Design of Mechatronic Systems
Professor Prasanna S. Gandhi
Department of Mechanical Engineering
Indian Institute of Technology, Bombay
Lecture – 42
Fundamentals of Sampling

I know what happens to the system when we sampled the system. So, all we know from our analysis, our practically implementation that we doing the sampling. We are doing these three steps reading the sensors, then computing the control, and then third one is implementing to the actuator. This we are doing again and again in some kind of sampled kind of a manner. Now, then the fundamental question comes is what happens to the signal when I sampled the signal; that is a first question. So, we will now see these questions in more details.

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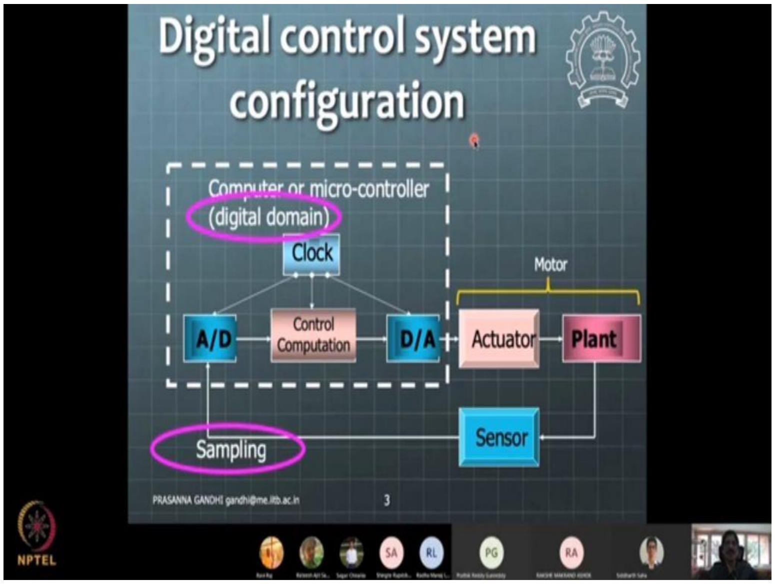

So, schematically something of this sort is happening in our system. So, you have your plant here which is say your, sorry, some mechanical system; which is inertia or something, which is actuated by some actuator, some kind of a motor or something. So, in your case like know your motor case, you will have a disk as a plant, and your motor as a actuator; or motor inertia plus disk inertia as a plant. And then the magnetics going on in the motor as a actuator, like one can visualize in some sense. Then encoder is getting your feedback as a sensor, and then you are sampling.

Now, here we do not have A to D here; we just have a encoder interface to do this convert, I mean get this reading of the encoder directly into your microcontroller. And then we do not have even digital analog interface; we have this PWM interface here to actuate your actuators. So, so this loop is continuously getting executed, every sampling time you read your sensor here; then you process that in your control computation; and then you implement it on actuator. Again, you read further keep on doing that in the infinite loop so to say. That is how your actual control will happen in, in any of the systems. Now, the question is, because we are implementing something; so this activity happens in the digital domain.

So, once it is implemented at say whatever sampling some instant this actuator, or PWM duty cycle has got updated. And from that update to, now all these things completion all these other activities happening with one sampling time come completion. And again it is updated, there is a lag of or there is a delay, or there is a time lapse of one sample time. So now, the way this would happen for my system is, I would; let me sketch it here. I would see the system would see some sort of; let me get a new page here.

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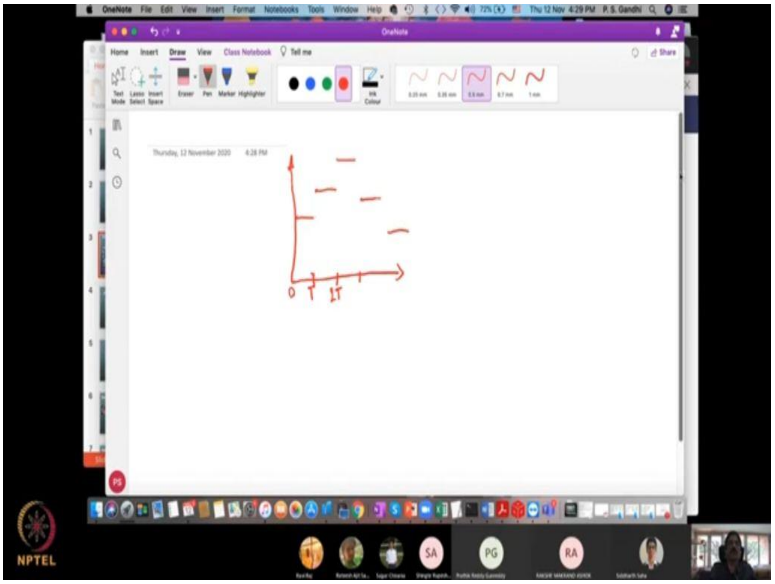

