Design of Mechatronic Systems
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Lecture – 44
Signal Processing

So, far we have seen like this kind of a signals clone so may, and now will see its effect on the system. So, this is how so one can ask this a fair question to ask.

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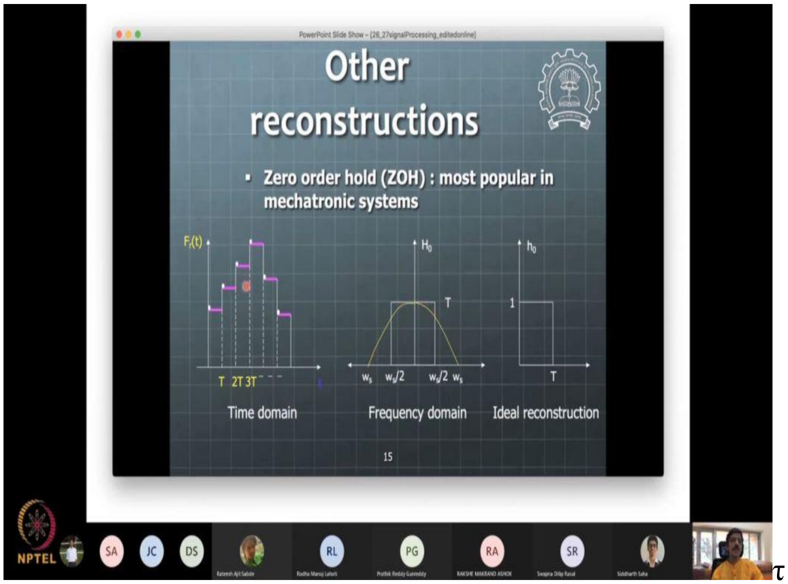

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If we are saying that the Shannon reconstruction like reconstruct the signal back in the frequency domain by using this kind of a filter, which is the T is a scale on a signal; and it is ideal kind of filter. So, rectangular filter which is sharp cut-off kind of filter. If we see this in the frequency domain, this kind of reconstruction, what is the filter that we are applying to reconstruct this back. It is nothing related to a signal has nothing what are the samples; it has something to do with how we reconstruct the signal back. So, this is in the frequency domain represents some filter of this sort; so this is filter is having this kind of representation in the frequency domain.

How that comes will not get into; but just use some field and it is not. If it is not ideal reconstruction, then what it is; then this is an uncertain field. It is reconstruction of this sort and it is kind of allowing some back beyond this ω s to also to; so something if it is existing for a signal when it is sampled; you may have something coming in this domain. Even if the signal is not aliased, then this some part of the signal coming here; that signal will get also getting some representation, when it is in this low order kind of a. that is a kind of a meaning one can think about. So, this is a filter, so while has to have a little bit of sense of when we talk of filter.

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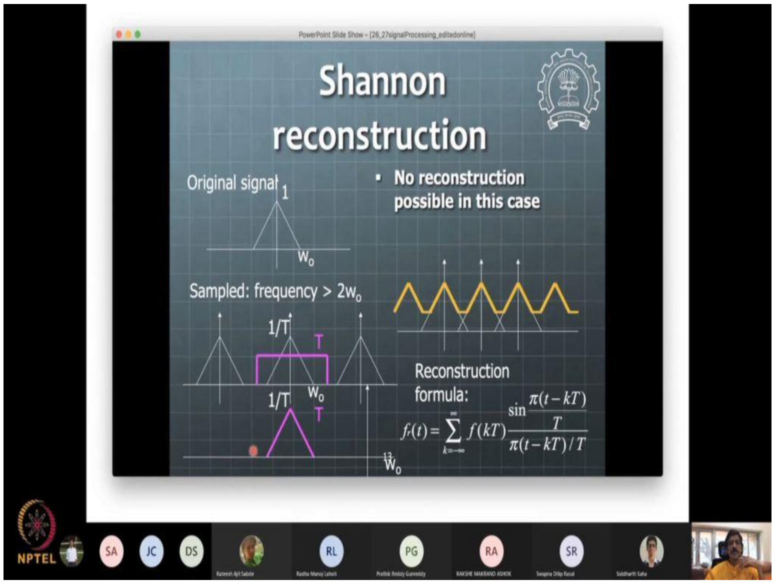

