Design of Mechatronic Systems
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Lecture Number 37
Application of Control design for Linear Systems

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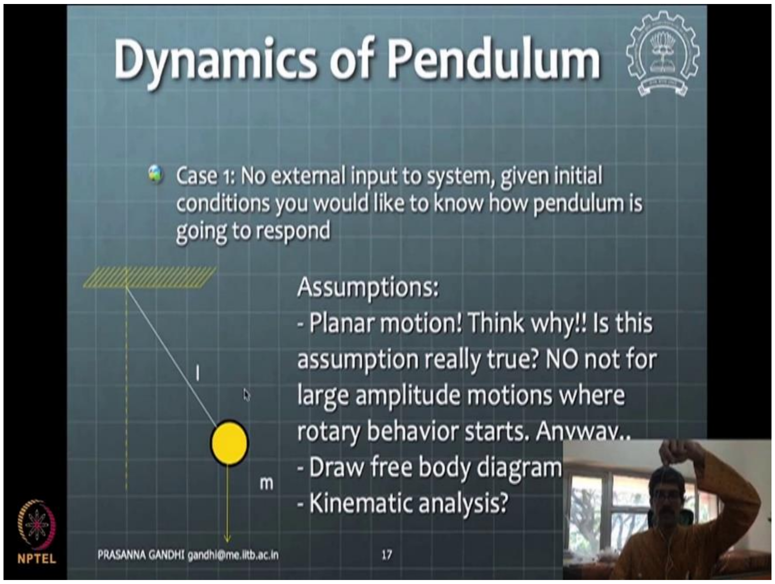

So, we start with our example now for development of control. So, we will develop some of these concepts here mainly based on known differential equation kind of analysis. And that discussion and also with the through this discussion we will develop some of the interesting concepts what, how do we kind of see what can be considered as a control input.

So, let us begin with these dynamics of pendulum. So, for this system if you see the model that we have done already in previous classes in previous courses you all are know very much familiar with this model. So, that helps because you already have some kind of understanding of this pendulum, maybe I will just get a pendulum long So, that I can demonstrate you some of these things I have the pendulum here for you to watch also. So, you can see the motion.

So, you all know about these but the there is some of these of control aspects we can see through some kind of examples. We will do here. Some experiments. So, this pendulum you can get the model if you have not done already by doing free body diagram or kinematic analysis of circular motion of a point mass. So, I am not going to getting into the details there.

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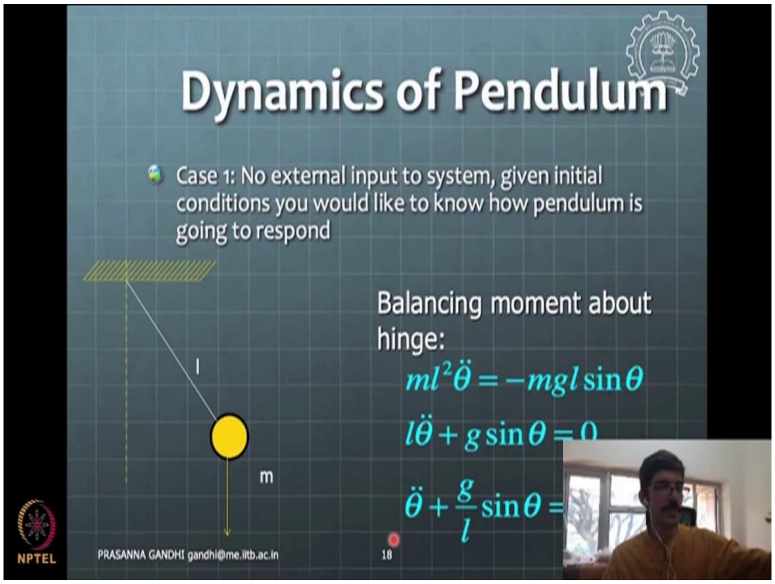

So, you can see this model of a pendulum here. So, this model is giving you this full nonlinear form of equation considering ϴ which is the angle of the pendulum here to be more than small angle approximation. So, with this model you can see that this has a harmonic kind of a behavior and that too without damping.

But if you see the actual pendulum, this the oscillation is damped down. If I keep on holding and let no letting it go then oscillations slowly damp down they come down to 0. There is some kind of a damping that is existing.

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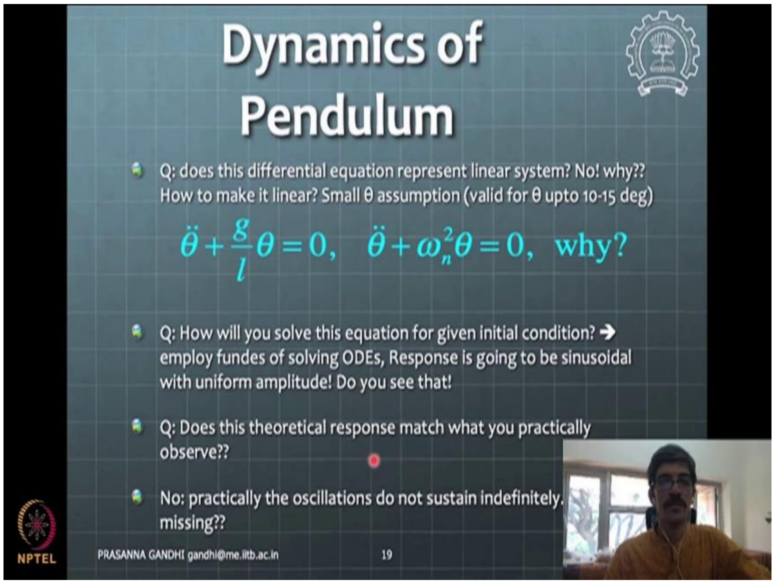

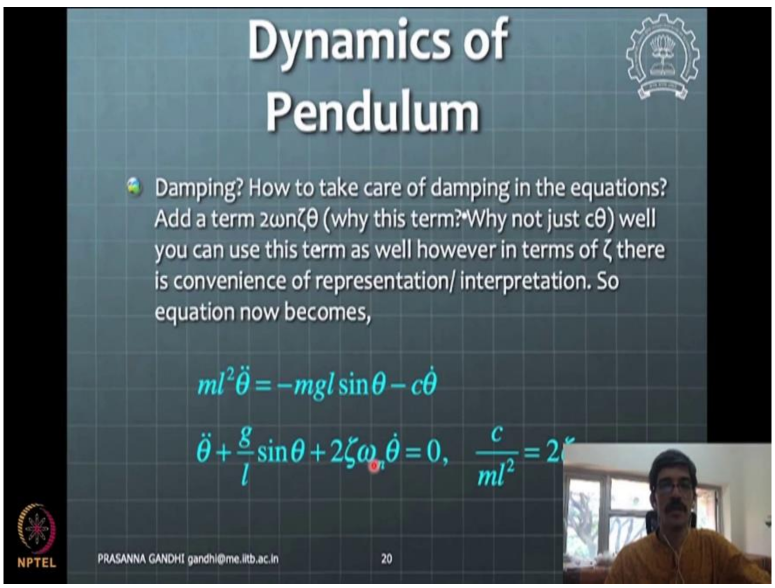

So, we will consider that damping into the system and then that will be more appropriate kind of a representation of the system. So, this what is missing here is this damping and we incorporate this damping term into the system here c Θ dot term and then convert this forms of equation into again the standard form. So, see we always seek to get the system in the standard form of second order system which you remember $s^2 + 2\xi\omega_{ns} + \omega_n{}^2$ that is a term for the standard form of a second order transfer function.

Whenever this ${\omega_n}^2$ over this kind of term. So, we seek that kind of a form to have for the equation also. And that form in the equation turns out to be where s^2 term does not have anything multiplying it. So, whatever is multiplying these $\ddot{\theta}$ Θ double we divide by that entire other parts of the equation and then no see the see these terms accordingly. So, $2\xi\omega_n$ multiplying $\dot{\theta}$ this term will be $2\xi\omega_n$ multiplying Θ will be now we do not have Θ we have sin (Θ) term here.

But for the low θ approximation this sine times equal to θ this is a term this will be ${\omega_n}^2$ kind of a term. So, that is how likely we try to seek in wherever possible many systems this may not be possible to seek some kind of a normalized form of the equations. So, that we can see through some of the details, for example, if ξ is found to be some value here, we know that this is this pendulum system is some underactuated system with this ξ value and that speaks about its nature.

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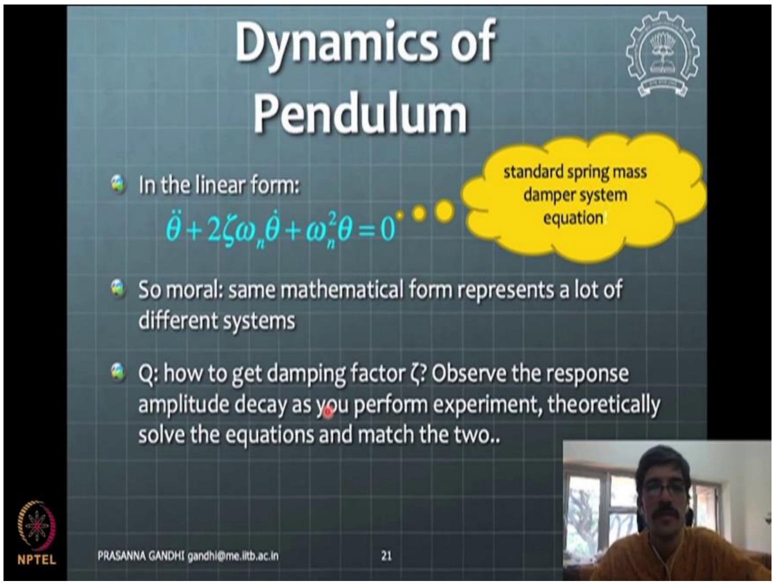

So, this is about dynamics. Now, one can kind of do some experiments to see what is how do we get damping factor that is ξ. So, you can think about if I timeout how much time it takes to get the amplitude to some half of its original value, something of that sort, then one can add number of oscillations based kind of counting of change of amplitude; that kind of thing will give you by logarithmic decrement, what is this damping factor ξ?

So, this can be a, these kind of things can be carried out by studying the dynamics of the system and carrying out appropriate experiments to get the parameters.