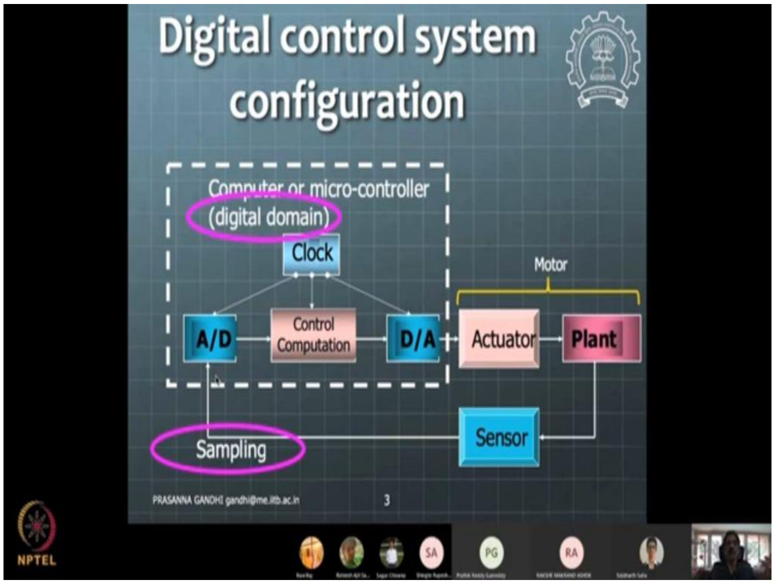

So, I would see some, some kind of by control input is, is if I see just plot the control input as a part of as a function of time. So, this is my first sampling time, this is second sampling time, this is third sampling time and so on; so, I have first duty cycle coming up here up to this point. Then it will get updated, and it will become some something here; and now over two sampling instance is going to remain constant, that is a important thing here. So, 0 to T, it has some value here, then from T to 2T; there is the other value this is how is control going on my system. So, the next question comes like that if my system is receiving these inputs in this kind of successive steps kind of a thing.

So, it is receiving these kind of successive steps keep on receiving like kind of input. How my system is going to respond to this kind of input? So, that is a question that is addressed; for example, when we talk about this digitization or like digital control of consideration. Normally, what we do is we just implement control, we developed some controller in analog domain; and we say look with enough fast sampling time, I would be able to implement it faithfully in a digital domain; and then our job is finished. We said I have enough sampling ignore fast sampling; so that this controller will behave as if it is continuous system, fair enough; that is also a good argument to go.

Many times people use that and they do not bother about getting into digital analysis itself. But we need to be aware as, as well as we as a mechatronics engineer need to be aware that at some point I may not have that flexibility of having control possibility. So, faster implementation possibility is not there as if the system will be consider as a continuous. In Under that situation, we need this analysis of this digitization; so we will have these two parts coming up. One is like your sensor, typically in our case it is anyway a digital sensor. But analog sensors will have this issue that you sample this sensor signal. So, sensor is read again every sampling time, actuator is given input every sampling time.

So, when the sensor is read every sampling time. Now, I need to know that whatever samples that I am getting from the sensor, are representative of the signal that I want. So, what happens to the sensor signal when I sampled it? That is another kind of mathematical question that one can have. So, one first question is, what happens to my system when the input is in this kind of steppe form? And other question is what happens to my signal when I sample it? So, when I see the samples of the signal, how do I see them representing my signal well not or whatever?

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So, there are this other kind of issues, so some practical issues are there; but some of them are related to sampling. Say for example, if you maybe with this will become more apparent when we start looking at something.

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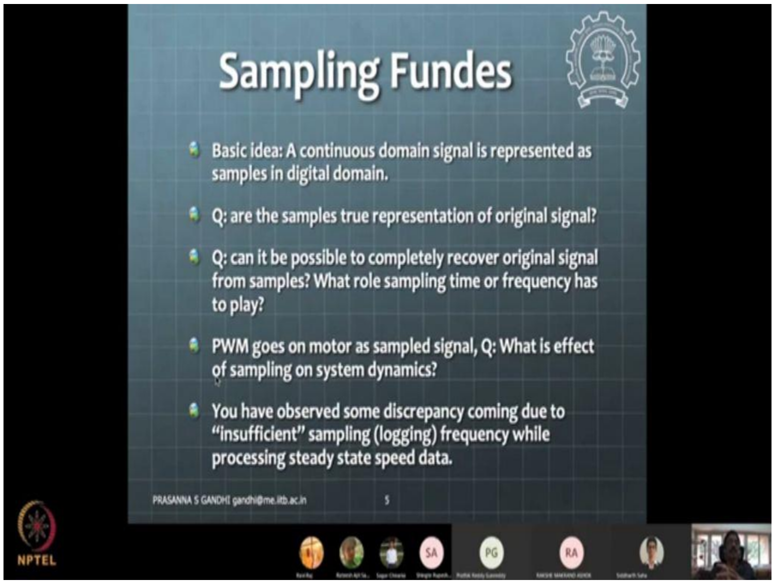