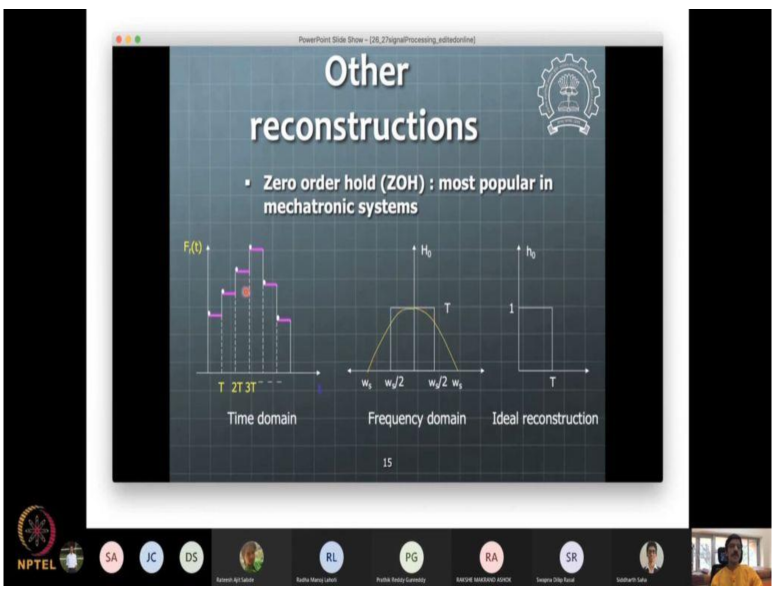

See, this is a sense that we talked about. The filter is filter in the frequency domain, allowing some signals to pass through; and some signals are completely not allowed to pass through. So, say when I say the filter is of this form, and like rectangle type form; then this particular filter function in the frequency domain, multiplies by frequency domain response to produce the output of the filter.

That is what, that is how one can think about. So it is like a input, which is going into a transfer function; and then transfer function is modifying that input and producing some output.

So, every system from that perspective can be seen as a filter also; so, this is nothing but a frequency domain that is response of the filter. So, transfer function, what is the frequency domain and response? When you have a transfer function what is frequency domain response of a transfer function, or frequency domain representation of a transfer function? Or when you substitute s is equal to j ω in the transfer function and get a magnitude and plot it, what you get?

Student: it is bode plot.

Professor: It is bode plot, exactly; this is like bode plot of a system. So, this is this filter is nothing but the bode plot of the transfer function of that filter basically. If we consider response frequency domain response of this filter is nothing but the bode plot of that filter. I mean this kind of a sharp cut-off filter is it is impossible to realize in the mathematically completely. But if it has been there like that it is an argument that Shannon has given actually. So, this filter transfer function, its frequency domain representation is giving you these kind of plot, which is bode plot of that whatever transfer function; that is a filter transfer function.

And when you pass this sampled signal to that filter, means we are multiplying the frequency response of that sampled signal, by the frequency response or frequency representation of a filter function or bode plot of that filter function to produce this. That is the idea of filtering, or is the same idea, when you have a input even to this system in the frequency domain; and you pass it through the transfer function, or this transfer function as input is given to the transfer function; and it turns produces some output. Then this, say suppose the input is a sinusoidal frequency, simple sinusoidal input; then it will just get scaled by the transfer function. And that scaling will be dependent upon what in this response, or bode plot of that transfer function.

That is the kind of idea that is there for the filter is the same idea, as you have an input representing the frequency domain. Transfer function representing frequency domain and output coming as a frequency domain representation. So, they two are not different, the filter is not different from any kind of transfer function kind of a thinking input plot domain; so, that is a important concept. And we say I want to filter this signal, so we need to think moment you say filtering of a signal; you need to typically thinking in the frequency domain, first kind of get my filter. So, we will talk about little bit about filter little later in more detail.

So, right now we are just looking at the signals, and we are filtering these some of these signals. And to correct to, to show that Shannon reconstruction will be reproducing this signal completely back. But now, if you see instead of Shannon reconstruction, I had this, the reconstruction which is zero order hold. Then, see this is ω_0, this $\omega_s/2$ frequency here, and this is ω_s here. You remember that this representing domain this much was going up to ω_s; so some part of this function will also come up in my filter signal, or in my signal represented as a zero order hold form.

So, see some of these are one has to be very clear in the mind about which domain you are thinking. If you start thinking in mixed up that domains of thinking, then typically as seen the confusion arises. To here clear here, we are thinking only the frequency domain; and in the frequency domain, we know that zero order hold has some kind of a form that is plotted up here. This is a frequency domain zero order hold thing will go like that. And when it is like this, then what will happen to my signal is what I am thinking in the; so, I will get some warping of this triangle to happen.

So, instead of this triangle getting produced, you need to reproduce completely like this; I will get this kind of warp, because this is not really a rectangle kind of a window; and then the additional part from the signal also chopping my form. So, one can think this is how it will get plotted in the frequency domain; rest of frequency domain representation of signal which is having a zero order hold.

So, see this is a (free) this is again a frequency domain representation of a signal which is sampled just sampled; so, this is just sampled signal. And now we are seeing from the sampled when we do some kind of a reconstruction; what is its frequency domain representation is what we are considering here.

So sampled signal we already know has this kind of a multiple replica of the original signal frequency domain response is coming up. But what we are looking at further is that, if I now have the samples represented as zero order hold; then what is going to be the (frenetic) reconstruction, what is the frequency domain representation.

