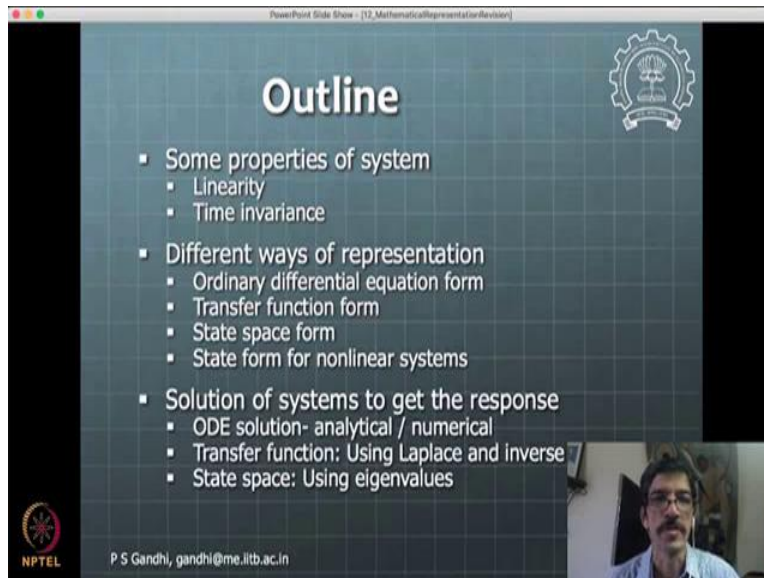


Design of Mechatronic Systems
Professor Prasanna S. Gandhi
Department of Mechanical Engineering
Indian Institute of Technology, Bombay
Lecture 35

Mathematical representations of systems for control

We start with now little bit of a revision and some kind of a consolidation of some of the concepts and we will see how things are for the mathematical representation of control systems. So, we have found like we have already done some of the parts, but now we are just consolidating these in some way. This has been done some part of, this has been done already in the past in your classes. So, we just recall some of the concepts and move on from there.

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The slide is titled "Outline" and contains the following bulleted list:

- Some properties of system
 - Linearity
 - Time invariance
- Different ways of representation
 - Ordinary differential equation form
 - Transfer function form
 - State space form
 - State form for nonlinear systems
- Solution of systems to get the response
 - ODE solution- analytical / numerical
 - Transfer function: Using Laplace and inverse
 - State space: Using eigenvalues

The slide also features the IIT Bombay logo in the top right corner, the NPTEL logo in the bottom left corner, and the text "P S Gandhi, gandhi@me.iitb.ac.in" at the bottom center. A small video inset of the professor is visible in the bottom right corner of the slide frame.

We will get going with this kind of outline. We will see some of these properties of the system, the linearity, time invariance. Then we will see these three major different ways of representation and fourth one comes for the non-linear systems. For linear systems, you have ordinary differential equation form, transfer function form and state space. These are the three major forms and then how do you get solutions in each of these forms. So, these are the concepts that we have already seen.

The non-linear systems one is what we are dealing probably with new. Some of you might have done some non-linear systems handling in system control kind of courses. So, we will not get into

too much of a depth, but again here idea is to kind of see from mechatronics application perspective, how we can deal with these different concepts.

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System Properties

Linearity (VIMP)

- Principle of superposition
- Power ODE is one & linear terms
- Example: Pendulum system

Time invariant

- Start at $t=0$ or any other t_i from same initial conditions \rightarrow the response is same
- Constant coef for TI systems
- Time dependent coef: time varying system
- Example: space vehicle

LTI system: linear time invariant

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So, we begin with the simple properties which you all already know. So, the linearity of a system, the principle is important first like many systems. This principle of superposition is valid for the linear systems. Meaning like the input, if a system is given input u_1 and produces output y_1 , input u_2 and output y_2 then if these inputs are scaled and added or subtracted correspondingly the outputs will get scaled and added and subtracted that is a kind of a concept that you have here.

Time invariance is another very important concept. If you start at $t = 0$ or any other time for the same initial conditions then your response is same that is what your time invariant principle says. So, these coefficients typically would be constant for time invariant system and in time varying systems the coefficient will be functions of time. So, if you see carefully your regulation problem or tracking problem, regulation problem does not change the system property of time invariance actually, but tracking problem introduces the time trajectory the $\theta_{desired}$ or $x_{desired}$ kind of a trajectory which is explicit function of time in the system.

So, the moment you start talking about the tracking problem in control, it will have a time varying system considerations to be given. So, you have other examples also for a time varying system apart from this tracking problem is space vehicle. Many of the, mass of the space vehicle is changing with respect to time. Those kind of things we will have to deal with a little bit in a

mathematically different form than usually we handle the time varying system, time invariant systems.

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PowerPoint Slide Show - [12, MathematicalRepresentationRevision]

Types of Systems

- Linear time invariant: represented by a linear ODE with constant coefficients
- Linear time variant: represented by a linear ODE with time-dependent coefficients
- Nonlinear: represented by nonlinear ODEs
- SISO – Single-input single-output
- MIMO – Multi-input multi-output
- LTI – Linear Time Invariant

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There are these kind of different types of systems, I mean, in there are kind of more kind of name that is given or definition that is given. So, linear time invariant system typically are represented by linear ODEs, so LTI systems. Then you have time dependent coefficient for a linear time varying systems. Then you will have these single input single output systems. Again, they are typically terminology in the linear system domain. And multiple input multiple output kind of systems MIMO systems, SISO systems, and then you use this LTI term for linear time invariance. And the non-linear systems are typically represented by non-linear ODEs or the state form of the representation that we have seen for getting the MATLAB numerical simulation going.