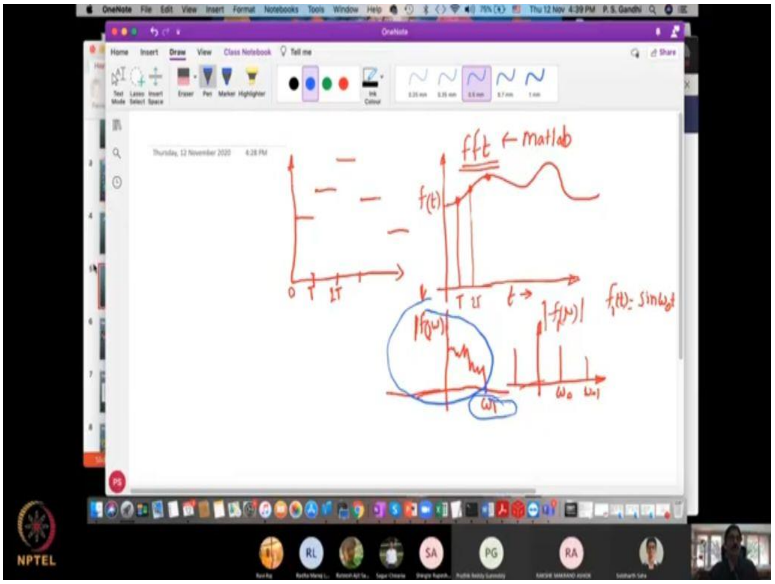

So, basic idea here is to diagnose here as a continuous domain signal is now represented as samples in the digital domain. So, you have this signal which is continuous in nature; so let us come back again to our schematics here. So, now here we are considering for the sampling and considering a question where you have this signal; so this signal has some signal.

This is a f (t), continuous domain function of time; this is my time here. And I sample, I take the samples of the signal; I sampled this signal at say some sampling rate. So, these T to 2T samples are coming at these different times. So, if I represent this signal only in terms of these discrete points, instead of this continuous; in computer I cannot represent continuous signal. I always will have only these discrete data points which are coming in some kind of a form of an array.

So, if I represent signal in this kind of sample form, what is that I am missing? And how these samples are representing my original signal, how faithfully? Those are the kinds of questions that come up mathematically actually. So, another thing like this signal has some kind of a frequency; so we need to be familiar with something called frequency content of the signal.

So, you have learned this Fourier transforms in mathematics. So, if I take a Fourier transform of some signal, so the signal will have a frequency contents up to certain frequency, say ω_1 here. So, now what I am having here f (jω), if you recall your Fourier transform, this is a magnitude of f (jω), So, this will have some magnitude, some variation, up to this ω_1. Now, just to give you some sense, if your signal is sinusoidal.

If your signal is sinusoidal of say frequencyω_1, then you think what will be your f (jω), is going to look like. So, f (jω), for f (t); so let us call this f_1 and some f_1 , is $\sin(\omega t)$, $\omega_0 t$ let us say. So, if you recall your Fourier transforms, you will find that it will have some kind of implicit presentation here at frequency ω_0. So, so we may need a little bit of refreshing of your Fourier transforms to really understanding it in really great depth. Or, you can just have a feel for like say look my signal has; say signals sinusoidal frequency signal, a single frequency of ω_0.

In the frequency domain I am not going to see anything other than ω_0; that is kind of a physical sense you can have. So, if the signal has two frequencies ω_0 and ω_0 ; then I in the, in the thing I will see these two peaks here. Instead of just one peek you ω_0 I will have another peek at omega 0 1 something like that. So, like that I can see my signal to be having this multiple kind of frequencies, depending upon what is the nature of a signal. So, that is a little bit from the physics perspective; and then to kind of get to precise, how this comes, you need to invoke mathematics of Fourier transform.

If you use a Fourier transform formula, then you will start like getting the similar kind of results. And, for a signal there are techniques by which you can numerically take Fourier transform; it is called discrete time Fourier transform. So, for that numerical Fourier transform there are, this very efficient algorithm called FFT algorithm; so this is algorithm called FFT Fast Fourier Transform. So, if you see in MATLAB also you will have this command called FFT; so you can use that command and see how it given a signal. So, I give a signal like this, and I fed in these discrete points of the sample points of that signal, which we know what is the sampling frequency.

