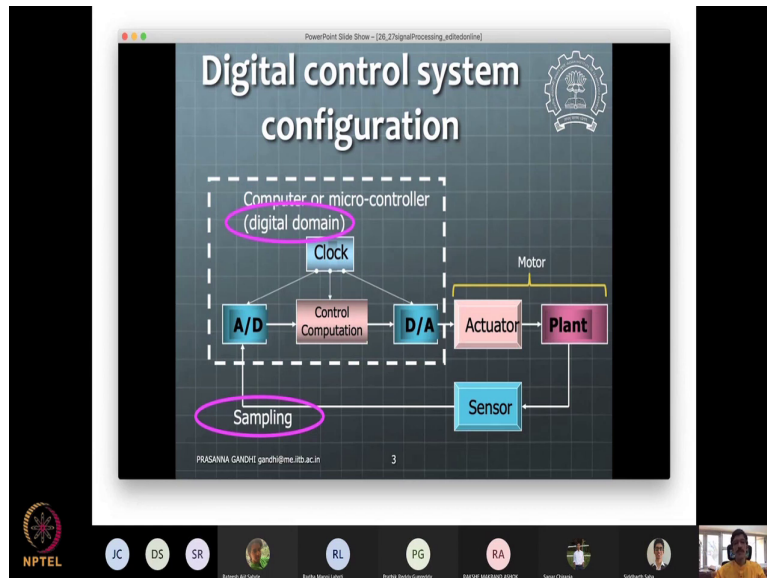


**Design of Mechatronic Systems**  
**Professor Prasanna S. Gandhi**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Bombay**

**Lecture - 43**

**Shannon sampling theorem and signal reconstruction**

(Refer Slide Time: 00:16)



And then you have your actuator, which is basically in the analog domain and plant analog domain. Sensors may be analog to digital depend upon whatever is sensor interface. And then you typically have this setup for any mechatronics system. And you may have feedback in some cases, like you may be missing the feedback also, the sensor feedback may not be there, some assistants are like that. So, but in general like you have this feedback and close loop control happening in typical, especially high, what we say, high fidelity mechatronics kind of a system.

And then you, we saw some basics of like what happens when the signal is sampled, especially when you take the sensor signal, which is allowed to made, and then you sample it, and you want kind of to represent it in the digital domain. And now we are looking at how the signal is faithfully representing what it is supposed to be, to the contents of the signal in some sense are preserved when you sample it. That is what we are discussing.

(Refer Slide Time: 01:35)

The image shows a screenshot of a video conference session. The main part of the screen displays a PowerPoint slide titled "Sampling Fundes" with a gear icon in the top right corner. The slide contains the following text:

- Basic idea: A continuous domain signal is represented as samples in digital domain.
- Q: are the samples true representation of original signal?
- Q: can it be possible to completely recover original signal from samples? What role sampling time or frequency has to play?
- PWM goes on motor as sampled signal, Q: What is effect of sampling on system dynamics?
- You have observed some discrepancy coming due to "insufficient" sampling (logging) frequency while processing steady state speed data.

At the bottom of the slide, the text "PRASANNA S GANDHI gandhi@me.iitb.ac.in" and the number "5" are visible. Below the slide, there is a video conference interface with the NPTEL logo on the left and several participant icons labeled with initials: JC, DS, SR, RL, PG, RA, and others. A small video window on the right shows a participant.

And for that, this is the main theorem, is about the *shannon* sampling theory. So, a couple of these questions you should be keeping in mind and seek answers for. How do we kind of say that the samples are the true representation of the original signal or we recover the signal back from the samples. Are there any method, is there any method or under what conditions, we can do this complete recovery back. And how is it applicable to our case, like say, the speed of them is going to the motor, I say, sampled signal, so now you may have a little bit of a clarity at here.

See, this PWM, is itself has this switching signal, it is quite positive and 0. At very high frequency, when we say sampled, so what, for all practical purposes what we consider is PWM signal when implemented on a motor is that it is giving some average voltage to your motor. And that average voltage is now given as a sampled form. So, this average is one value, and after some time, some sampling time, it is changing its value. That is how we can visualize this or look at this.

Otherwise, if you say, my PWM itself is sampled but it is at high frequency, and if I want to consider that also into *account in* analysis, then it is a different story. What we are assuming here is, especially for PWM signal, see otherwise if there is an analog interface to control motor, then absolutely no problem, that analog signal is at some value. *During the sampling instance it is at* constant value. And then like in the next simultaneous, it is a different value. So, what is this effect of this type of way of actuator receiving these signals, this kind of step

form. Every *something* time, the step value changes. So, we will be looking at that in due course.

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# Sampling Fundamentals

Example: movie with horse cart: observation about wheels

Cart is going forward

Wheels appear rotating in opposite direction → aliasing of signal

Q: why is it so??