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Control System Representation

Different ways to represent

- Differential equation form: equations governing system dynamics
- Transfer function
 - obtained by using Laplace transforms (linear systems)
$$G(s) = \frac{N(s)}{D(s)} = \frac{Y(s)}{U(s)}$$
- State space form:
 - obtained by reducing diff. Equations by defining appropriate state variables
 - Linear system only
 - State form for nonlinear systems
$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$
$$\dot{x} = f(x, u)$$

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So, this representation of the systems in different forms is what is important for development of control. So, first is like our differential equation form which we already seen in feedback lecture. You start with a differential equation form and start applying some of the control fundamentals and analysis of control in that form only.

Then we consider the transfer function form where you have poles and zeros defined for the system and based on the poles and zeros you kind of look at the open loop kind of a stability of a system and we look by many, many different ways you look at the closed loop stability of the same system by considering say, for example, a root locus kind of analysis or bode plot kind of analysis, frequency domain methods those are all kinds of things which you have part of like these Math codes in theory that you have studied some of these kind of tools and techniques.

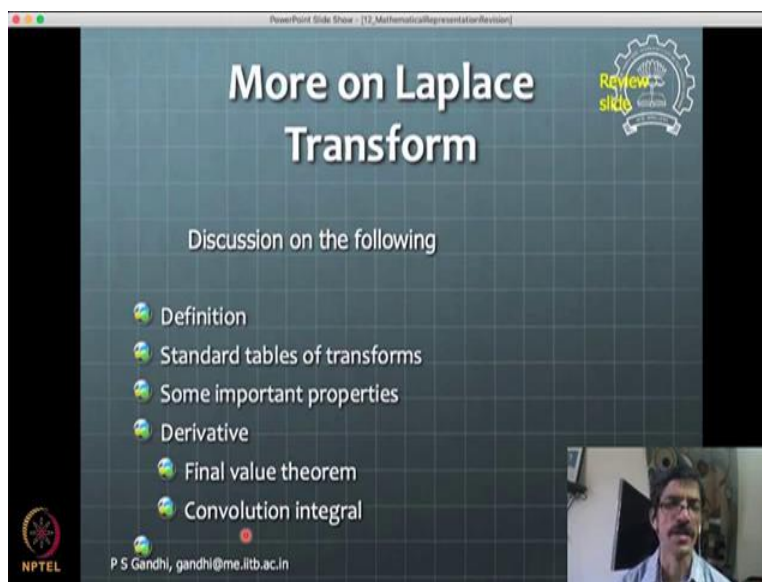
So, we will not get into the details of those tools and techniques. You maybe if you want you can just little bit kind of a refresh them. We will not get into a lot of depth of those. We see some of the applications if at all about those techniques, especially in the case of placing the pole at appropriate location kind of problems.

Then you have a state space methods. So, state space kind of a form is this $\dot{x} = Ax + Bu$ and $y = Cx + Du$. This form is used typically. So, now, this A, B, C, D matrices are basically as you know all constant matrices. And now how do you solve system in each of these forms is the next question. And how do you obtain this form this again you are aware about like you have seen already in the

MATLAB kind of simulation. While doing the simulation we anyway need to get the system represented in this $\dot{x} = Ax + Bu$ form.

When the system is linear and this part here $f(x, u, t)$ will reduce to some matrix A and B and can be separated out in like x related terms and u related terms. So, that is how like these different forms of the representation of the same system can be there. And you can choose to kind of, if you have a linear system, you can choose to work with any of the forms that you wish for development of control. And we are just going to see now a little bit of a brief or revise the little bit of a brief of this Laplace domain representations and state space domain representations and then we will see how do you solve the systems in these different domains and that will conclude this part of the lecture.

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PowerPoint Slide Show - [12, MathematicalRepresentation/Revision]

More on Laplace Transform

Discussion on the following

- Definition
- Standard tables of transforms
- Some important properties
- Derivative
- Final value theorem
- Convolution integral

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So, we use the Laplace transform standard tables of transforms are basic fundamental properties to get Laplace domain representation done. So, you need to revise this definition of what is the Laplace transform and how do you kind of take Laplace transform of a given system and things like that. And some properties of the transforms and some kind of, these are all also properties like the final value theorem and convolution integral.

This convolution integral is an important concept to be looked at. Although we may not use like for purpose of application or development of control. We may not use this convolution integral too much. But if you want to have a solution of Laplace transform system done, you may need that

convolution integral concepts. For a given, any given general input is there you may need. It is not mandatory, but you may need that.

(Refer Slide Time: 09:25)

Example: spring mass system
Zero initial conditions

$\dot{x} = Ax + Bu$
 $y = Cx + Du$

$x_1 = x \quad \dot{x}_1 = x_2$
 $x_2 = \dot{x} \quad \dot{x}_2 = \frac{1}{m}(-cx_2 - kx_1)$

$A = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix}$

Differential equation form:
 $m\ddot{x} + c\dot{x} + kx = u(t)$ Newton's method
 Transfer function: Input $u(t)$, output $x(t)$
 $G(s) = \frac{X(s)}{F(s)} \quad G(s) = \frac{1}{ms^2 + cs + k}$
 Usually: zero initial conditions
 State space form:

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So, this is an example of our standard differential equation of spring mass damper system. And you use a Laplace transform here considering, so for Laplace transform you need to kind of define what is your input and what is your output. These input output definitions are your own definitions like you need to define or whatever requirements our system will dictate what is input and what is output. Then you take Laplace transforms and represent like this is output of the Laplace transform divided by input of the Laplace transform of this system will give you this kind of a transfer function.

And usually in transfer function we assume 0 initial conditions. Suppose there the initial conditions are non-zero then you cannot get this transfer function form. This is very important, because it comes from the definition of or the property of Laplace transform for a derivative. So, typically the derivative will give you some kind of, derivative Laplace will give you some kind of a non-zero initial condition representation in the transform. So, to avoid that we assume initial conditions to be zero and then like the system has this kind of a nice form of like just Laplace division of two Laplace and polynomials.