And you feed them it into this function in certain kind of way; and this MATLAB can give you this the FFT plot, or this frequency domain representation of their signal. So, you can get actually this kind of signal. And typically, all the signals that be considered in the in the mechanical domain, will have some kind of a limit with this ω_1 here. So, you will have these signals which are limited in frequency; they are not like you cannot have a signal which is having infinitely many frequency contents. Frequency contents will be limited to by a certain higher frequency; because mechanical elements in the system cannot move faster than certain speed.

So, so this is what we will consider mostly for signal analysis, that our signal is band limited is called some band limited in terms of some jargons actually. So, this band limited signal by ω_1 frequency, and then we represent this signal in either time domain, or in the frequency domain. And we should be able to get go back and forth in time and frequency domain. That is what would happen based on the Fourier transforms; we will use of Fourier transform analysis to do that. Now, the questions are the samples true representation of original signal? So, further question will be, if it is true representation, then can it be possible to completely recover the original signal from its samples?

What is the role that sampling time or the sampling frequency as to play in this business of getting the recovery of entire signal. Now, this is the question that we have said PWM, which is going to the motor as a sampled signal. So, what is the effect of like some sampled input going to the in a steppe form, it is going to my system? What is its effect on the system dynamics? So many times you will find that discrepancies that are coming in actual practical stuff, and what you are getting in simulation will be because of this insufficient sampling, or sampling frequencies is not proper. So, we will get to these in little more details now.

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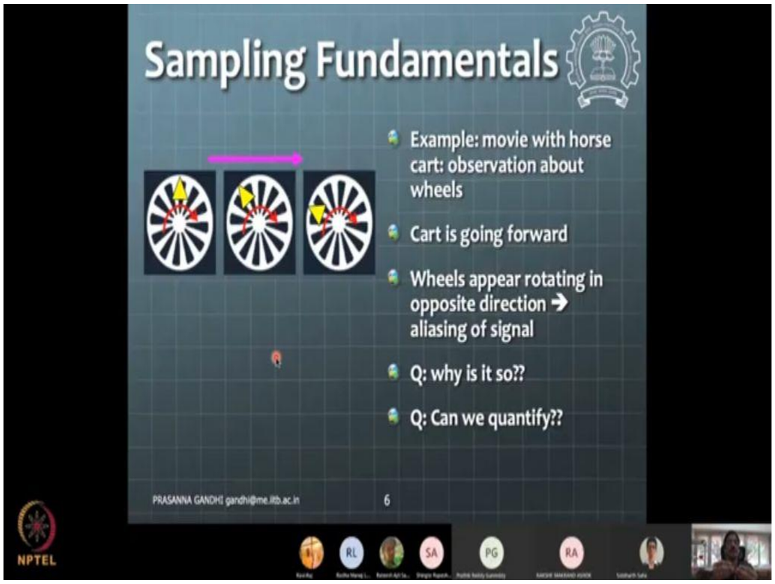

So, so first let us understand what is sampling; so and how this sampling can be visualized for simple cases. So, you all have might have, I do not know you might have observed some old movies, with the horse cart kind of the thing, old really black and white kind of movies. They used to have this kind of film to record the movie, and then they replay the film to play the movie. There is no kind of a digital camera for recording videos and things like that; so this is like a film based recording of the movie. And if you observe that suppose there is a say, typically this horse cart is moving in the forward direction; but then the wheels will appear to be moving in the opposite direction, and that is very funny.

I mean, when I was a child, so I then I was wonder why such things are happening there; and then, you might have seen your fan is turning. I do not know, now, there are tube lights which are having flicker; there are tube lights which flicker. When there is a flickering, and the flicker like the fan will appear to be moving in some different direction; those kind of effects basically. So, you have a stroboscope. Nowadays that even mobiles have this apps, which can give you a stroboscopic kind of effect; I might I have one. So, this is like nowhere I can set like some frequency, and I can start producing this kind of stroboscopic signal here.

So, now I can flash these signals against some kind of object which is moving, and it will freeze the images at the time the light is flashed on the surface. And if the frequency of motion and frequency of the strobing matches, then even that moving or vibrating object will look like as a stationary. Those kinds of effects are all like related to this sampling. For example, if you see this cart example like I take, say this is moving in a forward direction. And look at this pointer, right now it is pointing this. Now, it rotates actually a complete rotation and comes in the in this position in the second part that is shown here; sorry, let me get them.

