

Engineering Thermodynamics
Prof. S. R. Kale
Department of Mechanical Engineering
Indian Institute of Technology, Delhi

Lecture - 43

Applications. Problem Solving: Problem solving: Closed system

So, I am going to take up where 2 problems one of closed system and one for an open system.

(Refer Slide Time: 00:23)

Q: Substance in a cylinder-piston arrangement; constant pressure process

$D = 150 \text{ mm}$
 $L_1 = 100 \text{ mm}$
 $L_2 = 2 L_1 = 200 \text{ mm}$
 $p_1 = 20 \text{ bar}$

* Water
* Nitrogen

Work, Heat = ?

IITDC, IIT DELHI

So, we look at a closed system, where we have a cylinder piston arrangement and what this thing is that, we have a piston which is over here. Which got length is L_1 over there and what we do is we heat it and at constant pressure the piston rises from here to here. So, we are looking at a constant pressure process and the whole thing the piston does work. And the that geometrical sum here that values are given here that diameter of this which is circular with 150 millimeters length is about 100 mm this length L_2 is double that of L_1 which means it will be 200 mm. And we look at a case where the pressure inside here is 20 bar. So, p_1 is 20 bar.

We will solve 2 problems; one where there is water in this and second where there is nitrogen. So, we learnt that we can have a such a (Refer Time: 01:24) which we get treat as the vapour or we can treat it as a ideal gas. So, that is the differentiation we are doing and what we want to know is what is the work transfer and heat transfer in these

processes. So, the process of solving problems should begin always by making a sketch and defining the system boundary.

(Refer Slide Time: 01:47)

Sketch system boundary ${}_1Q_2 = W_2 + (E_2 - E_1)$

Closed system C.M. approach
 ${}_1W_2 = ?$ ${}_1Q_2 = ?$

Cons of mass
 $m_{cm} = \text{constant}$

Cons of energy / 1st Law
 ${}_1Q_2 = W_2 + (E_2 - E_1)$

Assume $\Delta KE = \Delta PE = 0$

${}_1Q_2 = W_2 + (U_2 - U_1)$

Diagram: Two states of a piston-cylinder system. State 1 (left) shows a piston at a lower height. State 2 (right) shows the piston at a higher height. A system boundary is drawn around the cylinder. Annotations include: $U + PE + KE$, $\int p dv$, $p(V)$, $p \text{ const.}$, and $pV^n = \text{const}$. The final equation is ${}_1Q_2 = W_2 + (U_2 - U_1)$.

So, that the sketch in state 1, this is the sketch in state 2. And where is this system boundary in this say that system boundary here is this one and in state 2 the system boundary we get like that. System includes all the substance which is enclosed in this arrangement excluding any of the mechanic parts or any other parts that are there.

So, this is what it is and they are going from state 1 to state 2 pressure constant. And to do that we know that we will have to reach it and then this will go. So, the first step was to define this system and get the system boundary current. So, after looking at this it generates crossing the system boundary at all in the same mass which is expanding. So, important thing here is that we know this is a closed system. This is from what you have learnt in the very first module.

And then we realize that this is the closed system the type of equations the form of the equation that will be there is to have the controlled massed approach which means that you have to write the first law equations for this here we will write Q_{12} is equal to W_{12} plus E_2 minus E_1 . A looks like a very simple thing to do, but I find that they were many people who make mistake right here.

Instead of writing this equation, they will write $Q \dot{c} v$ is equal to $m \dot{h}_1$ plus $m \dot{h}_2$ of that. So, if we go wrong at this point everything else is completely wrong. So, be very clear that to understand what the systems we are looking at make the sketch actable this is what it is and this is why we will use this particular type of an equation. Now we are on the correct path here and what we have been asked for is what is W_{1-2} and what is Q_{1-2} . That is our objective. And we then proceed the same way, we said what is conservation of mass telling us it will be tell that the total mass of this and this is the same there is no addition there is no subtraction.