To convert this system into state space form you need in this kind of a form you need to define states first $x = x$, $x_1 = x$ and $x_2 = \dot{x}$. So, $\dot{x}_1 = x_2$ and \dot{x}_2 will be equal to based on this equation 1 upon m times minus cx2 minus kx1.

$$\dot{x}_2 = \frac{1}{m}(-cx_2 - kx_1)$$

So, this is a typical process for getting the state space form. You define the states, because state space form has only single derivative but of a vector. So, you need to kind of add, introduce more variables in the system to represent the system into this form.

So, this is how like you represent system in these different forms in Laplace transform form and then the state space form. And then you can handle this system in either way. There are control tools and techniques available in each of these domains to think of the system properties and develop control or propose control algorithms further.

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Example: spring mass system
Nonzero initial conditions

- Differential equation form: $m\ddot{x} + c\dot{x} + kx = f(t)$ (Newton's method)
- Transfer function: input $u(t)$, output $x(t)$

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

$$ms^2 X(s) - ms\dot{x}(0) - mx(0) + csX(s) - cx(0) + kX(s) = F(s)$$

$$X(s) = \frac{F(s)}{ms^2 + cs + k} + \frac{m[s\dot{x}(0) + x(0)]}{ms^2 + cs + k} + \frac{cx(0)}{ms^2 + cs + k}$$

$$G(s) = \frac{X(s)}{F(s)} \rightarrow \text{Transfer function representation for non-zero initial conditions}$$

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

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So, this is some kind of example of a non-zero initial condition. So, you can see how they show up in the Laplace transform stuff. So, you can get a solution with non-zero initial conditions no problem by using Laplace transform that is no problem at all. You can use this kind of a method, Laplace transform method to get a system solved. But if you see here, you cannot represent it as a transfer function. So, only when the initial conditions are 0, this part will go to 0 and then you will

have this input can be brought here to kind of define a transfer function. But in state space form there is no problem of these zero initial conditions. So, that is how things go.

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PowerPoint Slide Show - [12, MathematicalRepresentation/Revision]

Finding solutions to systems represented in the forms discussed

Discussion on the following

- ODE solving : Basic mathematics
- Inverse Laplace transforms: Standard tables of transforms
- Using matrix exponentials for state space systems
- Nonlinear systems : closed form solution may or exist so we need to resort to numerical simulation techniques for dynamics and Lyapunov theory f

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Then next thing is finding a solution to these different forms which we have discussed. So, ordinary differential equation solving again I do not realize here the basic mathematics of ODE solving. You all know that I presume from the previous background. So, if you not like we can just finish up some of the fundamentals of homogeneous part of the solution and particularly integral and that those kind of things.

Then for Laplace transform representation you typically like substitute whatever is input that is given like Laplace transform the input and get the expression for Laplace transform output in the Laplace domain. And once that expression is available, you apply the inverse Laplace transform by method of dividing the polynomial into its roots. So, multiplication of multiple roots is a denominator and you represent these each of the roots separately. And you then use the standard tables to do the inverse transform.

So, this method you, I am presuming you all know this and you can go ahead and revise if you want to. Then the exponential for the state space representation of the system \dot{x} is equal to Ax plus Bu , you typically use the matrix exponentials. So, we will go into a little bit more details about this method. And then for non-linear systems, typically, some systems you may get a closed form

solution by using this typical ordinary differential equation solving methods. But many systems may not admit that kind of a closed form solution or it may be too tough to get that.

So, under this scenario, we can we will resort to the numerical simulation that we have seen and use those simulation techniques for understanding the dynamics and further for developing of, development of control one can use this Lyapunov theory. So, we will go partly through this Lyapunov theory and come up with some kind of universal controller for mechanical systems. So, we have seen the mechanical systems in, with Lagrange formulation.

So, for those kind of systems, rigid body mechanical systems, we can get a very interesting like a controller which is applicable for all systems of that sort. And that is for the tracking kind of a problem. So, regulation is very kind of a specific case of the tracking problem and one can kind of attempt to develop. So, one can have that controller used and all the rigid body control problem would be solved. So, this big domain of systems we can do the control of these non-linear systems by using some Lyapunov theory based techniques. So, let us get into a little bit more of this state space system solution.

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Solving State Space form

Similar to a first order system solution

- SS model $\dot{x} = Ax + Bu$
 $y = Cx + Du$
- Homogeneous part $x_h = e^{At} x(0)$
- Particular solution $x_p = \int_0^t e^{A(t-\tau)} B u(\tau) d\tau$
- Complete solution $x(t) = x_h + x_p$
 $x(t) = e^{At} x(0) + \int_0^t e^{A(t-\tau)} B u(\tau) d\tau$

More in the lecture to follow

So, you use what, you use matrix exponential. See if you see here the homogeneous part, so as we know for linear systems or any differential equations solution have a homogeneous part and a particular solution. In this case, the homogeneous part will be given as $e^{At} \cdot x(0)$. This is zero initial, this is like an initial condition. So, if these initial conditions are 0 in the state space case,

then this homogeneous part will be 0. But in general it will be like represented in this kind of fashion.

Now, this is like exponential of a matrix here. So, we will see how to get to that. And then particular integral or particular solution will have this kind of a form, e to the power A times t minus tau. So, this is like a convolution integral with the control input of the homogeneous part of the solution. So, this is $e^{A(t-\tau)}$. Bu, Bu is like your control, I mean, the non-homogeneous part or control input part. So, these are the, this is like will, this, addition of these two will give you this complete solution of a state space system and you use this finally for the state space system solution.

So, we will use this also for in the case of digital systems. When we talk the sampling of the systems we may come back to this form a solution. So, this is important for you to kind of know this form of a solution would exist. And now how do you find e^{At} .

