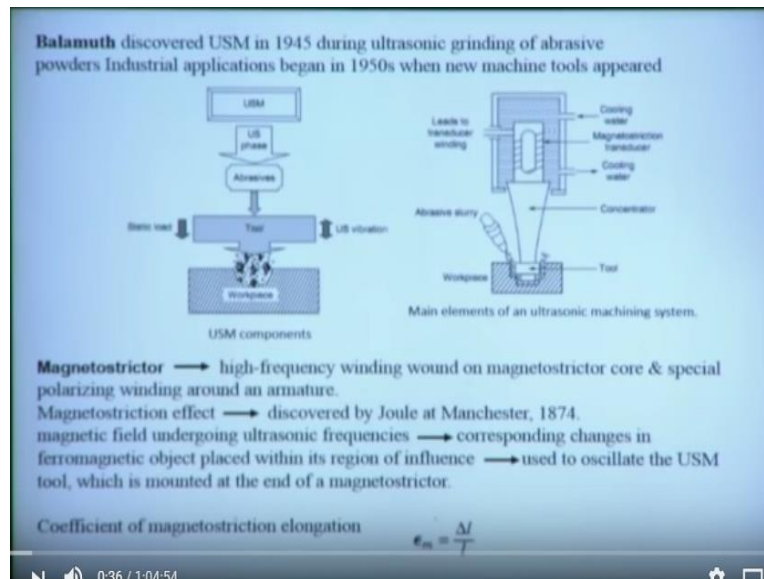


Advanced Machining Processes
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Module - 01
Lecture - 03
Ultrasonic Machining Part II

Welcome to the course on advanced machining process. Last week we had discussed about this mechanism of ultrasonic machining process and different parametric studies of ultrasonic machining process. Today, I shall continue again this ultrasonic machining process and few applications of ultrasonic machining process and few modification and hybrid machining process by using ultrasonic machining.

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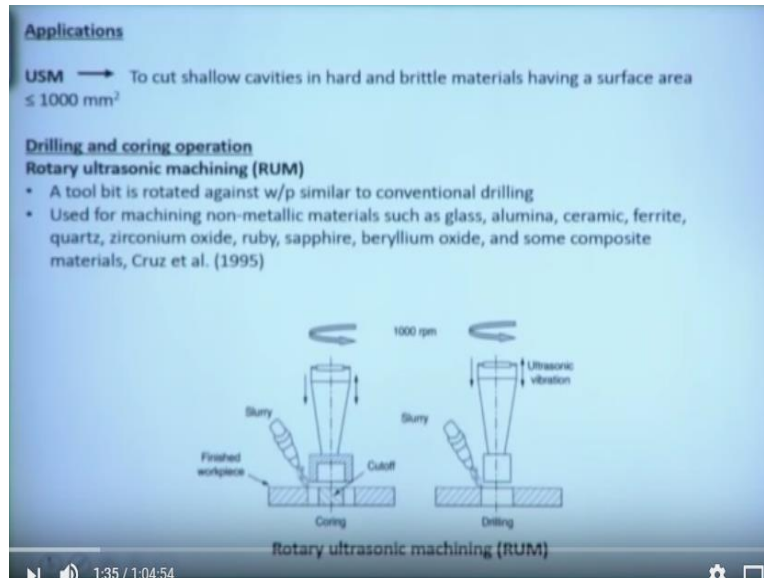


As I told that it was ultrasonic machining process was discovered by Balamuth in the year of 1945 during ultrasonic grinding of abrasive powders, industrial application begins in 1950 when new machine tools appear okay. So we have discussed that there are 2 types of magnetostriction transducers are there, one is magnetostrictive type, another one is the piezoelectric type okay, 2 transducers are there.

So in magnetostrictive type transducer this coefficient of magnetostrictive elongation when he keeps on ferromagnetic material under a bearing magnetic field there is a change in length. So this change in length is Δl and total length of this magnetostrictor coil is l okay so then this

coefficient of magnetostriction elongation is $\Delta l/l$. So it was discovered by Joule at Manchester 1878.

