Design of Mechatronic Systems
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Lecture 33
Concepts of Feedback

So, in the last class we have seen concept of how do you get going about selection of actuators and sensors for a mechatronic system. And now, we will just get more familiarity developed into the concept of feedback, the sensors that we use in mechatronic systems are mainly for feedback. Now, how do we process this feedback or sensor input to further drive the control application or to get a desired kind of application going that is what we are going to see now in the next part of the course.

Although this might be somewhat familiarity, familiar to you, you have already, some of you might have already seen the feedback in the context of your mech course or any other automatic control fundamental course, it is important to get a firm grip on this concept of feedback, so as to kind of use it in the case of the systems that we have not yet been exposed to for example, this nonlinear systems or Lagrange formulation based systems or any even linear systems, but added with the non-linearity of the friction or backlash.

So, in such cases how do you think about using the tools that you have learned already say for example Laplace transform, Laplace transform if you say they are not applicable, because they are valid only for the linear systems case, so then how do you go about doing such a analysis? So, that is where like we need this fundamental understanding of this concept of feedback, so that you can apply it with the tool of this ordinary differential equation to many different kinds of systems, I mean maybe it will linear or non-linear. And get to know what is a way you can think about in the case you are not given that the system is linear and the tools in the linear domain what you have learned are probably not applicable. So, let us develop this understanding for that now.

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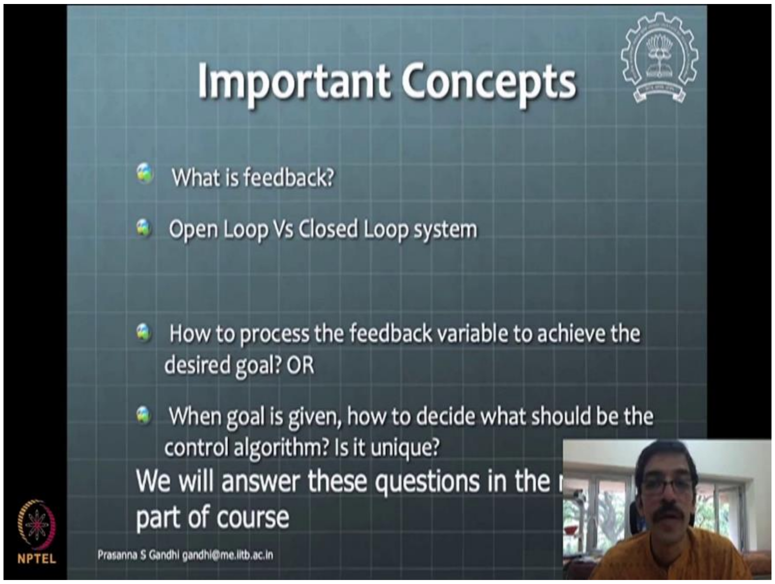

If you see what we are going to see is basically the what is feedback, how do you kind of what is called as feedback and open loop versus closed loop kind of systems, you may feel this is like a trivial thing, but nonetheless you should go through some part of it or make sure that all the parts of this are understood, so that you can apply them in the cases of nonlinear systems as well.

So, how do you process the feedback variable to achieve desired goal? That is another kind of a very, what you say more like a design question. So, there are many different ways in which one can process feedback variable who achieve what goal that is given to you, some part we have already some kind of a feel based on our intuition or based on common sense, so to say, we know how do you process this variable, but how do you convert that understanding into some sense of mathematics and use it further, this is another question that we need to bother about, as we go along we will start slowly unfolding answers to this kind of questions.

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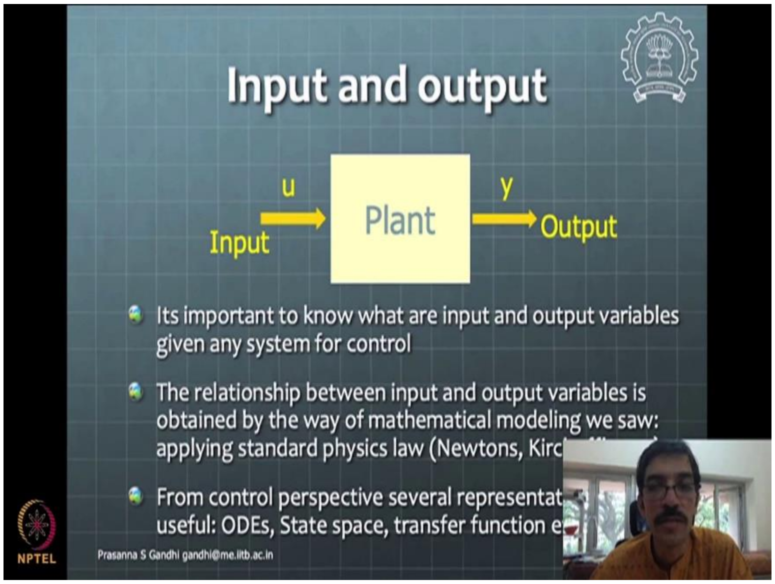

So, now let us take an example, but before that we need to be sure about like say given a plant, what are the inputs and outputs, you should be very sure about okay this is my input or this is my output, based on input and output like the kind of development of control and its properties may have a lot of changes that will happen and then establish relationship between input and output which is what we saw in the mathematical modeling will give you that.

