

Design of Mechatronic Systems
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Lecture 29
Control formulation: Regulation and Tracking

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General form of Equations
“Mechanical Systems”

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau$$

$$c_{kj} = \sum_{l=1}^n \frac{1}{2} \left\{ \frac{\partial d_{kl}}{\partial q_l} + \frac{\partial d_{kl}}{\partial q_j} - \frac{\partial d_{lj}}{\partial q_k} \right\} \dot{q}_l$$

Important mathematical properties:

- Matrix $D(q)$ is symmetric positive definite
- Matrix $\dot{D}(q) - 2C(q, \dot{q})$ is a skew symmetric matrix

These properties hold for mechanical systems rigid bodies and time invariant mass

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Energies in Mech
Electrical domain

<ul style="list-style-type: none"> Kinetic energy $T = \frac{1}{2} Li^2$ Inductance Potential energy $V = \frac{1}{2} CV^2$ Capacitor $C = \frac{\epsilon A}{g}$ g = gap here Resistive loss 	<ul style="list-style-type: none"> Kinetic energy $T = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$ Motion of mass Potential Energy $V = \frac{1}{2} kx^2$ Spring $V = mgh$ Gravity Damping loss
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Just give you some glimpses here for generalizing some part this Lagrange formulation for electrical domain also. So, this is the way kinetic energy and potential energies are defined in the electrical domain. So, if you use this and for a circuit and like do independently for the circuit, you will get this your circuit expressions, whatever expressions you get in cosloss. But the difficult part is when you have this electromechanical coupling between the two domains, as we saw in the motor model, then things are little different.

So, then see here when we talk of this inductance here Li^2 term it is only the self inductance we usually consider. When, we are used to consider in the self inductance only. But when we talk of our motor, it will not have, not only have a self inductance, in fact, the self inductance part we say di by dt term, Ldi by dt term many times we ignore that term.

So, the main important term in that case is going to be your coupling or back EMF giving term. What is a term that will give you back EMF based on like, so that is a mutual inductance kind of a term that we need to consider. And how it is to be considered we will discuss separately when we talk a little bit more about the actuators and motors and things like that.

So, we will not get into that. But you can give a thought to that based on our expressions for back EMF, how this mutual inductance can be considered in the kinetic energy so that finally when I derive the Lagrange formulation that time I get appropriate expression for equations of dynamics which is matching our Newton's formulation dynamics. So, that is, so one can kind of like see this application that opens up like if you use these kind of expressions that opens up your Lagrange formulation application to this electrical or electromechanical domain systems as well.

So, we will consider only this kind of inductance part later, where we have to, see we have to understand some more fundamentals about the electromechanical parts before we delve into this. So, you need to revise some of your electromagnetic, the magnetic circuits and things fundamentals of that sort to kind of get to really how do we consider these in different-different kind of cases. So, we will do that later.

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The slide is titled "Regulation problem" and features a diagram of a 2-DOF manipulator in a 2D Cartesian coordinate system with axes X^* and Y . The manipulator consists of two links of lengths l_1 and l_2 , with joints at angles θ_1 and θ_2 . The end effector is point P . The initial position is P_s and the final position is P_f . A yellow path is shown connecting P_s to P_f . The text on the slide includes:

- Task 1 → Lets say "we want the manipulator to move P from P_s to P_f ", Not bothered about path
- Think what steps you will take to achieve this point-to-point movement

The slide also includes the NPTEL logo, the name "PRASANNA S GANDHI", and the email "gandhi@me.iitb.ac.in". A small video inset shows a man speaking.

So, we will close this today's part by defining these two problems. One is called regulation problem in control. So, these are the definitions for a future kind of a control discussion that we will do pretty soon now. So, these regulation problems are the problems where you are given this, like starting point and end point, final point. So, P_s and P_f are given. So, now, I am showing it with the example of this to 2-R manipulator, but in general this can be for 1-R also. For θ , it is given as θ_1 initial and θ_1 final. It can be as simple as that.

So, you are given these two points in terms of the vector state of degrees of freedom. So, it can be also the general, what do you say, not just generalized coordinate, but they are derivatives also. You can give, you can be given a full state of system at initial point and full state at the final point.

And then you are required to go from like initial state to the final state. So, we typically consider for these manipulator kind of systems that initially it is starting at 0 velocity from the starting point and like it ends with the 0 velocity. So, typically the velocities of start and end points are 0. So, that may give you the generalized coordinate velocities also to start and end to be 0.

So, you can see that the, this problem does not give you any information how do you want to move from here to here or in other words the time expression for this θ_s to go from here to here. The desired, like I do not care like whether θ_2 in between is one value or the other value. It can be any, taking any value in between, but I want like the finally the expression should go, I mean, the value should match. So, finally, when it goes here like the point P should fall in the P_f .