Q: Can we quantify??

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So, we started off with these fundamentals of sampling and this is an analogy one can use our physical thinking can be done based on these. All concepts that we see later can be easily understood if we can connect it to this analogy, because this gives you some kind of a visual kind of a way to think about stuff.

And we will see formally as we go mathematically and how things work out, but we will not derive the equation, we kind of use some of these sampling theorems and the reconstruction part if we derive. So, as we saw, if you sample when this arrow is at a point before the 180 degree position, so if arrow is here, and the next arrow position, if it is before 180 degree position, we can see that, we can faithfully kind of see the wheel direction and velocity as well.

Now, you can see that, see if I sample it here and it completes one rotation and goes beyond, and again I take a sample, second sample, that is also a possibility. But that will indicate you a speed, which is not reality although the direction is same. You can see that arrow is pointing here, it completes one rotation and comes here and I take that as a sample. So, even in that case, the direction is proper but the speed is, speed information is not proper.

Further, if I say, it completes from, starts from here, completes one rotation, completes second rotation, and again comes to the same that position, that is also one possibility. So, but that will give you like the same speed, very but actual speed has happened like the weaker two rotations in between. So, this is information that is getting in some way. So, how to see

that in the frequency domain is, slowly, we will look at that. So, the frequency domain representation would like give you some all these kind of possibilities, that are there for such a sampled signal. So, how that happens like, we will see in a while.

(Refer Slide Time: 06:30)

The slide is titled "Sampling Fundamentals" and features a graph of a sine wave with a sampling rate of 1000. The x-axis is labeled "TIME" and ranges from 0 to 1000. The y-axis ranges from 0 to 0.5. The original sine wave is shown in black, and the sampled version is shown in red. The sampled version is a lower frequency sine wave, illustrating aliasing. The text on the slide reads: "Example: Similar thing for sampling a sine wave with sampling time T", "Original sine wave is lost", "We see new sine wave at completely different frequency → aliasing of signal", "Q: why is it so??", and "Q: Can we quantify??"

The slide is titled "Sampling Fundamentals" and features three wheels of a cart moving forward. The wheels are shown in a sequence from left to right, with a pink arrow indicating the direction of motion. The text on the slide reads: "Example: movie with horse cart: observation about wheels", "Cart is going forward", "Wheels appear rotating in opposite direction → aliasing of signal", "Q: why is it so??", and "Q: Can we quantify??"

So, same thing extended now to actual signal and its time samples here. So, you can see that here, some hurdles kind of a things, see sine wave is coming up. So, you have original sine wave, which is going at very high frequency and then when you are sampling it, like low frequency sine wave is coming up in a signal.

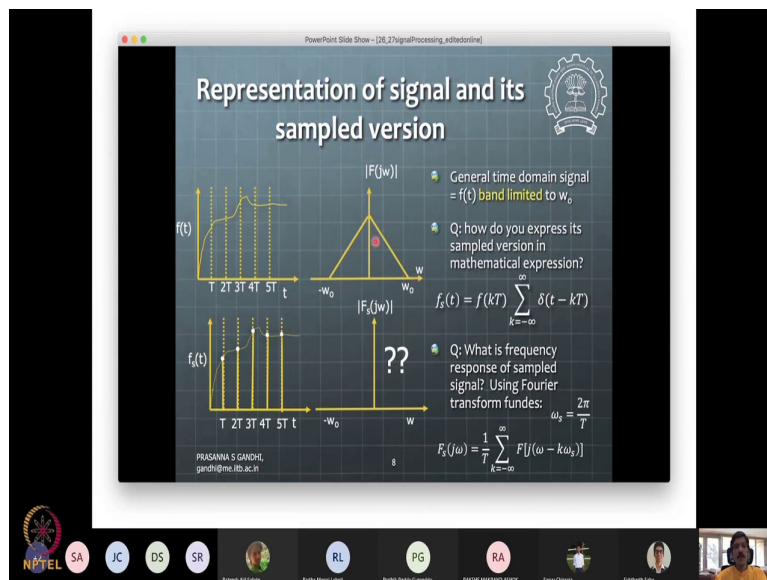
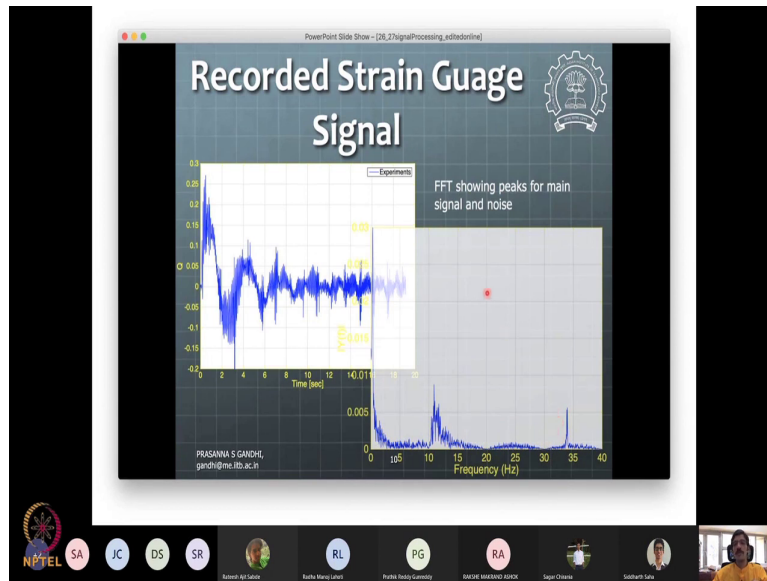
So, if you, if you see this problem is similar to like these wheel kind of a problem. Okay? So, the frequency at which you are observing the wheel to be moving is much slower and in