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The slide is titled "Solving SS form" and features the IITB logo in the top right corner. It contains the following text:

- How to determine e^{At} ??
 - Method 1: Using Laplace transform
$$e^{At} = L^{-1} \{ (s\mathbf{I} - \mathbf{A})^{-1} \}$$
 - Method 2: Using Caley-Hamilton theorem

More discussion in lectures to follow

NPTEL logo is in the bottom left, and the email address P S Gandhi, gandhi@me.iitb.ac.in is at the bottom center. A small video inset in the bottom right shows a man with glasses speaking.

PowerPoint Slide Show - [12, MathematicalRepresentation/Revision]

Solving State Space form


Similar to a first order system solution

- SS model $\dot{x} = Ax + Bu$
- Homogeneous part $y = Cx + Du$
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$x(t) = x_h + x_p$ $x(t) = e^{At} x(0) + \int_0^t e^{A(t-\tau)} B u(\tau) d\tau$

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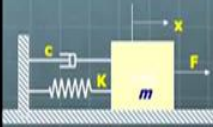
More in the lecture to follow



PowerPoint Slide Show - [12, MathematicalRepresentation/Revision]

Example: spring mass system

Nonzero initial conditions



- Differential equation form: $m\ddot{x} + c\dot{x} + kx = f(t)$ (Newton's method)
- Transfer function: input $u(t)$, output $x(t)$

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

$$ms^2 X(s) - ms\dot{x}(0) - mx(0) + csX(s) - cx(0) + kX(s) = F(s)$$


$$X(s) = \frac{F(s)}{ms^2 + cs + k} + \frac{m[s\dot{x}(0) + x(0)]}{ms^2 + cs + k} + \frac{cx(0)}{ms^2 + cs + k}$$

$G(s) = \frac{X(s)}{F(s)}$ → Transfer function representation for non-zero initial conditions

$\dot{x} = Ax + Bu$
 $y = Cx + Du$

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State space form: No problem



So, that is here. So, how to determine this form e^{At} ? The first method is like you use Laplace transform. So, if you take a Laplace transform of the system which is having like the homogeneous part here, this control input is zero so $\dot{x}=Ax$ and then look at this. So, what you need to look at is, so there are some steps involved here beyond these. So, let us, we will not get into that.

But finally what you get, see this \dot{x} will give you Si here. So, see although there is no i here, we have to kind of, when we start subtracting we need to assume that there is i here. And also it will give you this by the way up derivative property, you will get this kind of initial condition zero initial condition into the system when you take Laplace transform of \dot{x} .

So, by simply taking that and doing some kind of a mathematical simplification of homogeneous part of this solution you can arrive at this formula. So, you get a Laplace transform solved and then like you need to kind of bring that Laplace on the other side and then you will get this solution. So, you try it out and then like if you get into any trouble like we will see again in the class if at all. So, this is a typical kind of a form that you get for doing the, obtaining this e^{At} . So, you can use this.

So, if you see that this is particular matrix which is which typically we use for Eigen solutions and that matrix inverse times that matrix, you need to invert that matrix and get its Laplace inverse. Then this method 2 is based on Caley-Hamilton theorem. So, this Caley-Hamilton theorem you need to again devise. It basically allows you to express any higher power of A to the n, A to the power n which is higher than the dimensions or n and, suppose n is a dimension of a matrix n by n. So, A to the n and its higher powers are expressible in the form of a polynomial up to A to the power n minus 1. That is what this theorem says basically.

So, this comes from the fact that every matrix A satisfies its own characteristic equation. So, you know characteristic equation of a matrix which you will get by $\lambda I - A$. So, that is a characteristic equation in terms of λ . Now, if you replace by λA then that equation is also satisfied. That is what the Caley-Hamilton theorem says. And based on that you get this property which I said that A^n or A^{n+1} or A^{n+2} everything can be expressed as a polynomial up to A^{n-1} .

So, if you see this exponential by the way of series expansion, you have this a lot of like this can be expressed as a powers of generally like the infinite series, polynomial series with different powers for n, different powers for A and we use Caley-Hamilton theorem to kind of express this as a summation of powers up to, the coefficients will be different from what you have seen for the exponential series expansion.

So, we use the general coefficients and then coefficients are to be determined. How do we determine coefficient by using like, so that equation whatever you get will also be satisfied by when you substitute lambda there. So, we use like the property of Caley-Hamilton theorem only that, this equation will be satisfied by if you are expressing this A equation then it will be satisfied by the lambda also like the Eigen values also.

So, by using Eigen values in the equation which expresses e^{At} e to the power At in the form of say like series or polynomial up to A^{n-1} , one can now get the coefficients, n coefficients which are determined by substituting n Eigen values in this algebraic equations. And once you get those coefficients substituting them will give you the value of e^{At} . So, you try it out and check it out actually for some simple system and that will kind of develop more understanding. So, these two ways are there for solving, getting the e to the power, matrix power.

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PowerPoint Slide Show - [12_MathematicalRepresentationRevision]

Concept of stability

- Very important characteristic of the transient performance of the system.
- An LTI system is stable if the following two notions of system stability are satisfied:
 - When the system is excited by a bounded input, the output is bounded (BIBO).
 - In the absence of input, output tends towards zero (the equilibrium state of the system) irrespective of initial conditions (this is called as asymptotic stability).

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Then there are these concepts of stability we need to talk about or understand. So, for any of these forms you have a definition for concepts of stability. For system which is LTI, this is basic definition of stability, the one of the notions is, so this is like a fundamental notion. When the system is excited by a bounded input, the output remains bounded that is one notion of the stability for, especially for linear domain systems. And other kind of a notion is in the absence of inputs output tends towards zero or equilibrium state of the system irrespective of any initial conditions.

So, this is like kind of a notion of some kind of asymptotic stability. So, there are many, many notions or definitions of stability you will find. So, for linear system typically like basic fundamental definition is bounded input bounded output and from there one can derive the conditions for stability. So, this concept or notion of stability and then conditions of stability they are two different kinds of things. You do not say we get the conditions by applying these basic fundamental mathematical notions of stability.