So, here if I take second image and here I take at the same same speed it is going forward. And again, it completes one rotation with little less degrees, and I take third snap. And now I replay the snaps what will appear is to kind of these two be moving in the opposite direction. We will start moving in opposite direction, and this is called aliasing of the signal; this is like some kind of a loss that is in the signal. So, I have lost a direction of motion, and I have lost the speed also here. So, the speed actually is so fast, but now I am getting some signal which is now visualizing; it is very slow speed, but in and also it in the opposite direction. So, this is called aliasing of the signal; so your signal is lost here.

Now, the question is can we quantify this effect? So, one can see now that if this phenomena is happening like is a pointer moves from here, just to up to this point, no problem. Up to this point, I am talking of now only the one successive cycle. So, from here it is moving here, no problem; 90^{θ} motion, no problem. If it again further down here, they still there is no problem. It comes here, I will have some issues coming up; you see that pointer is going from here; and I take next snap when this is like a vertically down position. Then I am on the verge of like losing something.

If I go here, then I have already lost that; because I know it did appear that I will move in this direction. So, so you can see that there is also sense of that kind of speed of sampling needs to be like say wheel has some rpm, or frequency ω for motion ω_0 let us say. And if I sample two samples per second per cycle for this one cycle, I take two samples; then I will have one pointer here and other pointer say here, I will have some a lossy thing. But, if I take now so so, if I take more than two; then I will immediately start seeing that, this comes.

So, so what you are talking about is the number of frames per revolution; we are not talking in terms of the time here, not per second kind of thing; so, you do not get confuse. So, per with one revolution, if I have say, if the pointer comes here; that means in one revolution I am taking one, two, three and four samples for summarize. So, I begin here one, two, three, four 4 samples I will take, if I have 90^{θ} degree motion happening here; that is a kind of a sense. So, so one can always transfer that in terms of the time with considering the frequency of motion to be ω_0 . So, if my frequency of sampling is 2 ω_0 ; two times means, that means I am taking two samples in one rotation.

Then the possibility can be only this sample and other sample is 90^{θ} then I am in some confused state. If it is more than 2, I am on this side; then it is like I am safe, I can still determine. And if I am on other side, then also I can say I am confused completely right. This is less than 2, so I have one sample here; and next sample maybe coming up here here. So then again, I will have a loss of data. So, from that perspective, one can see that the frequency of sampling, if it is more than 2 or less than 2 or equal to 2; we will have a different kind of interpretations of the sampled signal coming up, is like a more like physics here.

Student: Sir, for less than 2 also, if it is less than 1; then I think we can still get the direction right.

Professor: So, if it is less than 1, so you have one sample here; and next sample coming up here, say for example. how we can get it.

Student: It is towards the right of the first sample.

Professor: So, for a sample is here.

Student: Which means, after one rotation, after some time; so one rotation plus some delta t getting it again; which means, there is a lot of delay, but it is.

Professor: So, that is correct. But, so, what we are talking about then; so, you know, loss of, we are talking of this loss; not only of the direction. So, I am giving this example for direction loss; but like in that case, you would have a speed loss. You will have one complete cycle is lost already; the speed has lost so much there, so, this is what we call it a harmonic. You have one complete cycle has passed and then next cycle may forward direction you are started measuring perfectly fine. You are getting the direction sense right then; but still there is a loss of the, it is not like the original data.

It is, I have lost so much of so many samples in between, or so much of the data. In a sense, I do not know now the speed that is I am getting is correct or not; I am definitely not have correct speed then. So, so this is just to give you a feel of what is happening. When we, so we can do like a lot of mathematics about it; that is that follows. But, if you have this understanding good here, then you can see that look even with samples little less like little larger than 2 omega 0 frequency. I still have a chance to get my reconstruction or signal represented completely; so that will see how mathematically it can come up little later. So, still we have 10 minutes to go.

So now, how do we quantify some stuff like? So, if I give you these frequencies you may be able to come up with this various speed, that I am going to see actually by doing this, that kind of analysis that, that we are talking right now. But are there any other kind of formal ways of doing these or seeing this in terms of the signals eventually? That is what we will see.