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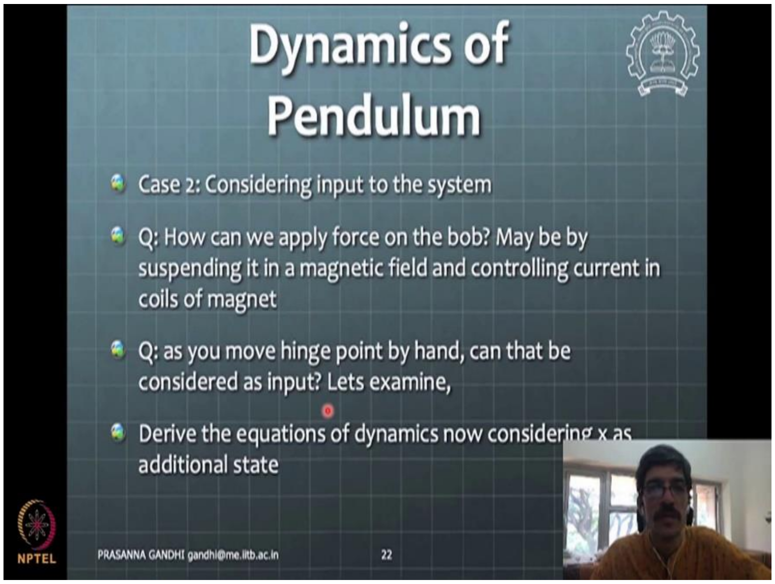

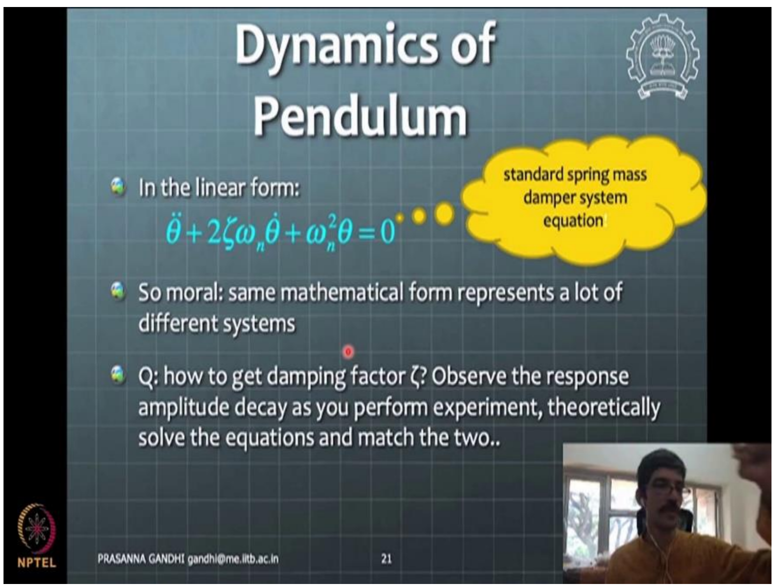

Now, this is mainly again so this is a dynamics without any input to the system and then if you have an input to the system what we will consider as input. If you think I want to apply some kind of torque on the bark or kind of a force in the bark. So, in which direction you can apply the force? We can apply that needs to be in the direction of Ɵ to have it represented up here or if we apply something else what will show up here that those kind of things we need to discuss and figure out.

If you see this is also likely similar to the system that you had with the hard disk drive in that we had a case where the head moves on the disk in a horizontal fashion it moves on the disk but one can see if that disk is not there only head is there and at the place of the head we have a big mass attached then in the vertical plane this will act as a pendulum kind of a system and at the backside you had the coil and the magnet.

That is the actuator which is giving some torque on this system that is a kind of a pendulum system that we can have. So, that is a one of the actuator as possible in the direction of Θ. So, but that is a little bit cumbersome to create such a system, so or based on your actual practical requirements there may be some other kind of considerations that may be coming in place for input. So, we considered here that okay a pendulum is mounted on a cart and the end of the pendulum is movable by the cart.

So, if you have this pendulum, it is mounted on the cart means this point where it is hinged that point is kind of is moving by the cart, the cart is able to move that point. So, if it is initially in motion now I can affect the motion my actually the moving the top point. So, that is how my input I am giving to the pendulum. So, I am not really applying any force here but I am just kind of moving the top point to give the input to the pendulum that is a kind of idea.

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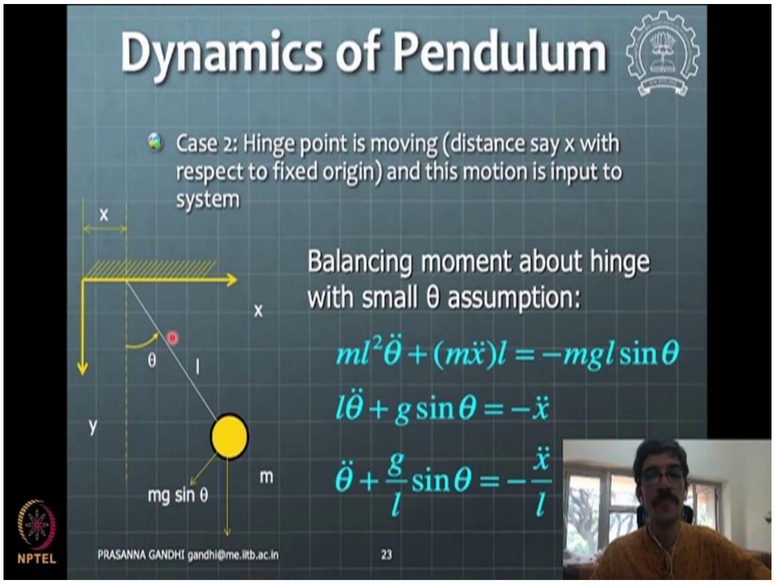

So, we can consider these kind of inputs to the system and when this input is considered here the hinge point motion by putting this hinge point in a cart. And then the say we can say I apply force to the cart or I can say I am directly applying this displacement to the hinge of the pendulum. So, these are different ways I can define my control input. So, although I have a cart here and I can apply some force on that cart I may choose to define that control be let the control be input x here or input $\ddot{x}$.

So, this choice of what I defined to be a control input to my system has somewhat flexibility that I can have. So, this is very important concept here. So, I can say for the system say there is a cart actually here for the system, but I am ignoring the cart dynamics, I am saying that I am applying this control input in such a way that somehow I am able to maintain whatever desired position for the cart to be x. And I applied that as a control input to my pendulum system.

So, as you will see later for such a definition, there are some good advantages that happened here. So, you can see what those advantages are by saying putting these equations together here. So, this equation now with this input here, not get into the details of this how this has come but you can check out by using Lagrange formulation or using your simple Newtonian approach you can simply see these terms.

So, you can see here for the same system which was there previously of course there was a damping here that we can add anyway for that this $\ddot{x}$ term appears here. So, this $\ddot{x}$ upon l entire of this term is some input which is in the direction of the generalized coordinate Θ because is a Θ equation, Θ generalized quadratic equation.

So, by doing this kind of input to my system I get a term for my control which is directly in the direction of Θ as if I am giving a torque on this string so normally giving you torque on the string is not possible. But because I am able to apply this a desired acceleration it is as if I am giving a torque to the string in the direction of Θ. And then this helps us to develop nice controllers later.

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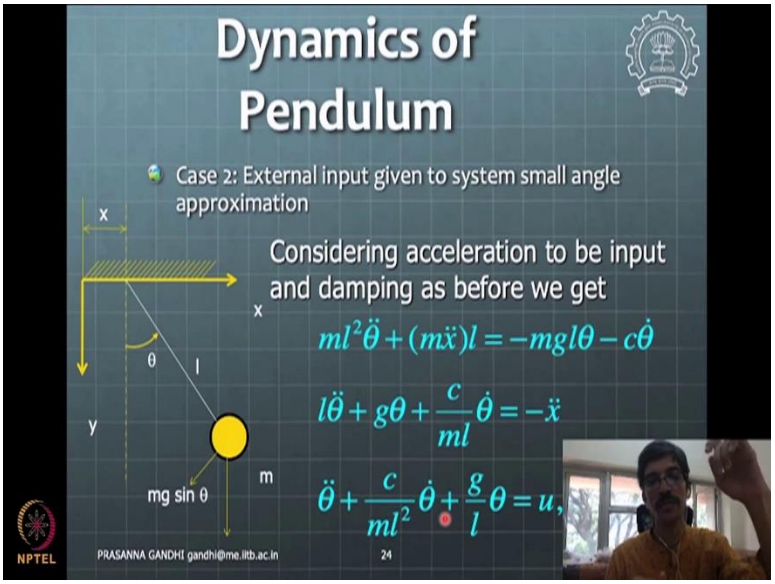

So, with this term consider this acceleration is considered to be an input to the system and for the damping is considered in the system you get no this kind of equation. So, this is now simple spring mass system, you can see the second order spring mass system with some kind of a force external force applied on the system. And with this force now I see I can define a control problem that develop this u in such a way that this Θ goes to 0 or initially oscillating kind of a pendulum Θ goes to 0 in some given amount of time.

Normally the pendulum is going to take a lot of time, lot of oscillations to carry out and after some oscillation it will stop anyway, but I want to kind of go to this final point in a very short amount of time and how do I carry out that job. So, where is this kind of utility of such systems one thing that comes to my mind where we have these overhead cranes. They are carrying some big amount of chunk of mass from one place to another place which is hanging out of the string and there is a cart which is on the top of the overhead crane.

You might have seen these construction cranes they have these. So, you do not want this mass which is with a long string attached and it can by the way and it can kind of oscillate, it can have some kind of emotion possible, we do not want that motion to happen, oscillation should be damped down otherwise it may be dangerous for the personnel life.

So, in that case, this control could be kind of very useful. So, what we do here is now for such a pendulum kind of a system we demand that we do not want any oscillation or if there are any oscillation they should be get damped down very fast.

