Design of Mechatronic Systems

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Lecture Number 40

Application of Lyapunov
Stability Analysis

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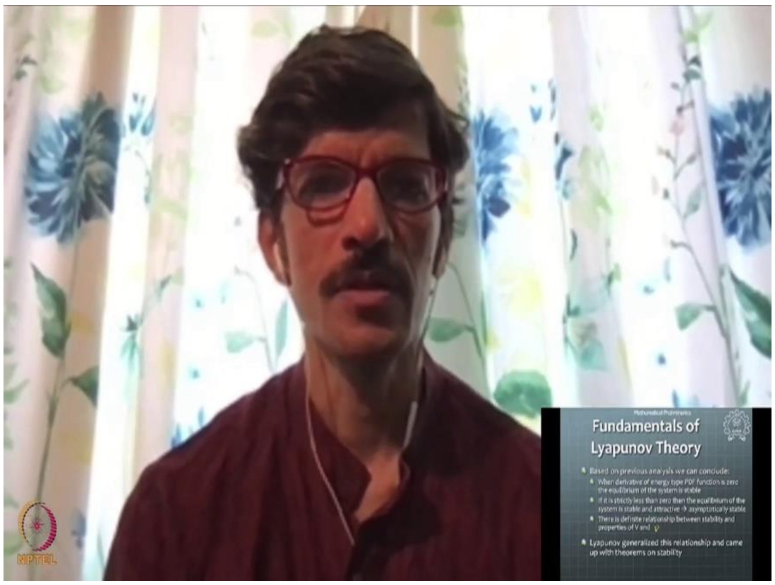

So, far we have been looking at some of the fundamentals of Lyapunov theory some preliminary background and some physical insights into how Lyapunov has come up with different different theorems. And these are some kind of a intuitive understanding about it. So, if you recall we had seen this stability proofs based on some definition of some functions which are energy based functions and their derivatives along the system trajectories.

So, we define some mathematical preliminaries and come up with this idea which about Lyapunov has come up with. And now we will see more formally some mathematical kind of stability theorems will not get into the proof of them. But we will just see the mathematics a little bit more in detail and apply it to the case of development of control and providing the proof for stability proof for any control that is proposed. So, that process we will go through.

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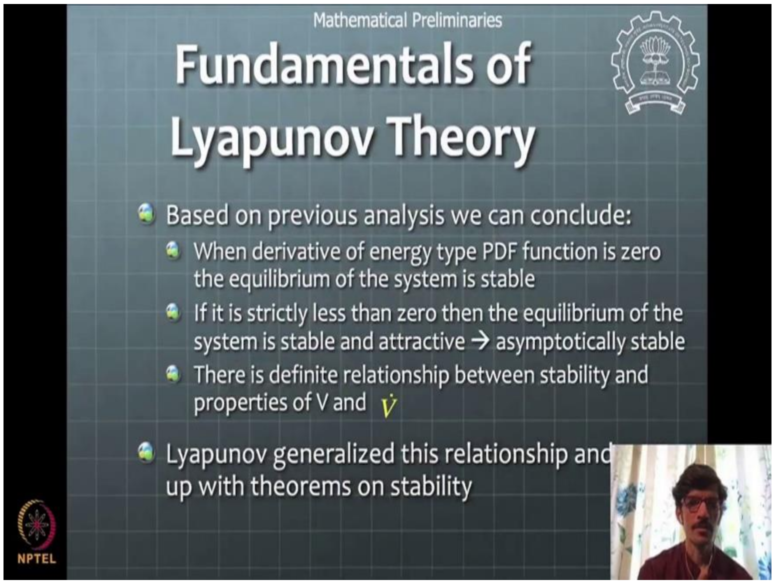

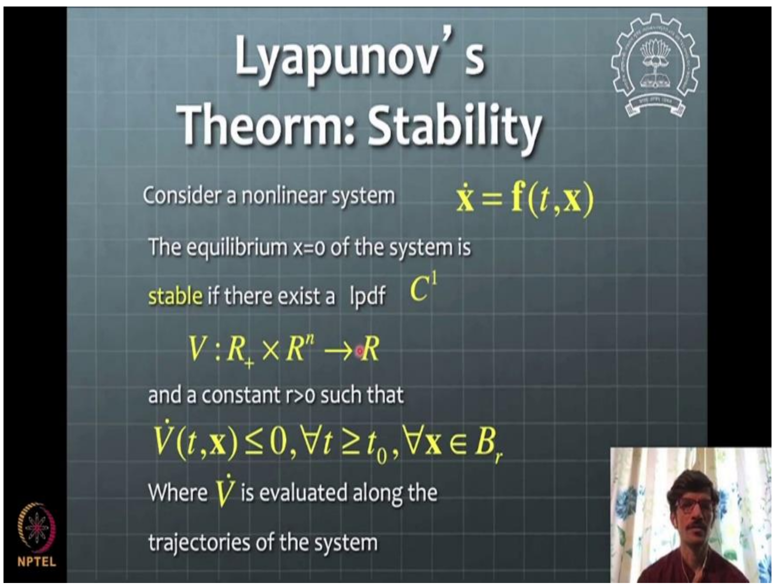

So, let me get to the slides. Now, to show this first theorem we will just recap that we saw was about the stability. So, stability of a system. So, we are considering this system which is

$$\dot{x} = f(t, x)$$

,where x is a state x is a so if you have n state system typically say if you take a 2R manipulator robotic system that robotic system would have 4 states. Two degree of freedom typically will have 4 states.

So, that kind of system if we have then this x will be a 4 vector. So 4×1 kind of a vector like that f will be 4×1 kind of a vector. And it is a function of so it takes R^n and R to space R n. So, that is how this is f is defined. Now, for this system this first stability theorem says that the equilibrium x = 0, this is a x vector, 0 vector of a system is stable if there exists this is now locally positive definite function C1.

C1 means continuously differentiable at 1 time and locally positive definite function we have seen what are the definitions of that. So, lpdf C1 function which is different like typically designated as V. So, V is Lyapunov function some function, if it satisfies this property lpdf and C1 then we call it Lyapunov function candidate. This V is Lyapunov function candidate if it is satisfying these properties.

So, this V is taking R plus means is a time in a positive value to and R n to R space. So, this is a real valued function. This is a lpdf function V and constant small $r > 0$. So, this is defined to have some kind of a finite radius ball or finite radius r here. So, we are defining a local definitions here, we are not kind of considering the global stability of a system, we are considering local stability of a system.

So, it is sufficient to have these properties hold true in small kind of hyper ball of this radius r. So, when this $\dot{V}$ of this function which is Lyapunov function candidate $\dot{V}$ is less than or equal to 0. So and this should be valid for all $t > 0$ or greater than t_0 and within this ball of radius r, x belonging to this ball of radius r. And $\dot{V}$ is a function which is again real valued kind of a function is evaluated along the trajectories of a system.

So trajectories of a system you remember like $\dot{V}$ will be actually $(\frac{\nabla V}{\nabla t} + \frac{\nabla V}{\nabla x})\dot{x}$. So, this is like typically given as expression for $\dot{V}$. So, we have seen what is this derivative along the trajectories and where this $\dot{x}$ is substituted as this function here. So, we will see through some examples how this is done and all these procedures but these how this stability theorem is defined. And if this is valid for like entire R n space then it becomes like a globally stable system and this also needs to be valid entire R n space.

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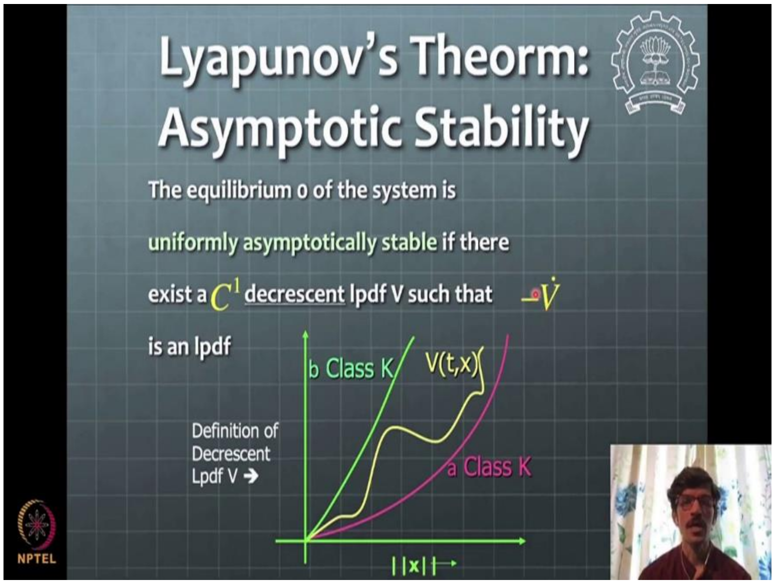

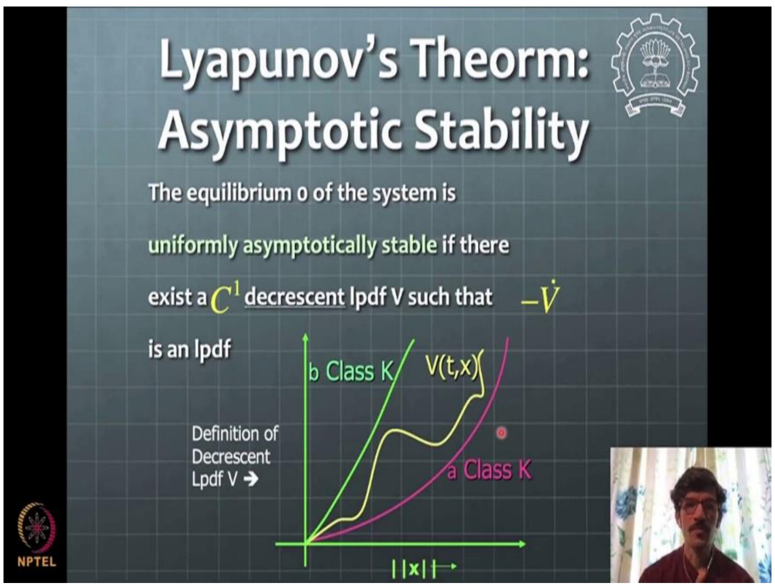

Then you have this notion of asymptotic stability we have seen. So, now this is asymptotic stability theorem for so we need now we say for the same system same equilibrium is uniformly asymptotically stable if there exists even decrescent now this is a decrescencies additional property that is put here. So, decrescencies is this upper bound by class K function. And then you have this lpdf we this this conditions are same.