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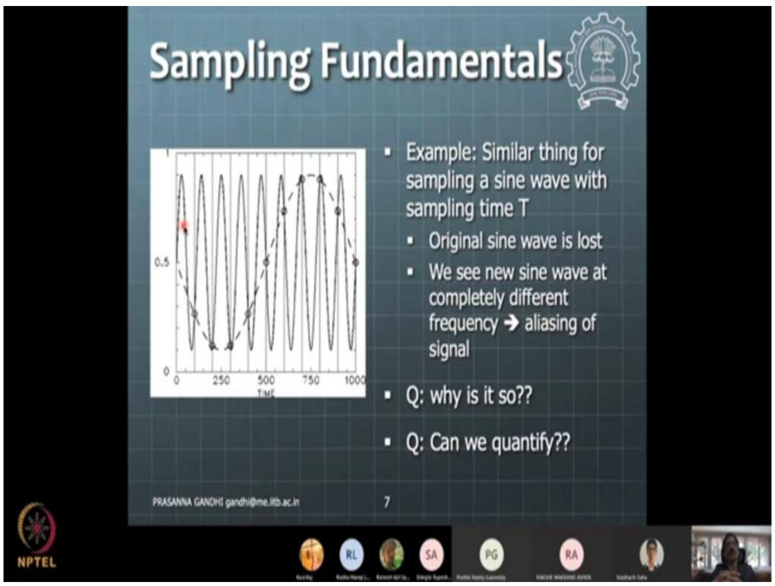

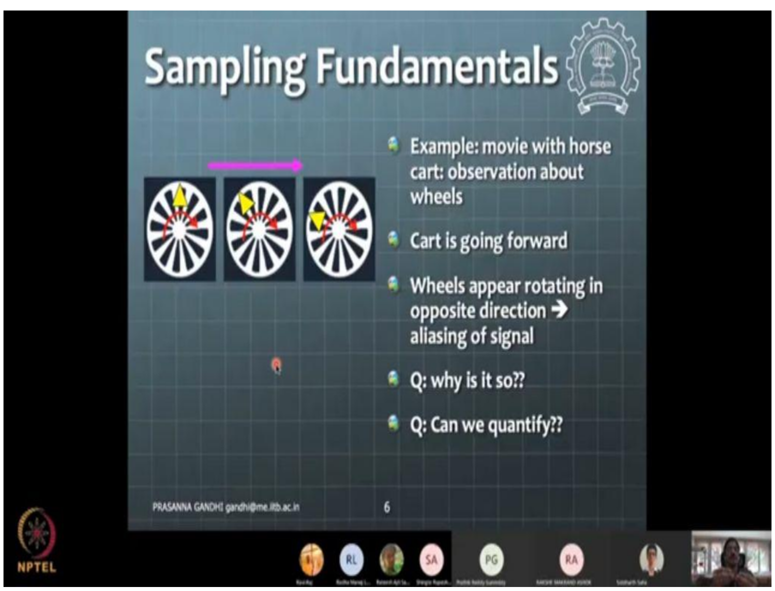

For example, now this is a sinusoidal signal. For that sinusoidal signal if I if I am sampling at certain frequency; which is now, so this is one period here. So, same thing that is happening here, I am now representing it in terms of the period; so this is one cycle is one period. So, this one cycle is corresponding to this one period here; so now I am considering like vibrating object. For example, so to say if you want to connect to the physics; it is like a vibrating object. And whatever these points that are just like snapshots that I am taking, when I do this stroboscopic measurement on that vibrating object.

That is a kind of a sense if you want to understand these in terms of some kind of a physics associated with this. But, in general this is now exactly what is precisely would be happening, when we start taking a signal, and we sample it. So, one can see that sense in terms of physics, by using a signal vibrating object; and these are the points where you have a stroboscope applied to that. So, if you see now these are the equispaced points; but which are beyond this 180 degree phase. So, this is the total 360^{θ}, this is 180^{θ} up here; and beyond that I am kind of putting somewhere first sample, and now equispaced other samples.

And if I start plotting them, I will see the some other sinusoidal emerging out of this. I will see the speed of actually, the speed of vibration was this; or a the speed of oscillation or frequency of the signal was this. And what I am seeing actually the frequency of sampled signal is much much lesser there.

So, original sine wave is completely lost here; we are not representing original sine wave in this sampled scenario. So, under what conditions we can see our original sine wave? So, if I say my first sample is here next sample is exactly at 180^{θ} ; then all the samples will be straight line here.

That is complete loss of; that is also not some kind of a correct representation of the signal. But, if I have some sample here, next sample is somewhere here; and then so on and so forth coming other samples follow. Then I apparently see the signal to be somewhat like not really sinusoidal; but by using those samples I can now generate original signal back.

Based on my arguments that look from that wheel I can see that, if I am taking this sample which is less than 180^{θ} rotation for the wheel; or, less than 180^{θ} phase for the signal. I will be able to reconstruct, I will be able to see my contents are original sense of that signal along with the direction; and the frequency or a speed that entirely is preserved. So, how do we quantify that?