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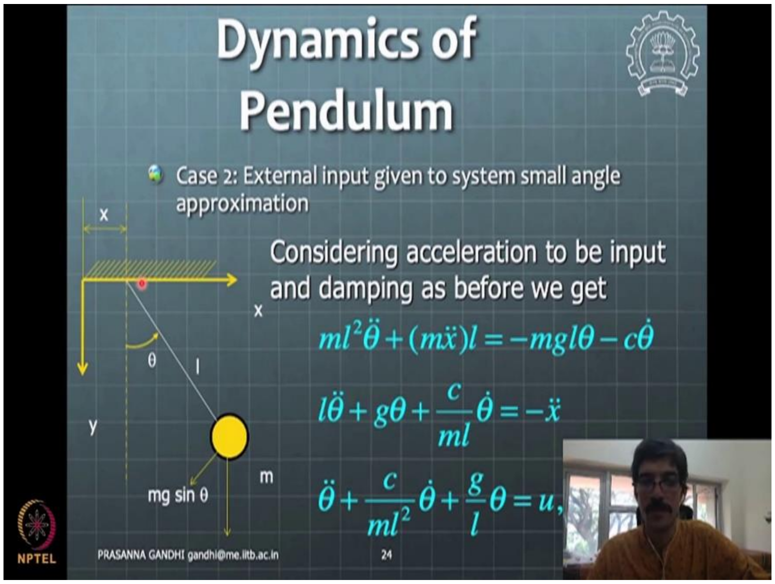

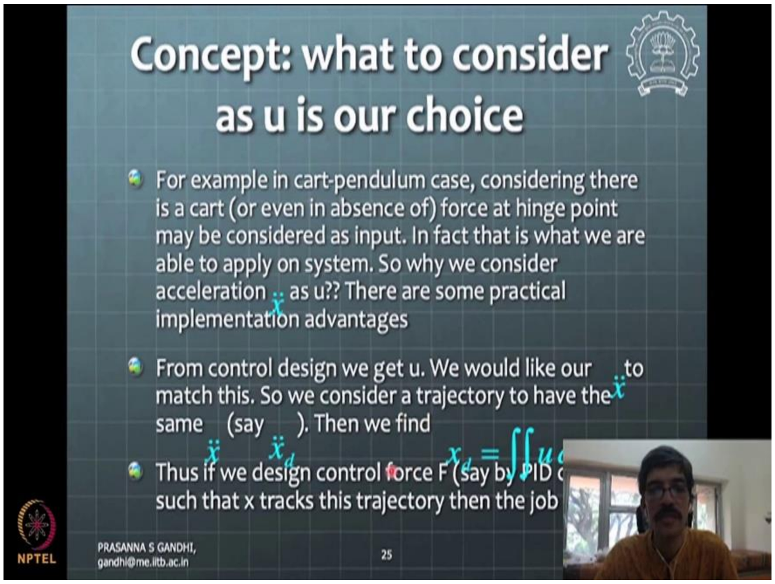

So, how do we do that now, so how do we… So, this is our control and control problem defined and suppose we find such a input u then how do we drive the cart such that this u is applied to pendulum that means we need to satisfy this equation. So, u is there and then $x_{desired}$ now, this is not x here, $x_{desired}$, we can make it as $x_{desired}$ here. So, my $\ddot{x}_{desired}$ should be that u and I integrate this u twice with the scaling factor l.

Then I will be getting the actually the $x_{desired}$ as a function of time trajectory. And that $x_{desired}$ if I go along that trajectory $x_{desired}$, then I am guaranteed to apply this u that is that was desired here. So, u itself if you see will be a function of time as we propose some kind of a controller which will be a function of this Θ which is in turn that function of time and then you integrate that twice to get x and that x is what I will say my hinge of the pendulum will be moved along that x, that is how I can plan my whole control implementation.

So, from control design we will get u and we would to like our $\ddot{x}$ to match this. So, we consider trajectory to have the same $\ddot{x}$. So, this $\ddot{x}_d$, then we find $\ddot{x}_d$, from that $\ddot{x}_d$ equal to something, by twice integration of u I have missing the scaling factors. You can put those scaling factors. So, now we can control force F suppose there is a cart pendulum, cart also is there in addition.

Then I can kind of use this force in the cart to be F by some means we can apply that force by superior PID controller tool track the trajectory which is coming out of this equation. That kind of idea can be possible for implementation. There are quite a bit of advantages of this kind of implementation which you will not discuss for now here. But later on maybe if you are interested I will kind of show very very good advantages of such a implementation.

Rather than considering directly this force to be my input and considering my force applied on the cart and cart in turn is having a pendulum. So, there is a cart here and the force f is on the cart and the cart in turn is carrying this pendulum along. So, that is not very great, practically good strategy to implement because there will be a friction between the cart and its wheels and the places its sliding part. So, this is a kind of a way we implement this, we propose to do this kind of control.

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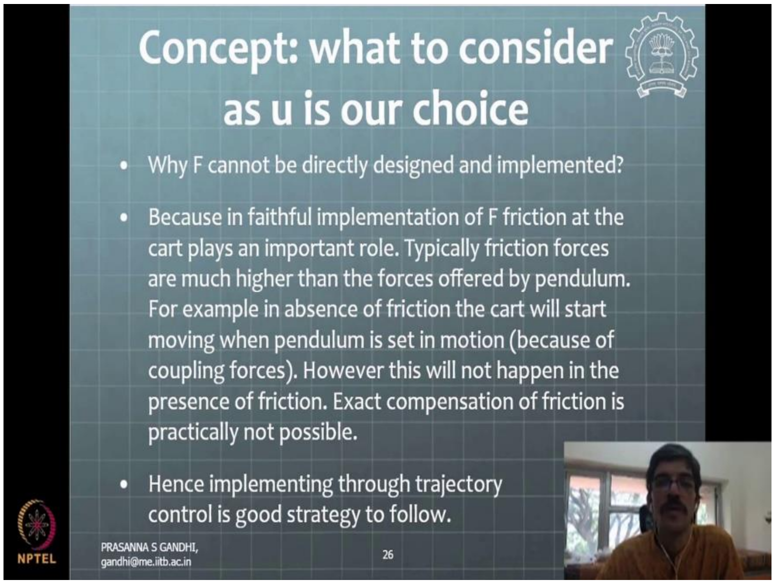

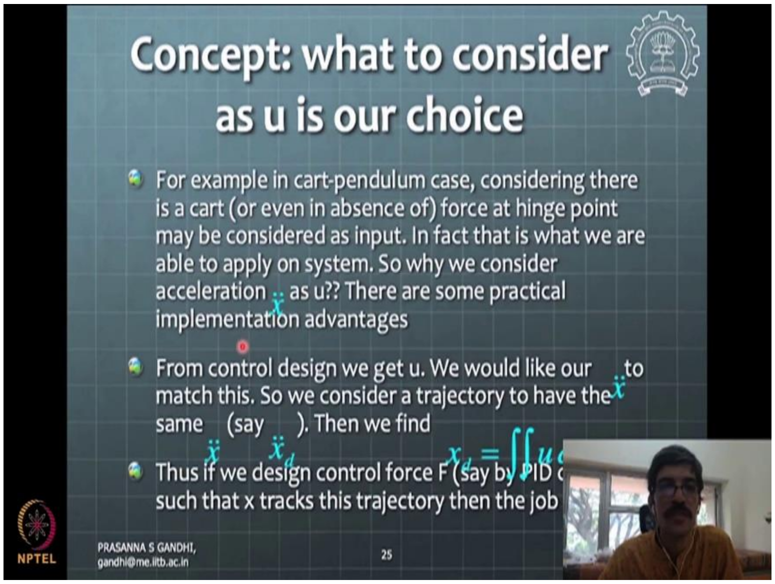

So, this is a discussion about why F is not directly designed and implemented because of this fiction that is there in the systems anyway. So, you can do some kind of equations development of you know what will happen in the presence of friction how I consider control with friction in the case of cart pendulum system and then things will unfold a little bit what I am saying here. And the friction is major calculate to faithfully implement F that will make my control better.

I can tell you something physically here. See, if the friction had not been there any pendulum motion here. If the pendulum motion happens here and if there is a cart without friction that is there some mass is there of course then the cart would respond to the motion of this pendulum as a reaction forces acting on the cart at this point will start moving the cart here and there. And imagine the worst possible case when the friction is so high that it cannot move at all.

Then even if the motion happens here the cart is not moving. So, if the cart is not moving at all then your feedback is somehow lost this so you cannot affect pendulum our pendulum is not able to affect the motion of the court in some way. And that is a big hurdle for getting any kind of feedback based on this force implemented faithfully that much I can tell you the physics part.

But to really get to this you need to write the equations in the form of a pendulum cart total system that will be two degrees of freedom system the cart degree of freedom and the pendulum degree of freedom Θ and the force consider as a input.

So, single input but two output kind of a system will be there and with the approach that we are discussing here right now those are all complications of under actuation will be called here if I consider u to be this kind of input and I implement this u by using the using these double integration ends and say some kind of a PID controller.

Now, there are some kind of mathematical questions that one may pose that how do you guarantee that we will be able to track this xd trajectory very well by using some PID controller with those kind of more kind of a finer mathematical aspects would be there which we are not when we say that our PID controller is fast enough to do that control so that it is able to kind of follow. The desired trajectory in a very appropriate sense. So, these what is this discussion about this force here.

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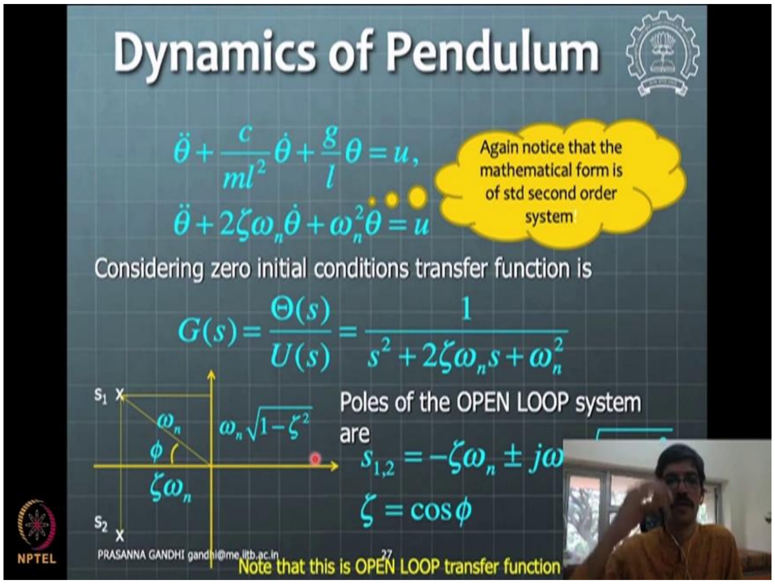

So, we come back to the system now we in the standard form it looks this and then can say write a transfer function of it. And then see the open loop poles are here then now we can propose some controller to move these poles around and things like that. So, that those are all kind of things can be done now.