So, mass in the system boundary or the controlled mass this is constant. It can be calculated from specific volume if that is there though, where is the question there we will come back to that questions later let me just do this part the conservation of mass. Then we have conservation of energy. So, the equation the same that I have written there and if there no assumption that is coming up in this part. And we make an now, we say that what are the reasonable assumptions in this application? We say that these 2 are such that kinetic and potential energy changes are not happening, it is stationary the centre of gravity of the system rises very slightly and that we can neglect.

So, we explicitly say that we assume that $\Delta K E$ is equal to $\Delta P E$ and this is 0 which means that this E which was U plus $P E$ plus $K E$, these 2 terms will drop out. And we now get Q_{1-2} is equal to W_{1-2} plus U_2 minus U_1 . And what we can see from here is that W_{1-2} , this can be calculated by integrating $p dV$ for which we need pressure as the function of volume. In this case we have been given p equal to constant, but that need not be the case we could even solve the problem by saying that $p V$ to the power n is constant, where n has the certain value we could found that. And then this integral can be evaluated.

But I need to go from state 1 to state 2 which means is that we need to fully have all information about state 1 and all information about state 2. Then these are the how do we calculate. So, that will give us the work, but to calculate heat we need to calculate this term also. And for that we need to know fully states 1 and states 2. We need to know all the properties in these 2 states.

(Refer Slide Time: 07:11)

Water State 1

$$p_1 = 20 \text{ bar}$$

$$x_1 = 0.8 \quad \text{sat}$$

$$T_1 = 212.4 \text{ }^\circ\text{C}$$

$$v_1 = v_{f1} + x_1 v_{fg1} \quad \underbrace{v_g, v_g, v_{fg}}_{\text{at 20 bar}}$$

$$= 0.0799 \text{ m}^3/\text{kg}$$

$$u_1 = u_{f1} + x_1 u_{fg1}$$

$$= 2261.5 \text{ kJ/kg}$$

$$V_1 = \frac{\pi D^2}{4} \cdot L_1$$

$$= 1.766 \times 10^{-3} \text{ m}^3$$

So, let us see what do we know about the states to do that we need to know first what is the working substance. And now I will solve the problem assuming that water is the working substance and for state 1 air being given that pressure is 20 bar. So, p_1 becomes 20 bar. We need a second property and let us say that the second property is given to us that the quality of water in that is 0.8.

So now, we need to calculate all the other properties there, say it is T_1 the first one specific volume and u_1 and v_1 . We start the last one which is straight forward that the volume initially is πD^2 upon 4 multiplied by L_1 and d is given L_1 is given we can do the calculation and this is 1.766 into 10 to the power minus 3 meter cube. Please make sure that you write the units at every step T_1 it means that this is the saturated state.

So, T_1 will be saturation temperature at 20 bar. And this is 212.4 degree Celsius ok. Then for v_1 we will get this is v_f in state 1 plus x_1 plus v_{fg} in state 1 v_{fg} is v_g minus v_f . So, although there are 2 terms looking here, but they are one and the same. All we needed to know is what is the pressure because we know it is the wet state and we can get those numbers from the tables and this number is 0.07909. So, we get v_f and v_g or v_{fg} and v_{fg} all are at 20 bar and we calculate this one.

Then we will need the specific internal energy and we do exactly the same type of a calculation. This is U_f at state 1 plus x_1 U_{fg} at state 1 and when we do the complete calculation this would be 2261.5 kilo Joules per kg ok. So, this is what we got now v_1

also. Now we say that to calculate $U_2 - U_1$ that we wrote over here we need this $U_2 - U_1$ can be calculated by mass times $U_2 - U_1$ ok. So, this is for that we need the mass and this could be mass is constant in the system.