And once we have that relationship and we know these inputs and outputs we are kind of more or less ready to kind of dwell into this development of control algorithms and there are several representations that can be possible for system, but right now we are going to look at mainly this ordinary differential equations kind of a representation which is a most fundamental form for the representation, the state space transformation they are valid only for the linear systems case.

For nonlinear systems you may have a state representation of a system that we will consider later for control development in a different way. But ordinary differential equations are the most fundamental form of representation that one can use for analysis of or propose any control and then do the analysis or synthesize the control based on the equations, how do we do that? We will see soon.

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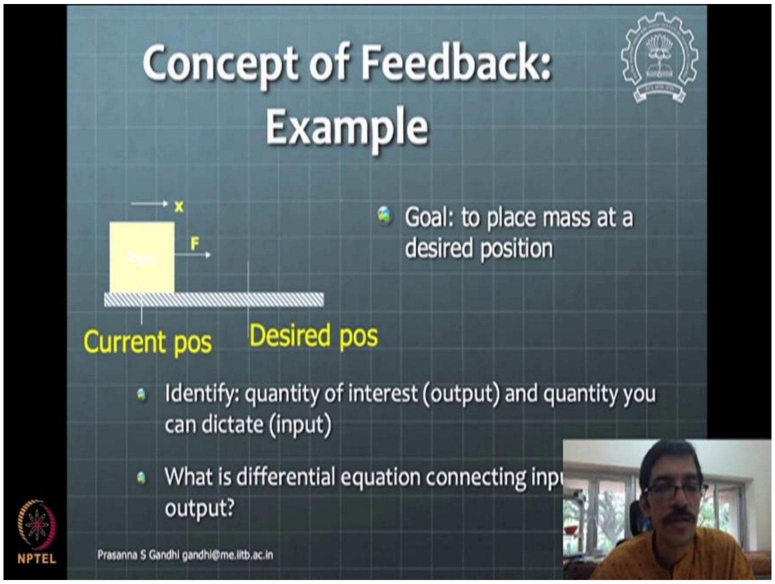

So, let us take this example of a block lying on the, this is our standard example, this block with mass m on the surface horizontal surface and it is currently at this position and it now this F is applied on the block. So, let us assume for now that there is no friction between the block and the surface and under such scenario we want to drive this block to the desired position shown.

So, what is the quantity of interest here? Quantity of interest is x, the desired position. So, position is a quantity of our interest. So, x is a quantity of an interest which is output and quantity that you can dictate is input to the system is force F. So, force F is can be any value that we want to apply. Of course, I mean when we talk of mechatronics system is F we have some certain saturation limit, but right now theoretical discussion we can hear okay whatever value of F is desired you are able to apply. So, this F is something that we want to dictate in such a way that we reached the final desired position.

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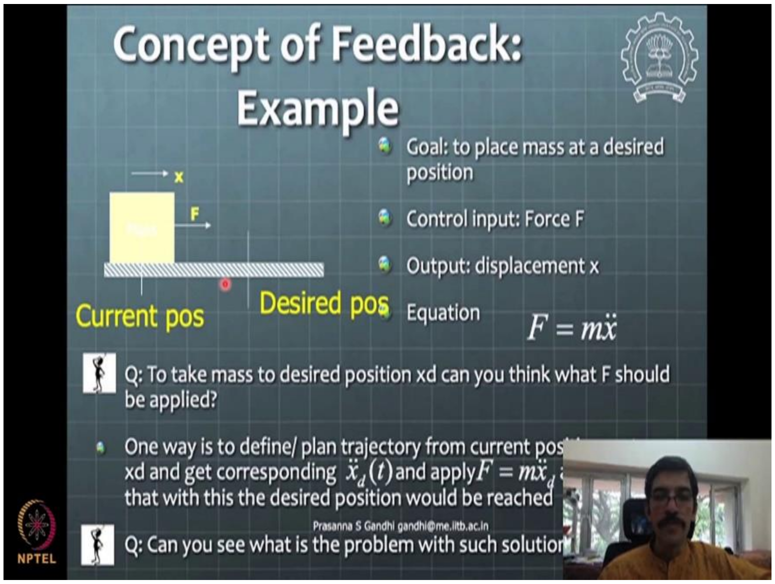

So, we see first the relationship between input and output which is standard F is equal to mx double dot. And now under such relationship if we want to kind of pose this question, to take the mass to the desired position x_d, what should be the applied F? So, there are many solutions to this problem.