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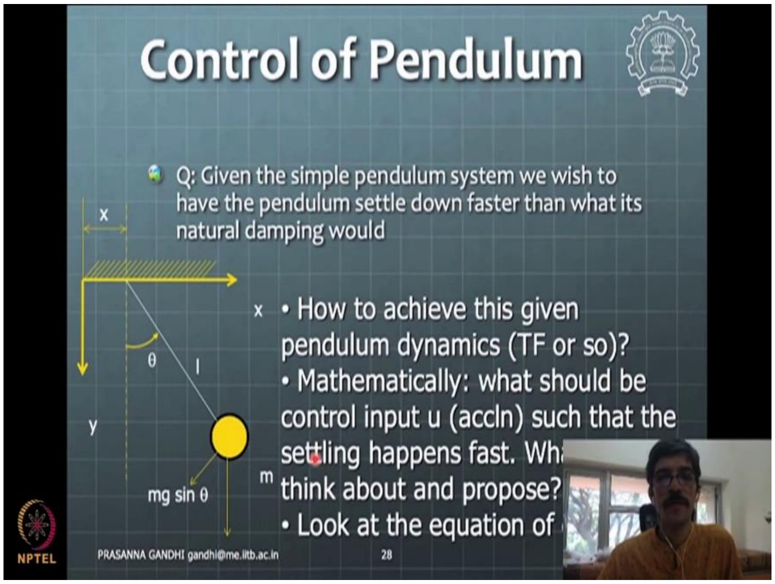

So, we propose say PD controller or P controller all these kind of different controllers can be done some discussion that you can find here it is pretty simple to follow through. So, the in the control problem is defined to have some say we just want to settle down faster than what it is its natural damping would allow it to do.

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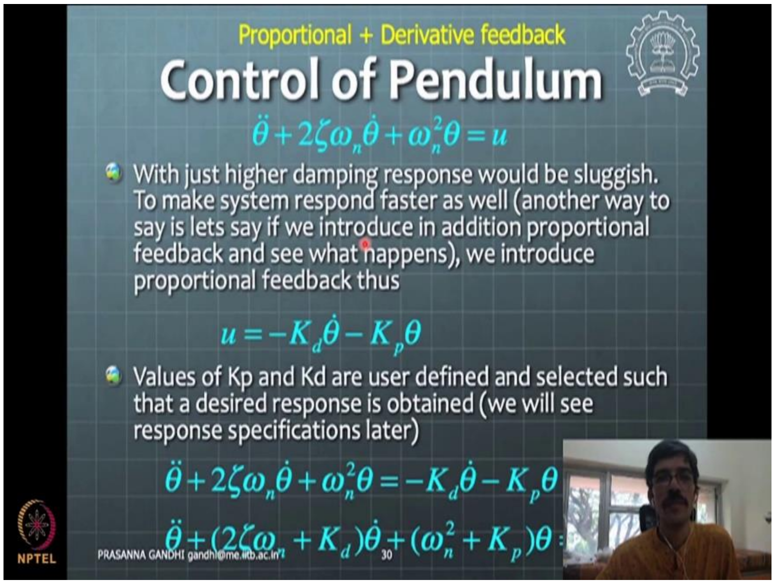

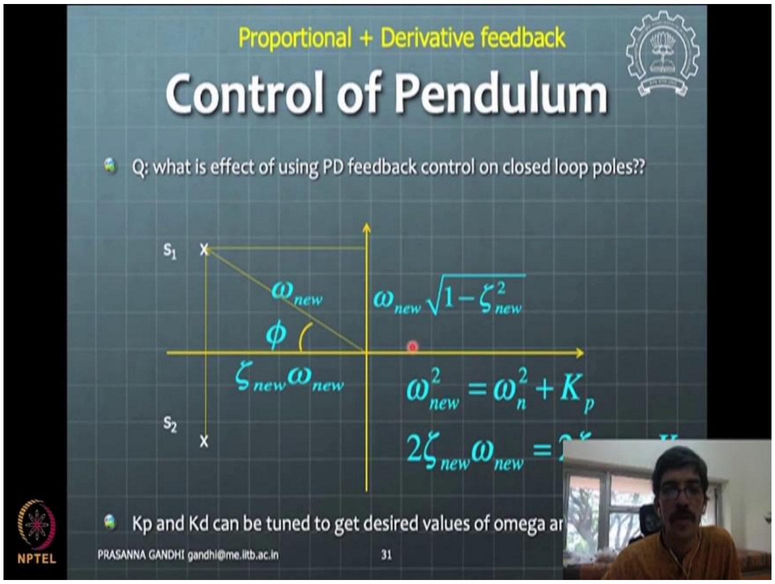

So, we may propose like the derivative control or complete proportional derivative kind of a control to achieve the task here we have two parameters available for us to tune. And so we are now seeing entire thing all this development only in the differential equation kind of domain. So, you see we have chosen to do this is, it is up to you to kind of consider that if you want to kind of do it on the Laplace domain you can do that and see.

Now for the new pole locations what should be this K_d and $K_p$again So, that I can move my post prepared location based on choice of K_d and K_p. So, this is a final form of equation in the differential equation domain see this is the error equation because Ɵ itself is a or error because we do not want Ɵ we want Ɵ to get damped out. So, Ɵ should be desired to be 0 then the error equation is same as the equation in Ɵ.

And with this we can now plan what we want how fast we want to be choose omega new based on that and so this is our omega new here omega new square and this is our new 2Ɵ ω_n complete term. So, we have some kind of a choice in both the cases to effect and can choose these scales to kind of have whatever desired omega new and Ɵ new and based on that our system response will evolve. So, you can use the standard system second order system specifications to design these values and your problem will be solved here.

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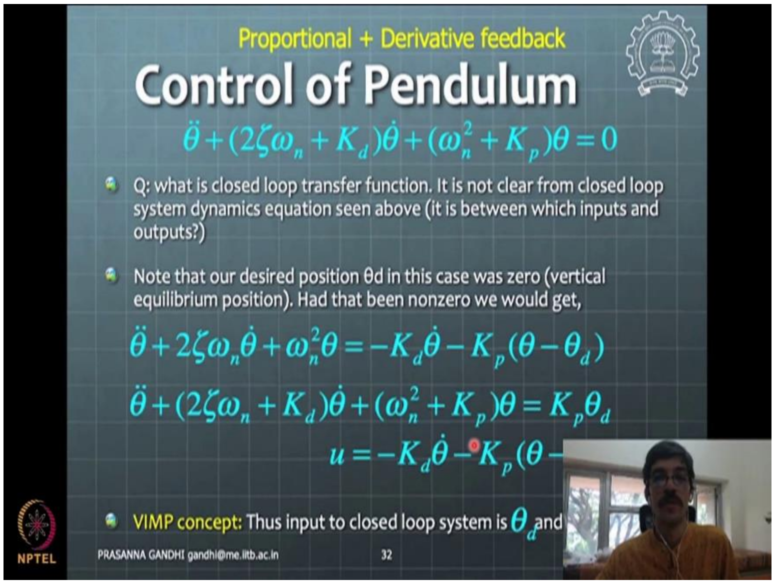

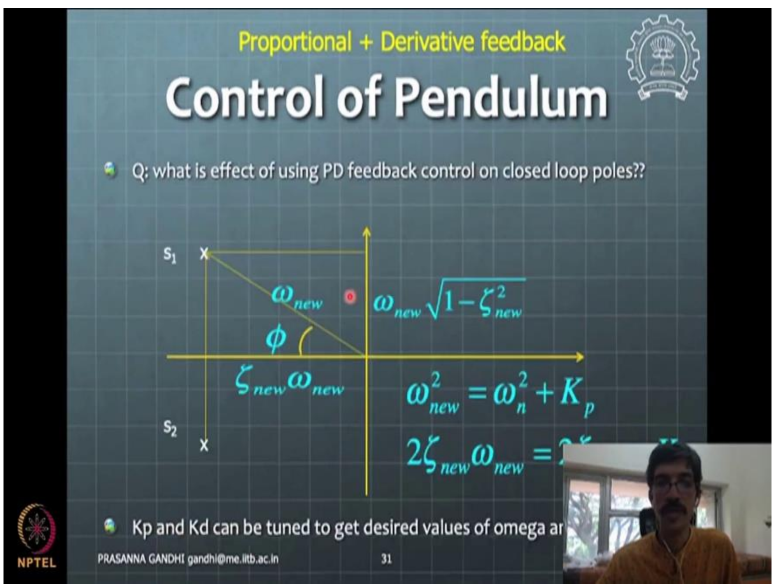

And once we get this ξ then your u ξ and this your $K_p$and K_d are sized appropriately then this is final u and then you can get this distributed. No this is sure with the Θ d is equal to so many.

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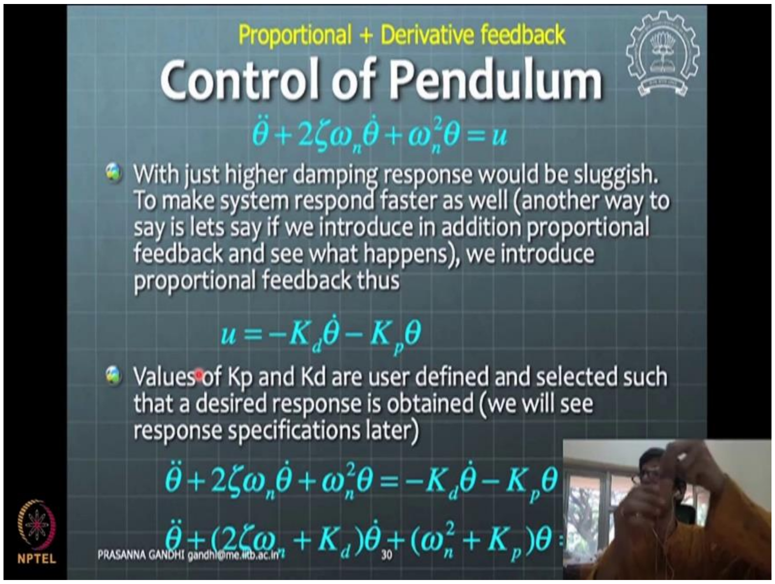

So, you get as u in this in this form here and then implement it by taking double integration of this and matching it to mapping it to x desired and now x desired is my motion of my hand here. So, if you see here let me change this view.