opposite direction for this particular kind of case. So, like that you have this frequency of this sinusoid is much lower and space is also in not in line with the phase of this original kind of signal. So, that is how one can kind of connect these two things to kind of think physically what might happen to your signal.

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Now, this is like more kind of a formal mathematical kind of a representation of what we are talking now, generalized to a signal. So, this is signal  $f(t)$  which is in general like evolving signal with some frequency content. **And what we are interested in is a signal for which the frequency contents are ((07:56)) in (( ))(07:57) kind of (( ))(07:57) of such a signal. So, (( ))(08:06) contents are obtained by using like the Fourier transform of this signal, and the numerical way of doing that is fast Fourier transforms.**

(Refer Slide Time: 08:27)



And those fast Fourier transforms would give you this kind of a plot of, we saw some examples in the earlier lecture, this signal, so it is of real signal. And **this is a fast** Fourier transform. And in the  $f(t)$ , we can see that this signal has a content, which is like some frequency here, and like some small frequency at would end up here.

So beyond that, there are **hardly any frequency content**, so this signal is kind of bad limited to frequency of low or like say 35 or that kind of a value. So, like that we can say that this signal typically represented this triangle as our, this kind of a straight line has no really great meaning here. I mean, it is just to signify that the signal has some content in this, in this domain, it may be like one kind of a single line or like multiple lines. We do not know what the contents are.

But typically, the signal is generally represented in this kind of a form. So, actually, if you want to really see what it is, then you need to take  $f(t)$ , but just to kind of like this for the purpose of the discussions, people would represent the signal like saying that okay, it has some content here, like represents its contents are in this type of triangle form. And now the question arises is that, suppose I sample the signal and get these two white dots, so these are the samples which are recorded in my computer, it is coming up as a vector array in my computer, then what is the frequency content of such a signal?

So, these frequencies can have multiple kind of values here. How that is happening is based on some mathematics of Fourier transform. So, this signal is represented for the mathematical kind of transforms, via transform or sample signals, the sample signal needs to be represented in terms of some kind of a map, and that is a mathematical representation of the sample signal in **Dirac delta kind of function**.

And then when you take a Fourier transform of this signal and do some kind of mathematical manipulations, you end up with these sample signal Fourier transforms. Now, for this sample signal Fourier transform, you can see that, just a moment, you can see that this  $F$  is either Fourier transform of original signal in the analog **domain**.

So, this sample Fourier transform is represented in terms of some kind of a form of original signal Fourier transform. So,  $f(t)$  is original signal and its Fourier transform is  $F(j\omega)$ . So, this is  $F$  here. So,  $F(j\omega)$  is this Fourier transform, but now when it is sampled, there is some kind of a scaling that is happening here, whenever  $T$  is some kind of a scaling factor coming.

And then there is a summation of multiple of, multiple copies of this  $F(j\omega)$ , which are replicated at frequency  $k\omega_s$ . So, for example, if  $k$  is equal to 0 value, you can see that this is nothing but a  $F$  of  $j$  over here, so that is one part. If  $k$  is equal to 1, then that means it is shifted by frequency  $\omega_s$ , that is  $\omega_s$  is the sampling frequency  $2\pi/T$ . So, that is how one can interpret these. And with these if I want to plot it here, say for  $k$  is equal to 0, this will simply replicate here as this signal  $F$  of  $j\omega$  is this value and, this will simply replicate here at  $F$  of, or  $T$  is equal to 0. And then for  $k$  is equal to 1, it will be replicated at  $\omega_s$  value.



So, you, I need to kind of find a way to omega s here at random axis, I need to again like replicate the signal as it is from here to F of, where k is going to 1, where omega is equal to omega s value.