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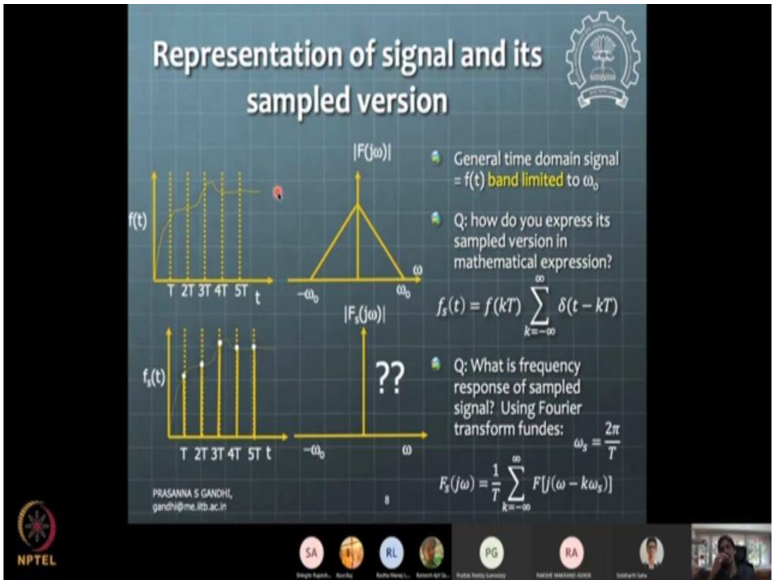

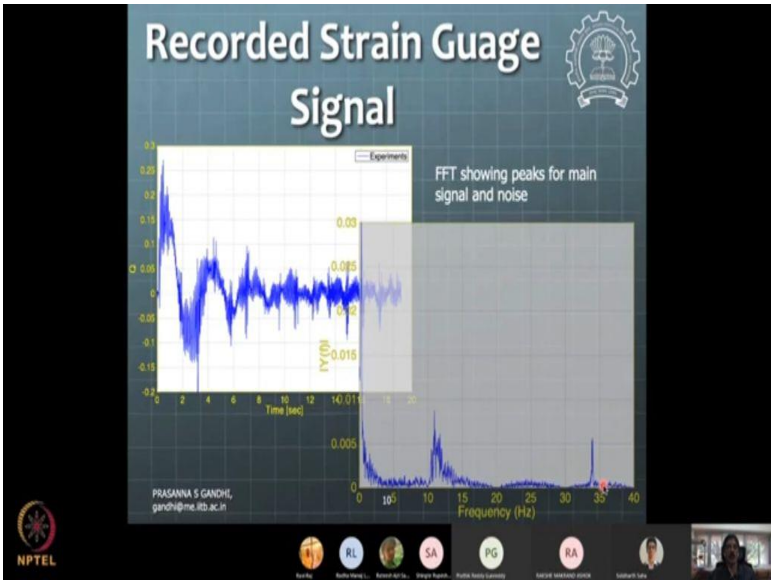

So, that is why we start now looking at a such for doing that. Now, again here we will not have much time or will not have time at all probably to get into the proofs of this theorem. So, we are trying to kind of understand this theorem and apply; so we are into that kind of a phase. So, the proofs of this theorem will follow by two parts here; one is like you need to have a understanding about the Fourier series, and other is understanding about Fourier transforms. And third thing maybe about this direct data function. So, these three are some base mathematical background requirements to get to the proofs. So, the idea here is that now as we have seen say some signal f(t) is there.

And as we saw all the signals mechanical domain or this mechatronics signals will have this limit of the band. So, we do not know what is there in between these two points; or here from 0 to this ω_0 what is there? We are just trying to represent it some by somewhat by this straight line. But, this is not a thing that is actually representation of a signal. This is just to show that your signal exist between this to this frequency. So, typically you give this f and let me say that it is a band limited signal; and then you construct some kind of representative of that band limited signal.

So, this means this signal is band limited by this ω_0 frequency. So this is, this may not be linear variation; it may have some very skewed variation in this domain. But nothing would exist beyond ω_0 No content for f (jω) would exist beyond ω_0; there will always be 0. So, this is how we consider the band limited signal.

What it means is the fastness of the signal is not more than omega 0; it can only be fast, as fast as ω_0 kind of frequency that is it. Beyond that, it does not have anything which is the faster variation than that speed. And this is very important concept for signal representation in time domain to frequency domain; so that we can see the different aspects of the signal better in the frequency domain.

Say for example, this signal has a band limit of ω_0. This aspect will not come by just seeing the signal in time domain at all. You see some variation in time domain, how do you know that this signal has no frequency contains more than ω_0. That is not, very apparently coming in the time domain. So, that is why we have this different mathematical representation of the signal from time domain to frequency domain and so and so forth. So, if you see the actual signal, I will show you some actual signal; actually we can see its transform here. See this is actual signal actual signal in time domain.

