will change this picture is how do you represent stiction in this part. That you can think about and draw that and then like we need to see what is a corresponding representation for the equation.

So, this is a stiction, so you have momentarily a very large kind of force called F_s coming up here and then like we decreases suddenly and you have this again our coulomb and viscous friction continue. So, this is little more enhanced model with the stiction considered. Now again, how do you represent this into how do you represent stiction now in the mathematical form pause again for a while write your own equations.

You create that kind of space in your mind to understand what is being said here otherwise, like you just read these slides or go through this as if you are, recording go through this recording as if you are just kind of listening to some video things may not make sense unless you do some stuff. So, please write down what is that you think as equation for this this case and then like we will see this equation coming up.

So, now this F_f will be F_c signal $\dot{x}$ plus c ($\dot{x}$) that is what we had previously, but that is now only valid for $\dot{x}$ is greater than 0. It is not valid when $\dot{x}$ is equal to 0. So, what is that is valid $\dot{x}$ is equal to 0? What do you should represent there? So, now, $\dot{x}$ is equal to 0, you cannot use signum of $\dot{x}$, but still there is a change in the sign of this F_s value if this is positive here F_s is negative here, how do you kind of consider that what kind of thing that you can use?

Think about this again. See how can you represent that and then like see this. So, what you need to do is you need to take F applied here now, see the system is not moving but you are applying force if F applied itself is 0 then again this F_f is going to be 0. But if F applied is equal to value which is more so F applied is 0, then signum of F_s signum of F applied we will have some kind of a no strong like a positive value.

So, till the time F applied becomes equal to F applied becomes equal to F_s, F applied is what is going to kind of be the friction force. Once F applied becomes equal to F_s. So, F applied is equal to F_s then like this friction force will raise reaches limit value and things will start moving, once the motion happens then w you switch from this condition to this condition and then w the friction value is now coming down to suddenly to F_c.

So, that is how this would be represented in the model form. So, till the time F is applied F applied becomes exceeds F_s value by something and then the system starts moving $\dot{x}$ changes its value then you get into this equation before that you are into this equation. So, F applied has to go beyond F_s to get into this form that is what it means.

So, you see this model representation. So, one can simulate this system also and then check out simple system like a mass see on the surface with this kind of a thing how the system is going to behave one can check. By the way, think about this in simulation can you represent it like that and hope to simulate or do you see any difficulty that may happen here?

So, think about these issues here not very trivial issues that then you may expect some difficulty when there is this kind of non-linearity or discontinuousness in the system, it has something to do with some basic conditions that need to be satisfied for the solutions of differential equation to exist. So we will see that. Now again, is this enough, think about whether this is enough again.

So, this captures most of the things but, the main thing that is happening that this F_s going from suddenly becoming F_c that is somewhere it is not, in the nature the things are not so sudden. So, for this to understand little more about this we need to like really go into macroscopic level of what is happening to this, how this friction is coming up in the first place.

And understand later on different-different phenomena that are happening there and then we can enhance this model further to see that, this F_s does not go to F_c in that kind of a sudden fashion here x equals 0 and x is greater than 0 and some sudden value coming up here, but there is like more like a smooth pattern that may be happening here. So, this smoothness that is happening here is the next thing to capture how the smoothness will come is here.