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## Sampling in Frequency Domain

If **frequency response** (fourier transform) original signal 'band limited' by Frequency  $\omega_0$  is represented as  $\rightarrow$

Then frequency response (Fourier transform) Of sampled signal when: frequency  $> 2\omega_0$  will be given based on Shannon sampling theorem as  $\rightarrow$

$$F_s(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} F(j(\omega - k\omega_s))$$

<http://dspcan.homestead.com/files/Sdtt/dtftalia.htm>

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So, this is what will come like this here. So, when the signal is like this, it's sampled form will look something of this sort. Here, there is 1 kind of there is a replication, then there is a second replication here, then there is a third replication at a minus (0)(15:59). You see if it is minus infinity to infinity k value, so it is keeping on replicating everywhere. That is a main crux of this sampling theorem that it says that your signal in the frequency domain will get like a mirror itself at multiple omega s values.

And that are some, since I was talking about like even after like rotation of one complete rotation you will have the frequency can be same. Some kind of a sense of that information is coming here. Here, your signal is getting repeated at multiple omega s values, and that is that is how your samples can be visualized. So, your samples are just discrete data points. If I see the frequency contents of those discrete data points, there they will be repeated in this kind of form.

So now, if I start changing this omega 0 also, my frequency content or omega s, I decrease the sampling frequency. So, I want, you can imagine now this sample is first replica is coming here, you can see the scaling also whatever T scaling is happening here. And now this part is going to come more closer to this as I start decreasing omega 0, omega of s. So, as

$\omega_s$  decreases, this will start shifting, in  $\omega_s$ ,  $\omega_s$  is here. And keeping  $\omega_0$  same, if I, decreasing  $\omega_s$ , then at some point like this  $\omega_s$  over 2, then these two will start touching each other. So,  $\omega_0$  becomes  $\omega_s$  over 2.

I still have these two samples, where this is these signals, they are touching each other. So, if you see there, these signals here still have the same kind of a form like a triangle form, that it was there for the analog signal, it is still preserved here. But once you see that my  $\omega_s$  decreases below a certain value, which is 2 times  $\omega_0$  kind of a value then my signals will start interacting with each other. And you see that there is summation of these things. So, when they start overlapping, you need to actually sum them up in the frequency domain, this is a frequency domain of this sample.

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## Aliasing in Frequency Domain

If *frequency response* (fourier transform) original signal 'band limited' by Frequency  $\omega_0$  is represented as  $\rightarrow$

Frequency response (Fourier transform) of "sampled signal" when: frequency  $< 2\omega_0$

Can you see Why? How?

$\rightarrow$  So what? What is implication of this? Sampling freq more than Twice the "bandwidth" of system

You sum them up in the frequency domain and then they start missing the information. So, this original signal or something of the sort, and the new signal has now lost that information here, new signal has beyond this frequency lost some part of their information, it is getting lost. So, that is what is called aliasing, and that is why the signal has to have sampling enough, frequency enough, so that we do not see this loss of a signal.

So, any questions so far? This is what is a crux of (16:17) one part, which is that the signal should be sampled at least twice its largest frequency content, so that it is represented in its full form, in its samples.

Student: Sir, in strain gauge Fourier transform...

Professor: Yes?

Student: There were not any repeating triangles. Right?

(Refer Slide Time: 16:41)

### Shannon reconstruction

$$f(t) = \sum_{k=-\infty}^{\infty} f(kT) \frac{\sin \pi(t-kT)/T}{\pi(t-kT)/T}$$

Given samples how to reconstruct the signal back as proposed by this formula?

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### Recorded Strain Gauge Signal

FFT showing peaks for main signal and noise

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PowerPoint Slide Show - [26\_2]SignalProcessing\_edtradrone]

## Representation of signal and its sampled version

General time domain signal =  $f(t)$  band limited to  $\omega_0$

Q: how do you express its sampled version in mathematical expression?

$$f_s(t) = f(t) \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

Q: What is frequency response of sampled signal? Using Fourier transform fundes:  $\omega_s = \frac{2\pi}{T}$

$$F_s(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} F[j(\omega - k\omega_s)]$$

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PowerPoint Slide Show - [26\_2]SignalProcessing\_edtradrone]

## Aliasing in Frequency Domain

If **frequency response** (fourier transform) original signal "band limited" by Frequency  $\omega_0$  is represented as  $\rightarrow$

Frequency response (Fourier transform) of "sampled signal" when: frequency  $< 2\omega_0$

Can you see Why? How?

$\rightarrow$  So what? What is implication of this? Sampling freq more than twice the "bandwidth" of system

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Professor: Right. So, Fourier transform would give you a signal, which is only up to the first like first part of the frequency contents. Okay? Only up to the limit here, see these are done like a digital Fourier transforms. Okay? So, this is done in a way that your maximum frequency here depends upon what is the sampling frequency.