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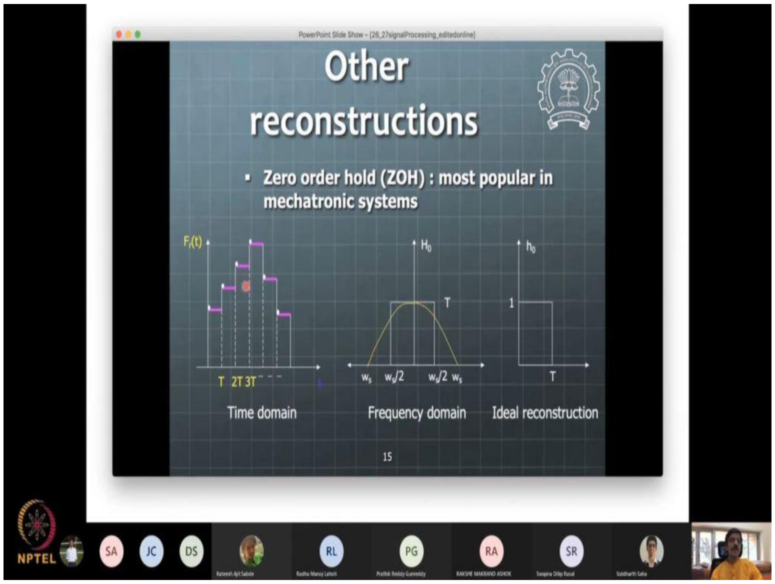

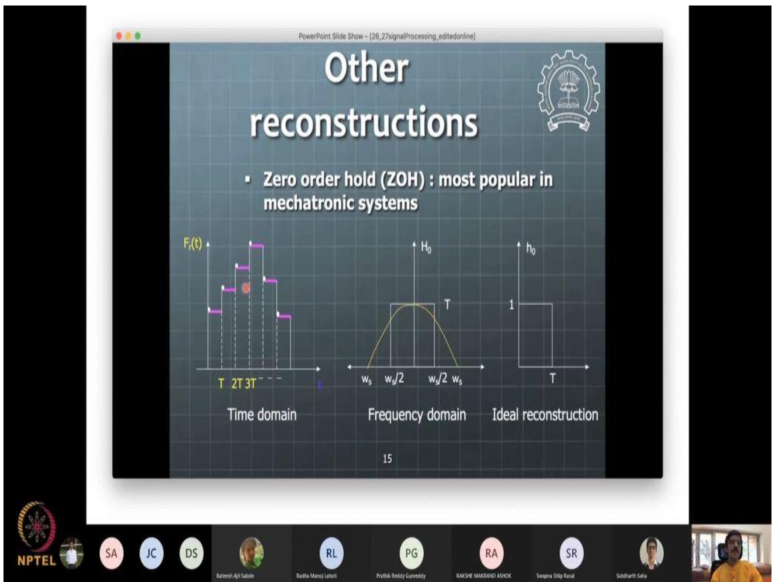

So, all that as not much of a physical value or implementation value in some sense; but it is just good to know that for this zero order hold; there is a the signal if it has triangle representation in a original frequency domain, analog signal; then it will get represented in some kind of a warp way, when it is sampled.

And note this sampled and made a zero order hold like that; so this is important, not just sampled but sampled and like reconstructed as a zero order hold; that is a very important statement. Otherwise, it is just a sampled signal; it will have just mirror images. So, that is it, there is a certain difference between the in the. So, the warping in the frequency domain is happening is somewhat like an indication.

I mean one cannot really get what you say, intuitively seeing that this; this means it will be warping. It will have some difference, it will have some kind of warping; but we cannot tell it exactly what will happen. Only by doing this mathematics, one can be able to say what exactly this part is getting multiplied by this the straight line; if it is, if it had been a straight line.

See this straight line what we are saying that triangle is just our sense of representation; that is the signal has contains up to that point. But even there those things are going to get somewhat disturbed. So, the signal contains at different frequencies are getting disturbed, in some sense where they say it is bad; so let us move further then.