(Refer Slide Time: 26:29)

The image shows a screenshot of a presentation slide. The slide has a dark blue background with a grid pattern. At the top center, the title "More tangible conditions" is written in white. To the right of the title is a circular logo of an institution. Below the title, there are three bullet points, each starting with a globe icon. The first bullet point states: "If all the roots of the characteristic equation (poles) have negative real parts, the system is stable." The second bullet point states: "If any ONE root of the characteristic equation has a positive real part or if there is a repeated root on the $j\omega$ -axis, the system is unstable." The third bullet point states: "If the condition (i) satisfied except for the presence of one or more non repeated roots on the $j\omega$ -axis, the system is marginally stable." In the bottom right corner of the slide, there is a small video inset showing a man with glasses and a mustache, wearing a light blue shirt. At the bottom left of the slide, there is the NPTEL logo. At the bottom center, the text "PRASANNA S GANDHI gandhi@me.iitb.ac.in" is visible. The top of the slide shows a window title bar with the text "PowerPoint Slide Show - [12_MathematicalRepresentation(Revision)]".

So, when we apply these notions, for example, for linear system, we can get like these more tangible conditions. So, the conditions are like roots of characteristic equation or poles, you know the poles of a system with the Laplace transform representation you can define poles and zeros of the system. And when the poles have negative real part, the system is stable.

So, if any one even only, even one root of this characteristic equation has positive real part then the system is unstable. And so system is marginally stable if these roots are under imaginary axis. So, we will see like a little bit more about how this relationship between the system response and the location of poles has, so this you need to have some kind of a feel for these relationships that will be good.

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PowerPoint Slide Show - [12_MathematicalRepresentation/Revision]

Poles and zeros Definition

$$G(s) = \frac{N(s)}{D(s)} = \frac{Y(s)}{U(s)} = A \frac{\sum_{m=0}^M (s - z_{0m})}{\sum_{n=0}^N (s - p_{0n})}$$

← zeros
← poles

Pole zero plot and animation

<http://www-es.fernuni-hagen.de/JAVA/PolZero/polze>

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So, these are the more details about poles and zero definitions which you I presume you already know. And then pole zero plots and animations you can see some of the websites are given here to try out. And I have some kind of things plotted here for you. So, let us, we will go through that.

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PowerPoint Slide Show - [12_MathematicalRepresentation/Revision]

Effect of Poles on system response

- The nature of impulse response $g(t)$ depends on the poles of the transfer function $G(s)$ which are the roots of the characteristic equation
- These roots may be both real and complex conjugate and may have multiplicity of various orders.
- The nature of responses contributed by all possible types of poles are shown in the following slides.
- In each case find whether the system is stable or not with the help of BIBO condition (i.e check whether area under the absolute-valued impulse curve is finite or not).

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So, effect of poles is there on the response. So, we will see the cases, different cases of these how these poles individually or like or two or three poles affect the response of the system.

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PowerPoint Slide Show - [12, MathematicalRepresentation/Revision]

System with single real pole in left half s-plane.

$G(s) = \frac{1}{s+a}$

$s = -a$

$a = 1$

■ Impulse response is exponentially decaying output)

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So, let us see it here. First you have a single real pole which is on the negative part of the real axis. This real axis is for the Laplace s . s is imaginary number. Its real part is here and like imaginary part is here. So, when s is equal to minus a , then it can be represented on this complex s plane in this kind of a form. And with that typically you get the impulse response. So, all these are like impulse responses which we have plotted here which is bounded or decaying exponentially.

So, the same kind of impulse response now we are going to observe for different, different locations of poles. So, just to get a feel for like, when the location is here or location is somewhere else, how the response is typically going to look like. See for example here real axis poles, you will not have any oscillations of that behavior.

(Refer Slide Time: 29:18)

System with single real pole in right half s-plane.

$G(s) = \frac{1}{s-1}$

$s=1$

Impulse Response

■ Impulse response is exponentially growing (unbounded output).

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But the moment you introduce the two poles on the, off the axis, real axis, you will get oscillatory behavior. Imaginary axis, imaginary part will be there for the S , then you get oscillatory behavior. So, here now we see that when the pole is shifted to the real side, positive real side, then your system response exponentially increases.

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System with a pair of complex poles in left half s-plane.

$s = -0.5 + j0.866$

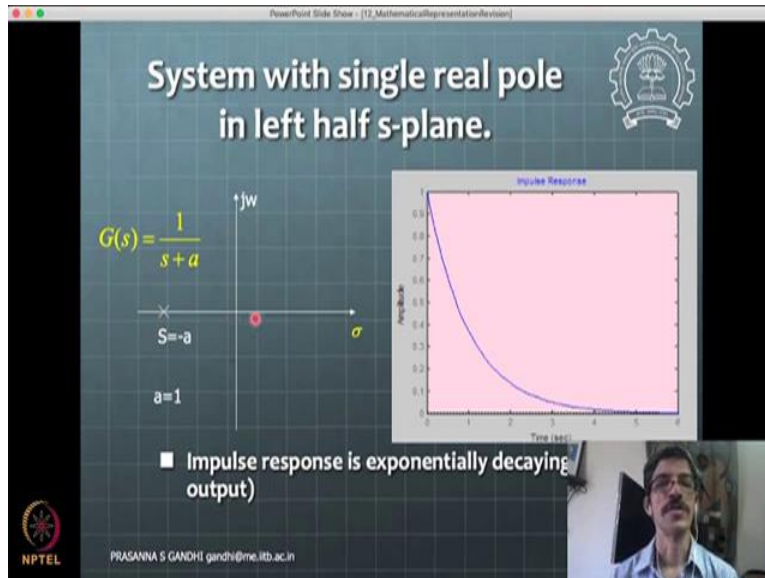
$s = -0.5 - j0.866$

$G(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

Impulse Response

■ Impulse response is exponentially decaying (bounded output)