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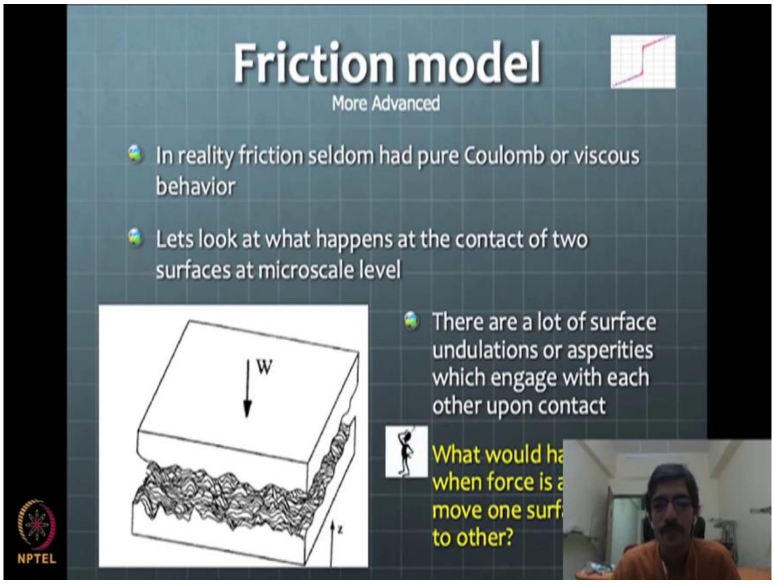

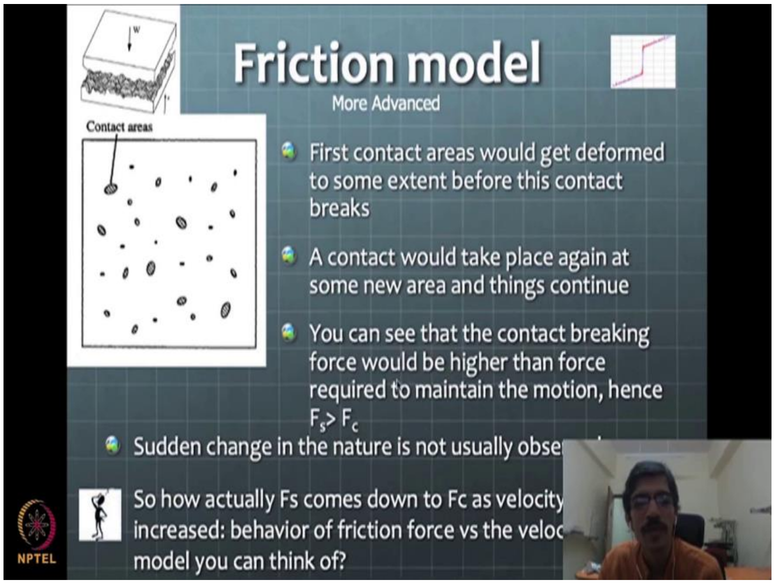

You can see this has at a microscopic level you can see some differences that is this is the expertise of these are in contact with each other undulation or asperities on the surface that engage with each other and what will happen. When the force is applied in the sidewise direction in these asperities on the surface will start deforming and then they will start kind of like breaking and at some other point contacts will happen or some other point or expertise will start coming up.

So, when they move related to each other like energy right now, this is a picture of the contacts of asperities that are happening in many different places as I start moving this contact will break from these places and it will start making up some other places. So, the contact areas will get deformed to some extent. So, this kind of phenomena that is happening will make that transition to be not so, sudden it will make that transition to be a little bit smoother.

So, this also explains like this breakage of the contact happens and then now new contacts come in like again they will break again new contacts with coming again they will break that is a kind of a thing that will happen continuously here. But once first time the contacts are broken like that is the highest form that you need for breaking the contacts next contact is made and like immediately it is broken it is not allowed for the for the settling of the system to in that contact position. So, that is why that force will be a little lesser than initial contact breaking force.

So, that is what you need to see that and slowly as a velocity increases you will find is this force will come gradually down. So, this F_s is coming down to F_c is in a gradual manner. So, you can use conceal some model for the gradual change and that will typically be based on some experimental observations. So, you get an experimental profile. And then like based on that experiment profile, one can kind of think about this, this kind of a model, what kind of model can fit his profile.