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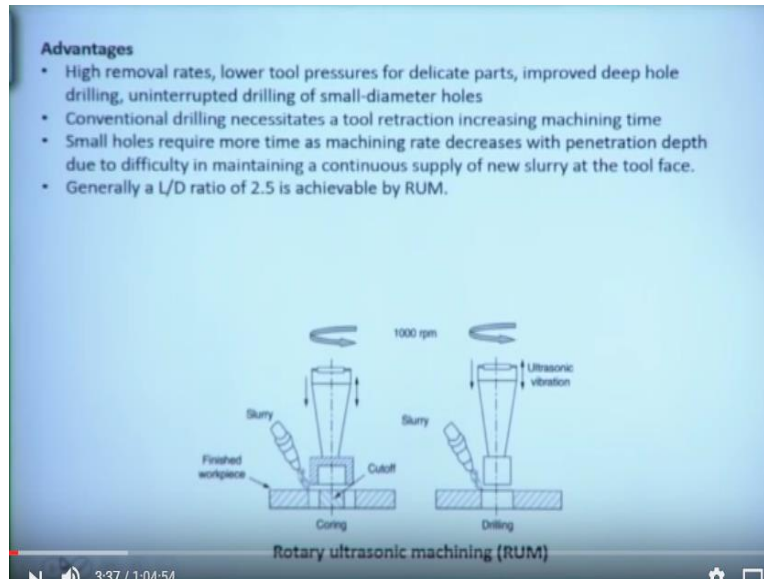
Now using ultrasonic machining there are few applications are to cut shallow cavities in hard and brittle materials having a surface area less than 1000 millimeter square. So there are another applications drilling and coring operation. So this drilling and coring operation can be done using a rotary type ultrasonic machining process. In normal ultrasonic machining this tool is diverting at a high frequency in the ultrasonic frequency but in rotary ultrasonic machining process this tool is tool rotates as well as as well as it vibrates.

So in rotary ultrasonic machining process we are giving some modification to this tool so it rotates during vibration also okay. So tool bit is rotated against the workpiece similar to the conventional drilling operation so used for machining non-metallic materials such as glass, alumina, ceramic, ferrite, quartz, zirconium oxide, ruby, sapphire, beryllium oxide, and some composite materials.

So you can see here this is the tool here in normal it is in the normal coring operation and this one is the ultrasonic vibration we are doing the drilling operation. This coring operation you can see that so this kind of tool is used okay, so this is a hollow tube here and in this case actually this this much material is not removed. So only this this many material this material is removed okay so by rotating this tool. So using this (()) (3:07) actually this machining is (()) (3:10).

But in drilling operation total volume of material total volume of material is removed okay. So this is the difference between coring operation and drilling operation. In both the cases this tool is actually rotated. So it is rotational ultrasonic machining process okay.

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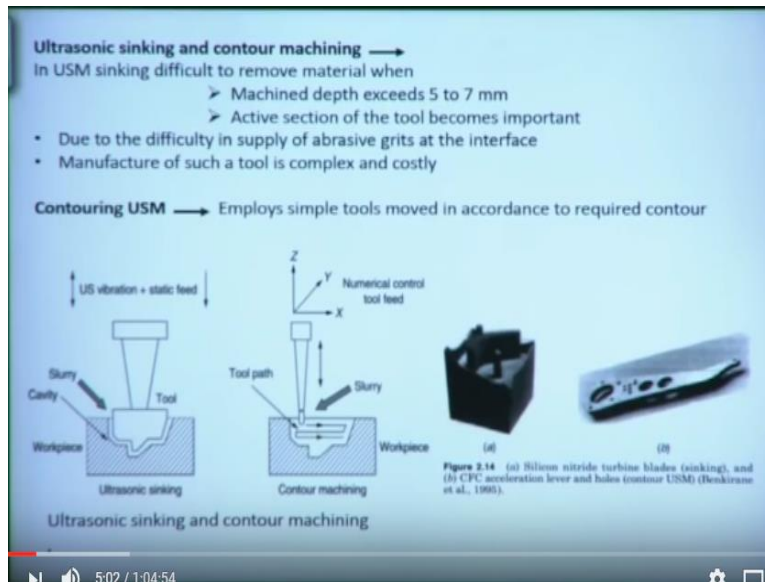


So these advantages are there due to this rotation high removal rate. So your removal rate increases. In normal case there is there is no rotation but in this case because of this rotation so you can expect more material removal rate in case of rotary ultrasonic machining process. So lower tool pressure, tool pressure also reduced.

For delicate parts it is very helpful and improved deep hole drilling okay. So in this case there is chances of getting higher length to diameter ratio. Uninterrupted drilling for small-diameter holes. Conventional drilling necessitates the tool retraction increasing machining time. But in this case we are using ultrasonic machining, rotary ultrasonic machining so we do not need any tool retracting like in unlike in your normal drilling operation.

Small holes require more time as the machining rate decreases with penetration depth due to the difficulty in machining a continuous supply or new slurry at the tool face. So that is the case that is covered for both ultrasonic machining and rotary ultrasonic machining process. So in this case rotary ultrasonic machining process generally L by D ratio is very high so minimum L by D ratio is achievable in this case is 2.5 okay.