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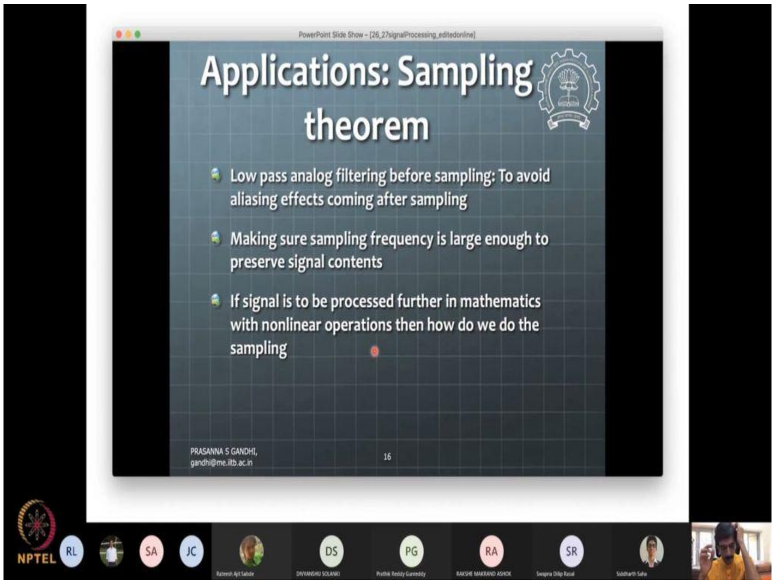

So, now we just see some small application of sampling, like how do we think about a system. Say for example, if you are developing a mechanical system; typically, before getting into sampling, you use this low pass filter. Now, you use this filter to make sure that beyond certain frequency, the contents are made zero. Otherwise, if you take a generally like analog main signal and see its frequency contents; you will find the frequency contents to go all frequencies they are represented in some way. So, to make sure that you higher frequencies are completely chopped down, you use this low pass analog filter.

This analog filter that you can use some simple RC circuit to get this analog filtering. And now, once you do this, then one can think about, now if I am sampling. See, if you want to really preserve all the contents of your signal, and make sure that you are representing your signal really in a better fashion; this is a way to do stuff actually. Use this analog domain as filter, but you cannot have analog domain filter with a very low cut-off.; because, with a low cut-off, you may get into. Either you may lose some part of your signal that you want to preserve; or what is happening is like the first order filter does not have a sharp cut-off.

These RC filters are first order filters, then you have a slope; so that slope will allow some frequencies in the higher than the cut-off, also to pass through. If you know one word τ x + 1 is the is a transfer function by the first order domain system; then it's bode plot is easily like you get by using this τ. Corresponding to τ, one of our τ is here like cut-off frequency; and beyond cut-off frequency, you will have a 20 degree magnitude slope cut-off will happen there. So, that slope is what is going to cause, some of the higher frequencies also to cut-off come beyond cut-off; those frequencies will come and get represented.

So, you will it will require ideally sharp cut-off filters; but then sharp cut-off means your cost is going to increase. You cannot use simple RC circuit to get sharp cut-off filters; RC circuits will give you only the first order cut-off. So, this first order filter you use and sampled now your signal; frequency is with a larger frequency, so that this is some of these signals, which are getting past beyond the cutoff frequency. They also get represented in this high frequency large sampling frequency will preserve those frequency contents. And then there is the signal is further processed for any operations; so that it will get completely.

So, it can have multiple operations that can be possible beyond this point. So, the idea here is to use first analog (metric); so you do not kind of directly get into sampling given a signal. Given signal, what is the frequency content that you are of interest to you; and use that as a cut-off for your RC circuit to develop? And use that circuit on board, before the signal is taken into your microcontroller; that is idea.

And once it is there in microcontroller, you sampled it at a relatively high frequency; and so that its all the contents are preserved. And then you can once it is contents are preserved and there you are; then you can take a call based on the frequency response, the representation of the signal what to do.

You can use some digital domain signal, digital domain filter to further filter the signal and things like that; some kind of further processing can be done easily. And one of the best is this.

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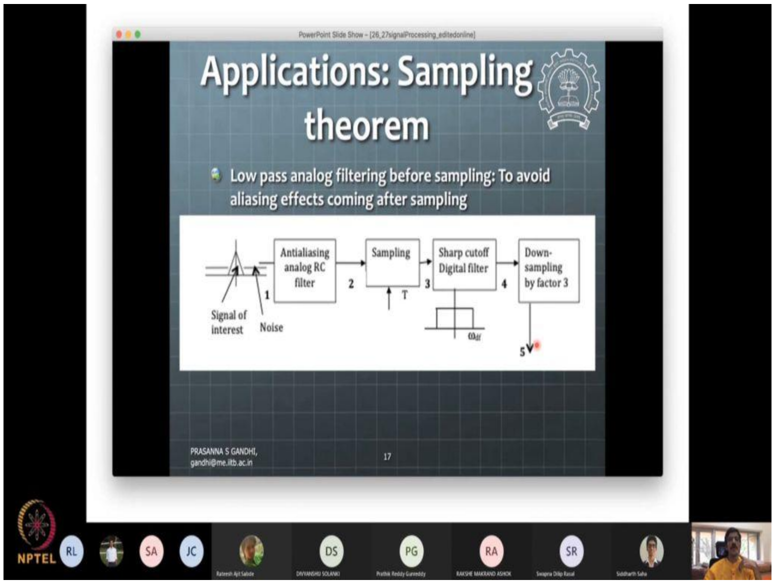

So, you have this signal of interest here is represented typically by our triangle representation. And then there is a void noise, which is our call frequencies; then you use this is called antialiasing filter. So, this analog domain RC filter is typically used, and then you can get the signal representation at the point 2.