One simple way if you remember we had done some kind of a planning for the trajectory, if you recall the trapezoidal plan for the trajectory that okay I will move, start moving the block in along some kind of a trapezoid trajectory such that the final point of the trajectory is taking me to this desired question. Mind this desired trajectory trapezoid trajectory is in the voltage, I mean is in the velocity not voltage, it is in the velocity $\dot{x}$.

So, you plan $\dot{x}$ dot along the trapezoid such that you accelerate first you move with the constant velocity and then you decelerate and you have reached in the final desired position with the velocity in the final position 0, and we also begin with 0 velocity in the starting position. So, if we plan a such a trajectory we get some $\dot{x}_{desired}$ first and then we get we if we differentiate that you will get $\ddot{x}_{desired}$x

So, if you see along the represented trajectory these are just pulse of acceleration, so is it trapezoid trajectory velocity means if you take a derivative of this then initially the velocity, the derivative will be positive, it is a positive slope of the trapezoid. And then for some time duration, whatever the time duration, then you apply 0$\ddot{x}$, and then you apply $\ddot{x}_{desired}$ which is reverse, so they saw that the trapezoid trajectory will define the desired acceleration.

Now, if you notice desired acceleration $\ddot{x}_d$, which now will be in terms of this pulses which are coming, so these two pluses, one is or the beginning of that trapezoid trajectory and which is positive pulse, $+\ddot{x}_d$, and then the end of the thing is like a $-\ddot{x}_d$, So, these two pulses are to be applied as F multiply they will be scaled by mass m and if we apply this pulses we hope that with these the desired position would be reached.

So, this is one of the mean kind of a trajectories of F that can take you to the final desired position. So, this particular way of doing things where you are really not using any information about where I am at the moment, along the track where if I there is no dependence of F at all on x it is directly function of $x_{desired}$, and $x_{desired}$ is what is I have planned. So, this is important to know that this F has no any kind of dependencies on an x explicitly.

So, under such scenario this F will be an open loop control. So, can you see what is the problem with such a solution where you have just to open loop some kind of force that is applied here I am not looking at what is my current x position and I am still up keeping on applying whatever F I have computed based on the knowledge of the system and I am hoping that I will be taken to the final position. So, this is some kind of open loop based control and we can immediately see that in the presence of friction or in the presence of some other disturbances that are there around here, we may not be reaching the final position. So, that is a issue with this kind of a solution that we have developed.

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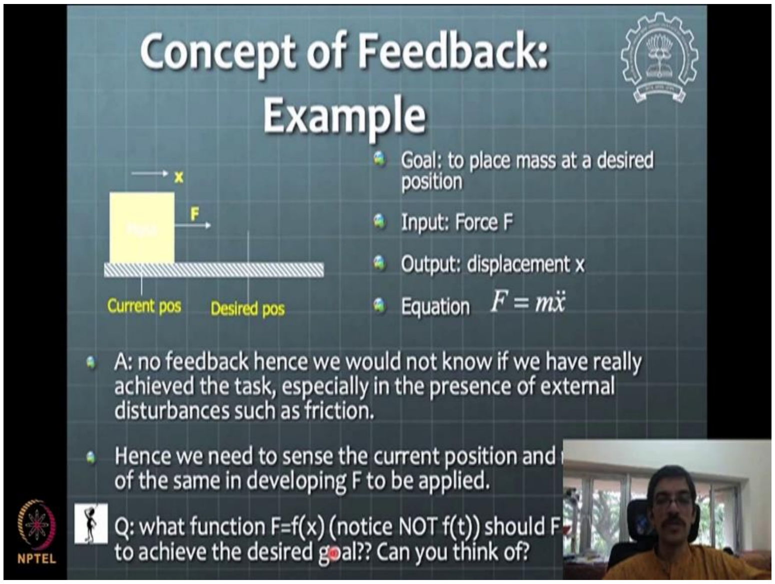

So, now the question comes that look here I mean I mentioned the same thing no feedback, so maybe I have not used to feedback of our input output variable x here. So, we will not know if we have really achieved the task. So, we have given this input friction I have forgotten about it. And then now if we look what we find, we are not at a desired position. So, then we will have to do something.

So, we are not really looking at whether we have achieved the final goal or not in real time. So, we need some sense of current position, make use of the same develop this F to be applied. So, mathematically this question can be posed, see say for example, if you are given this kind of a scenario with your eyes open and you are saying I want you apply force whatever you want to so this block for your mobile for example, x mobile to take it from one position to another question. What do you think of doing?