(Refer Slide Time: 10:32)

Handwritten calculations on a grid background:

$$m_1 = \frac{V_1}{v_1}$$

$$= 0.022 \text{ kg}$$

State ② $p_2 = p_1 = 20 \text{ bar}$ $V_2 = \frac{\pi D^2 \cdot L_2}{4} = 3.532 \times 10^{-3} \text{ m}^3$

$$v_2 = 2v_1$$

$$= 0.1598 \text{ m}^3/\text{kg}$$

Check state $v_2 \leq v_g @ 20 \text{ bar}$

p_2, v_2

NPTTEL logo and text: NPTTEL, IIT DELHI

So, to calculate that we now have the equation there that m_1 is equal to v_1 upon specific volume at one. And this we can calculate and this becomes 0.022 kilograms. Now, we go to state 2. Here we have p_2 is constant which is given as p_2 equal to p_1 . So, this is 20 bar and volume we are told has doubled. So, whatever v_1 we had or we can independently calculate this at p_1 square upon 4 into L_2 this becomes 3.532 into 10 to the power minus 3, which and from there we get that v_2 is equal to twice v_1 . And so, this tells us that this is 0.1598 meter cube per kg.

Now state 2 we got p_2 and we have v_2 give that independent property. So, everything is completely specified. So, we have to now go back and say how do I get the remaining property. Now temperature at, so, we make a quick check and say how does this v_2 compare with v_g at 20 bar which is p_2 . So, we go to the tables and pick up v_g .

(Refer Slide Time: 12:29)

$v_{g@20\text{bar}} = 0.099587 \text{ m}^3/\text{kg}$
 \Rightarrow state ② is superheated
 $T_2 = 435.2 \text{ }^\circ\text{C}$
 $u_2 = 3004.63 \text{ kJ/kg}$ | 'h' ✓
 $u = h - p v$
 Work, $W_2 = \int p dv = p_1 (v_2 - v_1)$
 $= 20 \times 10^5 \text{ (Pa)} \times (3.532 - 1.766) \times 10^{-3} \text{ (m}^3\text{)}$
 $= 3532 \text{ J}$
 $= 3.532 \text{ kJ}$ Work by system on surroundings.
 NPTEL logo and $v_2 \text{ (m}^3\text{/kg)}$ are also visible.

And it turns out that v_g at 20 bar, this is 0.099587 meter cube per kg. And what we have v_2 which is just calculated here was 0.1598 which is more than this. So, this means state 2 is super heated. So, we go to the super heated property tables and find out that at what specific volume at 20 bar what is the temperature that gives that T_2 comes out to be 435.2 degree Celsius and from that we can also get u_2 as 3004.63 kilo Joules per kg.

In many property tables U_2 is not given, but h will always be given. So, what we need to do is do an extra calculation. We get h there and say that u is equal to h minus $p v$ and v are also given. So, one extra calculation is involved if u is not explicitly given in the table or in the property details packages that we have. So, we got this and now we have everything that we needed for state 2. We can now move on and several in our big scheme we wanted to calculate work. So, let us write down what is the work in this process this integral one to 2 $p dv$ and this is p_1 or p_2 because pressures are both are same v_2 minus v_1 .

So, when we do the remaining calculation, this will become 20 into 10 to the power 5. Remember this is Pascal multiplied by where 2 volumes that we had 3.532 minus 1.766 into 10 to the power minus 3 meter cube. And writing the units at every steps. So, please keep that in mind because quite often people go off when there are mixed units. When we use Pascal and meter cube the answer will be in Joules. If it was a polytropic

work output was 3.532 kilo Joules. And heat transfer this is also positive this was also positive 19.88 kilo Joules ok. So, we have solved the problem.

The couple of things to think about firstly, where did we get this property data from? And we have many sources you can look up the web there are online property tables. You can look up the web there were online property table another book that also got property tables. And these numbers could be slightly different depending from the source that we have taken up. Positively for us for many substances all of them will have the same reference state. So; that means, for water at triple point temperature T equal to 0.01 degree C h_f is equal to 0 and s_f is equal to 0, but still because they all are based on data fitting and then making a curve on it.