So, you can think look with the RC filter, what will be then this signal representation in the frequency domain, at this point where you apply. Then for this is one of the examples. So, you should be able to given this filter filter function, we can see what is the frequency domain representation coming out.

So, these are all, I am not naming these access; but this is a filter frequency domain f of jω and of this signal; and this is ω s is something better. So, this all whatever is represented here is all in the frequency domain; so all these signals which are coming out will be represented as a frequency domain signals.

So, there here there will be a representation that original signal will be there; and then with a sharp cut-off, with the antialiasing RC filter; you will get this signal beyond this point take down this some value here. So, this 20 DB is sloping cut-off will taper the signal off from here to some value which is very less; it is not be a constant going over like that.

As the frequency increases this filter will have a effect, that it will nascent the higher frequency signals completely. It will completely chop off the higher frequency signals or taper of the higher frequency signals. And then when you sample it, then there is sample should be represented in some way. Then you can see that how these samples after sampling how this signal is going to look like, and how.

We got frequency actual sample, such that this tapering off part does not mixed with this signal itself. This signal of interest we need to preserve, under the sampling. So, after sampling is happened, you know that whatever this original signal frequency response it is going to get replicated or mirrored in multiple places, at ω s kind of a frequency.

So, we want those when that is happening, we do not want this higher frequency components to come into the domain of signal of interest; so, that is what we want to make sure of this. So, I would suggest at you ponder over this example, and actually start writing these or sketching this signal represented in frequency domain at these multiple locations. And then see whether how the things are making sense. So these are the different different operations, there is an antialiasing analog RC filtering operation is happening here, then sampling operation is happening here.

Then you can have some sharp cut-off digital filter is given here. This is some kind of digital filter. Ideally, I would say, it is not like practical filter; we cannot have a sharp cut-off practical filter possible in digital domain also. So, if that is there, what is a signal representation here; and there is a this something called down sampling.

So, this down sampling just means that you skip the samples. So, you have only signal sample let us say frequency ω s; then the down sampled signal by factor of 3, will be the same signal when sampled at frequency ω s factor 3. That is how, that is what it means actually the down sampling. So, what you do is out of these multiple samples, you take one sample; omit two and take third sample.

Then take omit two again, take next third sample; like that you can keep on doing that; so, you omit the samples in between. That is how you get a new signal, which is down sampled. Why you need a down sampling is to save the memory. So, if you have not down sampled the signal; you have to make cut-off samples will be stored into your memory. So, down sampling should not see this down sampling sometimes may create some aliasing effects; we do not want those aliasing effects to be created. So, so then you what you need would have this cut-off frequency for this sharp cut-off filter to be propagate this state.

So, those are all the engineering points to be worked out, as an engineer you will; you know this down sampling may not work by write down sample, only by factor of 2 order. Or, I have to cut-off of this too much lesser frequencies, so that this down sampling does not produce aliasing. Those kind of thinking is what I am calling as; some kind of engineering that needs to be done.

So, the fundamentals for that are on there so far we have talked about; its application here to see however can proceed this signal to finally get it in a form, that is no use in the control condition. So, beyond this of a or beyond this point even after sampling, you can start using this signal for control purposes, no problem.

You do not need to sharp cut-off these, and then do these operations at all. So, you do just analog filter and then sample it; high sampling frequency and you will be kind of go. Or, you may say, I need to remove this some parts on the sample, so that I can have a better processing possibility with the down sampling. Then, you do these additional steps and reduce your number of competitions. So, this is like one of the examples of how do you start thinking about sampling, and use that in a typical mechatronics

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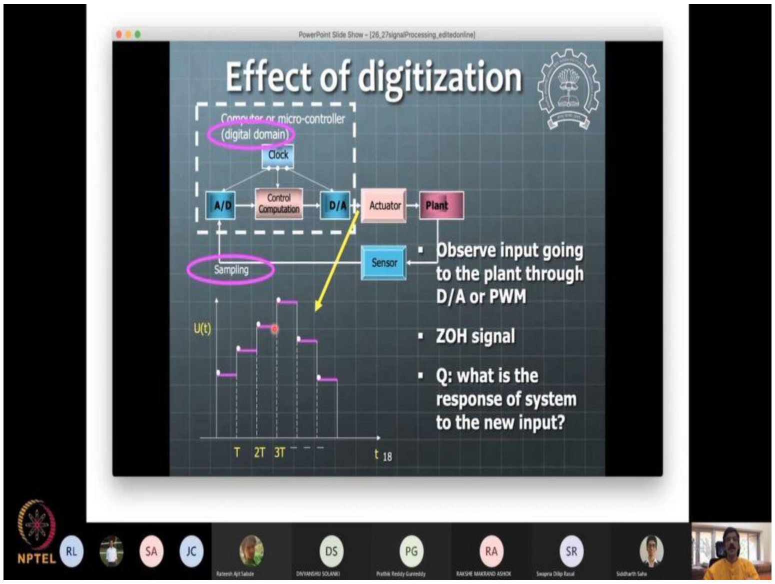