See what force you will apply? You will see okay I have applied some force, the mass is more and I keep on kind of monitoring as it reached the final position and it reach the final position if not I applied little more force. If I apply a little more force and it has over shooted the final desired position I will apply opposite force bring it back.

Can you see this kind of common, kind of common sense kind of way things will be happening inside your head to get it to the final desired position. So, the question is, how do you now convert this whatever you are thinking to get it to the final desired position inside your brain to give it some kind of a mathematical form. So, this is what is the whole synthesis of control algorithm is all about this question.

So, if I look at this question or if I am monitoring this desired, monitoring this current position how far is away from my desired position, I do apply some things inputs such that I would get go to the final desire position in some way. So, if you see what exactly we would do, suppose somebody asked you okay look there is a competition and you the person who takes this block from current position to desired position the fastest possible way would get gold medal. So, this competition is there.

So, now you are here thinking now like okay suppose I do like you know, so initially if error is high I would apply like large force, but if I apply large force I would go fast but then I have a danger of overshooting these, I need to apply some kind of force which is again reduced as I come closer and closer.

So, this is kind of a thinking that you would carry out in your mind, or in some place I may have to apply I may have to accelerate first and before I reached desired position I need to start decelerating so that I can control myself to kind of make sure that I do not overshoot, with this this kind of things may be happening in your mind about when you hear participating in such a kind of competition.

So, the question is how do we convert this whatever knowledge that we develop as a common sense you will thing into some kind of a mathematical form. So, the question is, what is a function F of some function which is a function of x such that, so this is not a function of time explicitly, it is an implicit function of time through x. So, F should be what kind of a function of x such that we achieve the desired goal. So, pause here and like propose some function, and then we will try to kind of see what we can do with that kind of a function. So, this function should be involving x in some way.

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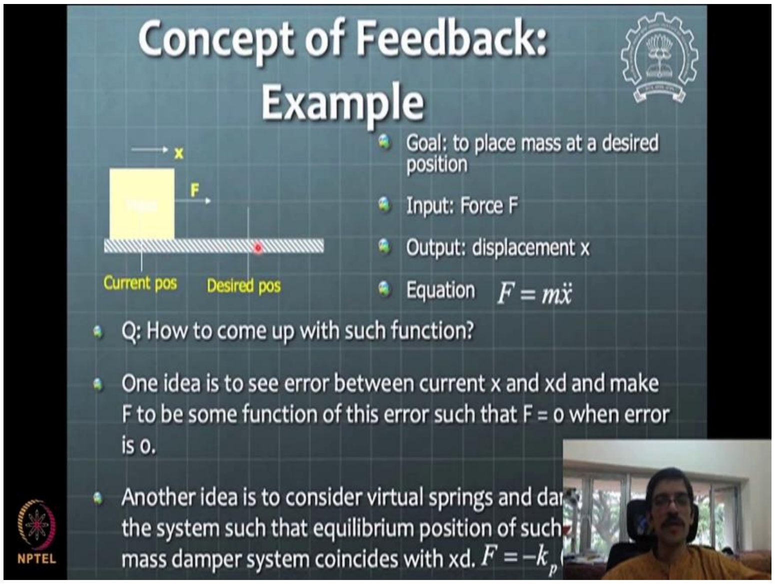

So, the simplest kind of a solution one can come up with is make sure that you see that error as I said, error between current x and x_d and make F to be some function of this error. So, we want F to be 0 when error is 0. So, once you go to the final position we do not want to apply any force, you will agree with me with on that, you do not want to apply any force once I am at the final position. But now if I just do this that apply F proportional to this error, error between current position and final desired position then what would have happened think about that.

Will I really reach the desired position and then F will be 0 there, although F is 0 will I stay there forever? Those kind of questions that we need to ponder over. That is one way of thinking, other way of thinking is to see if I put in some kind of a virtual system of springs and dampers such that the equilibrium position for such a spring damper, spring and damper is my final desired position.

So, that is so this is other kind of maybe philosophical or maybe conceptual thinking that we one do. So, you say okay if I put some kind of a damper and this is like a more kind of physical way of looking at things that I throw in some springs and dampers in a virtual way, they are not there in a practical system, but I am thinking of their putting them as a virtual kind of elements in the system which will finally try my system to my desire goal.

So, this is, these are the two ways one can think about and in either way one has to kind of do the analysis to make sure that we may verify or validate that whatever we are proposing is indeed doing the job that we would like finally to happen that is this our goal, place a mass at a desired position. So, here again this is like a regulation problem, if we are we have discussed the regulation problem and we are not interested in path along with this mass moves, all the straight line in time it can be different paths. So, but we are kind of interested only in reaching the final position.