Such that minus $\dot{V}$ is an lpdf function now this is not just less than or equal to 0 that equal to sign is not valid now. It is this needs to be strictly locally positive definite function minus $\dot{V}$. So, it can be 0 only at 0 no other point it can be 0. But in the previous stability definition this was less than or equal to 0 kind of side. So, now we want $\dot{V}$ to be strictly less than 0 for asymptotic stability. So, this is a more stringent condition this one's kind of a condition decrescency and then there is under stringent condition lpdf ness.

So, this is how this theorem comes up so we will not again as I said we not get into the proof of this theorem but we are more interested in application there is a there is an entire different course can be given about theory and its proofs and all the mathematical nitty gritties and details about the Lyapunov theory for utility in many many different applications. What we are looking at is their application in the mechatronic systems alone right now.

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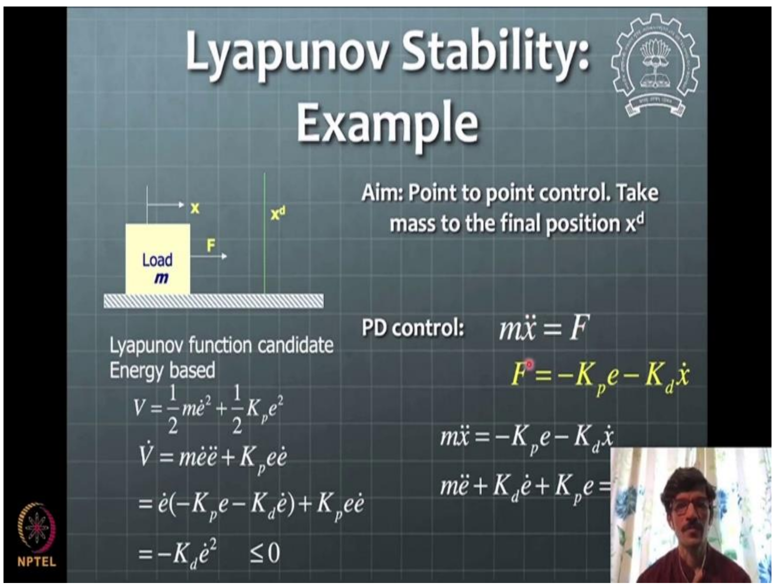

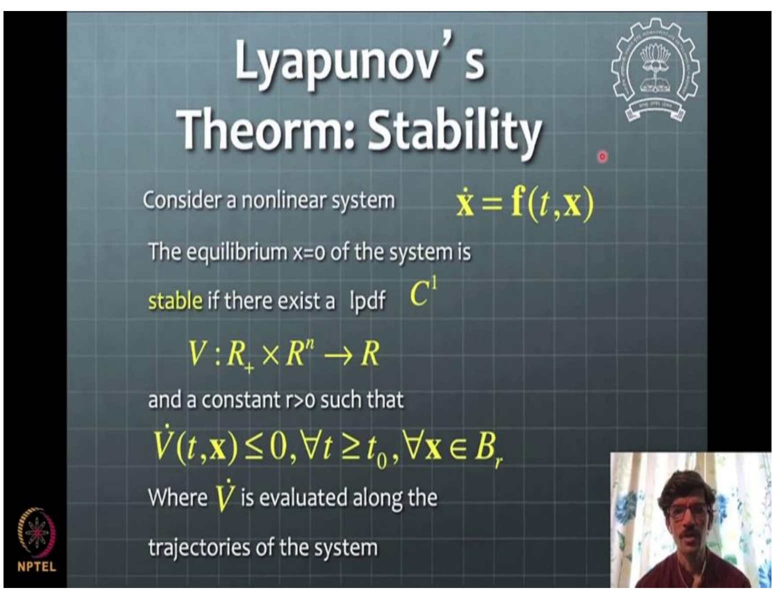

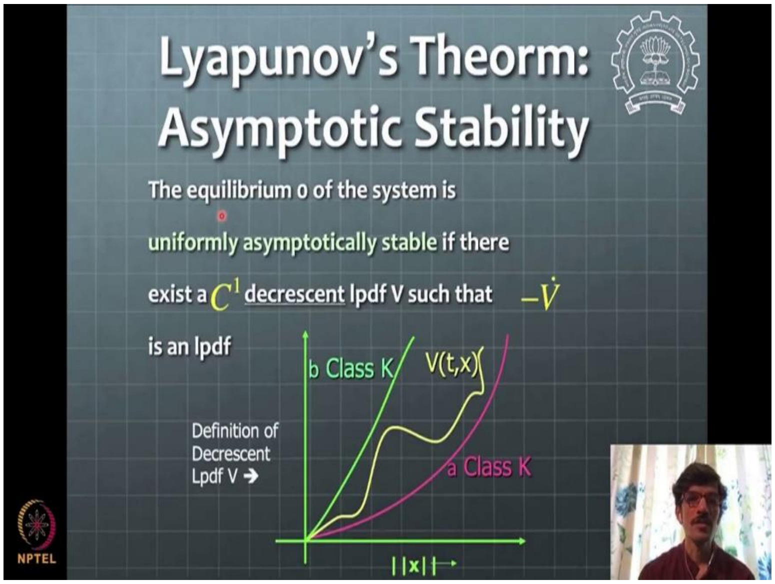

So, let us say with the example so we have this example of you want to control this mass which is resting on the surface and applied by the force F this force F is considered to be a control force I can change it in whatever way I want it to be changed and we want to know what is this force if it is specified like say in this case like a PD control then whether this is going to stabilize my system or not that is what my interest.

And I want to use Lyapunov theory to prove that. So, to say whether it is asymptotically stable or this is stable or what kind of a proof I can get that is what I want to know. So, what is the equation that is governing the system dynamics is $m\ddot{x}$ = F very simple. So, this is a two state kind of a system in terms of if you want to see it in terms of $\dot{x}$= f (t,x) you can convert the system into that form by saying x_1 and x_2 and x_1 = x and x_2 = $\dot{x}$ so $\dot{x}_1$ = x_2 and $\dot{x}_2$= $\frac{F}{m}$.

So this how I can define that system, but I can also work with, see for such a simple systems we can directly work with this basic system itself. And understanding that our state has x and $\dot{x}$. So, the system can be transformed into the error equation by considering like x t-, we want to take the system to x_t, so x_t-x is the error. And then we can convert this $m\ddot{e}$ = F will be the new system. Like that you will do like a lot of this little bit of intermediate steps to see that e double dot is finally your equation.

F and F is defined in terms of e as K_p e -$K_d\dot{x}$ $\dot{x}$also if you write x = e = x (t -x). Then $\dot{x}$ becomes = $\dot{e}$ rather x e = x - x t that is what you need to write. Then $\dot{e}$ will become equal to $\dot{x}$. So, this model nitty gritties are there to be worked out and see for yourself. Then no for such a system if we define Lyapunov function candidate.

So, typically for mechanical systems we resort to the energy of total energy of a system as the indicator of this Lyapunov function candidate it is not a must but many times it works very well. So, given some system you try to see in terms of this control and in terms of Lyapunov system dynamics what is total energy of a system. For example, here we know that for the system $\frac{1}{2}m\dot{x}^2$ will be its energy of this mass.

In terms of e it is to be half m$\dot{e}$ squared this a first term in this V. And then we will have this potential energy coming up because this proportional control here is acting as if we are connecting the spring here. So, the spring potential energy is K_pe^2 that is what we are we are doing this K_pe when we put then we are kind of putting up a spring which is getting deformed by the value of the error.

And that's the spring will have this e will be 0 when this mass reaches the final value x t. So, like that this term comes from the proportional control term. So, you will find this term you can take it similarly for many other kind of systems as well when you are using proportional control you can take this $\frac{1}{2}K_pe^2$ as a term for the energy corresponding to that proportional control.

So, this is like becomes like a total energy now this is this can be verified to be locally positive definite function by applying all the different conditions that we had put for locally positive definiteness of a function. So, it has both $\dot{e}$ and e both the states are coming here and that is why this is like a locally positive definite function I mean it is actually a globally positive definite function with respect to e belonging to R^2 space.

Then we differentiate this to get a $\dot{V}$ and then we get this e double dot here. Now this e double dot is to be substituted from the system dynamics equation here and whatever control that choice that we had so this m $\ddot{e}$ will become K_pe. So, this is x $\ddot{x}$ nothing but $\ddot{e}$ = - K_pe. into K_de. dot $\dot{x}$is also the same as $\dot{e}$. So, this e double dot is substituted here from the system trajectories m $\ddot{e}$ is this value.

And then we substitute that and simplify you get V = -- K_de is $\dot{e}^2$. Now, this is not having any term corresponding to e that is why this is less than or equal to 0 not strictly less than 0. That is why we can conclude that this is control PD control gives the system stability only we can say the system is stable or this equilibrium x = x_t is a stable equilibrium it is not asymptotically stable.