So, if you assume that the signal has motor aliasing that had happened already, and beyond that, like whatever is the signal content, it is produced only up to  $\omega_s / 2$ ,  $\omega_s / 2$  will be the frequency here. So, if you have sampled each signal at a frequency, I think in this case, it will be sampled it at a frequency of 1 millisecond or 1 kilo hertz, so this maximum value here on this frequency axis can go only up to 500.

And beyond that, it may be, we cannot represent because it is going to be replica of the same thing is, actually it will not come as a part of a, the numerical f of t method of like taking a Fourier transform. So, this understanding is implied here that this part is going to get repeated beyond this point and hertz frequency that is there for the  $\omega_s$  over 2 value.

Student: Perfect, got it. Yeah.

Professor: I mean, one can kind of set up your plots in a way that you want to kind of see them as plotted like that, but f of t algorithm does not by itself give you that.

So, this is the main part here. So, you see that when we take samples, although these samples are looking as these kind of points here as some impulses, in the frequency domain, they are not seen as impulses. That is the main thing that you should not confuse about. They are seen as a continuous signal here. When we do numerically some stuff, there is a different story, do not worry about that, but because it is sampled here it produces samples here also, no, that is not correct. It produces this actually in this, this is like a analog kind of a signal here in some sense like this is analogous means like it is a continuous signal here, it is in the frequency domain. So, see these are different domains of thinking.

So, frequency domain think like its different from time domain thinking, there is a transform that is happening in between. Okay? So, once you say, this is a sample set I am representing here in the time domain, that does not mean that in frequency domain also I will have only the samples. No, that is not correct. That is not main thing, one should not fall in that trap. So, clear this part?

Like we say, how like these signals getting mingled up or mixed up and gets messed up when we start sampling it with a lower sampling frequency. And now, one can see that, see if I have my signal in the form of that rotations, suppose we take the wheel rotation example, then  $\omega_0$  value is what is of my interest. So, there one kind of a, say what,  $\omega_0$  is the frequency of order or the speed of rotation.

So, that whole value will be represented here as  $\omega_0$  signal, and now if I sample it at multiple kind of different-different frequencies, one can see that if the sampling is not fast enough, then I see that this  $\omega_0$  coming up here, if I measure from here, see, this part is  $F_s$  here or  $F_s$  of,  $F_s$  replicated at, so this is my 0 basically. So, when k is equal to 0.

And now this is  $\omega_s$  value up here. Now, because  $\omega_s$ , it will get replicated here again and  $\omega_0$  from the side will come up here. So, this frequency is what I am going to see as a first frequency when I start replaying that signal. And this frequency is now, we can see that it is less than  $\omega_s/2$ . Okay? So, this is much lesser frequency than frequency original frequency  $\omega_0$  that I had here. That is what is going to be seen in this signal, which is not sampled enough actually, that someone can start thinking about this. And, okay, so like that one can see whatever that mutation way of doing things can we see now in formal mathematical representations.

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## Shannon/Nyquist Sampling Theorem

- Two parts:
  - Sampling frequency should be more than 2 times the max frequency content in the signal (called Nyquist frequency); follows from
 
$$F_s(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} F_j(\omega - k\omega_s)$$
  - Shannon reconstruction
    - Built original signal using the samples by formula
 
$$f_r(t) = \sum_{k=-\infty}^{\infty} f_s(kT) \frac{\sin(\frac{\omega_s(t-kT)}{2})}{\frac{\omega_s(t-kT)}{2}} = \sum_{k=-\infty}^{\infty} f_s(kT) \text{sinc}(\frac{\omega_s(t-kT)}{2})$$

T = sampling time  
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PowerPoint Slide Show - [D:\2\SignalProcessing\_sdfredonline]

## Sampling in Frequency Domain

If **frequency response** (fourier transform) original signal 'band limited' by Frequency  $\omega_0$  is represented as  $\rightarrow$

Then frequency response (Fourier transform) Of sampled signal when: frequency  $> 2\omega_0$  will be given based on Shannon sampling theorem as  $\rightarrow$

$$F_s(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} F_j(\omega - k\omega_s)$$

<http://dspcan.homestead.com/files/Sdttv/dttfala.htm>

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Now, Shannon has proposed further reconstruction. Now, the question is, okay like you have said that (( ))(22:24) of, if I sample it no faster than twice the frequency contents of the signal, the signal can be completely deep pressed here, I mean, the signal can, signal contents are preserved. So now, the question is how do I reconstruct the signal back. And there is this other definition, mathematical definition is more or the derivation like, how this (( ))(22:45) comes up from basic Fourier transforms and like how this expression comes up by considering the filtered version of that signal.

So, you filter this signal by using, so your sampled signal is like having now any such kind of replicas here. We want only first replica, which is at 0, the frequency, that value, that is what



we need to kind of preserve. And so, we filter the signal using this ideal filter which is a flat filter here with the scaling  $T$ , so that we cancel out this  $1$  over  $T$  by this scaling factor, and then we can reproduce this signal back. That is how one can think of reproduction of the signal in the frequency domain. And by using those kind of fundamentals one can work out the, what is happening correspondingly in time domain, and that is where like where you get this formula for time domain representation of  $(())(23:49)$ .