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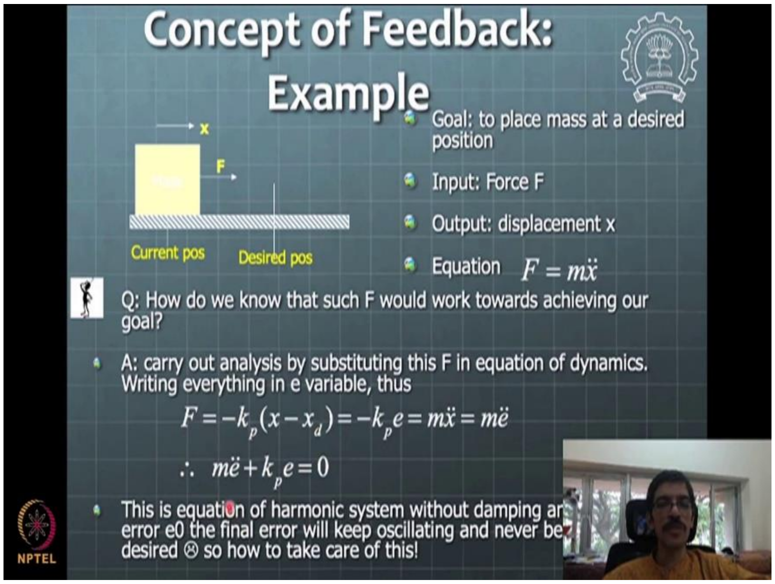

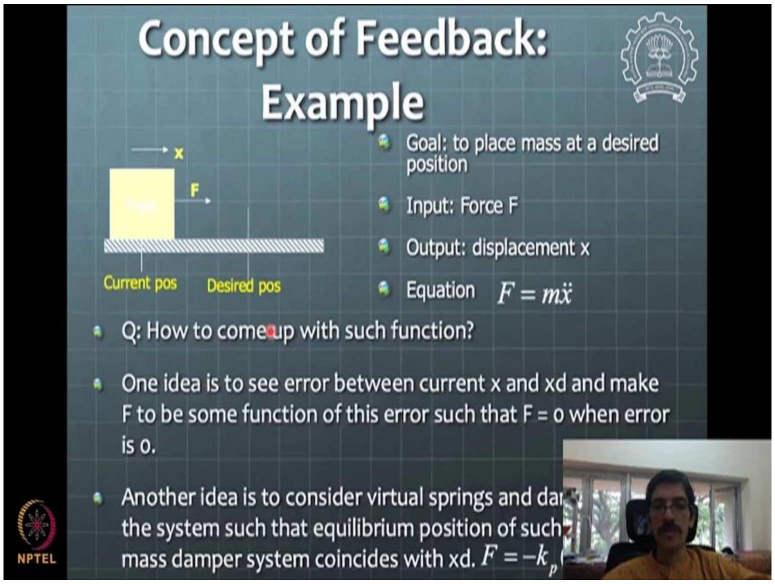

So, let us propose something say we will be proposed some function then we need to know how this F could work towards getting your goal achieved. So, let us propose this. So, this is F that is proposed here, this is some kind of a error multiplied by some proportionality constant which you may be familiar with as a proportional kind of a control action that is happening, so kp times e is giving you the final form of F proposed.

And which when you use this how do you validate that okay this will work or not work, if you see if you put this expression into this equation of dynamics that is going to give us what is going to happen. So, I put this F into the equation of dynamics and we find the equations which govern the aerodynamics this is a very important step that you find what is the equation that is governing the aerodynamics once you propose some value for some function for F, some function for control input if it is given then what is your final error equation that you will get is very important to kind of get to this equation.

Now, by observing this equation we can see here this is a harmonic system, so the error there is no damping in the system, so initial errors suppose it is e0, the final error will keep on oscillating between the values e0 to minus e0. So, the error will oscillate that means your mass is going to kind of oscillate around these desired position continuously.

Can you see that? With this equation that is happening and once I apply this kind of a force like the other force gets 0 and desired position inertia built a system in such a way that it is kind of pushing me, pushing my system at a velocity is not yet 0 here so my system we are getting pushed to another side again it will come back again it will go back the other side it will come back like that it keep on oscillating around the desired position. So, how do you take care of these? Now, you can observe this equation and see what we want this equation desired form for this equation to be.