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So, I have this pendulum mob here you can see I will use a shorter length. So, with this if it is initially in motion and I want to stop this motion I need to kind of say move till it stops immediately. Otherwise it will continue a lot of oscillations to move I move this my hand in in such a way that it gets to the stop condition very fast. So, the I am doing something in my head to do that you can also try out and you will be able to also do.

So, whatever is happening in my head okay in some way is getting captured in the in the mathematics that you see on the slides. So, I can kind of move this quickly to do the job. So, I will not be able to do as quickly as for example these equations can do for a tune gains.

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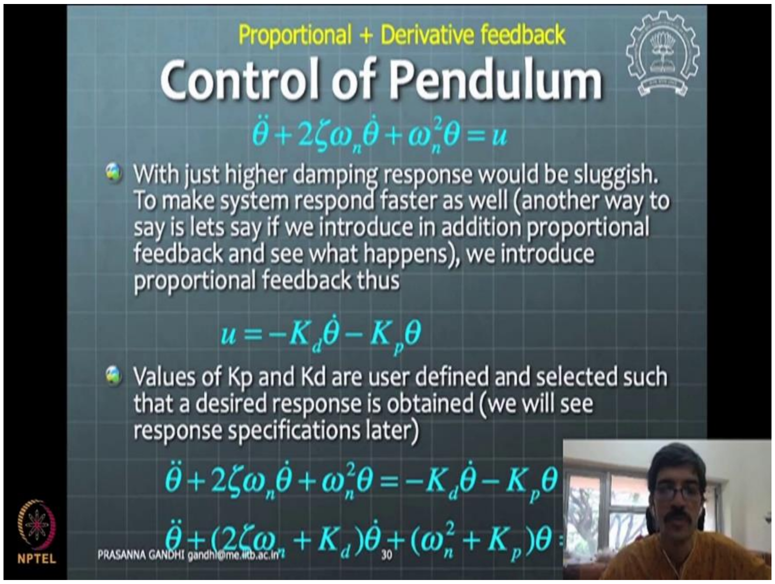

So, that is the beauty of using the mathematics to some kind of a nice way developing control over such a kind of a phenomena. In one of the examples. There are many such examples one can kind of start developing and do that. And this has immediate practical applications in the overhead cranes kind of cart that moves in a overhead cranes.

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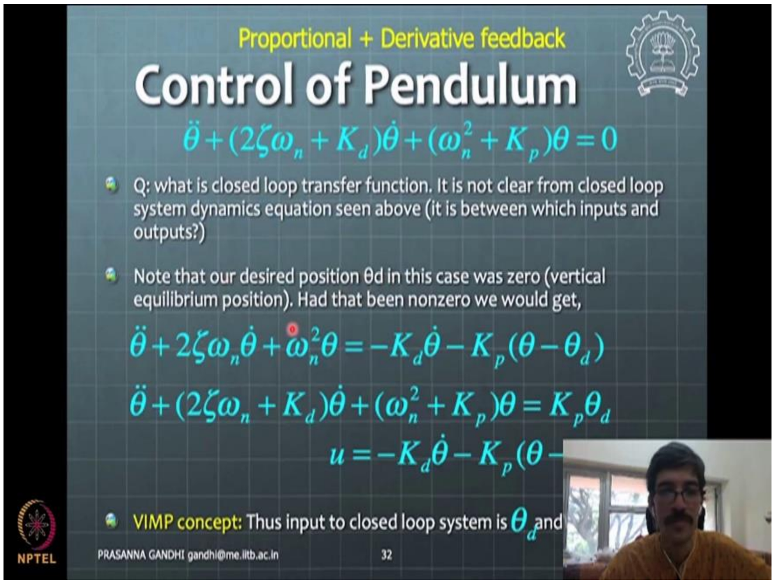

So, let us move on from here to the other example. So, you can get to more detach if Ө desired was not 0 then we have some difficulties here.

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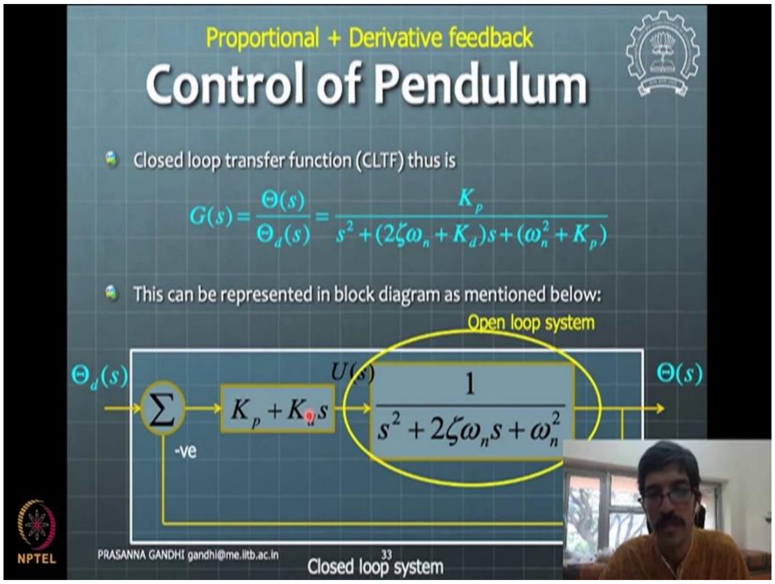

So, now one can see this whole in the in the Laplace domain how things will be presented and with this one can kind of see this feedback some kind of a unity feedback here with Kp Kd as here then you can as we have discussed no one can do root locus analysis of such a system. And that also kind of helps to kind of tune the gains or move see the things in a different kind of approach.

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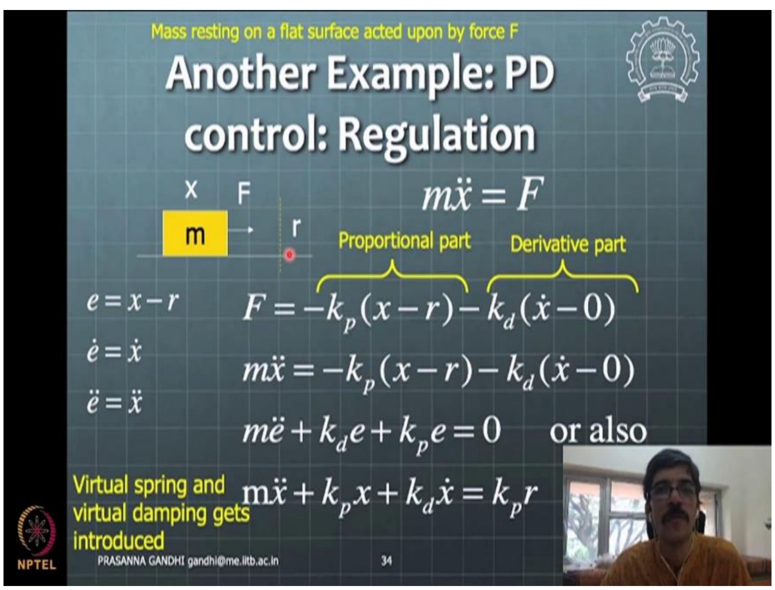

So, this is another example about the simple PD control for a regulation purpose. So, this is much simpler form which we have already seen. So, it is just again PD control applied here and you can see this in the error form here. So, the idea here if you see from these error dynamics what happens is that there is a virtual spring as if virtual spring is attached to this mass which will have equilibrium at the desired reference position.

That is what is one can interpret this controller as if we are actually attaching some virtual springs and dampers to the system to get some kind of a physical inside in to.

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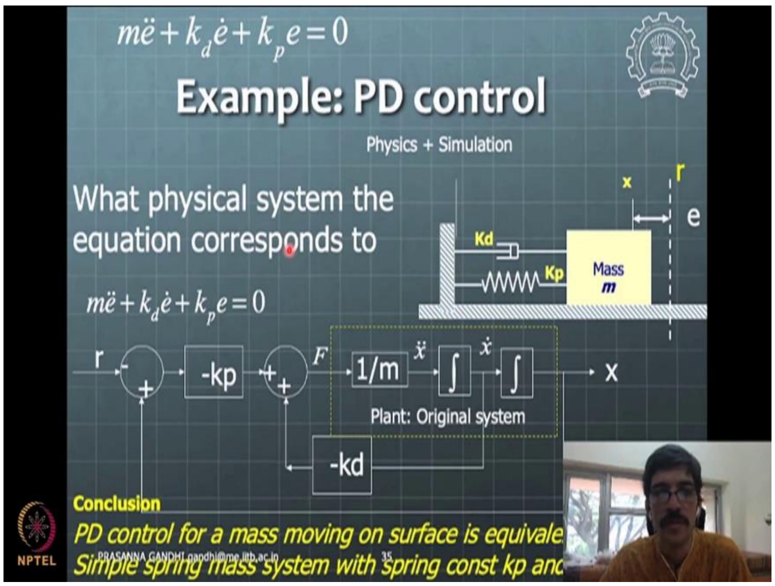

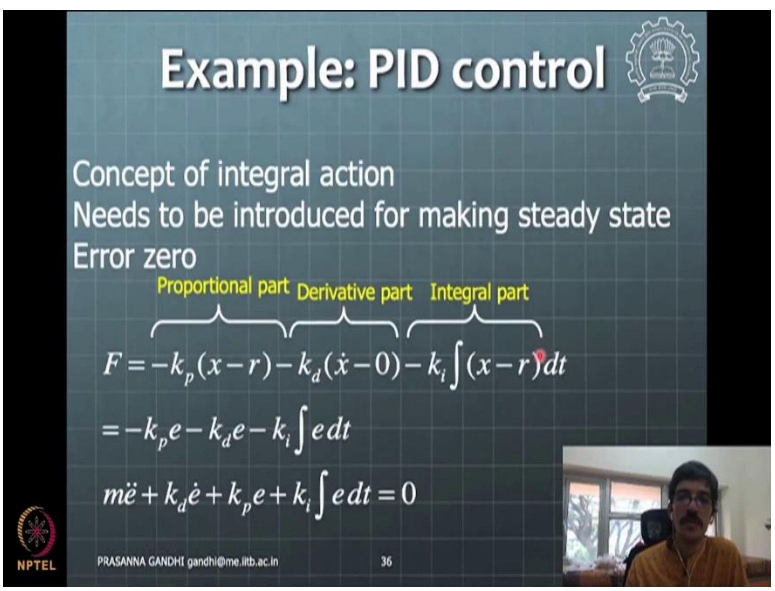

So, this is a same kind of discussion in the domain of this example. Now, when we introduced this integral part whenever we have the steady state error not going to 0 that time we introduce this integral part. So, you have seen in your simulation of the simple single link attached to motor kind of a system that when you try to oscillate this pendulum it does not kind of come to the vertical position because of the friction.