The values given by different programs will be slightly different. So, instead of this it could be within say plus minus 2 or 3 kilo Joules per kg. That is as long as throughout the problem we use the same property source. And I am writing this because the solutions that I have provided are based on one particular source that I am using. You could be using a different one and the numbers could be slightly different that does not mean that you are making a mistake the one important is that after the point of formulating the equation if you are same and then you are on the right path the numbers could be slightly of here and there, that does not matter, but they will not be largely different.

There are only a couple of cases where the numbers will be drastically different. And that is for certain refrigerants where the reference states have been chosen differently by different sources. So, what will happen in the calculation is that values of h and s and u which are all dependent on the reference state they will be different values of v will be the same everywhere that is not going to make issues these will be different, but do not worry too much about it because in most of the problems whether it is heat transfer or work transfer or say totaling process.

Here always dealing with difference of 2 properties. And the difference of 2 properties will be pretty much close to one another irrespective of what the reference temperature was. So, you could have slightly different values of individual properties, but the difference in the properties will not be very much. So, need not worry too much about it at that point the difference should be matching with what we have. The second issue

would be there is how much round off should and here again if you round off at the first step itself from 3 decimals to 1 decimal and then calculate it the final answer could be somewhat different from what is there.

That does not mean that you did the big mistake you have processed up the first step was right you are in good check. The numbers you did there would be slightly different round off you did differently that could be slightly different that is ok. The thing to remember is that when you look at property data or any property source, the number of decimal places that they give are there because they are confident that many decimal places are ok. The next decimal place is in doubt. So, if something gives value of say specific say value of R as 0.2978 kilo Joules per kg per Kelvin.

It means that it is the next place of decimal you are not sure about. So, if you are multiplying it by mass and your calculator give you 8 places of decimal nothing beyond this decimal actually makes sense. So, you can safely truncate it or round off up to that many places of decimal that your original numbers had, but if you are multiplying this by 0.22 and now you have a problem or you are multiplying it by 0.5 then these the next decimal place is in doubt.

So, even if you get 4 decimal places from here this multiplication tells you that you can round it off and keep only that many decimal places as the highest value that you have. So, again it tells that you round it off at every step to the best possible number of significant places. And the final answer we can put it there. In the real world this is would mean we are able to discriminate 1 kilo Joule in 1000 kilo Joules rarely do we get this type of exactness in the real world of experimentation and so, you may even actually end up making some more rounding off from (Refer Time: 23:05), but in this course keep that in mind make round off at every step as appropriate.

Please also make sure that you write the units at every step that you do. That when you will avoid there quite a few people look at that when you do p v type of a calculation pressure they have taken bar, whereas, you should have been taken in Pascal the complete answer goes (Refer Time: 23:31) proportion is your at least 99 90 percent correct whereas, number would be only a 10 percent.

(Refer Slide Time: 23:36)

Nitrogen Ideal gas $p_1 = 20 \text{ bar}$
 $pV^n = \dots$
 ${}_1W_2 = \int p dV = 3.532 \text{ kJ}$
 State ① $p_1 = 20 \text{ bar}$ $T_1 = 212.42^\circ\text{C}$
 $= 485.57 \text{ K}$
 $m_1 = \frac{p_1 V_1}{R T_1} = 0.0243 \text{ kg}$
 $R = \left. \begin{matrix} C_{p0}, C_{v0} \end{matrix} \right\} \text{constant.}$
 $m_1 = m_2$ $\frac{p_1 V_1}{R T_1} = \frac{p_2 V_2}{R T_2}$
 $T_2 = 2T_1 = 971.14 \text{ K}$

Now, we go back and say that instead of if I had nitrogen. So, nitrogen is now at different (Refer Time: 23:47) together. And we say that nitrogen is a the geometry of the device is the same pressure is same 20 bar, question is for knowing state 1 either I will give you the mass of nitrogen or a density of nitrogen or specific volume or temperature. So, what we will do is we will assume that temperature of nitrogen is same as what was there in the temperature of water. Does not have to ignore, but that is what comparison says if it tells us that these are the same pressure and temperature microsecond is to the water what happens to this.