Now,  $(())(23:51)$  this is how likely the signal can be reconstructed from the samples. Now, let's understand what is it, this part is doing here. These are the samples, this is  $f_s kT$  is a sample of the signal and sample. And then each of the sample is multiplied by some kind of a function here. And then they are all added up together, is what gives you this reconstructed signal pack. So, and in this particular function is of the form sine theta upon theta. So, we will see what is this function looks like in  $(())(24:33)$ , so let's kind of see that.

(Refer Slide Time: 24:34)

The slide is titled "Shannon reconstruction" and features a gear icon in the top right corner. It contains the following content:

- Original signal:** A triangular frequency spectrum with a peak at  $1$  and a bandwidth of  $W_0$ .
- Sampled: frequency  $> 2W_0$ :** A yellow sawtooth wave representing the sampled signal.
- No reconstruction possible in this case:** A bullet point indicating that reconstruction is not possible when the sampling frequency is greater than  $2W_0$ .
- Reconstruction formula:**

$$f(t) = \sum_{k=-\infty}^{\infty} f(kT) \frac{\sin(\pi(t-kT)/T)}{\pi(t-kT)/T}$$

At the bottom of the slide, there is a navigation bar with several icons and names: NPTEL, SA, JC, DS, RL, PG, RA, SR, and a small video feed of a person.

So reconstruction part is having this kind of a formula what we have seen. And now, we will see how one can construct the samples back, the signals back from the samples. So say, these are the samples here, so these are different coloured kind of things started thinking, samples  $T, 2T, 3T, 4T, 5T, 6T, T$  they have.

Now, these function as we are saying its sine theta upon theta kind of a function, and if you know from your like mathematics, this function will have a, if you see sign theta upon theta limit as theta tends to  $0$ , its limit is actually  $1$ . So, this function is called sometimes sinc

function, since  $\theta$ . And it has some representation of this kind. So, if you plot it against  $\theta$  when  $\theta$  takes value of  $\pi$ , then this function is going to go to 0 again.

So, at 0 value, all this  $\sin \theta$  upon  $\theta$  in the limit as  $\theta$  tends to 0 is 1. And when  $\sin \theta$  is equal to  $\pi$ , this will take value 0 and so on so forth for like no integer multiple of  $\pi$  values, that is what is going to happen here. And this is a way, the amplitude will go on reducing because as  $\theta$  increases, the overall value of this coming as  $1/\theta$ , so amplitude is  $1/\theta$ , which gets reduced for a sine function. So, these are all like, this function evolves in  $\theta$ . Now,  $\theta$  in our case here is actually  $\pi t$  minus  $kT$  upon  $T$ . So, what is, what does it mean?

Suppose, we say  $k$  is equal to 0, then this is like  $\pi t$  upon  $T$ . And then like this  $\theta$  would take this value  $\pi t$  upon  $T$ , and this function will have now variation coming in time. So, now instead of  $\theta$  it will be coming as a time function behind. And when the time becomes equal to capital  $T$ , this  $\pi t$ , then becomes equal to  $\pi$ . So, this value will be equal to capital  $T$ . You can see that. Right?

Like now we say,  $k$  is equal to 0, so this term is not there, and now your  $t$  becomes like a capital  $T$  value, then  $\theta$  becomes  $\pi$ . And we know that at  $\theta$  is equal to  $\pi$ , this function takes value 0. So, when it gets can kind of multiplied to this, inside this scaling factor here, additional scaling factor is about  $kT$ . So, every sample will scale this signal. So, this, suppose this is a sample, so instead of this 1 value add whatever like this value.

Other thing, let's see when, what happens when  $\theta$  is equal to 0, when  $\theta$  equal to 0 part here this is  $\theta$  equal to 0 position, which is this  $y$  axis for this function plot. At point,  $t$  becomes equal to  $kT$ , that means like for each sample like this function is going to get placed here at a sample value such that this one matches or gets scaled by this sample value itself. So, I will have this say, for say,  $k$  is equal to 1, if I want to put this here then  $t$  will be equal to capital  $T$  here. So, at that point, my  $\theta$  is going to be 0. So, this this value 1 will get scaled by the rate of  $kT$  factor and then it will come here.

So, this signal if I start now drawing, and I know from here if I go  $1/T$  apart, I will leave it,  $\theta$  is equal to  $\pi$  value. So, this signal is going to come first as like this. So, for  $k$  is equal to 1 value, this function is going to look like this, and it will have some type of variation. Is this this is part clear? Any questions about this? So, we are just considering one case,  $k$  is equal to

1 and then like seeing the, how this mathematical form or this function it's represented as for  $k$  is equal to 1.