So, we are not bothered about this path to go from here to here. When I say path it becomes like a time trajectory for θ_2 . So, we are not bothered about like what trajectory of time we follow for θ_2 and θ_1 to go from this place to this place. So, that those kind of problems are termed as regulation problems. We are just interested in the initial position and final position that is it.

So, one can now see to go from here to here like you just evaluate, at this position what are the θ_1, θ_2 , when it goes at this position what are the θ_1, θ_2 . And between now so say this is θ_{1s} and θ_{1f} and then like we have similarly θ_{2s} and θ_{2f} . And then we say for going from there to there, I just need to kind of drive my θ_1 from θ_{1s} to θ_{1f} and θ_2 from θ_{2s} to θ_{2f} and my job is done.

I then like I say I will first move θ_1 to that position, θ_{1f} position and then after that I move θ_2 to who cares. I mean, you can move them together, you can move them one after another or whatever, but you can take it to the final. So, that is what is a regulation problem. So, you can, typically, then for saving time we will move them together like to start both the motors simultaneously and then you will reach the final point. But we are not bothered like what is the path it takes to go from here to here.

(Refer Slide Time: 07:44)

The slide is titled "Regulation problem" and features a diagram of a 2D coordinate system with axes X_1 and Y_1 . A starting point P_s and a final point P_f are marked. A path is shown from P_s to P_f . Joint angles θ_1 and θ_2 are indicated relative to the axes. The slide also includes a list of steps and mathematical expressions for joint angles.

Regulation problem

Steps

1. Find out desired joint angles corresponding to P_s and P_f by inverse kinematics

$$\theta_s^d = \theta_{1s}^d, \theta_{2s}^d$$

$$\theta_f^d = \theta_{1f}^d, \theta_{2f}^d$$
2. Command your motors to move joints simultaneously go from starting point to final point

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PowerPoint Slide Show - [10, Mathematical Modeling]

Regulation problem

Questions

- What is meant by command?
- How much should be the motor torque? What does it depend on?
- What if we want robot to take P to the final position faster?
- How do we make sure that after reaching final position links stay there?

Answers proper de...
understanding cont.

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So, you have found like in this what I was talking about finding these initial and final paths for the, initial and final values for the generalized coordinates and command your motors to go either simultaneously or one after other or whatever ways to go from starting to the final point.

So, like how do you kind of this command your motors. So, this is like topic of further discussion for control. So, we will see how we can control like. But in regulation problem is when you want to go from one position to the other final position or one state of generalized coordinates to other state of generalized coordinates without bothering what are the in between states. So, that is our regulation problem. So, we will further, we will discuss this further when we talk, start talking about control very soon.

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PowerPoint Slide Show - [10, Mathematical Modeling]

Tracking Problem

Task 2 →

- Lets say "we want the manipulator to move P from P_s to P_f in a straight line (desired trajectory)"
- Think what steps you will follow to achieve this *trajectory tracking*

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And then other kind of a problem that we need to understand is tracking problem. So, in tracking case, you actually want that my point P should go from this position to this position along say circular path or along a straight line, something like that we give. Then what it amounts to defining the desired values of θ_1 and θ_2 is they should be at exactly some place in the, along the path in time. So, do not confuse like if you have single link manipulator, what is the tracking problem there. There it says path is always going to be circular. So, how can I define tracking problem?

No, no, it is not that. It is like the values of θ which should be dependent upon the time in between, so I want to go from this point to, say for example, for a single link, forget about this link, if I want to move from this point to say some other point in the circular path there also I can move like slowly there or fast there or say starting acceleration and then slowing down and go there like that I can have many different time trajectories for going from θ_{1s} to θ_{1f} only for single link manipulator.

So, that also is a valid tracking problem, although I am moving on a circular path only, because there is only possibility for this single link manipulator. When you talk of two link manipulator it is little easier to understand, because there are multiple paths that can be possible which you can see. So, see what we are seeing here is like space kind of a variation of the path, but time variation of the path is not seen when you kind of look at this to 2-R kind of a manipulator case or one, single link manipulator case I would say.