The friction is what is not allowing it to settle into desired position or in the vertical position. So, this friction is causing some kind of a steady state error in the system. And even if you if the system is to move in the horizontal plane. The same thing would happen has would happen for this mass which is in a horizontal plane where the spring and damper kind of a case. So, these are anyway the spring and damper are actually the virtual springs and dampers that are put in the system.

So, if in the presence of this virtual dampers in the system one can see that if I keep this mass oscillating it will stop at some point which is not really matching this r completely in the case when there is a friction. So, friction the system will not allow it to reach r it will have some finite error that will be there in the system that is a kind of a point. So, in the case of that kind of error then you introduce this integral control action.

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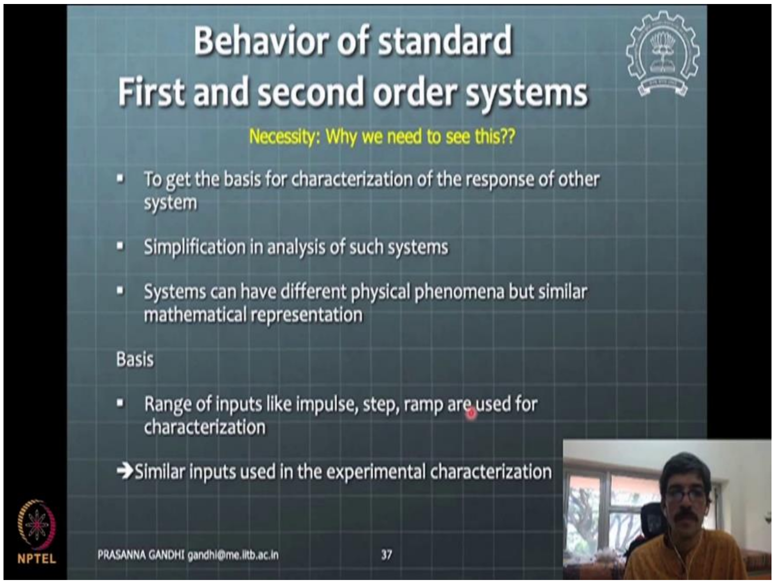

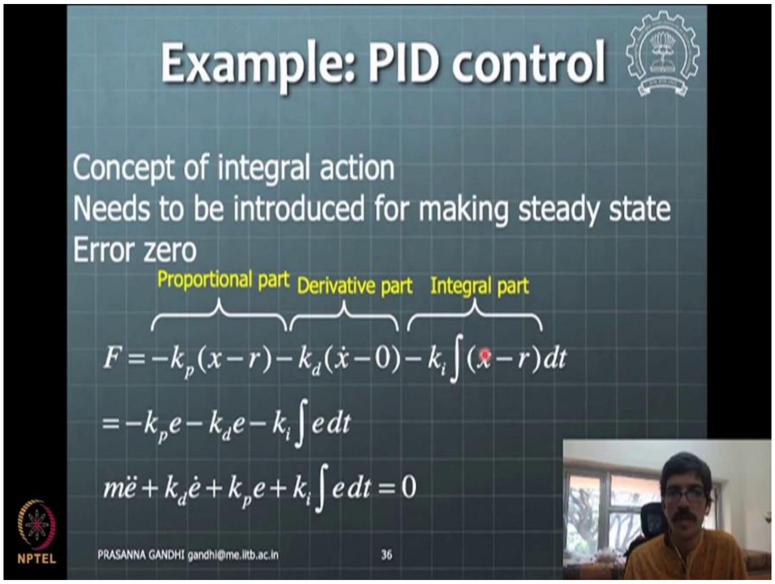

And the system has this steady state error you see there pause to the PD controller and you see that there is some kind of a steady state error then you introduce the integral control action. So, we will talk about this integral control action part I think this main kind of a thing or maybe we can give a summary of like when to use these different kind of control actions in generally in the system so this is be good to kind of know.

And then as we have seen we use this make use of this standard first and second order system responses which we have seen already.

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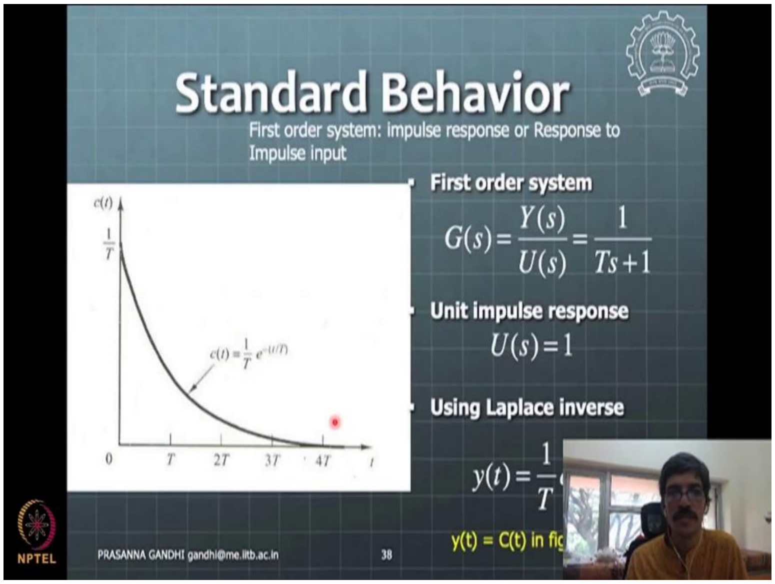

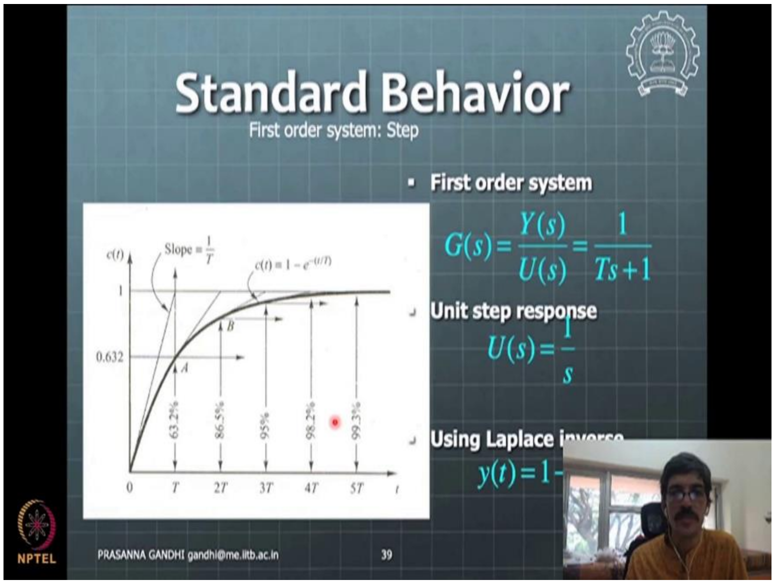
Standard Behavior
First order system: Step
First order system
$G(s)=\frac{Y(s)}{U(s)}=\frac{1}{Ts+1}$
Unit step response
$U(s)=\frac{1}{s}$
Using Laplace inverse
$y(t)=1-$
Slope = 1/T
c(t) = 1 − e^(−t/T)
c(t)
1
0.632
A
B
63.2%
86.5%
95%
98.2%
99.3%
0
T
2T
3T
4T
5T
t
NPTEL
PRASANNA GANDHI gandhi@me.iitb.ac.in
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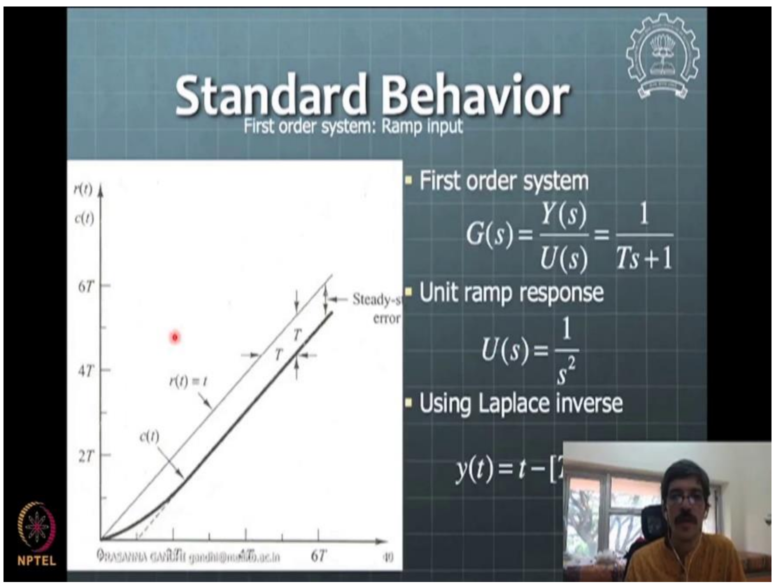
Standard Behavior
First order system: Ramp input
First order system
$G(s)=\frac{Y(s)}{U(s)}=\frac{1}{Ts+1}$
Unit ramp response
$U(s)=\frac{1}{s^2}$
Using Laplace inverse
$y(t)=t-[T$
r(t)
c(t)
6T
4T
2T
r(t) = t
T
T
Steady-s
error
0
6T
NPTEL
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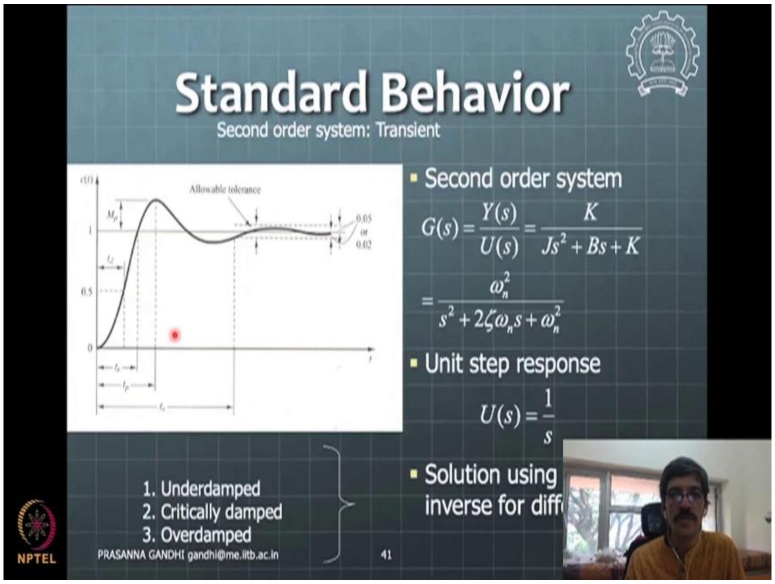

I am just flashing the same slides here actually. So, these are the standard first and second order system responses with this is this information should be handy to us for the application in several cases. And then we will have this formula for the second order system behavior formulae for maximum overshoot settling time this should be also handy to you.