And then we need to consider that now that for  $k$  is equal to like minus infinity of infinity (( ))(29:27) but we do not have samples before 0. So, we consider like a sample starting from 0, that first we get is maybe 0, then how this signal is recorded, how this construction is seen. Okay? So...

Student: Sir.

Professor: Yes?

Student: Not clear, why did you choose this form of function? Because there will be interference between the consecutive sample reconstruction. Right?

Professor: Right-right. So, it is not insert interference but this so say if I now do it for  $k$  is equal to 2, I will get other kind of signal which is sketched up here like that. Okay? So, this is why...

Student: This is...

Professor: We produce 2 signal. So,

Student: Sir, this is...

Professor: Yes?

Student: Still an approximation of an original signal. Right?

Professor: So, I am just kind of considering  $k$  is equal to 1,  $k$  is equal to 2 only. I had to consider  $k$  is equal to minus infinity to infinity, then it is giving me complete signal. It's not an approximation, it is, see if I limit myself to like some few values of  $k$  alone, then it will be approximation. If I consider all the values of  $k$ , all the samples that are like possible for the signal to be considered, then it is not an approximation.

Student: But essential, you are still fitting it between the points. Right? Its just that if you have lot of points, then it might look like an original signal.

Professor: No-no. Its not that. So, look, so its not that you need to have in between points also. No-no, that is not what I am talking about. I am talking about that it has all the samples are there. So right now, I am talking a little bit of a, from the mathematical perspective.

So, in that tape, I have all the 7 samples known from from  $t$  is equal to 0 to  $t$  is equal to infinity or  $k$  is equal to 0 to  $k$  equal to infinity, all the samples have, I have available or beyond certain point and my signal is 0 for example, and I know that okay, all the samples are there. Then I will be able to kind of reproduce my signal back completely.

Student: So, my doubt is if you evaluate this function at suppose  $t$  by 2 somewhere in the middle point more or less there will be a slight error. Right?

Professor: No, that is what I am saying like, so we will come to that. Like say for example between these two values  $T$  and  $2T$ , like you have this signal coming up right now and this signal coming up right now. So, they are getting added and they will produce some kind of form here. Okay? Then if you see, if I consider next sample  $k$  is equal 2 sample or  $k$  is equal to 3 sample, I have this signal coming up. So, this is also contributing to, this part is also contributing to something between  $T$  and  $2T$ .

Like that every sample, let us say, I consider this blue part, a blue part has also something to contribute between  $T$  to  $2T$ . Then if I consider next part, like that if I consider all these parts, they will have something to contribute between these  $T$  to  $2T$ . And that is what this is a beauty and problem both for, I will tell you why beauty and why problem. Okay? So, this Shannon reconstruction consists all the samples. And like every sample has some representation between these two values.

Student: Okay, so if I...

Professor: (( ))(33:19)  $2T$  to  $3T$  again, like I can see where your concern is, the previous samples are also contributing something and future samples are also contributing something.

Student: Okay. Ideally  $k$  is going from minus infinity to infinity, but if I put it in a numerical software, then it depends on how many samples I am choosing for the summation. Right?

Professor: Yes. Right-right. That is why we usually have, if you are doing the numerics, anything, anytime, you will have some approximation, but it will be much better construction than like saying, "Okay, I am just holding this sample here, and like I am waiting till the next

sample to happen, and again, I am retaining their value and holding it here”, I mean like that. Its much better approximation than that. Okay.

So, or if you have this (34:03), some linear kind of interpolation between the two samples, then the other kind of a construction that can be possible, but that is not still less of a reconstruction because the signal is not smooth like it is not getting it reaches exactly what it was. But if I, even if I use like, say, for this sample, I use two future value and two past values, and reconstruct that itself with a Shannon kind of a formula, it will give you, give me much better kind of approximation.

It is a smooth function that is going to come up about like that. Your signal is actually in something getting reproduced in a very smooth manner. And your signal is actually in some sense getting reproduced in a very smooth manner. And if  $k$  is equal to infinity, they it will be like, exactly (34:50). But now the problem is that for any reconstruction here, I need the future values, all the future values. And a mechatronics system or real time control system, its impossible to get future values beforehand. Right? Because your feedback is happening, based on that feedback the future values will come.

So, you cannot have the channel (35:20) possibility in the mechatronics control systems at all. So, this is a very important aspect of the circuit, whatever reconstruction that we need to have, that need to be causal in nature, which should be using only the previous samples values. They should not be using like the future values, which we do not know. So, a standard reconstruction is not a causal reconstruction. It is using all the future values to construct the signal pack, that is where like the problem is. So, that is why you cannot use standard reconstruction or it's ideal kind of a reproduction of the signal for mechatronics or control domain system.