This is important here see with this we cannot get it is to be asymptotically stable but is it that a system is that way that only e dot is going to 0 and e cannot be going to 0 no. We know from the linear system analysis and our actually a perfect solution that both $\dot{e}$ and e are going to go to 0 when the PD controller is applied. So then but Lyapunov theory is not yet able to establish that. So, this is how like we can look at these results so Lyapunov analysis is not saying that it is not asymptotically stable it is just saying that it is stable system for now.

For asymptotic stability we need to do something more to prove. And remember again these rules of Lyapunov theorem are only like the sufficient conditions. So then if these conditions are satisfied then you say the system is stable if they are not satisfied that does not mean anything you cannot conclude nothing about it.

If these conditions are satisfied then like you have this result available. So this is very important thing to know that because you are not able to find out any V which will satisfy these conditions that then the system is not stable or you cannot conclude that otherwise result is there only sufficient conditions.

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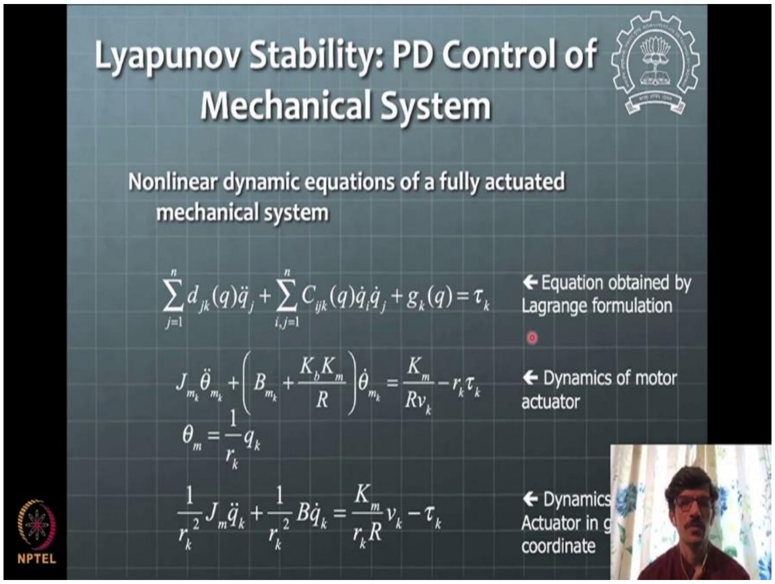

Now we can apply this to entire domain of this mechanical system so to say which are obtained by using our Lagrangian formulation applied to fully actuated mechanical system. Mechanical system I mean the rigid body mechanical system. We could restrict our discussion to rigid body systems alone here although some parts can be extended to flexible bodies also later but right right now for this course let us focus on only the rigid body systems multi body systems and fully actuated systems.

So, number of degrees of freedom are equal to the number of actuators in the system. So, this was the equation if you remember that was obtained by the Lagrange formulation and we introduce the motor dynamics into it. So, motor dynamic equations if you remember they are also in the in this form but now this this torque k or this is kth joint whatever we are considering here this is actually coming from this equation or this is what is a connection between the Lagrangian formulation.

Which is giving us the external force in the generalized coordinate in the direction of generalized coordinate as torque will be provided by the motor, so this is coming as a load of torque on the motor dynamics. So, with this some kind of a gear reduction here. So, r k is some kind of a gear reduction. So, Ɵ is a motor angle and then these are some of the motor parameters and this model can be incorporated through these kind of a connection tau k.

Now this τk if you substitute from these you get some simplification here and you get some kind of equation or we can convert these motor dynamics also into, so this Ɵ m can be substituted by 1 over r k into q k as, so everything will be in terms of the generalized coordinates of your Lagrangian formulation that we have obtain and this tau k can be substituted here and now everything becomes into the generalized coordinate q k.

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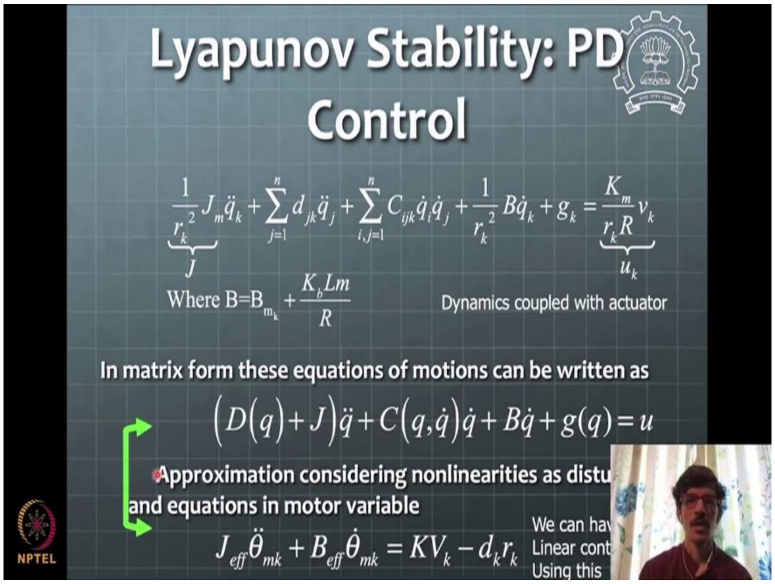

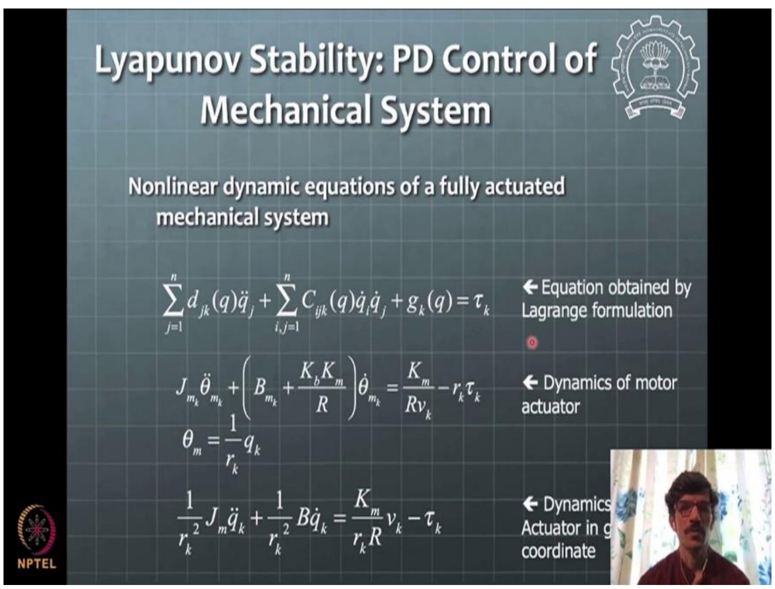

And this equation will look something of this sort. So, this has some kind of complications coming up here additional kind of inertia terms are getting added and that is what is happening to this equation here. Now, in the matrix form these equations will finally look like this. So, this entire thing can be put into some kind of a matrix vector form. So, then it will it looks the complication that you usually get by using this Cijk kind of terms and this is kind of gone here.

So, you see that this is one kind of n by n matrix where n is a degree of freedom the system or n is number of actuators in the system same fully actuated system and you get this D matrix little bit modified because of the motor parts coming in. And then this u vector is basically all the inputs to the input voltage to the motor which is in defined this way.

So, V is actually voltage going to the motor but this constant getting multiplied u we get our u or control input to u in the actuator. That is represented as a vector up here then this g vector is coming B vector is coming and then C was a big many different terms here that will be kind of collected in some kind of a matrix form so C q and $\dot{q}$C is a function of q and $\dot{q}$multiplied by $\dot{e}$ here. So, this D + a is this is similar to our inertia matrix kind of term here.

So, now this so this one can have another approximation to this system by saying that you have the motor equation alone but all the terms which are coming on the motor equation you remember that the motor equation was something of this sort.

Here is a motor equation in terms of Θ and in terms of these now these can be considered as a disturbance and we can directly write some control here that is another kind of a possibility that can exist if you want to avoid or we are not interested in too many we are very high performances in the in robotic system or mechanical system then we can do that we are just kind of considering this as a all nonlinearities as a motor as a disturbances.

And then collect them all together as these d k terms is nothing but this tau k term. And we can use linear control on this equation also. But now right now we are interested in doing some nonlinear analysis with this equation.