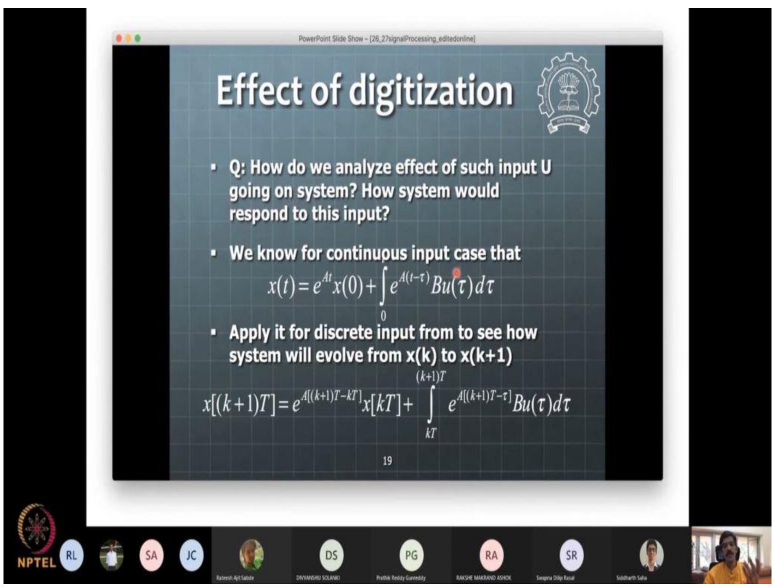

So, I think we may not have much time till we go today; but. So, next part is what we talk about is, say suppose we have this; so I will give you flavor of this. We will not get into lot more details here; and we may come to the point, where we need this e to the power. We had this question somebody has asked that e to the power a matrix power of exponential of a matrix, where do we need to, where we want, where we need to use that. This is a place where we have used that. So, question here is really, if your actuator is getting the signal of this sort. Now, we know why the actuator will get a signal of this sort.

So, this signal is received by actuator, then actuator will be implementing this signal on the plant; just amplified version of this signal on the plant. Now, under such scenario, how my plant is going to be. So, a plant is not really receiving a continuous input; but input is now received in this sample and zero order hold form. So, this question is what we need to answer here. And for that we consider our system solution trajectories. The systems in the continuous domain, if we represent this signal a system as a state space system; so, we know what is the state space condition now.

So, with this state space representation of the form of $x(\tau) = Ax + Bu$, and $y = cx$. Only after that we do not consider Bu kind of a form in that. So, $x(\tau) = x + Bu$ and $y = cx$; in that form, if I sample this signal, or the input is given in this form to that signal.

So, when I say, I sampled this system means my input is going into this form; that is a terminology that typically people will use. This is a sampled system, or this is a continuous system, or this is sampled system or digital domain system. So, if I have my input going in this fashion on my system, then how my system is going to behave is what I want to derive mathematically.

And that is why I start off with the solution of a system in this form; you have this homogeneous part of the solution, and then you have particular integrated part of the solution; these both the forms are there. And X_0 is like your initial condition, and this is how your solution of the system of the form $x(\tau) = x + Bu$ will be.

So now, when I have, so this x is typically a vector here; and B is again a vector, and we are considering only single input to this system; Bu is a single input that is going into the system. Now, if I want to see that this form is implemented; U is of this kind where U is constant over the time period 0 to T, T to 2T, 2t to 3T like that U is constant.

So, this is I can call this as say, U1, U2, U3 like that I can give some discrete values to U. Under that scenario, what is going to happen? So, now my system is at some point here, in general x(k) value; and I want to see what is my system, how my system evolves over with the constant input given here as uk till that point, like x(k + 1).

So, what is my x(k + 1)? X k plus 1 of T. When I start at x(kT); so this is my x (kT). So, I use the same form, I have started with initial condition x (kT) here; and then solution from now time over time, which is T starting from 0 to T here. Now, which is kT to T here; so, this(k + 1) T - kT will be there. So, kT to (k + 1) T is what will be evolution having over here.

So, here evolution is come T is equal to 0 to T is equal to small t. So, like that when this is equal to kT to t is equal to, final T is equal to k plus 1 T; this will be the evolution that is what I am representing here. This is a homogeneous part and the interesting thing will happen over this part.