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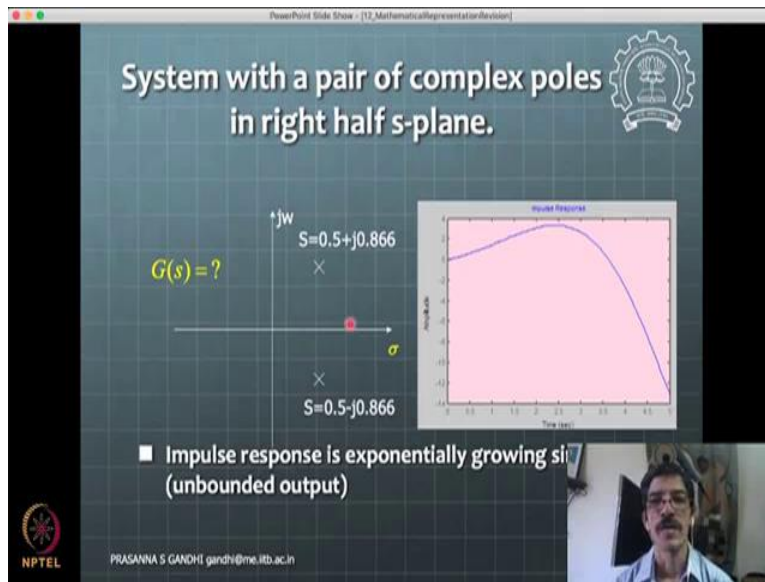


And this is a case where like you have two poles which are having both real parts and imaginary parts. When the imaginary part is added to the system, of course, you will need to consider, these poles will always exist in the pair by the way. You cannot have only one kind of pole shown up here and no pole corresponding that scenario is not there. So, you will get this kind of oscillatory behavior in this case.

See other thing is like if these poles are further shifted towards left side then these oscillations will start decreasing. These oscillations will decrease little faster than this. And if the pole is on the real axis then the exponential decay also will be faster. It is far away in this plane then exponential decay also will be faster. So, if you see here, this exponential decay is proportional to this e to the power At times, like, so A is larger you will get like the decay which is faster. So, this decay will be faster and faster happening if the pole is far away shifted from here to like negative side.

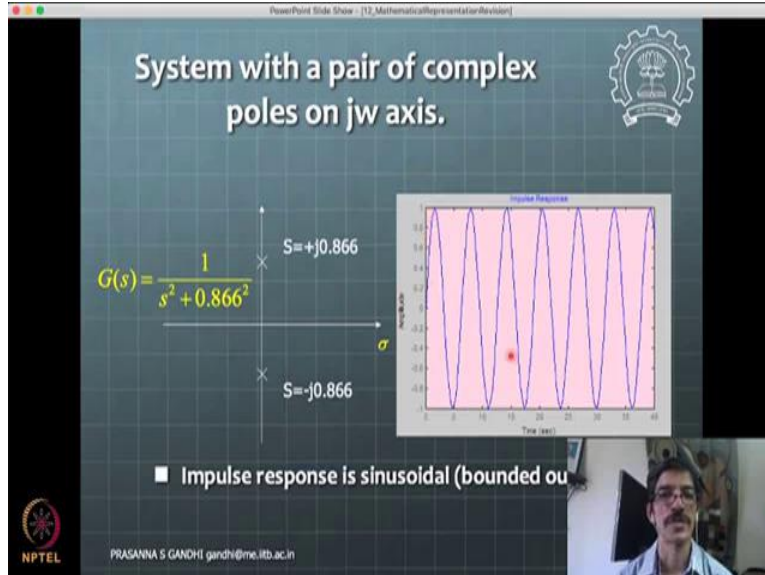
Now, these are very interesting connotation or influence or implication in thinking about multiple poles. So, if you have multiple poles coming up on the, on this side negative real axis which will be there for many kind of real life systems, the poles which are far away would contribute lesser to the response than poles which are close by. So, if the poles are place or they are far away in the system, one can understand that those far away poles will not have much of influence on a response of a system. The response because of those will decay faster than response, because of the poles which are close to the imaginary axis. That is the kind of important understanding we should take away from these discussions.

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Then pole shifted to the right half like you will see that this response will be kind of continuously growing with oscillations. So, imaginary part of solution is also there.

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Then poles are on the imaginary axis, you will find that this is sinusoidal kind of oscillations, harmonic systems basically without damping.

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PowerPoint Slide Show - [12_MathematicalRepresentation/Revision]

System with a double pair of complex poles on jw axis.

$s = +j0.866$ 2poles
 $s = -j0.866$ 2poles

■ Impulse response is linearly increasing sinusoidal (unbounded output)

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Then double poles there on the imaginary axis. Amplitude of the oscillations will keep on growing. So, that is what happens.

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PowerPoint Slide Show - [12_MathematicalRepresentation/Revision]

System with a pole at origin

$G(s) = \frac{1}{s}$
 $s = 0$

■ Impulse response is constant (bounded output)
 ■ Marginally stable system

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Then you have a single pole at origin, you will get this kind of a straight line behavior in the response, constant behavior. Impulse response is a constant.

(Refer Slide Time: 32:56)

System with a double pole at origin

$$G(s) = \frac{1}{s^2}$$

Impulse response is linearly increasing (unbounded output).

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So, then if you have the double poles then the input again goes unbounded and then this is unstable system.

(Refer Slide Time: 33:06)

Effect of zeros on response

Recall the impulse of the system with a pair of complex poles in left half s-plane.

$$s = -0.5 + j0.866$$
$$s = -0.5 - j0.866$$

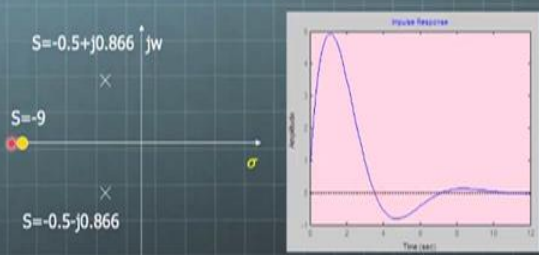
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So, now if you add zeros to the system like the response will have some small variations that is happening.