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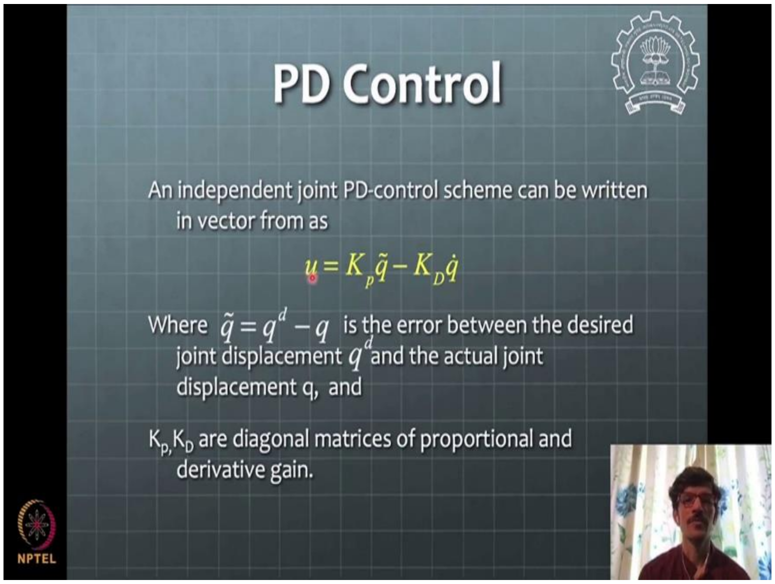

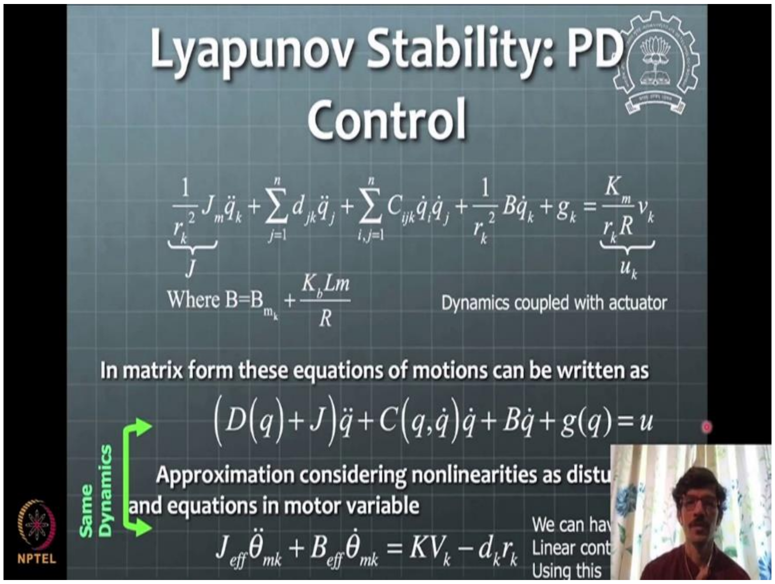

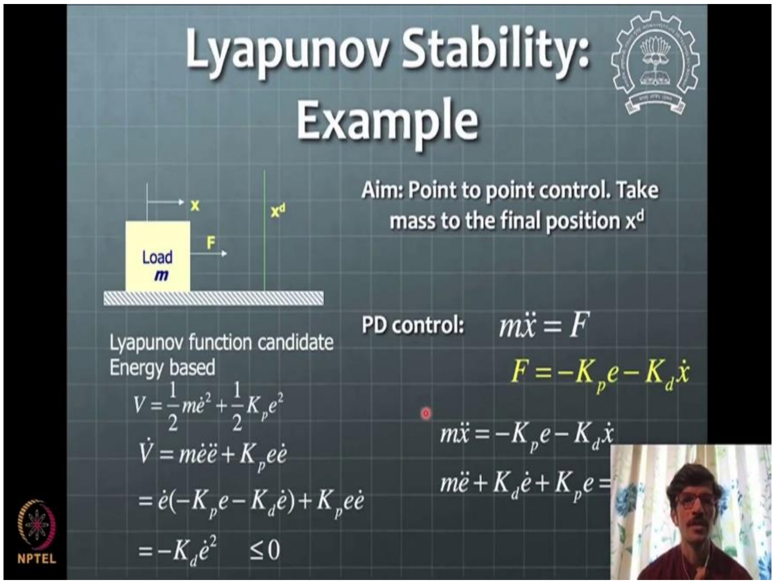

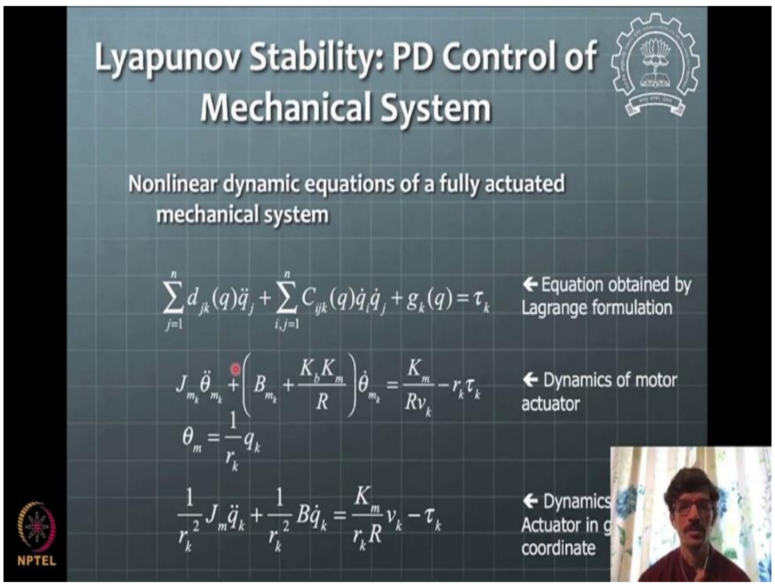

Now, we want to see that we propose this PD control on the joints. So,q is a generalized coordinate in terms of typically if it is a n degree of freedom robot then robot joints or generalized coordinate. So, in the direction of generalized coordinate you define this u where K p and K D are matrices they are kind of typically diagonal matrices. And $\tilde{q}$ a is defined as q d minus q here.

So, desired kind of a generalized coordinate minus actual generalized coordinate that is an error term that we are defining here. So, this u = K p times error and K D times error dot is what we are defining here. And these are now in the vector matrix form. So, this u is again a vector which is used here. So, this u is here and now we are interested in looking at how do we do this analysis using Lyapunov analysis.

So, what is what is it we proposed as a Lyapunov function candidate here can you think about that. So, see what we did for the simple kind of a mass resting on the surface we used some kind of energy based Lyapunov function. Now, we can think of on the similar lines. So, what is the energy of such a system where you have fully nonlinear dynamics of n degrees of freedom robotic system happening here right now.

So, this kinetic energy of these is based on the D J K terms or D matrix in this equation. So, this will give you kinetic energy.

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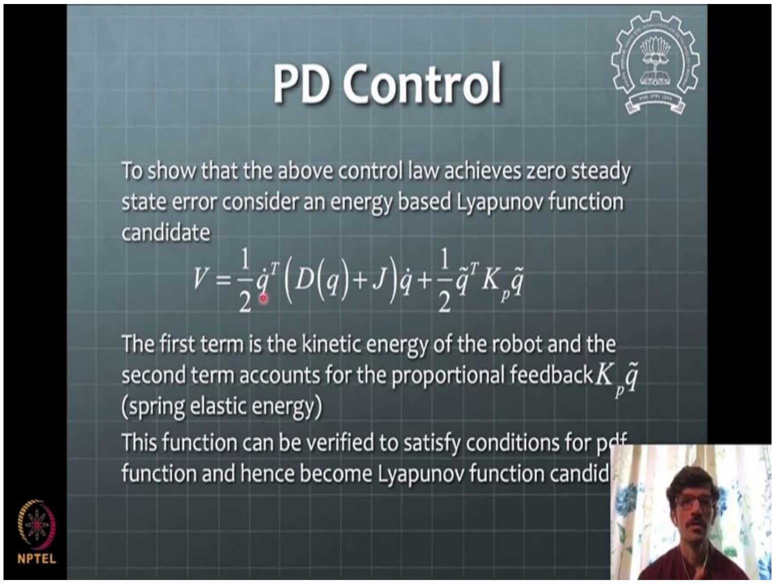

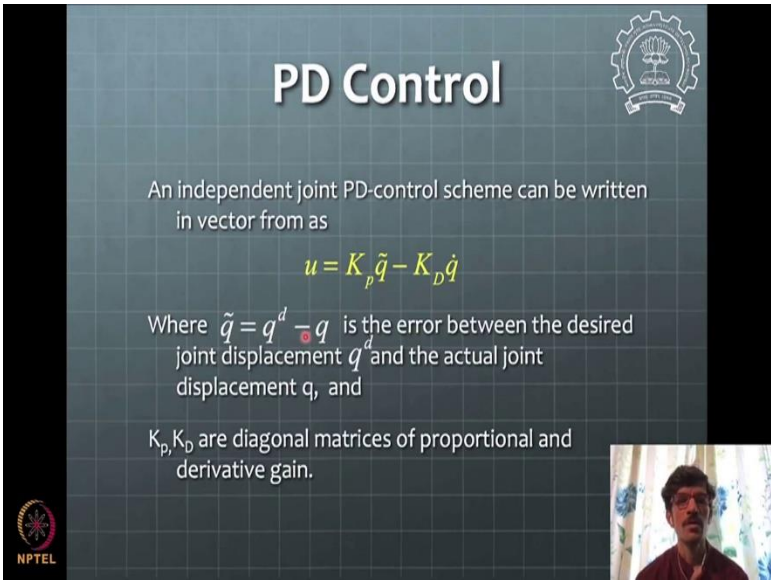

So, typically the kinetic energy will be given as $\dot{q}$ $\dot{q}$ transpose this D matrix now we have additional term coming from the motor inertia also and $\dot{q}$q transpose d q $\dot{q}$transpose d $\dot{q}$will be your kinetic energy. And potential energy term similar to the potential energy that we defined for this for the proportional control consider as the spring.

We have now because this is a matrix form is $\tilde{q}$ transpose q K p times $\tilde{q}$ that will be the total potential energy corresponding to all the variables or all the errors $\tilde{q}$ is a vector here is a matrix and this is again vector. So, first term here is a kinetic energy and the second term is accounting for the proportional feedback in this in the form of a spring elastic energy.

So, now we have these terms corresponding to $\dot{q}$ also and $\tilde{q}$ are also. Now this entire be converted into $\tilde{q}$ alone by using this relationship q d dot minus, so q d is a fixed quantity, we want to be able to a fixed desired position.

So, we considered for this discussion right now that q d is not a function of time, it is a fixed point where we want to go. So, this is a as we saw that is this regulation kind of a control not a tracking control. So, for this q d is constant so $\tilde{q}$ dot will be equal to minus $\dot{q}$ that is what we can do here, substitute for this here.