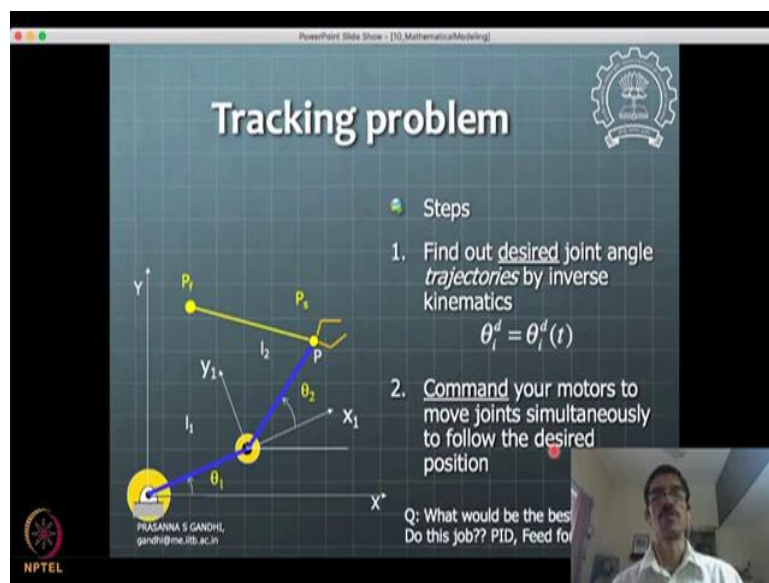
So, now, for doing this you can have multiple paths possible. So, say let us say, I am defining like know this path along which I want to go, so not circular path, but I want to go along by a straight line kind of a path. Now, moment I say that then I have to define at every point on this path then θ_1 and θ_2 gets defined. They will have some kind of specific value only. So, and then we should have that this θ_2 at this point of time θ_1 should be this and θ_2 should be this for to, for my point to be on the path. That kind of definitions would come.

So, we would in general get definition for θ_2 in terms of time and θ_1 in terms of time to go from θ_{1s} to θ_{1f} . This, starting and final positions are still same. But now I have to define a time trajectory for θ_1 and time trajectory for θ_2 to make sure that I move along this path alone. I do not kind of deviate from this path. So, that is what is a tracking problem. So, this is called a trajectory tracking kind of a problem.

And this problem has been explicit time dependence, because θ_2 desired is a function of time here or θ_1 desired is function of time here. Both are a function of time. And in the previous case θ_1 desired was just a final value of, like final value θ_{1f} and θ_2 desired was θ_{2f} . I did not have any like consideration for like what happens between like from 0 to or 1, initial position to final position.

So, that is difference between the regulation problem and the tracking problem. The tracking problem will have a explicit time dependence. Regulation problem will not have explicit time dependence.

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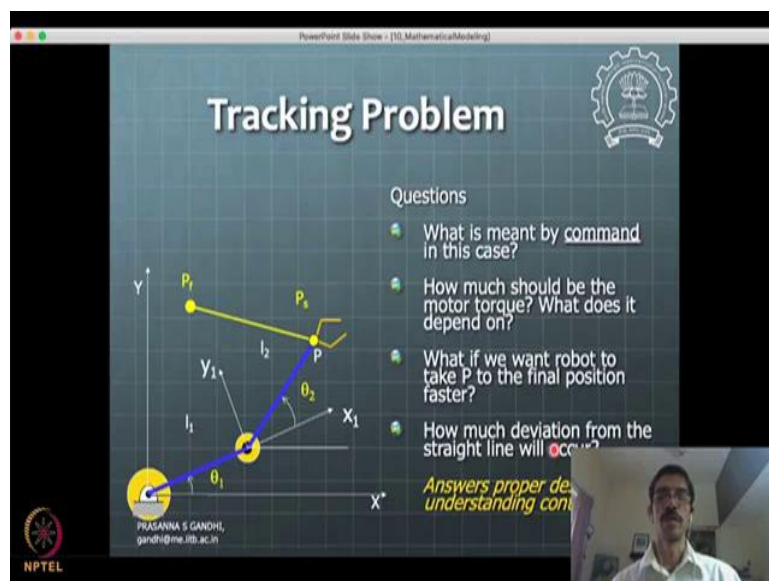
Tracking problem

Steps

1. Find out desired joint angle trajectories by inverse kinematics

$$\theta_i^d = \theta_i^d(t)$$
2. Command your motors to move joints simultaneously to follow the desired position

Q: What would be the best way to do this job?? PID, Feed forward?



Tracking Problem

Questions

1. What is meant by command in this case?
2. How much should be the motor torque? What does it depend on?
3. What if we want robot to take P to the final position faster?
4. How much deviation from the straight line will occur?

Answers proper depends on your understanding of the problem.

So, these are the steps you will typically follow as I said earlier. Use, for getting this θ desired trajectory, you use inverse kinematics of the manipulator. Inverse kinematics you will get from

kinematic relationships. And again you have to kind of use some control to do this motion along the path. So, these two problems are there in the, typically in the control domain, and we will see how we can now make use of the fundamentals from the control domain to get to these problems. I think we will stop here for now.