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PowerPoint Slide Show - [12_MathematicalRepresentation/Revision]

Add a left half s-plane zero to this system



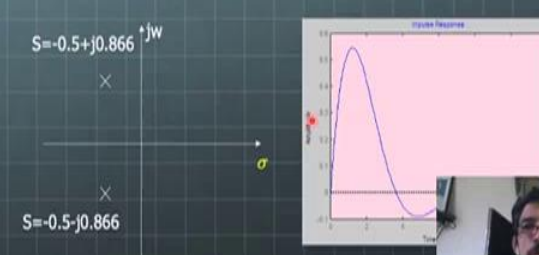
The slide shows a complex plane with poles at $s = -0.5 + j0.866$ and $s = -0.5 - j0.866$, and a zero at $s = -9$. The impulse response graph shows a damped oscillation with a peak amplitude of approximately 1.0.

- We see no change in stability of the system
change in the shape of the transient.

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PowerPoint Slide Show - [12_MathematicalRepresentation/Revision]

Effect of zeros on response



The slide shows a complex plane with poles at $s = -0.5 + j0.866$ and $s = -0.5 - j0.866$, and a zero at $s = -9$. The impulse response graph shows a damped oscillation with a peak amplitude of approximately 1.0.

- Recall the impulse of the system with a pair of complex poles in left half s-plane.

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So, this is, now we are considering this system with this kind of oscillatory behavior. We have a zero added. So, without zero and with zero added you see hardly any change in the response, only this amplitude here changes when the zero added on the left half.

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The slide features a title "Add a right half s-plane zero to this system" and a pole-zero plot. The plot shows poles at $s = -0.5 \pm j0.866$ and a zero at $s = 9$. A red dot on the real axis indicates the damping ratio σ . To the right, a graph titled "Step Response" shows a curve that rises and then settles to a steady-state value. A small video inset of the presenter is visible in the bottom right corner.

■ We see no change in stability of the system
change in the shape of the transient either
or left half s-plane zero.

And then when you add zero in the right half then there is some kind of a small deviation of this one, but the overall behavior of the system does not change. So, these zeros they do not affect the stability of a system that we all know that mathematical property, but this is just to kind of see what is the effect of zero is typically for second order system.

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The slide is titled "Behavior of standard First and second order systems" and includes a section on "Necessity" and "Basis".

Necessity

- To get the basis for characterization of the response of other system
- Simplification in analysis of such systems

Basis

- Range of inputs like impulse, step, ramp are used for characterization
- Similar inputs used in the experimental characterization

A small video inset of the presenter is visible in the bottom right corner.

So, then other important idea that you already have probably studied and maybe we need to recall here is about the first and second order system standard kind of response. So, every system like system can be looked at purely from a mathematical perspective without kind of giving I mean the

keeping the physical kind of interpretation apart for a while. So, what if you are able to see the system from that perspective, then like the basic mathematical principles that are there for the systems as a standard kind of a system responses they are applicable.

For example, we have seen our motor. So, motor with the speed as an output is a first order system. And if you know the first order system response behavior, you can get to this directly the some of the interesting parameters for the, especially the time or the characteristics time for the system immediately seen and can conclude about like whether that dynamics is important to be considered or not important to be consider all those kind of things we can we can talk about.

So, we are not then there we are not specifically like dealing with a motor system alone. Now, for the same kind of concepts would be applicable for, remember, this tank filling system with the flow input with a tank and then some kind of a drain output. So, that kind of tank filling system also is a first order system. Then heat transfer systems, they are all first order systems. So, first order systems you have some kind of characteristics response known.

For example, when you run a fan with a motor, the fan will not go to the speed more than the maximum speed and then come back to the maximum speed that kind of a oscillatory behavior would not be observed in the first order system response. So, this is, these are some of the very important kind of concepts from the understanding of the system from purely mathematical kind of a perspective and then interpretation in terms of physics that can be done very nicely if you, at least this behavior of the first and second order standard systems.

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Standard Behavior

First order system: impulse

First order system

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{Ts+1}$$

Unit impulse response

$$U(s) = 1$$

Using Laplace inverse

$$y(t) = \frac{1}{T} e^{-t/T}$$

$c(t) = \frac{1}{T} e^{-t/T}$

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So, this is a first order system impulse response behavior. So, now this is given in terms of these characteristics time T . And this T comes in transfer function in this kind of a form. So, if you have some other kind of a terms on the top, you just kind of divided them on the bottom side and like produce this part, sorry, wait, not on a top side, you need to have $Ts + 1$ form here first.

So, this, whatever is, if this term is non-zero you divide it by this term and this term. So, you get this kind of a, some kind of a scalar multiple here. So, that will you multiply this response here. So, this, otherwise this will be 1 over T value. But if you have some scaling factor here in place of 1, it will be like that scaling factor times 1 over T . That kind of scaling will happen to the response also. That is the only kind of a difference.

So, this is how one can get, your any kind of a first order system transform into this form and observe the behavior and see what is this time T . So, you can see that at some one kind of a time T the response goes down by roughly about 66 percent. So, this is important kind of understanding can come up with these time constraints. So, characteristic time constraints for different systems you can see and then see what is the dynamics that is of importance, what is dynamics, what is of not so much of importance, and then like we can take some call to simplify the system behavior.