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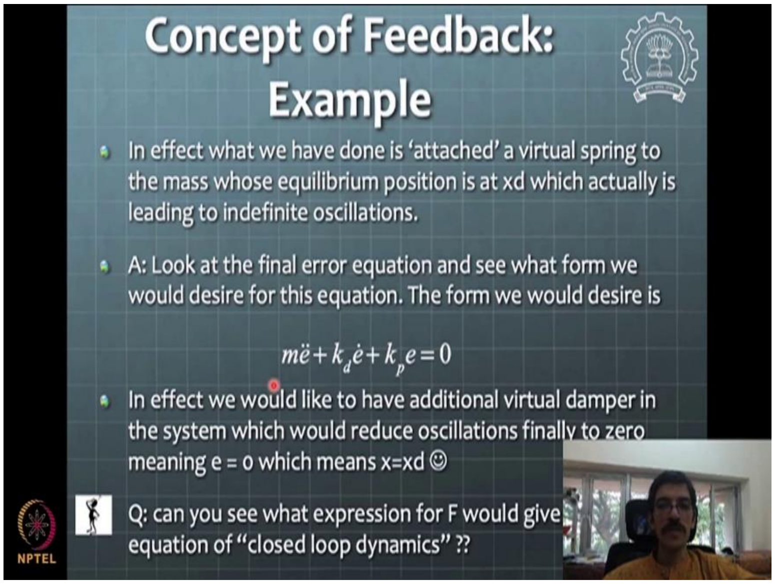

So, we need some kind of additional limit which is a damping here. So, we put that additional element in this desired form, so this is my desired form I would like to have, then if I have I would like to have this kind of a desired form, what is control input which will gave me this form or what is my F which will give me this form, eventually.

So, this is how one has to think about in the space of ordinary differential equations to realize systems, so I am taking this very very simple example to not like you know get a concept across this can be extended to whatever complicated dynamics that you are looking at, see this is a pure kind of a differential equation kind of a form of analysis, so even if you do not know what zeros poles nothing no concepts are known, you can just do this kind of simple analysis to get to the thing.

In the presence of, especially in the presence of friction and other place other kind of nonlinearities if you do this analysis that it kind of give you much nicer insights into the into what is going to happen, then you Laplace transform or other like you know Linear forms of analysis that you might be aware about. So, we can use those forms also, it is not a big it is not that okay we have to use this so what I am saying is that we should be aware that whenever those forms are failing or they are unable to give you enough information especially in the presence of nonlinearities, we need to have some backup tools available to us to carry out this analysis.

So, now you can think about this expression for F is not much shared difficult to think about, it is just a simple mathematics if you work out and you work it out actually and then see what is a form for that F that you get and find that again it is your well known kind of a PD control that you probably are aware about in your mech courses. So, this is what is the main concept now, let us switch, we will see some kind of some of the problems here and then you can think about those do some kind of analysis and if you have any doubts or questions about that we will take up that.

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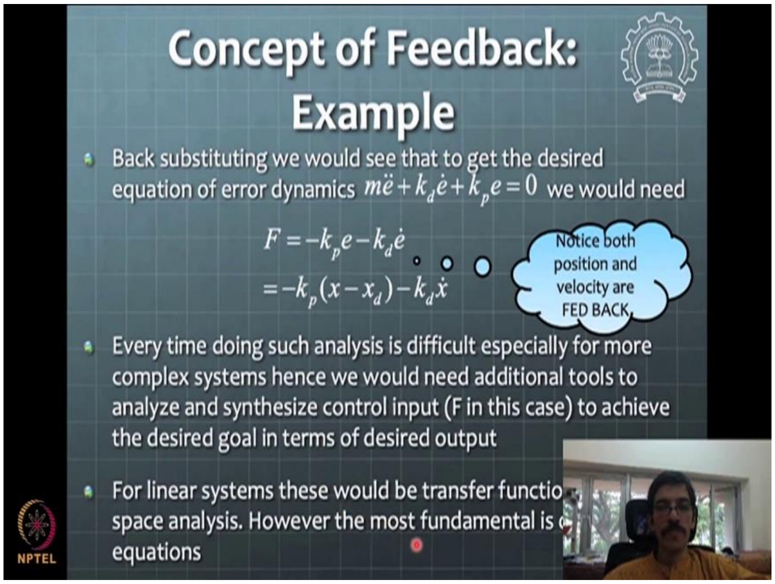

So, you can look at this this is a PD as I have said this is a PD kind of a control feedback. Now, based on, the beauty is that once you have this and this error dynamics known you can play around with these parameters k_d k_p to kind of see that error response is controlled in a way you desired and that that will help you tune k_p and k_d so as to get like to whatever desired response. So, there are so usually you have I, as I said other tools that are available one can use those tools directly or one can come to this depending upon whatever is easier or applicable that is how one can look at this.