So, we take that this is given to us. And in ideal gas equation of state remember all temperatures have to be in absolute value Kelvin. So, we immediately convert this into 485.57 Kelvin. This is we need values of the gas constant for nitrogen may be C_p 0, C_v 0 these are all given in property tables. And we take these to be constant for the entire states that the process undergoes ok. So, then we start saying that the equations that we derive they are still exactly the same.

They do not depend on the working substance that we have ok. So, was here it is, what it is this part of the problem solving had nothing to do with what were the working substance in it, but as everything to do with which equation we should be solving. So, this quotient is very important. And here we will say that so, nitrogen also it is exactly

the same thing work is also integral $p \, dV$ it does not depend on what the material was internal energy change is also $m U_2$ minus U_1 this is also there.

So, this sheet or this part of the solution in the problem is practically independent of all the working substance was that differences started coming in later on. So, let us see what happens now with nitrogen. It tells is that integral $p \, dV$ is going to be exactly in the same here a constant pressure process. And so, this we can write as 3.532 kilo Joules. If this was $p \, v$ to the power n equal to constant, then we would have done exactly the same thing I am written that downwards here.

It depends only on $p_2 \, v_2$ and $p_1 \, v_1$. So, we will now also there need to know what is the final pressure and volume in the case of nitrogen and there we will try to calculate over. So, over that we would know what is state 1 pressure is given 20 bar mass we can calculate from there $p_1 \, v_1$ upon $R \, T_1$ we are assuming that this is got an ideal gas we were and stop the process in every state the gas behaves like an ideal gas. So, that is what we will we are taking here when you put this in there this is 0.0243 kilo grams. The mass remains same in the system instead 2 this become m_1 is equal to m_2 .

And so, from here we can write convert m into $p \, v$ upon $R \, T$ and we said $p_1 \, v_1$ upon $R \, T_1$ was mass at state 1. This is $p_2 \, v_2$ upon $R \, T_2$. And when we solve this we get that T_2 is equal to 2 times T_1 t one has to be in Kelvin. So, this becomes 971.14 Kelvin or you can write 971.1 that is the it will work ok.

(Refer Slide Time: 27:40)

Handwritten calculations on a grid background:

$$U_2 - U_1 = m(u_2 - u_1)$$

$$= m C_v (T_2 - T_1) \quad C_v = \frac{J}{kg \cdot K}$$

$$= 8.12 \text{ kJ}$$

$$Q_2 = W_2 + (U_2 - U_1)$$

$$= 12.45 \text{ kJ}$$

Additional notes: $C_v = \frac{kJ}{kg \cdot K}$, $\frac{kg}{J}$ lower case

So, we got T_1 we got T_2 and what we now need is U_2 minus U_1 . The formula for this is same as what we had earlier this is $m U_2$ minus U_1 and this is equal to $m C_v (T_2$ minus $T_1)$ C_v value will come from the property tables.

And we will do the calculation and substitution this is 8.12 kilo Joules. Please remember that units have to be maintained in this calculation when you write everything mass has to be in kg C_v is say Joules per kg per Kelvin. And both temperatures in Kelvin, so, if this was Joules per kg Kelvin here U_2 minus U_1 will be in Joules. If we initially we took it as kilo Joules you must consistently keep kilo Joules and make sure final answer is also in Kelvin's. So, this is we get change in internal energy and substituting that finally, in Q_{12} this becomes W_{12} plus U_1 minus U_2 minus U_1 .

And this is calculated as 12.45. Please remember that when we write this or we write kilo grams this k is already a lower case. It should never be a capitalized. So, writing kilo Joules as k J or kilogram as kg both are wrong ok. So, we solve the problem and we can see that this solution methodology was different we used a different set of thing the main thing was equation state and the same process for 2 different substances we have been able to solve it.