So, this is time per second, it actually recorded from our own strain gauge measurement, for a beam which is vibrating. This is the strain gauge is at the bottom of the beam and the beam is vibrating. So, you can see that this has some kind of a, by seeing the signal; how would you know? You say there is somewhat some frequency base frequency is there, and some additional frequencies are there. But, we only can see that; we do not know whether this will be having some kind of a band limit or it is happening, what kind of frequency is contained in this signal, for actual signal. So, that is where this FFT of the signal is very very handy.

So, what it shows here is that around 12 hertz, we have one frequency coming up; and then there is a mother kind of a component of frequency which is coming up around 34 hertz frequency. So, these are actually two modes of vibration of cantilever beam; the first mode is here and second mode is here. And then there are these small noisy components are there; so, these are safely ignored. You just need to see what are the major peaks for the for the FFT signal. So, beyond is now, there is no other frequency coming up, as at least in this signal; which has excitation given to some extent, because it is having this kind of response.

And in this response now the signal is band limited to say this is the frequency of up to say 35 for example. Like that one can see the actual signal in the similar as we have seen this. So, you can see that the signal is no way like this triangular representation at all. And for the sake of mathematical completeness, you actually define these on the other side also.

So, we not get into more kind of when you get into proofs, you will see that this part is actually needed to be considered. So, that is coming mainly from the mathematical precision of discussion. It has this part has no real physical significance; what is negative frequency will not have a physical significance at all.

But for mathematical completeness, it needs to be considered. When it, when you do the further analysis, then this will start using on the other side what is happening. So, now I would leave you leave you with this question that, if my sampled this signal to represent this kind of sampled points, what will happen to the signal in the frequency domain? That is a key question. How do we see that? How do we see the sampled signal appearing in the frequency domain? That is.

Student: cut-off in the frequency domain, maybe at 1 by.

Professor: at some point it will be cut; here the (freq). So, that is what, that is where we have to see. See, when these are the samples seen in the time domain, what is that you will see in the frequency domain is given that when there. This is a continuous signal, the frequency domain signal looks like this as a frequency content of that continuous signal. Now when sampled, what is going to happen is a question. And when you, so see normally we have a notion that something is sampled here; it will be sampled in the frequency domain as well no; that is not correct notion.

That is what we need to wash out that notion that; something is sampled in the time domain, does not mean that in the frequency domain also. There will be some samples that you will see only these kind of samples here; that is not true. So, what happens will this mathematical analysis is this, we will see what is this interpretation maybe in the next class. But maybe I will post these slides, you can try to see up to this point; maybe to guess what is happening here, it is not trivial to guess that. But see if we can give some kind of a thought to that, based on whatever your knowledge about Fourier transforms and things; you can apply that to such a signal, which is represented as a discrete kind of a sample.

So, see this Fs is a mathematical representation of the samples of the signal; they are represented as the summation of these all these direct delta functions, which are coming up at each of the sampling time. So, we do not get into all these details; but we will be certainly need to understand what happens here. By doing this mathematical analysis, what is final conclusion is what we need to be very very careful about to understand it; and apply to some situations. That is what we will do in the next class.

Student: Sir, what is the frequency even mean in a discrete sampling setting? For a continuous signal I can think of a summation of sin terms; so they each have an omega.

Professor: Right, so for a continuous signal, you have the same; you can say if I need to represent the signal, say for example by using; so, I would I would give you some sense of thinking. If this signal is has been, had been a periodic signal; let say this signal is periodic signal. What you would think? You would think in terms of representation of periodic signal, in terms of Fourier series.

And in the Fourier series, the coefficient of Fourier series would give you the contents of the signal at each of the Fourier components; these are sine and cosine components there. So, the each of the frequencies, how much is that signal contributing to will, is what I am getting in the Fourier coefficient.

So, this is like a Fourier coefficients that you are plotting here. So, you have a continuous signal here or periodic signal here. But, when you see the Fourier coefficient, Fourier coefficients come as a discrete here. Now, when I say in the limit that my period of the signal goes to infinity; then it this becomes like a continuous signal. And that time this instead of these discrete parts, I get also this kind of a continuous domain continuous representation here; so that is a kind of a sense one can think about. So, this Fourier transform is nothing but the Fourier series in the limit as period of the Fourier series goes to, the period of the signal goes to infinity.

This is now this period is infinity, means is not periodic signal; here non periodic signal will have this infinity period for that signal. So, that is a kind of a sense we can see for the Fourier transform that is happening. Now, if I say in the similar kind of a way, like if I need to represent these multiple peaks that are coming as these direct data kind of functions here. How can I represent them by using a Fourier series? That is maybe start thinking and you will get some answers there.