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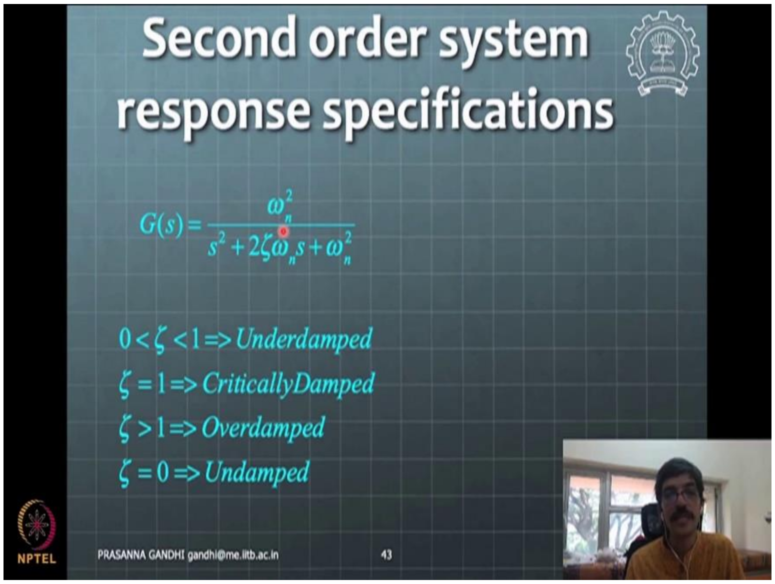

So, that as we saw in the pendulum problem for example we can use them to set up this omega n and ξ to have a desired value or when you do the pole placement problem the two dominant poles which you are placing close to the imaginary axis they can have some values taken by this thing.

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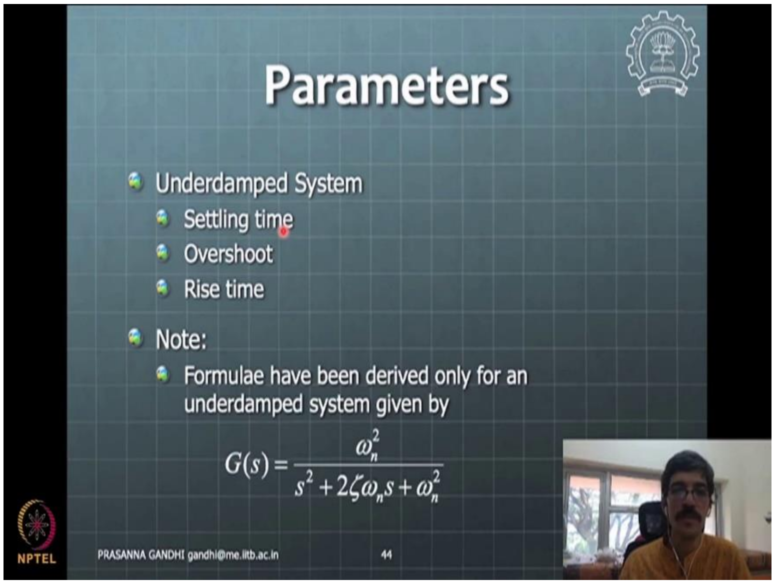

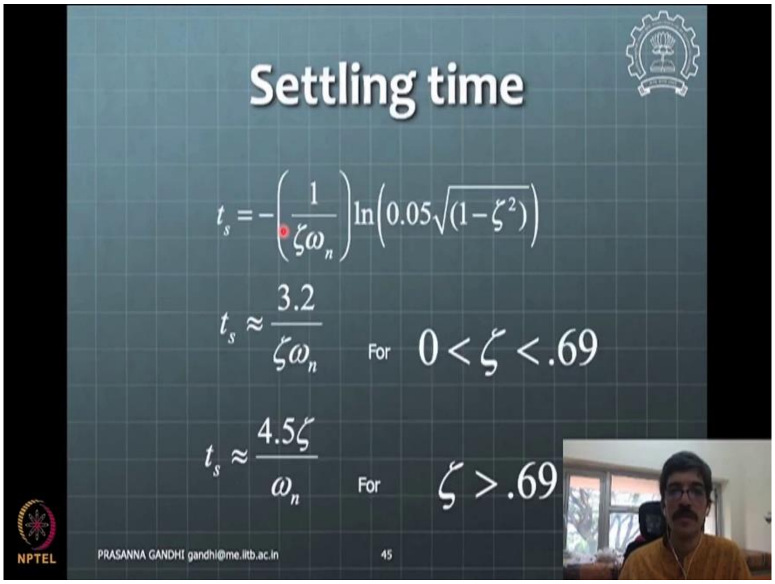

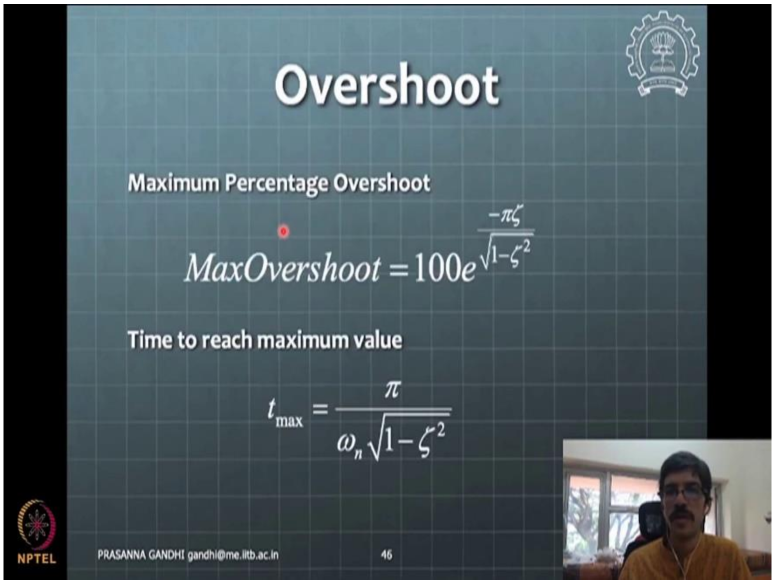
Overshoot
Maximum Percentage Overshoot
$MaxOvershoot = 100e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$
Time to reach maximum value
$t_{\max} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$
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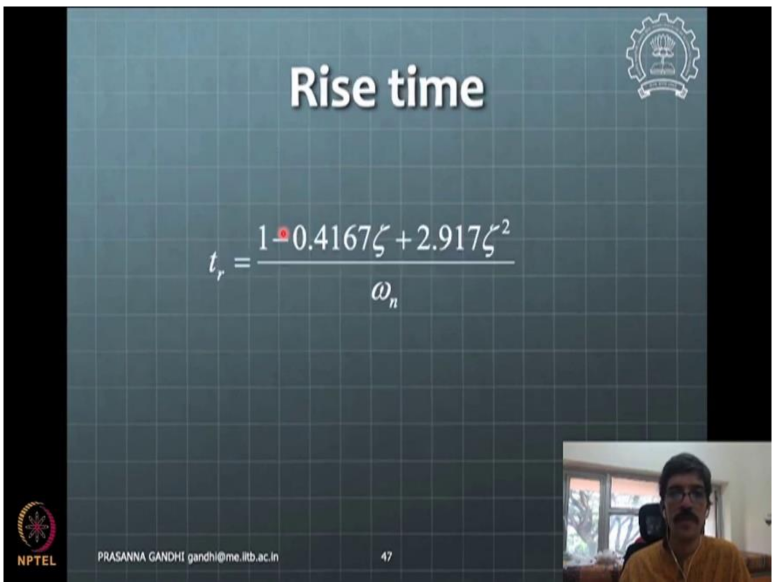
Rise time
$t_r = \frac{1-0.4167\zeta+2.917\zeta^2}{\omega_n}$
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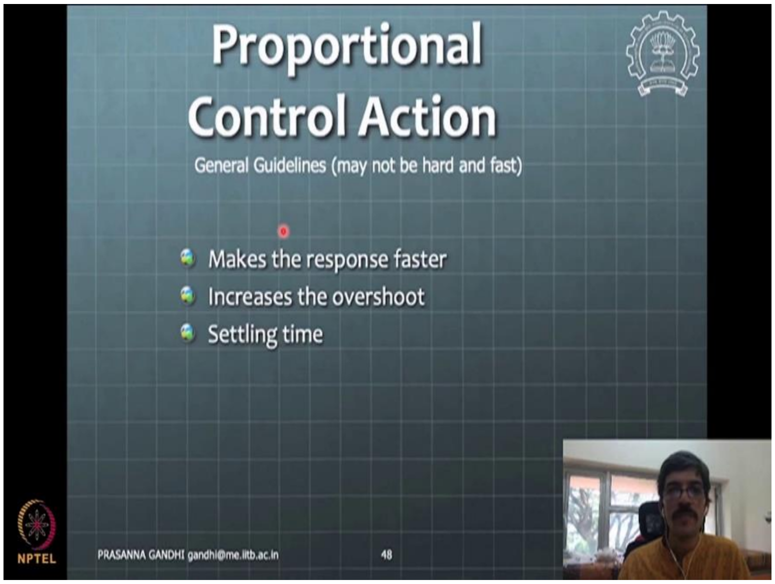

So, these are the typical for parameters that we define. So, this is this is a form that is given in the slides you can just go through them to make use of them in the pole placement problem or in simple second order system problem. These are some of the general guidelines that one should be aware about that proportional control action will typically make the response faster increase the overshoot and increases settling time. So, if you will use Kp gain higher and higher it amounts to increasing the settling time also all the responses faster it will settle in time also increases. So, for the settling time to decrease we will use the K_d gain.