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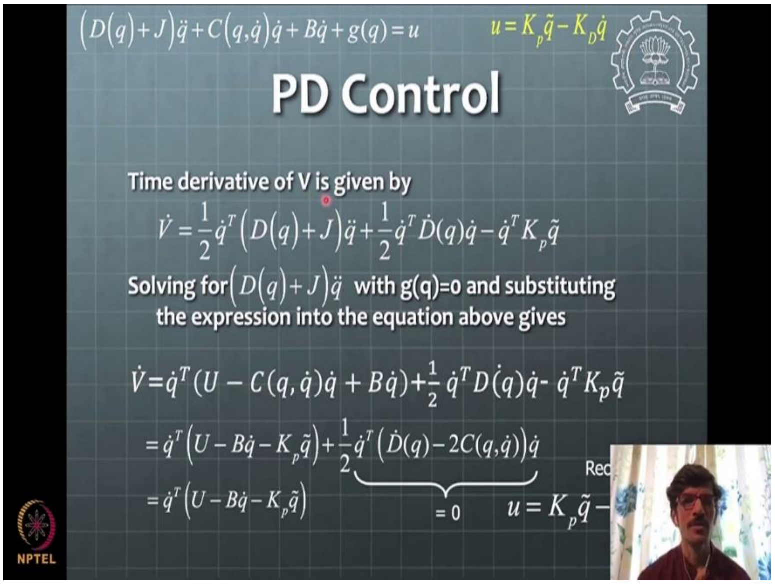

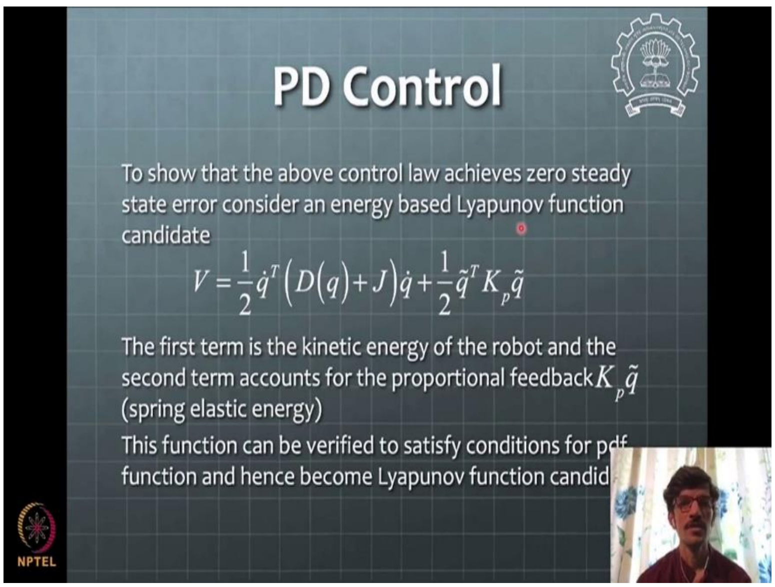

Now, if you see this time derivative of V so we can use this $\tilde{q}$ according to wherever we need so finally it is right now kept in $\dot{q}$ terms only. So, now time derivative of V if you see here if this is V can you write the expression for time derivative of V just try it out. Pause here, try it out and then proceed. So, it will have this terms which are derivative of these times these plus these times derivative of this.

So, this is like a chain rule that we are applying here. So, this chain rule, when you apply what is a derivative that results here is this. So, this is $\dot{q}$ transpose d q plus j $\ddot{q}$ here. So, first derivative of this term is taken and then half $\dot{q}$ transpose d dot q. And then there will be a term which is same as this term. So, this half will actually go away. So, this half is coming half should go away here.

The way you have this $\dot{q}$transpose K p $\tilde{q}$. So, here also this $\tilde{q}$ dot transpose K p $\tilde{q}$ is coming because half will be gone because like $\tilde{q}$ transpose K p $\tilde{q}$ and $\tilde{q}$ dot transpose K p $\tilde{q}$ and $\tilde{q}$ transpose K p $\tilde{q}$ dot these both are going to be same terms and they will add up together and you will get this half cancelled out. And so you can see this negative sign will come based on the transformation between $\tilde{q}$ and $\dot{q}$.

$\tilde{q}$ and q actually or $\tilde{q}$ dot and $\tilde{q}$. Now, solving for this this we know that now from the equation of dynamics here that D q plus J $\ddot{q}$ this part of the term will be equal to this whole thing shifted on the other side and substituted for u okay you will get this term here as u minus C q q U C of q and $\dot{q}$ times $\dot{q}$ plus B $\dot{q}$. Now, we are setting g q to be 0 here for this discussion right now.

So, we can add and we will have some compensation to be done in control for that term but right now we will not worry about that term to make our first understanding easier here. Then you have this this term. So, this is this half is not there. So, I am put that half here. So, you get this term first then this term as it is and so you have derivative of D happening here. And then here you will have this same term coming up here.

Now, if you see that we collect these terms in a very specific manner. What do you see still we have not used to u here u is kept as it is and then this B $\dot{q}$ term and K p $\tilde{q}$ So, this C term is moved out of this place. So, this C term is taken out and this K p $\tilde{q}$ term is put here in. And here u minus the C $\dot{q}$ minus B $\dot{q}$ this this should be minus sign here. So, this is that is why this will become minus B q. Then you use this d dot minus 2 C term here this 2 is coming because this is half outside here. So, d dot minus 2 C $\dot{q}$ then $\dot{q}$ transpose.

So, we are collecting these terms here for a reason because we know that d dot minus 2 C is a skew symmetric matrix for the mechanical system as we have seen in Lagrange formulation and that is why this is a quadratic form will yield a 0 here. So, we do that and we get this expression for $\dot{V}$ as $\dot{q}$ transpose u times B $\dot{q}$ and minus K p into $\tilde{q}$ is coming. Now you propose this V = Kp $\tilde{q}$ -K D into $\dot{q}$.

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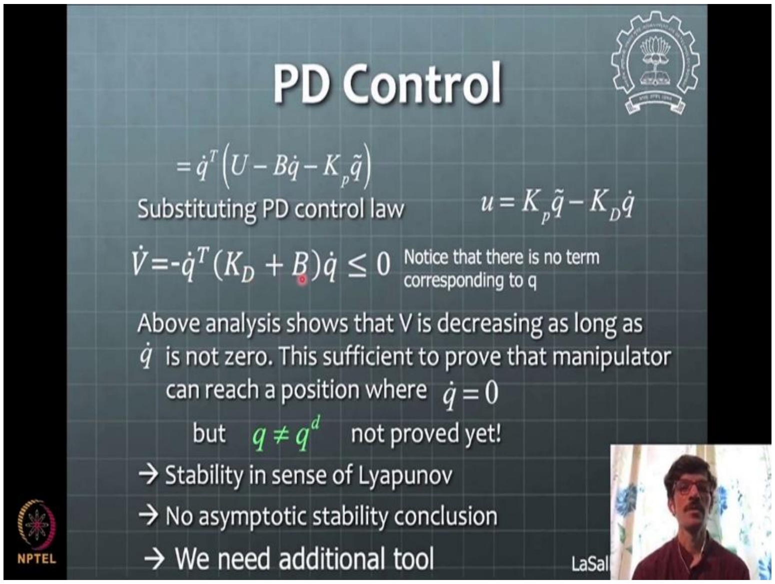

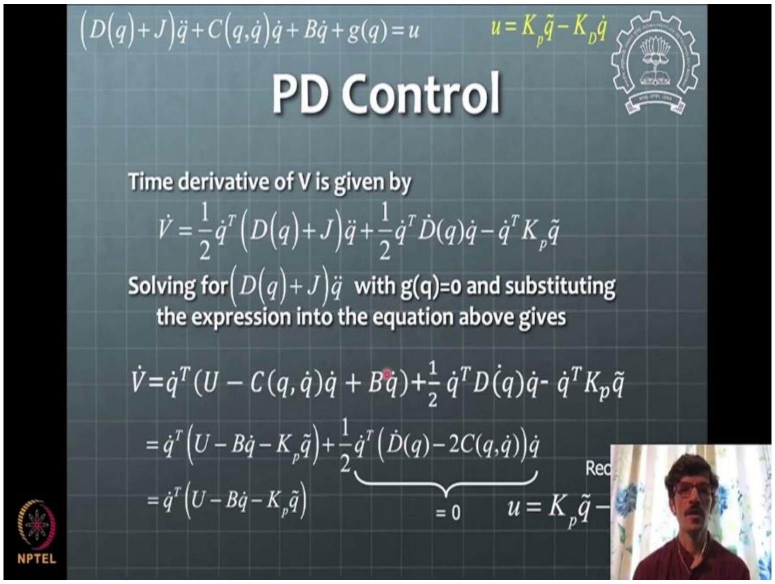

And substitute it here we get this expression here to be this. $\dot{V}$ = minus $\dot{q}$ transpose times K D plus B $\dot{q}$. So, this remember this K D is a diagonal matrix B is a is a is a damping matrix in a system that also can be a diagonal matrix here. And then you get this final expression. And this is again as only $\dot{q}$ kind of a terms here. So, you if you see this these relationships happening are exactly similar to what is happening in the scalar form for our simple mass on the surface kind of a system.

So, again we get this is less than or equal to 0 that because there is no term corresponding to q. So, for some values of q this q which is nonzero this term is still going to be 0. So, if $\dot{q} =$ equal so that means it is violating this pdf ness of minus $\dot{V}$ kind of a term that is why this is less than or equal to 0. So, this $\dot{q}$ will be made this will this condition we can make u dot equal to 0 but q is not equal to q desired yet.

So, this is not proven yet so $\dot{q} = 0$ means $\dot{\tilde{q}}$ also will be equal to 0. So, this to prove this we need some more kind of tools. So, this is this equation is giving just a stability in the sense of Lyapunov and no asymptotic stability here. So, we need some additional tool and that tool is basically LaSalle's theorem.