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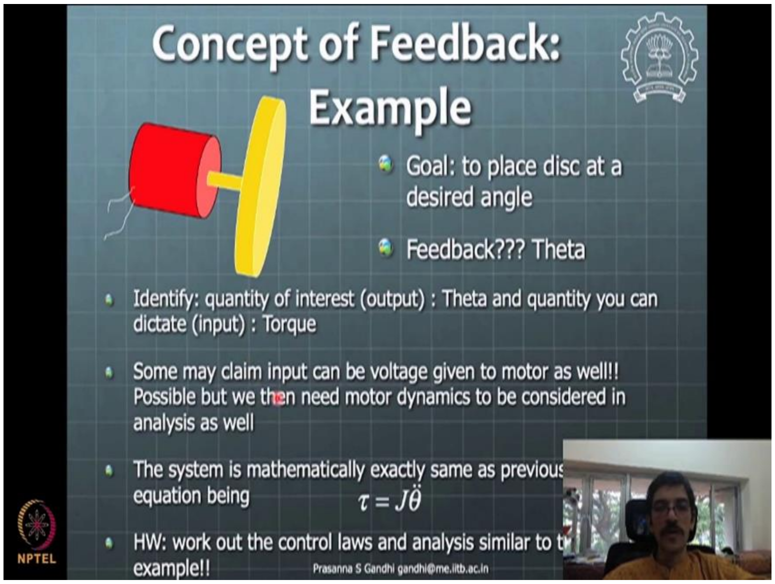

So, now let us see this other problem. So, this is the same problem but now further rotary kind of dynamics, so you have the motor, so this exactly like you know your motor but nowadays this disc is added on top of that. And we have the encode at the back which will give you feedback theta.

Now, our quantity of interest you want to place is disc as the desired angle or quantity of interest is theta and quantity that you can dictate is input is torque, you may like you know want to take voltage also to be voltage as a input as well, especially when you know now your moral of the motor and you are actually applying this PWM duty which is average voltage that is given to the motor from 0 to 100 percent that you are giving this voltage to your motor.

And that can be considered as input. So, when you consider this as input then we can take the motor dynamics part also into account and get your entire model ready. Now, this model without friction and without motor dynamics will be similar to previous set of equations. But now, I would like you to add let us your work at home that you would add motor model to these without considering the inductance in the motor.

So, that your equations are, order of equations remain the same and you can get now that how this voltage and theta relationships are coming up, that dynamic equation involving voltage will be coming or you will be getting and then now for such a system you this like you design now what is a feedback control that you would apply, so that you can do this desired goal of taking this mass from position one to position two finally. So, this is what your standard control a problem could be.

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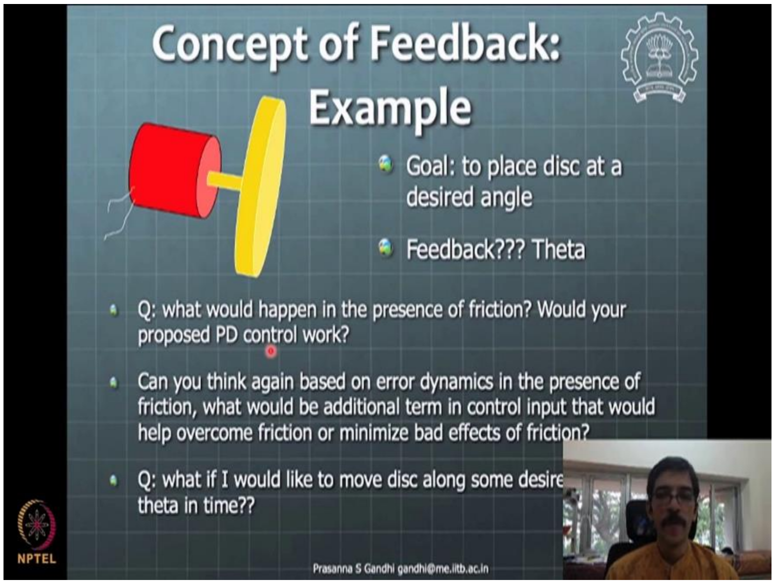

So, now practically when you start doing that, you will find that there is a additionally the friction in the system. So, now the question is how do you analyze whether in the presence of friction, my proposed PD control feedback will work or not? That is where now you see that okay there is tools of this aerodynamics analysis which we saw for basic concept of back and fundamentally od analysis equations they would come handy to you.

So, I suggest that you incorporate some model of friction simple model of friction say to begin with some simple coulomb friction model you can put, into your equations, we have seen on the products of friction. So, if you put just a coulomb fiction model what would be like result of that? So, in the presence of friction can you work out whether my final error will be really 0 or not, we will find that there will be some kind of steady state error you will not be able to compensate.

So, that steady state error will be there in the presence of PD control. And that steady state error value will be some function of your gain and friction value. So, find out what that function is okay what is steady state error means. And once you know that I am like you know this particular control, so this is what is going to happen in practical case also scenario.

So, you start commanding your motor to the final desired position, but whatever we will not go that position integral of the friction So, you can say oh you know this the expression the expression of the error one can get some kind of idea that okay if I tighten my gains further I may be able to go to the final position. But that is about it you will not be able to kind of make that 0.

So, now you get error dynamic situation worked out and think about that okay for some other desired aerodynamic equations what should be, to get to that what should be the modifications in the control algorithm that you are proposing? That you think about and then see with if we apply that can you compensate for friction in some way or minimize the bad effects of the friction.