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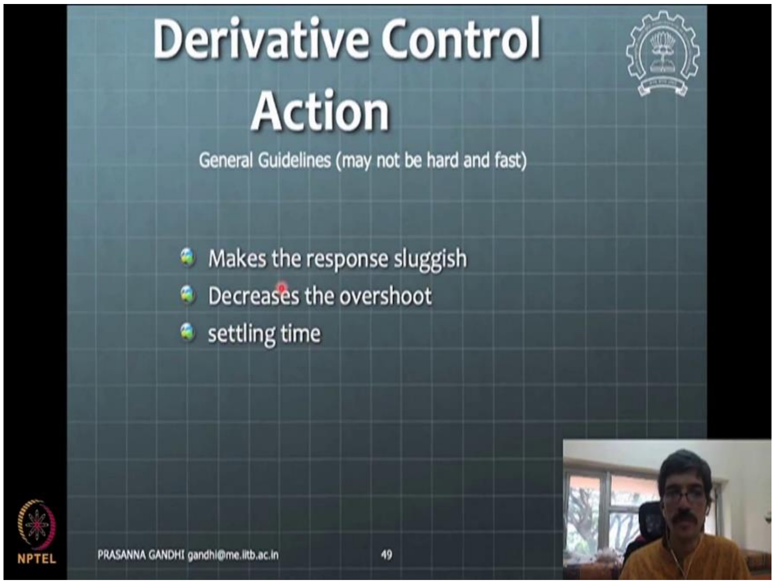

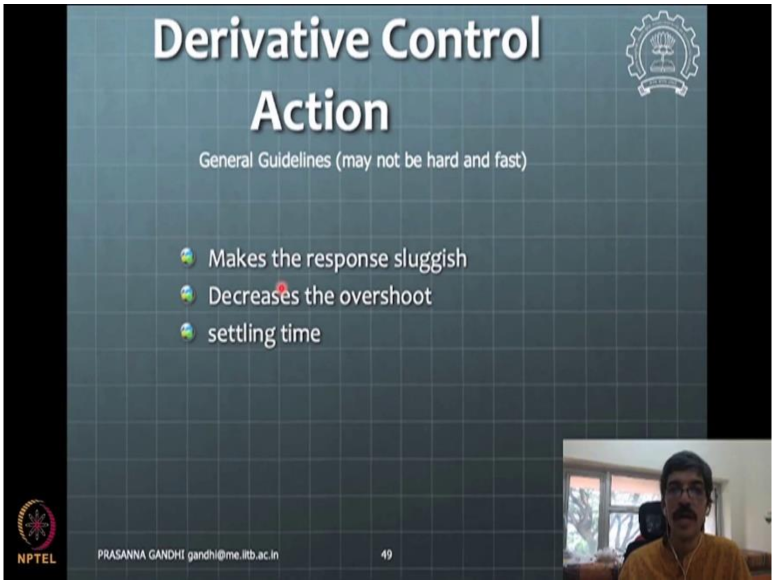

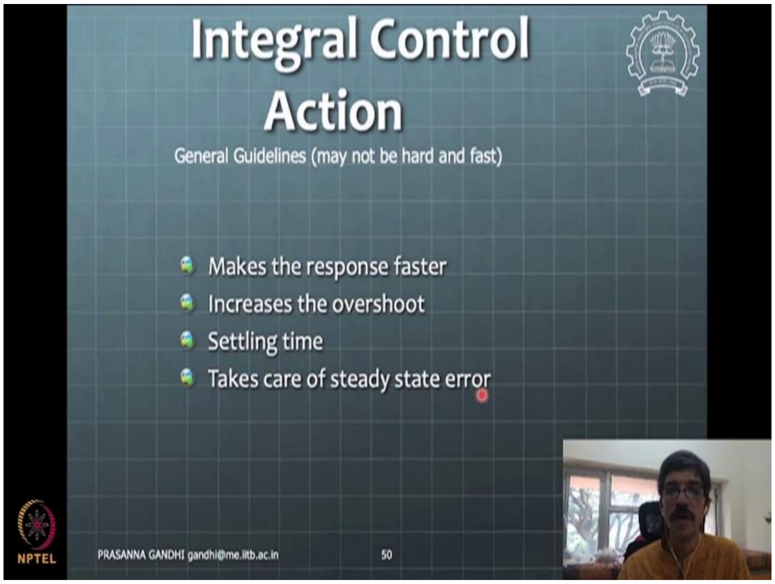

This derivative action decreases the overshoot and decreases also the settling time. And but the response is a little bit sluggish. So, the combination of Kp and Kd would give you some kind of nice response possibility. And integral control action again it makes a response faster it actually introduces some 0 close to the origin in the controller transfer function not really for the system thing but controller transfer function will have a 0.

So, it may amount to have 0 in the system also we but we do know it depends on the kind of a form of a system in a close loop. So, it makes the response faster increases the overshoot and settling time but it takes care of the steady state error. So, imagine if you have a steady state error like this here and integral control action is their derivative because Ɵ dot is 0 derivative control action is stopped proportional control action this Ɵ is constant.

So, proportional control action will keep still keep going but it is not changing Kp times this error will be a fixed kind of amount of input that will be going into the system. But at the input is doing nothing to the system. Because say for friction for example even if you apply force player the system is not moving that kind of a case will happen here. So, in this case the integral control action will help.

Because if you put an integral control action if you see from this point just the integral is starting from this point you can see the integral control action will be the area under this error. So, I get between these two lines will be the integral control action that area will keep on building up at some point it will exceed the friction value and depend upon gain that you have used and this error will be taken care of.

So, there is a chance that with this integral control action in the presence of friction this buildup happens so quickly that with certain application on that force the system moves on the other side. And again there this integral control action will have to come in picture to move it further back and there are certain tones for this process that system will keep on hunting between the two values rather than settling into some final value.

So, one needs to be careful about choice of the integral gain to prevent this hunting. So, how that is to be avoided and those kind of things are more kind of mathematics can be worked out for such kind of a case by considering say your fiction model or there are many other techniques also available. So, you use this friction model and use something called describing function method to come to what is the frequency that that your system will be keep hunting and do.

So, that these are not the matters of our discussion here. But one can use these methods for such phenomena if at all you find somewhere in your future application of control this thing is happening. So, this integral control action can have possibility of system getting into hunting.

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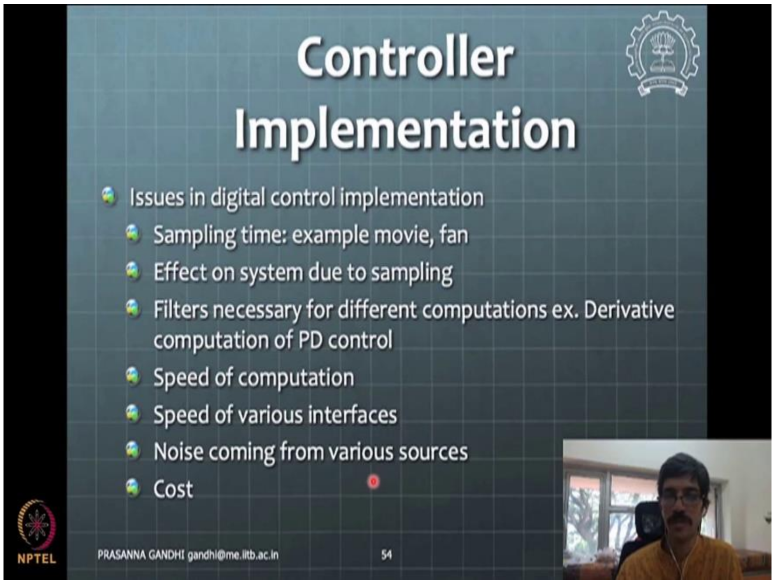

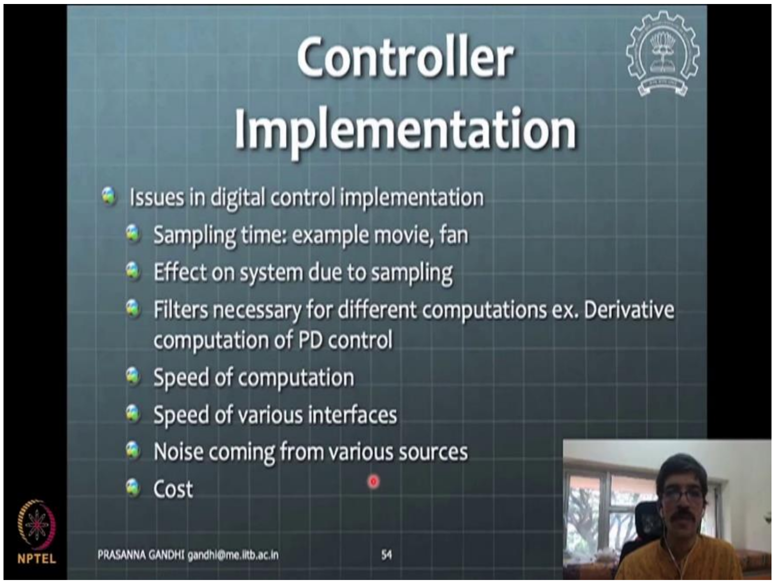

And how do you tune this PID gain typically is given in this procedure called Zeigler Nichols procedure it is just a matter of some kind of steps that you follow you can see these more details and this there is no point in getting into and explaining those steps or minutes simple information you just kind of follow the steps and your gains will be tune. So, you do look at this for practical tuning of the cities based on only know the experimental system is ready.

And I am now going to tune the gains or your simulation system is ready and I am going to tune the gains this the way you can tune the gain you can try it out on some of the systems that we are discussing is a part of assignment in our class. So, this is I think we will stop here for this discussion and these implementation aspects we have already covered some of them and there are some more issues listed here.

Sampling time we have seen already. Then effect on system due to sampling we are yet to see. And filters these are all related to your sampling and then speed of completion or how much is a competition that we need to do that is also important consideration that we should finish computation of control within the sampling time and speed of various interfaces also is a matter of things action that has to be completed within a sampling time.

And so these are many some of the aspects we have seen, but you should have this list to see or check or put like a checkmark you have taken this aspect in so pettish consideration this is a kind of a checklist you can do for controller implementation we have considered all these aspects or not you can check that. So, maybe we will stop here for now.