Now, this u of τ is going to be a constant value here; between the values kT to k plus 1 T, you have u(τ) to be constant. So, say 2T to 3T, u of t is equal to u (2T) is equal to constant. So, that is a value that is constant here; and that constant will come out of this integral. And I get this in a much simpler form like this.

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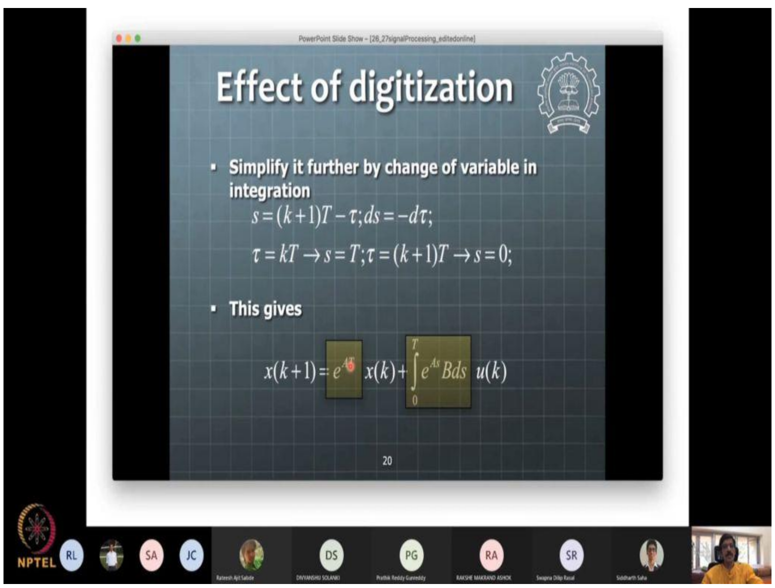

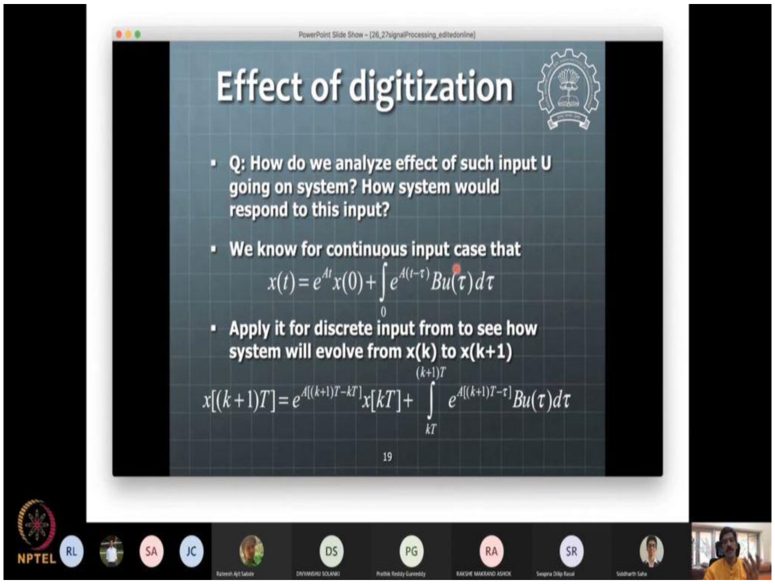

So, what I am going to get here is; so e^{AT} here. So, this is sorry, this is a matrix if you see (k + 1) T - kT, which is going to give you e^{AT} and x (kT) here. So, now I am dropping this capital T to make the simpler. to make the life simpler. So, instead of this capital T we just keep it as k + 1. So, x of k + 1 is equal to e^{AT}. x (k), plus now this integral from 0 to T; if you shift this the time axis shifted, then you will get this interpretation here 0 to T e to the power some variable s here As, B and ds. So, this s, some other variable that is used to replace this τ.

So, this is and this is given here; what is this s variable and all these things, here we now take here. So, you get these two matrices here; and this is where like you get this e to the power AT function here. And one can now start representing this x (k +1), as some function; which is now characteristic functions for the system; this does not have anything related to k. And this is also does not have anything related to k; these are like characteristics functions, characteristic matrices of the system, which are going to give you the system in the form.

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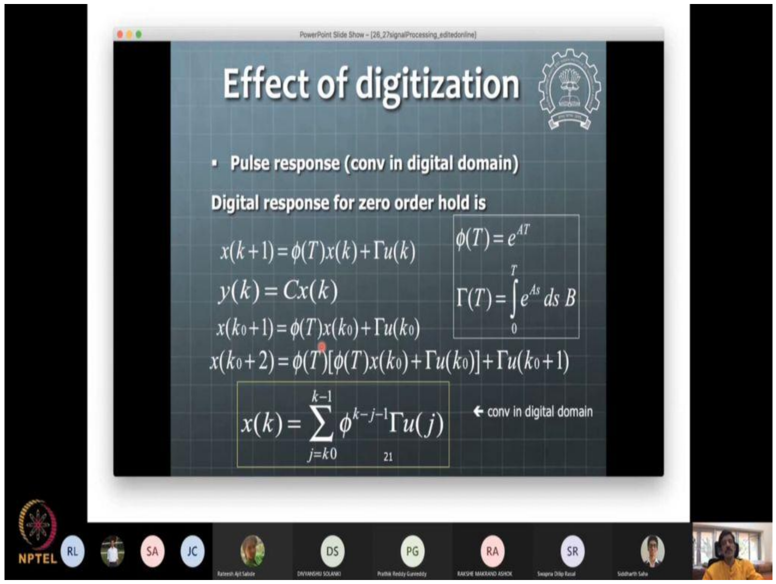