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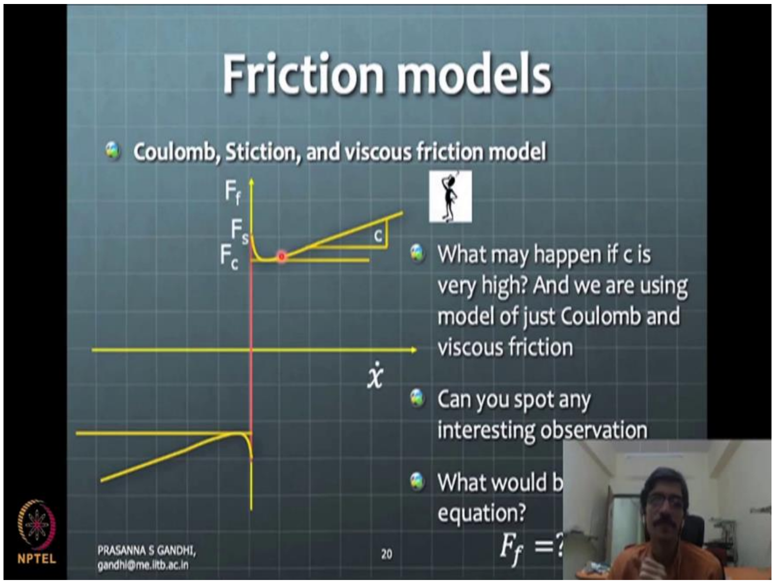

So, this is an experimental profile for example the F_s will come down to F_c and then like we will have this slowly the coulomb the viscous component the friction kicking in to kind of get a get this again higher value. So, this is a kind of a gradual change that will happen in the actual scenario. In some systems metal to metal contact this may not be the key situation.

So, you need to kind of see this as a situation where is it is talked about in general for the entire model of the friction now, some phenomena may be dominant in, some cases some phenomena may not be dominant in some cases. So, we need to consider that appropriately say for example in your system like distinction and F_c will be there but this change may not be, so gradually then you it may be more of a sudden change also possibility.

So, the slope of this coming down here maybe a very large slope here. So, that may be possibility and then it will rise again in some cases they will come like a very gradually down and then go up like that you may have different-different possibilities depending upon the surfaces and materials that we are using, but I am giving you all these different kinds of a model possibility.

So, that one can choose one of these models suiting to the system at hand and that is how one can think about and get appropriate model to appropriate details you may not be see if you are always using motor at little bit higher speeds, you are not worried about this part at all you are not stopping and starting motor multiple times and you are not interested in very fine positioning into the extent.

See if you have a positioning system you would definitely stop, but you are not interested in fine kind of a positioning, then you do not worry about this part. You leave this part to your integrator module to kind of handle if at all. But if you want to kind of capture this small integrities you have these models available and they will be useful in the cases when you want to go extremely high precision positioning considering the friction into account.

But the scenario will not remain for so long because as a system where centers, these curves are going to shift or they are going to change. Now, you think about this part like what will happen if this c is very-very high and now what is the governing equation for such a kind of behavior? So, then you need to know, what is how gradually you are coming down from F_s to F_c.

Whatever that gradually you are coming down that part will be factored into your equations in some way. And then so assume say some Gaussian kind of a way of behavior coming down. So, you think about if it is a Gaussian variable, the equation of Gaussian curve if you do not know you just Wikipedia may search karo, you will get the equation of Gaussian curve.

It has some e to the power negative $\dot{x}^2$ square kind of a relationship in this case coming up something like that. So, if you want to use that kind of model to go from F_s to F_c. So, you are on the part of the Gaussian curve coming down, we are not worried about a negative part of the Gaussian only positive part of from F_s high value we are coming down with a Gaussian variation to F_c value that is what we want to do.

Then how do you kind of write the equations of friction here, so think about that. What is this F_f going to be under such case, pause, write down your equations and come back.