(Refer Slide Time: 38:20)

Standard Behavior
First order system: Step

First order system
 $G(s) = \frac{Y(s)}{U(s)} = \frac{1}{Ts+1}$

Unit step response
 $U(s) = \frac{1}{s}$

Using Laplace Inverse
 $y(t) = 1 - e^{-t/T}$

The graph shows the step response $c(t) = 1 - e^{-t/T}$ starting at (0,0) and asymptotically approaching 1. Key points on the curve are marked: at $t=T$, the response is 63.2%; at $t=2T$, it is 86.5%; at $t=3T$, it is 95.0%; at $t=4T$, it is 98.2%; and at $t=5T$, it is 99.3%. A tangent line at $t=0$ has a slope of $1/T$. The initial value is labeled as 0.632.

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For this same first order system this is step response, sorry, 66, 63.2 percent it is going. So, similar, I do not know in impulse response also there will be some value. I think it will be similar 63.2 percent. So, this is, these are some of the important things to kind of note that the response you know already if you know the system like that. You do not need to solve the system. If it is a first order system with this particular kind of representation then this is a response for that system.

So, this response characteristic we will be able to use in the sense of this characteristics time constraints and other kind of parameters which are defined for different, different order systems. So, first order system has this T as a characteristic time constraint.

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Standard Behavior
First order system: Ramp

First order system
 $G(s) = \frac{Y(s)}{U(s)} = \frac{1}{Ts+1}$

Unit ramp response
 $U(s) = \frac{1}{s^2}$

Using Laplace inverse
 $y(t) = t - T$

If you come to the second order system, so this is you can go through a little bit more detail for ramp response of the first order system. Some of kind of a response would be shown here.

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Standard Behavior
Second order system: Transient

Second order system
 $G(s) = \frac{Y(s)}{U(s)} = \frac{K}{Js^2 + Bs + K}$
 $= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

Unit step response
 $U(s) = \frac{1}{s}$

Solution Laplace inverse

1. Underdamped
2. Critically damped
3. Overdamped

For second order system you have many more parameters to define. And those again you will consider the standard form of this kind omega n square upon s square plus 2 zeta omega n s plus omega n square.

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_{ns} + \omega_n^2}$$

This is a standard form of a second order system. So, if your system is not in this form, you basically like do some division and multiplication whatever you want to do and get it into this form, then you may find some scaling factor may exist on the top that can be scaling the response. Instead of 1 this value will be that scaling factor.

And you have these different cases coming up here. So, this is typically for an under damped system you will get this kind of response. And then there are formulae directly for maximum overshoot and this rise time, so 0.5 or 50 percent of the maximum value that it is reaching in the rise time. So, these are some of the specifications that are important from control perspective.

One may say okay I want to get my system settled in the, in say the settling time is specified to be say five seconds or two seconds or whatever or I do not want any overshoot for my system. For example, robotic systems we do not allow any overshoot typically. The system should like you can see that okay your arm should go finally and stay there. It is very like annoying to see that arm is going like this and then settling to the prime position and again going like this and settling to the prime position.

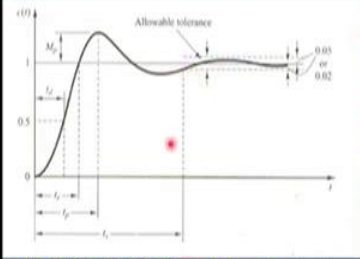
So, that is, so that to avoid that like you will have to have the specifications given in terms of the second order system response. Although robotic system is like a non-linear system, you can give the specifications of these and then like see how we can match those specifications.

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PowerPoint Slide Show - [12_MathematicalRepresentation/Revision]

Standard Behavior

Second order system: Transient



The graph shows a step response $c(t)$ over time t . Key parameters are labeled: M_p (Maximum overshoot), t_r (rise time), t_p (peak time), t_s (settling time), and t_c (corner time). A horizontal dashed line indicates the 'Allowable tolerance' at 0.05 and 0.02. The NPTEL logo is in the bottom left corner.

Response terms

- M_p = Maximum overshoot
- t_s = settling time
- t_r = rise time
- t_p = peak time

These can be found by using the re Standard form in book

Use: for designing system Parameters/ control satisfying Certain constraints in response terms

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And then these are like some of the formulae I will get for this. So, standard formula you can check some books and our website also this form would be available. And we can use this to design the system to satisfy some of these kind of constraints that are given. And typically this transient response time will be specified in terms of like 5 percent or 2 percent of maximum value if you want to settle into that how much is time. That is how one can define.

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PowerPoint Slide Show - [12_MathematicalRepresentation/Revision]

Important Concepts

- What is feedback?
- Open Loop Vs Closed Loop system
- How to process the desired feedback quantity?
- When goal is given, how to decide what should be the control algorithm? Is it unique?
- How to account for control input saturation!

We will answer these questions in this part of course

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So, these are like, in summary, some kind of important concepts. So, what is a feedback is, so not summary of this lecture, but I am saying generally these are the important concepts to ponder over or like understand before we kind of start applying the, these techniques and tools to actually

designing control for mechatronic system. So, feedback is important which we have already seen. Then you know what is open loop versus closed loop kind of a behavior or system.

And the other thing is how do you process this desired feedback quantity. So, we have partly seen that in terms of like the simple PD control for ODE based system and how do you analyze it and things like that to come up with different kind of a control methods or algorithms. And then this is important. Like what is if the goal is given, how do you decide what should be control algorithm.

So, you use all these techniques and like see okay which of these representations you want to use or it is applicable for a given scenario and then choose a proper control algorithm to try it out and do the analysis and get okay this is satisfying my system response. You go ahead with algorithm. Otherwise you like iterate this process. You choose some other kind of control algorithm and then go ahead with that.

And this is another important thing is like you need to look at control input what is needed to achieve your goal. So, in mechatronic system typically you will have a limit on this control input. So, you need to kind of make sure that we look at this control input and especially if the control input is too high then the system response will get modified when the saturation is put on its control input.

So, like that we will look at some of these in little more detail for the especially like when the goal is given how do you choose what kind of control algorithm that we will discuss further in the linear domain, non-linear domain and take these discussions forward. So, this is what we will conclude for this part of the lecture and then we will continue. Thank you.