But then the question is, “Where is it is useful?” Can you, anybody knows where this energy construction could be used, useful in some way? Okay. This is useful for the cases of like you know you have heard about like you are digitally representing a sound. So, your songs are or like anything sound representation in a digital way if it is done, your only, your future and past samples are available. Right? For any kind of sound to be played back again.

So, this half, high fidelity sound systems, they could use this kind of future information about the samples to do some kind of reconstruction based on the future value, future values of the

samples to get the quality of sound produced in the same way as it was recorded. Its not analog recording, its like a digital recording of a signal.

And you want to produce really to a very close extent to the analog kind of reproduce that sound again, you can use these kind of concepts. So, communication industry would use this in some way. So, your digital high fidelity sound representation, they are reproduction of the sound again, they can use this channel reconstruction. But for our control systems, this is not feasible to do. So, what we do is we hold the values in a register.

Even you have this duty register in which like once you put a value, it will not change by itself till the time you put other value there. And you will have a chance to put another value only when you kind of like complete one sampling, time instance and come back again and like you put another value there. So, till that time, the value will be held constant. So, the way you are like reconstructing the signal is basically putting this signal constant from here to the  $T$  to the  $2T$  value. Then it is changed to this value and again that value will be held constant. It a changed to this thread value and again that thread value will be held constant.

And again, then it will be changed to blue value, it will be held constant. This is how we like maintain some kind of mistake form you represent at signal, as a reconstructed signal, especially for it, for implementation on motor or actuator. So, this is how like one can think of Shannon sampling theorem and make use of that for sampling the signals in a way to make sure that the contents are preserved towards its best form. Its just to think about how, what should be the function here, between one pair by this set of  $kT$  and this should be function of  $t$  minus  $kT$ , such that I now produce my this  $T$  function, this is the  $T$  formed here.

Student: Sir, for the stapler function, we can use the rectangle.

Professor: Yes-yes.

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## Other reconstructions

- Zero order hold (ZOH) : most popular in mechatronic systems

Time domain      Frequency domain      Ideal reconstruction

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## Digital control system configuration

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Student: But...

Professor: So, this is what we are talking about, this type of function there, and it is already here. So, these are frequency domain, okay sorry, do not get confused here, it is a frequency domain kind of a representation. So, maybe my writing, like some writing here is little confused, I guess, which is called Zero order hold (ZOH). I will just correctly send and repost the slides again properly. This is called Zero order hold or you may note down like this ideal reconstruction written here is not a correct thing. Just delete this part again because this is not ideal reconstruction at all.

Ideal reconstruction will be this part, where you have this  $T$  and like a rectangular window coming up here is an ideal reconstruction. This is a different part. This is like that rectangle

function that Siddharth was talking about. So, instead of this same function here, we will (( ))(40:50) this rectangular function will be coming up here. So, this function then like now representing the same form  $f$  of  $kT$ , this function  $t$  minus  $kT$  would give you, actually this reconstructions like that, it is called Zero order hold.

In this zero order hold, you will find in MATLAB some places, people have used this in the digital domain to A to D or D to A conversion. So, the system's speed want to transform from a domain to digital domain. Likely, you will be asked whether it should be used in bilinear transforms or it should be used in zero order hold kind of a form or what kind of a form that is to be used.

That time, you will understand that zero order hold means like I am value till the next sampling. So, if this is happening, and under this scenario, if I want to see what my system needs responding then that digital system conversion from the analog critical domain is what is based on this zero order hold kind of a form. Okay.

So, if I reconstruct signal pack like this, so we are not yet coming to the systems, we are just kind of talking about only the signal, how the signal is represented and reconstructed back from the samples. So, what we saw so far is like this signal getting sampled at this. And once, this is sampled what is happen to its frequency contents. And after that, like if I want to reproduce a signal, what are the different ways I can reproduce a signal.

And one of the ways is this, Shannon reconstruction, reproduce it back completely, but that this is a completely ideal case, we will be not writing in the control systems domain, we will rather do this kind of domain. So, now this is the way we are reconstructing the signal into the sample.

So, if you see the sensor signals also when they are sampled and, let me go to that slide, when a sample, so it is the first slide we are talking about. When sensor signals are sampled and put into a digital domain their value is held constant till the next sample comes for the sample. The way it happens for your  $2T$  cycle, the same way it will happen for the sensor value also. The sensor value will be held constant till the next time sensor sample comes.

Again, that value will be held constantly the next time sensor sample comes. Tall is kind of a reconstruction that is happening inside the domain, although we are kind of representing



these values at a discrete time values only and processing them for what other purposes, they are actually if you see, they are held constant, what, that new samples.

And then that holding constant over the two samples has much more kind of an effect when it is implemented on the actuator. It does not matter too much when kind of just process those values here but it has more implication than it is used on actuator. And how that happens, we will see in a bit.