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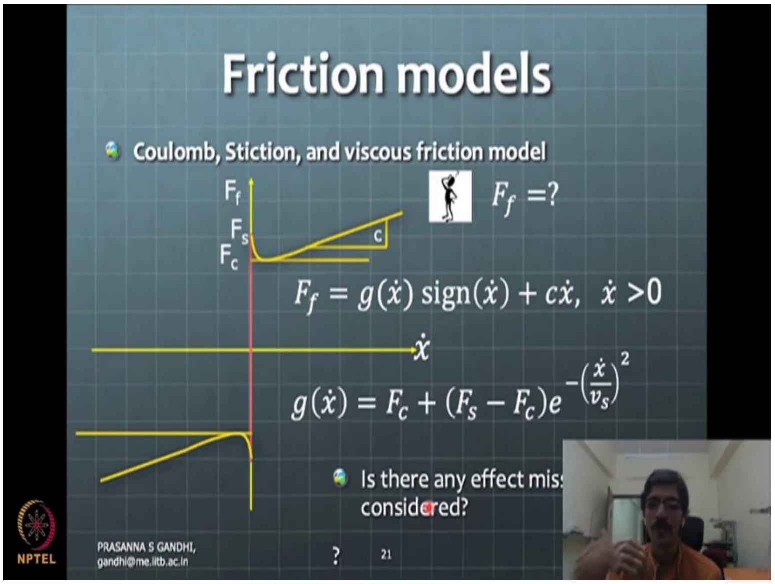

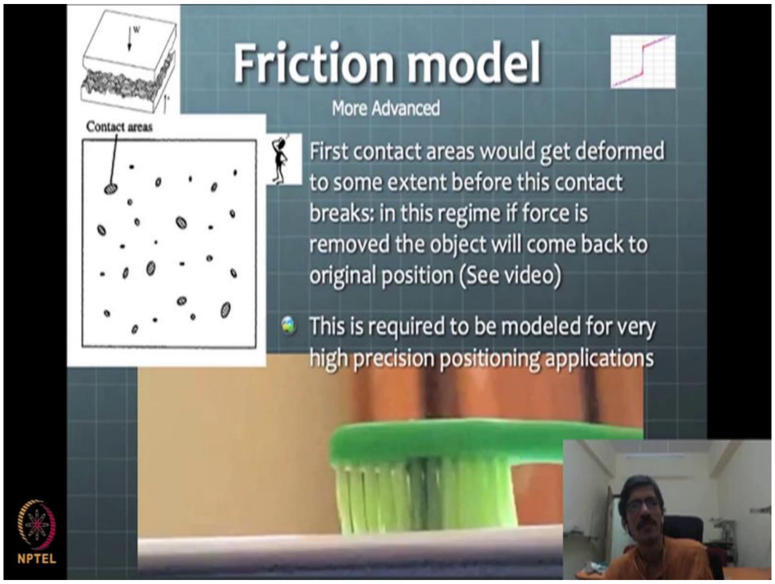

So, what is F_f is coming up here is you see this some function additional function that is introduced here is $g(\dot{x})$. Now this $g(\dot{x})$ function will come get a force from F_s down to F_c value and then you have the c $\dot{x}$ come as it is. And then this $g(\dot{x})$ term is like is given in this fashion.

You have some F_c some kind of like level constantly just shifted to kind of a Gaussian thing and then F_s to F_c difference is here, like these F_s to F_c difference and that is kind of coming becoming lower and lower as $\dot{x}$ increases and then v_s is in characteristics velocity called Stribeck velocity that is be used for, this is like a fixed property of the two surfaces and as these v_s is high or low you will change the slope of this coming down nature.

That is what will happen here. Now again is there further anything to be considered and the answer is, yes, if you think of a surfaces like a very soft rubber. So, very soft rubber when you are you are having these two surfaces in contact like very soft rubber will first have some motion, the trouble will get deformed kind of motion will be there and then the motion of the two surfaces related to each other and all this friction and other effects are going to happen.

So, this deformation that is happening before this other thing come into picture is what is missing right now. One can understand that based on the simple video that is there. So, you can see this is a toothbrush which is running on the surface. You need to carefully observe this area now the direction is reversed.

So, the bristles are getting deformed in other direction and then they start kind of making and breaking contacts on this paper substrate. So, look at this video carefully, it is moving in this one direction here, you can see that there is some kind of a bristles which are making and breaking contact up here and then they are kind of, when I reverse the direction they stop and then unless by my deformation happens beyond certain limit they will not leave that contact point.