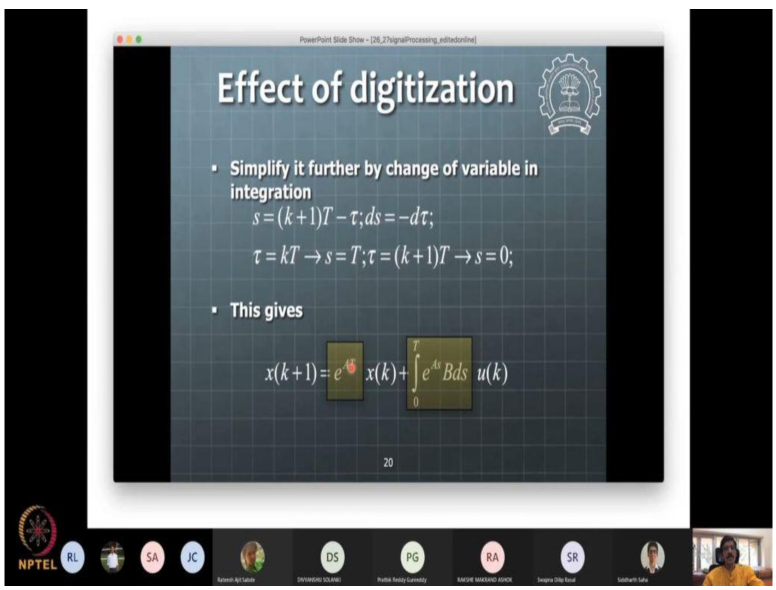

So, this is a form, the system is coming into; x (k + 1) equal to ϕ(T). Now, this function is a function of sampling time t; and then this is the Γ(T) u (k). So, this is a form that you get for your digital system; and y(k) is equal to C times x (k). This remains the same, there is no change in this form; and then these matrices are defined here. And now, this opens up like a completely new domain of analysis for or systems in the digital form; x(k + 1) is. So, you can transform the system from your analog domain to a digital domain by using the zero order hold form.

So, this is what is happening when you use in MATLAB also, while converting the system from analog domain to digital domain. If you use the zero order hold option, then it will do this precisely.

So, I think maybe this is enough for us to stop here. I mean, there is a good amount of discussion that can happen further about, how do you now use this digital domain.

So, this is the most important crux of, how do you go from analog domain to the digital domain; very nicely from physics, understanding of physics of it. I mean, many times like to you just use the zero matrix transfer function; but what is zero mean transfer function? How it comes? Where is it like genesis of it happens?

Those are parts are where this is, this part is very important. So, this x(k + 1), if we get into the C domain initial or like which is the z transform domain representation. This is z, z transform of x(k + 1), is z raised to 1 times x of 0 or some. So, we are not get into that in details; but basically this is a this some shift operator that is happening. Shifting of the signal with one sample; so x (k + 1) is equal to zx(k); that is how one can see in a C domain. So, that kind of operators can be introduced into this form to get new transfer functions in the discrete domain.

So, this, this is what is the next step that; and all that now we can develop this domain of digital systems. And all the tools that we have root of this analysis, or this Nyquist analysis to find some equivalent representation in the digital domain. That you can have a transfer function in the digital domain; and then you can define some new conditions for the stability of the transfer function in the digital domain. And all these things can be slowly developed for that. So, that is what this digital digitization of the system starts giving you all these multiple tools at your hand, to do the analysis.

So, if for some reason you cannot use have sampling time; see most of the time, when you have analog controllers; use high sampling time, and you will be fine. You will not please go back in implementation of controller and things like that. But, whenever you feel that I must use just sampling time; then you may need to trigger the digital domain analysis of their system, to make sure that whatever your designs are not hitting any worst performance of the system. So, one can think in a digital domain itself and design controllers in this domain itself, to make sure that your system will work even if your sampling is little less.

So, that typical figure for the sampling frequency to be is, it should be at least three to four, four to five times more than your close loop type frequency. That is like bode plot figure one can, based on experience. How it comes one can do whole analysis about that; but bode part figure is like bout five to six times more than your close loop bandwidth of the system should be your sampling. Then, your controller will get into that properly.