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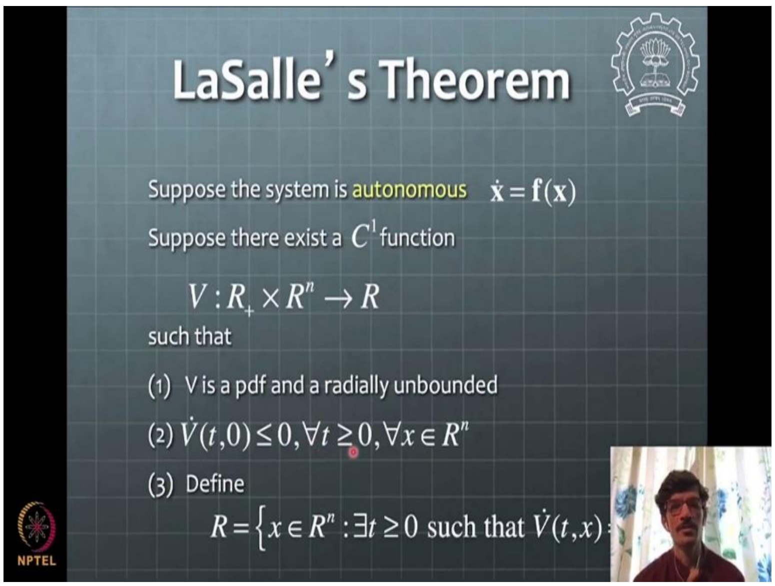

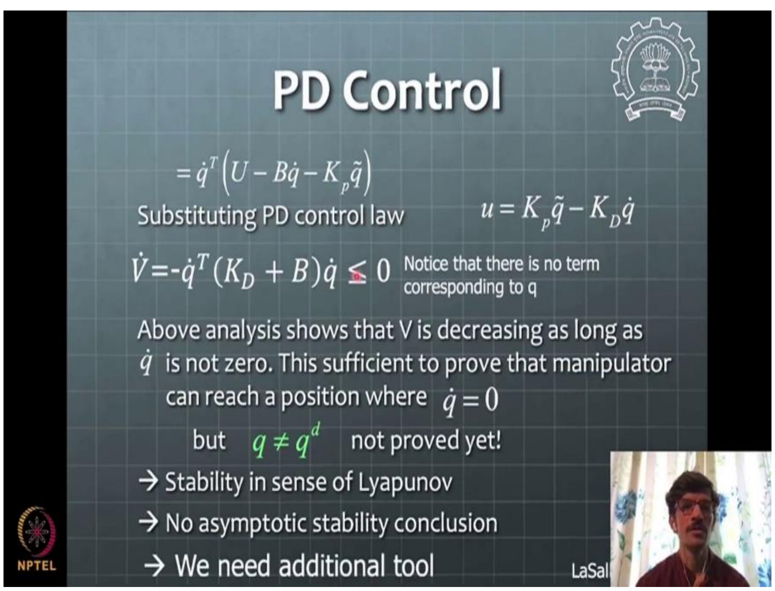

What is this theorem telling us is given here so this suppose the system $\dot{x} =$ f of x is there now see the t is removed here. So, this system is autonomous there is no explicit time dependence happening here now. So, this system is $\dot{x} =$ f of x only and there exists C this function this again our Lyapunov function candidate. And this V is pdf and radially unbounded.

So, there is a new property coming here for this thing is should be pdf and radially unbounded. And then $\dot{V}$ is less than or equal to 0 as we are getting in this case here. Let $\dot{V}$ is less or equal to 0 for all t greater than 0. And such that this x belongs to R n space here for all R it is for all x it is valid. Then we define this set R here are this set is define in this set is defined such that x is belonging to this R n space and for all t equal to 0 v such that $\dot{V} = 0$.

So, this x is such that $\dot{V}$ of t comma x is equal 0. So, this is how the set is defined. So, we look at this $\dot{V} = 0$ condition now. So, $\dot{V} = 0$ condition we need to look at. So $\dot{V} = 0$ condition means like $\dot{q} = 0$.

So, we look at this $\dot{q} = 0$ condition and see that x is belonging to this R n space or these q or q this x is belonging to this R n space such that $\dot{V}$ (x, t)= 0. Now this x is nothing but q in our case now. So, we will see with the example later but understand that we put this condition such that $\dot{V} = 0$ and x is belonging to this R n space.

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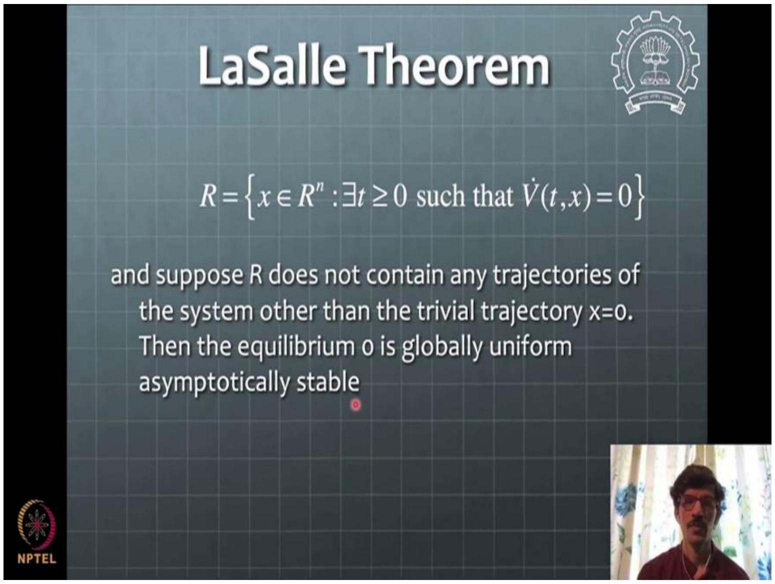

So we collect all such x together and observe them. And this such all these x or this set r does not contain any other trajectory of the system other than the trivial trajectory x equal to 0. Then the equilibrium x equal to 0 will be a globally uniformly asymptotically stable system. This is what is the LaSalle's theorem saying. So, this theorem is what we need to make use of in our case.

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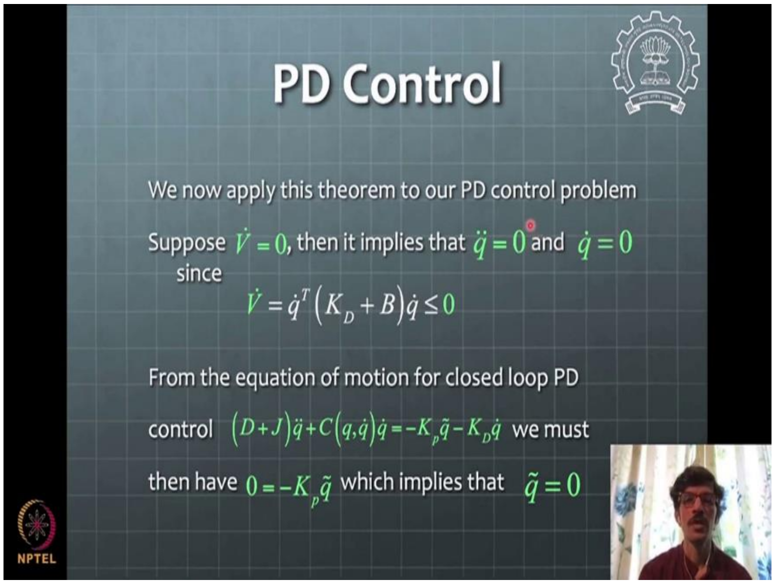

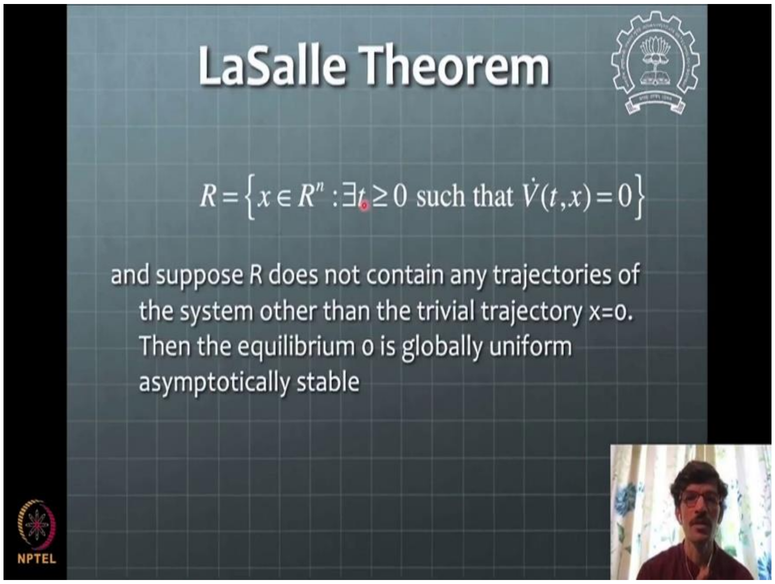

So, for PD Control we apply this theorem if we see that $\dot{V}$˙ equal to 0 means $\dot{V}$ is this. This = 0 that means $\dot{q}$ equal to 0. Then $\ddot{q}$ q when $\dot{q}$ = 0 I differentiate it once and I say that $\ddot{q}$ = 0. And this is because it is $\dot{q}$ equal not instantaneous it is for all time. So, we need to see that for all time t to 0 $\dot{V}$ = 0 that is a condition that is coming.

So then I see this in the system equation. So, in the system equation wherever $\ddot{q}$ is there I will put that to be 0 and wherever $\dot{q}$ is there I put that also to be 0. And then what remains is this control term here. And in that $\dot{q}$ again this = 0. This means that $K_p\ \tilde{q}$ = 0.