So, this is a kind of a phenomena that is going to happen. When you reverse the direction here you see that I have some deformation happening without anything happening at this and in this position if I leave this brush, if I leave the force the brush will come back to its original state because this are cantilever kind of a deformation, they have some stored energy that will push these things back.

So, this is a kind of a phenomena that is going to happen for the cases where you have very very soft surfaces are in contact with the hard surface for example and these needs to be getting modeled for a very high precision positioning applications. So, I gave you this example of soft surfaces, but even in the hard surfaces, if you are interested in nanoscale kind of a precision in control, then you will need to have these effect also taken into account.

So, that is what is important. Nanoscale is difficult, little bit difficult to kind of realize because there are some uncertainties that will govern this process, but people have developed these kind of models in the literature use as a mechatronics engineer, typically we will not have to use this kind of a depth of model. Rather for a very high precision positioning, more better solutions exist in the form of these compliant mechanisms or like Piezo based system.

So, will not use probably this advanced part of the model for the friction. But I mean, this is good for you to kind of know, and it is, I am not expecting you to kind of think about the model for this friction. I will just give you the model in the next slide to see and think about how people have captured this mathematics in a very elegant way into some of these model equations. This is maybe a development a few decades ago kind of thing.

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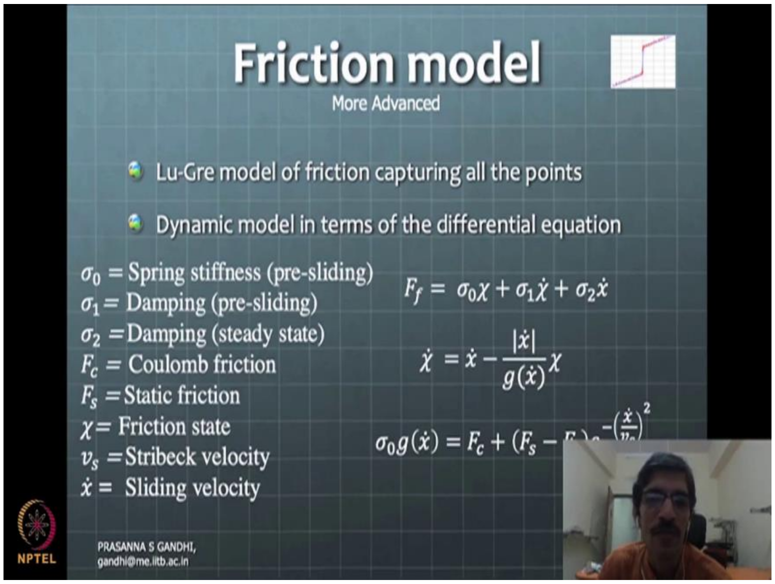

So, this is a model that is called Lu-Gre model, for two universities, Lund University and the Grenoble Institute. These two, some joint might have happened and it is called Lu-Gre model in depth of fiction and it captures techno all these effects that what we are talking about so far. And how it captures and what how these states are legally defined in this kind of fashion and all I mean, it needs a good amount of mathematical deciphering of things that people have proposed this model.

So, we may you do that mathematical modelling like really intrigued by some kind of mathematics, challenge for you to kind of figure out how this is representing, what effects and how this is really, the meanings of all these variables are given here. This χ is a friction state which is corresponding to these deformations are the bristles that are that, we show for the brush or that kind of a small deformation and then is corresponding spring stiffness is σ_0, in the pre-sliding regime this σ_0 is the spring stiffness and this is a force that will come into picture when you have $\dot{x}$ is equal to 0.

So, think about this these effects and this model how it is capturing what we have said the intent of this pistol based deformations and further continuation into normal fiction regime that is sliding regime. So, think about that, it is a good full for your brain, all analytical sharp brains here. So, give a nice thought and understand if you have any questions like maybe we will discuss in the discussion session. So we will end for now.