So, this is how one can start thinking when we, so see for example even the, in the presence of friction we will not be able to find transformation, then you will say okay oh what I can do about such a system with Laplace transform and other stuff, so there is there you will get into a little bit of difficulty thinking into the domain of functions, so that is where like this thinking purely in the ordinary differential equation domain would come very very handy. So, think about that, and slowly we will build upon these concepts.

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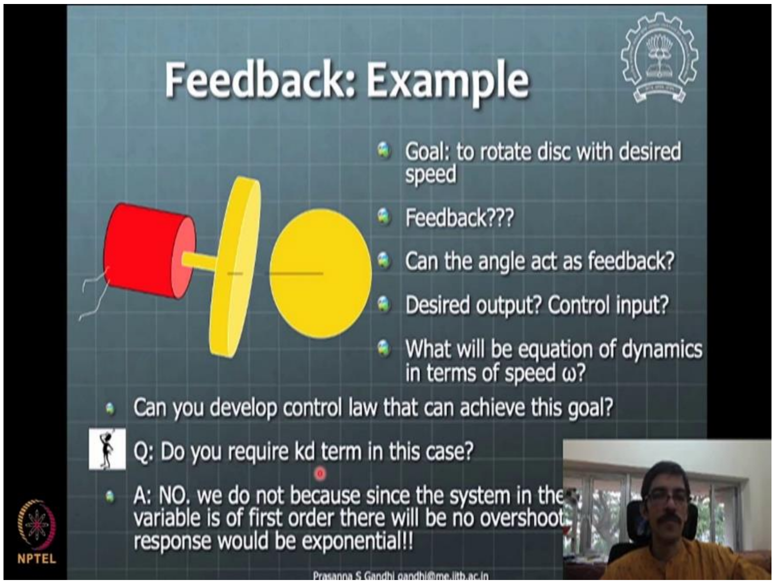

And then there is another problem that one can think about see now the goal is different, goal is to rotate this with a desired speed. So, for these know what will be your feedback? What is going to be your model of the motor? If you use angle as a feedback, let help for this, these are the questions you need to think about ponder over and come up with some kind of a design for such a so this is the other different problem.

So, our controller input is same, same voltage and desired output now is speed is already there output. So, then what is now this new variable which is desired variable output, what will be the equation which is governing the dynamics? So, these dynamics now in is terms of omega I am not really in terms of theta, so the order of equations will reduce by one.

So, now with that can you develop control now which will achieve this goal? Can you propose some law and do there analysis the header analysis, now you can think over whether now you will require now the kd term is required in this case. So, one can see that this system in terms of omega is going to be first order system instead of second order system, and then there will be no overshoot for such a system.

If there is no over shoot then probably like this stamping may not be required, typically you need require damping to avoid this overshoot that was happening in the previous case. So, like that one can start thinking about doing things and getting to control which will take us to this final desired speed. And now, that I want that to happen in the presence of friction as well.

So, what should I do? Those are kind of questions that one can ponder over or think about and come to the some kind of control expression which will take you to your desired goal and you prove that there aerodynamics you are indeed going to the final position in a way that you have learned.

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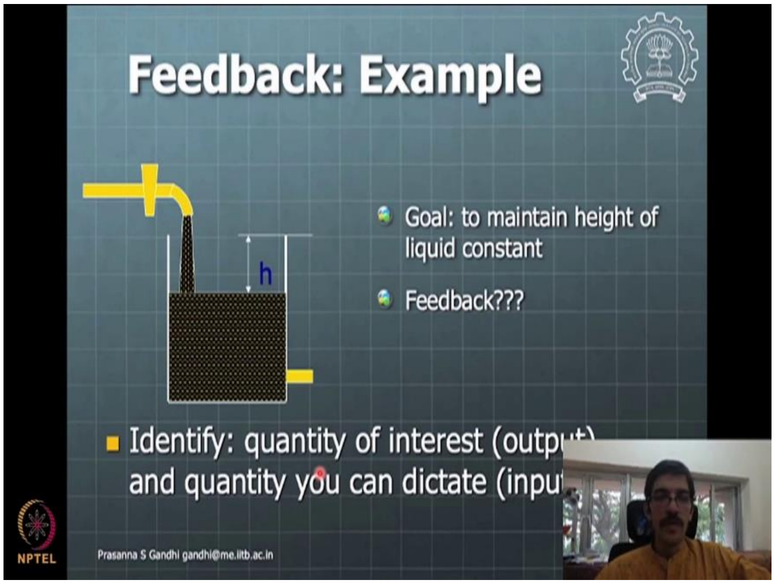

So, like that you can consider now these different different other systems, couple of things one or two examples and you will form up this concept in your head, so I think maybe we will stop here.