And this implies that our $\tilde{q}$ are should be equal to 0 and $\tilde{q}$ is the error between the desired and an actual trajectory. So, this is how we establish in the full mathematical sense that PD control indeed will take position and the velocity will go to the final desired kind of a value.

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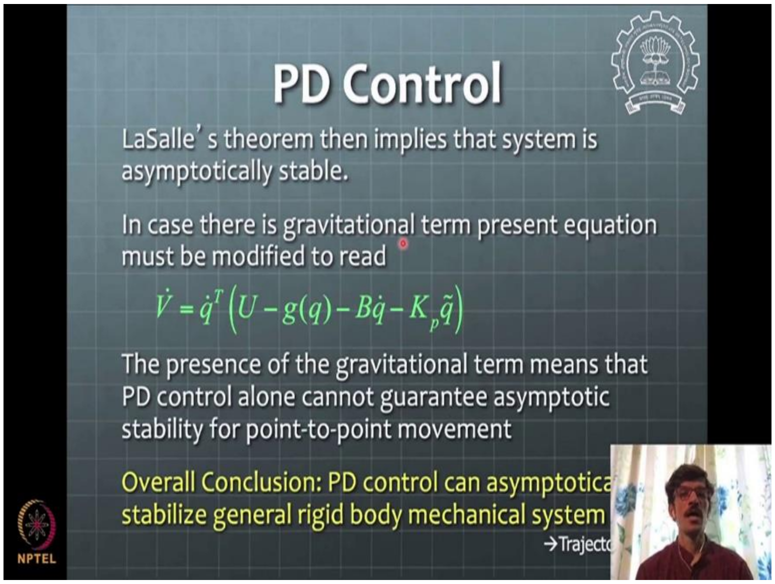

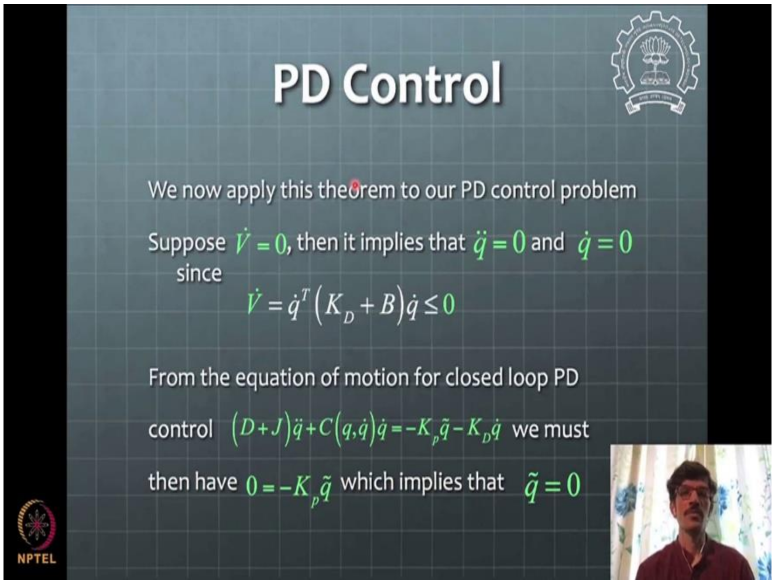

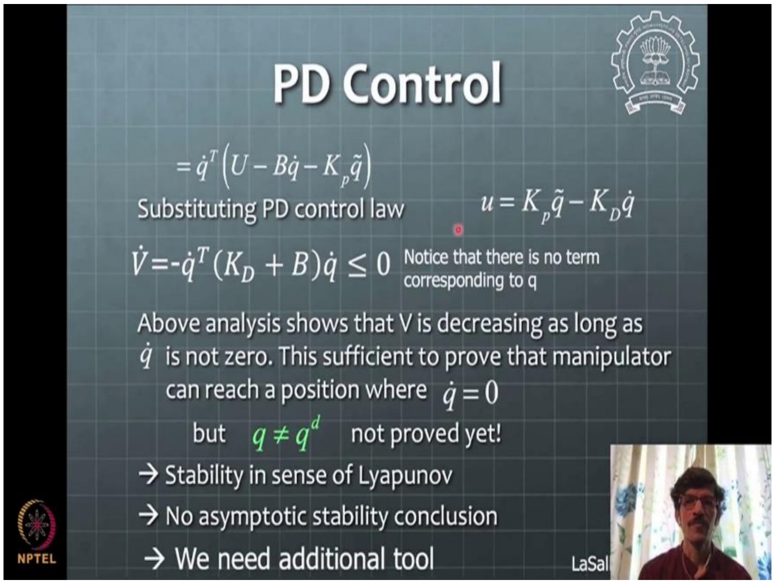

So, this LaSalle's theorem implies that then the system is asymptotically stable. And this is what like we make sure that our system is taken to the final position also. And final velocities are also to be desired value which is 0 the final position. Now, when the gravitational term is there then we need to compensate for that term. Because it is depend upon like the g (q) q means it is depend upon generalized coordinate that term can be computed and compensated for by the control.

So, you do this control competition and you introduce this term in the control to compensate for that. So, this is u minus if you without compensation it will come like this here. So, in u if suppose there is a term which is g (q) then that will get cancelled out in so we can have all the terms that were there in the in the in the u before this $K_p\tilde{q}$ - $K_a\tilde{q}$ minus plus now there will be a g(q) term coming up here.

And if that term is added to my u then this will compensate for that here in the $\dot{V}$ and again $\dot{V}$ expression will become similar to what we had before. So, these are like we go ahead and handle that. So, overall conclusion is that PD control can asymptotically stabilize a general rigid body mechanical system is very very important kind of conclusion here. So, if you use simple PD control on many systems it will they are fully actuated it will work.

It will take the system to its final value in the presence of gravity, if there is a gravity compensation to be done then we do that and then the speedy control will work. So, even without having all these kind of analysis one can definitely like control any rigid body mechanical system fully actuated by just using PD control and it should it should work. That is why these PID controllers are kind of quite powerful tools that are available in in the market.

And they work quite well. So, in the presence of so even if we do not know whether the system is a nonlinear or linear or as long as the is a fully actuated mechanical system there are no under actuations in the system then this PD control is going to work quite well. Now, the problem comes when we have found to do the trajectory tracking. This trajectory tracking is an important issue here.

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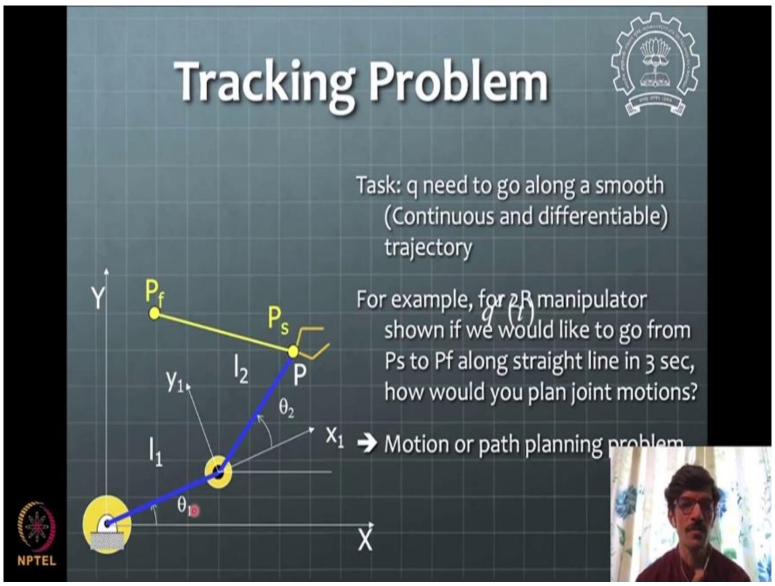

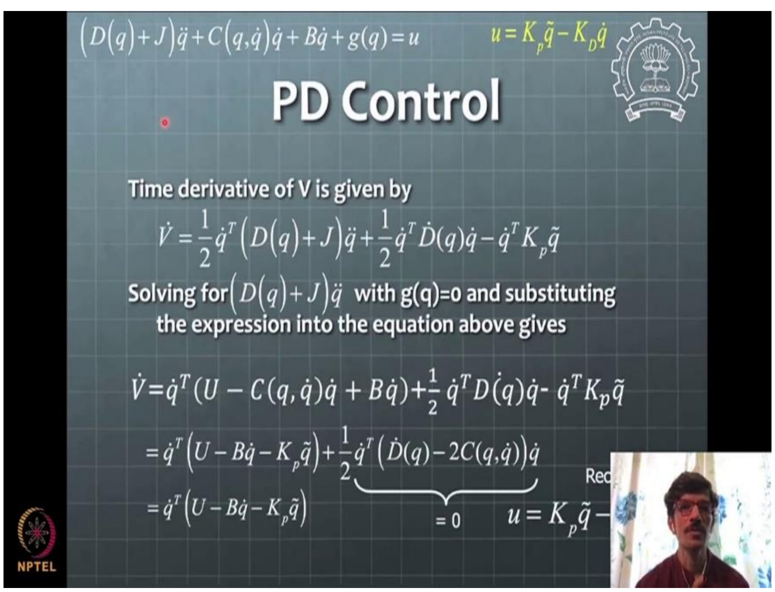

So, for that we need a lot more kind of thing to be done. So, the trajectory for tracking problem is simply like the suppose you have a tooling manipulator say I just explained by using this tooling manipulator. But we want to track this trajectory like say the problem is given that we will not like to go from Ps to Pf in three seconds along the along this path straight line path. This becomes like a motion or path planning problem.

And this the solution to this path planning problem will give you Θ_1 desired and Θ_2 desired as a function of time which is which will make this happen. So, we first get like this path trajectory kinematics done like if I know x position I know what is the y position and if I know x position in time I know what is the y position in time that is the kinematics of the path that is solved.

Then I can convert that x (t) and y(t) into Θ_1 (1)t and Θ_2(t). And then I get these are like my desired Θ's which will take me along that path. So, I want to now track the trajectory for Θ_1 to go along with Θ_1 desired as a function of time this is like a inverse kinematic problems given end effector position in terms of time how do I get like my joint coordinates in terms of time.

That is a motion or path planning problem that is typically used in robotics kind of iteration and similar kind of thing exists for any mechatronics system for example if I want to go if I want to have some desired kind of operation done in time as the trajectory thing then what should be my joint or my motor angles in terms of time. And once we know this $\Theta_{1desired}$. t and $\Theta_{2desired}$.t or $q_{1desired}$. t $q_{2desired}$ .t, how do I now track this desired trajectory that is a problem.

So, then computed torque controller is one of the ideas that can be used. So, this computed torque is basically idea where you know this trajectory you know this joint angles based on the derivative now so the joint angles desired are known the desired joint angle derivatives will be known and those one can use from this equation of dynamics. So, this is the equation of dynamics here.

You know this say suppose joint angles q are known the derivatives are known $\ddot{q}$ s are known $\dot{q}$s are known. So, everything here is known then you know what is control let any without any other disturbances you will know what is the desired control that is to be there right. Because q when I know that trajectory q I know $\dot{q}$ $\ddot{q}$ all are desired of course they are desire you put in the equation and you will get to know what is a torque that will maintain those desired trajectory in the absence of any other discrepancies.

Any other kind of so this is how you find that computed torque and use that torque in addition you use the control now should be that you desired plus this feedback u or you for feed forward and you feedback. And then this controller will work typically.

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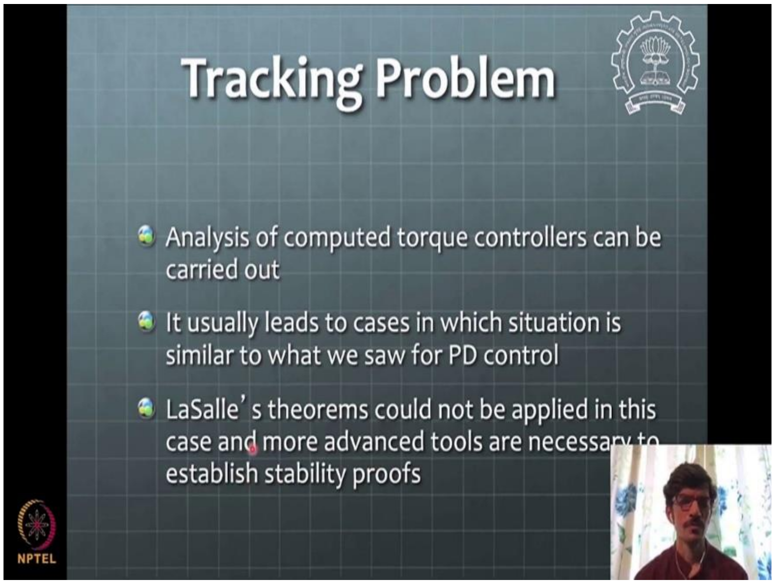

So that is what is written analysis of computed controllers can be then one can use these computed controls and see whether we can use our Lyapunov theory to kind of get something from there. So, the problem here is a LaSalle's theorem will not be you will not be able to apply So, I would leave this analysis you carry out this analysis then some things will make sense. You do this computed torque based control and try to kind of see a similar kind of a PD control plus feed forward the computed torque control term.

You can use as a control and start working out Lyapunov analysis and you will find at this LaSalle's theorem may not be applied in this case. There are some other kind of stability proofs that will be needed. So, will this is it out of scope of this course.

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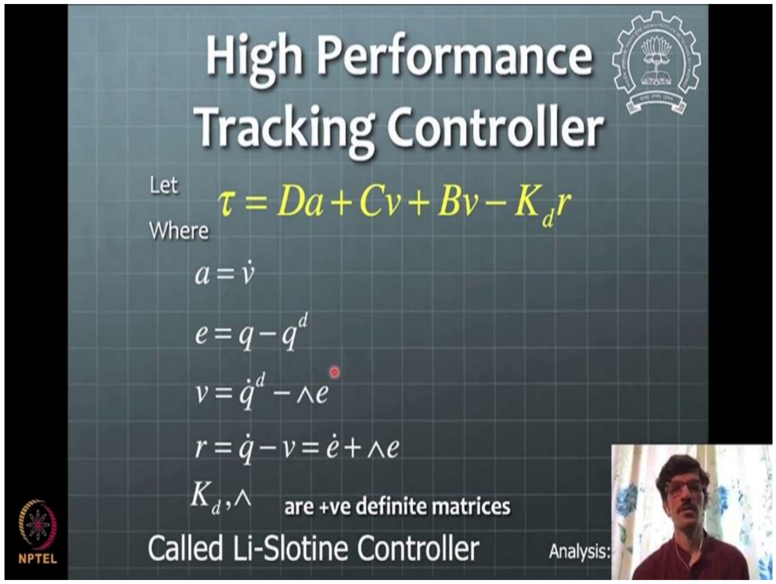
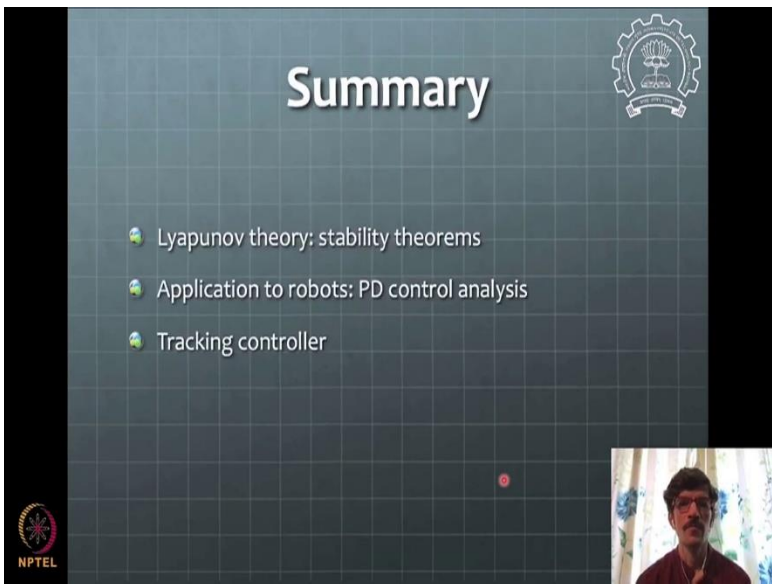

High Performance Tracking Controller
Let $\tau = Da + Cv + Bv - K_d r$
Where
$a = \dot{v}$
$e = q - q^d$
$v = \dot{q}^d - \wedge e$
$r = \dot{q} - v = \dot{e} + \wedge e$
$K_d, \wedge$ are +ve definite matrices
Called Li-Slotine Controller
Analysis:
NPTEL


Summary
Lyapunov theory: stability theorems
Application to robots: PD control analysis
Tracking controller
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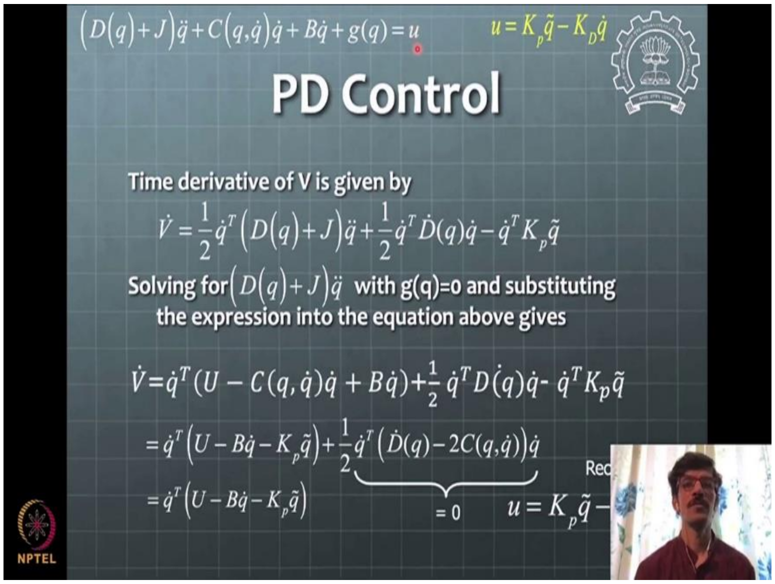

But there is a very high performance tracking controller that can be possible we will this expression of this controller is given here. So, these D C terms are coming from the system equation of dynamics that we have seen before and these other terms are new and a are defined in this manner here. This is called Li-Slotine controller. And this is a controller this can do the trajectory tracking also. See this q, $q_{desired}$ is not constant here which is can be function of time also.

And how this is able to kind of do the job we can prove by Lyapunov stability analysis that will be happening in the future classes to come. But you can ponder over this controller and see whether you are able to kind of carry out the analysis of this by using Lyapunov theory. And this control is applied in this in the equation of this kind so this is u is basically given by this u is basically given by that expression that you see as a tau expression in the control here.

This expression and gravity is not there here. Again, you assume that gravity is 0 g term is 0 in the equation of dynamics. And see whether you are able to kind of do something about proof of that. And the hint is you express everything in this variable r. And try to express everything in terms of variable r although your generalized coordinate are Q you express everything in terms of r and you will find some interesting thing happens in that current dimension. So, we will do that analysis in the future. Thank you. We will stop here for now.