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So there are another applications by this ultrasonic machining process. So this is called ultrasonic sinking and contouring machine okay. So this is the first case when we give this ultrasonic sinking operation okay so here ultrasonic vibration and we are giving some static field to this tool okay. So now this is the shape of this tool okay so we do not we want to make same shape of this tool into the workpiece surface. So it is called actually ultrasonic sinking operation.

Now this is the ultrasonic contouring operation. Here we are getting a very small tool okay. So this small tool it is moved to this finishing to this machining zone along the path to be machined okay. So you can move in a zigzag pattern or you can move in a helical pattern based on your requirement. So this is the small tool you are using here, very small tool.

This small tool it will move the workpiece surface and it will generate the contour. But in this case actually one single tool is used okay. So its the shape of this tool is shape of the workpiece to be generated and in this case actually small tool is used and this tool actually moved around the profile to be machined on the workpeice surface.

So this is the application here so silicon nitride turbine blade sinking is the sinking operation and after that this B is the CFC acceleration lever and holes. So this is actually your contouring operation it is done by using ultrasonic machining process okay.

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Production of EDM electrodes

- USM used to produce graphite EDM electrodes, Gilmore (1995)
- Small machining forces permit the manufacture of fragile graphite EDM electrodes
- Machining speeds, in graphite, range from 0.4 to 1.4 cm/min
- Ra ranges from 0.2 to 1.5 μm
- Accuracies of $\pm 10 \mu\text{m}$




Figure 2.15 Graphite EDM electrodes machined by USM (Gilmore, 1995).


6:42 / 1:04:54

So another application is the production of EDM electrodes okay. So this ultrasonic machining process is used to produce the graphite EDM electrodes. Small machining forces permit the manufacture of fragile graphite EDM electrodes. Machining speeds in graphite ranges from 0.4 to 1.4 cm/min and Ra ranges from 0.2 to 1.4 micron so which is very good and accuracy is achieved it is plus minus 10 micron. So this kind of graphite electrodes can be machined by this ultrasonic machining process which is used so this graphite electrodes are used in electrode discharge machining tool okay.

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Ultrasonic polishing

- Occurs by vibrating a brittle tool material such as graphite or glass into w/p at ultrasonic frequency and relatively low vibration amplitude
- Fine abrasive particles in slurry, abrade high spots of w/p surface removing 0.012 mm of material or less
- Surface finish of 0.3 μm can be achieved (Gilmore, 1995)



ultrasonic polishing for **1.5 to 2 min** to remove the machining marks left by a CNC engraving operation

Ultrasonic polishing of CNC machined parts (Gilmore, 1995)

7:31 / 1:04:54

So ultrasonic polishing. So ultrasonic machining process, the same process can be used for polishing purpose also. So in this case even the amplitude of vibration is very less okay. So

occurs by vibrating a brittle tool material such as graphite or glass into the workpiece at ultrasonic frequency and relatively low vibration of amplitude. So here amplitude of vibration obviously it is very less. So fine abrasive particles in slurry abrade the high spots on workpiece surface removing 0.012 mm of material or less. Surface finish achieved it is around 0.3 micron okay. So here you can see this is polished so before polishing this one and this is after polishing you can see that by using this ultrasonic machining you can get a better polished surface also okay.

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Micro-ultrasonic machining (MUSM)

- Utilizes workpiece vibration
- Vibrating w/p allows inferior tool system design without considering transducer, horn
- Complete system is simpler and more compact than conventional USM
- Micro-holes of 5 μm dia on quartz, glass, silicon made using **WC micro tools**
- Sintered diamond (SD) tool** having **high wear resistance** used to machine multiple holes using a single tool
- Can be used for machining 3D shapes

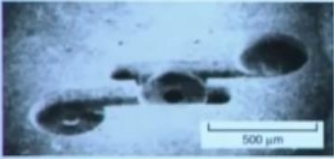
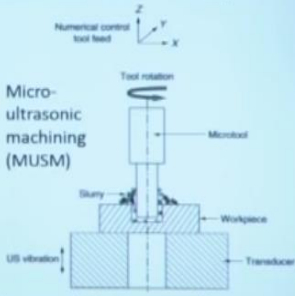


Figure 2.18 Micro-ultrasonic machined cavity (Masuzawa and Tinschel, 1997).



The diagram shows a cross-section of the MUSM process. A microtool is positioned above a workpiece. A slurry is applied between the tool and the workpiece. The workpiece is held by a transducer that provides US vibration. The tool is rotated. A coordinate system (X, Y, Z) is shown with the Z-axis pointing upwards. The text 'Numerical control tool feed' is also present.

Other applications

- Cutting off parts made from semiconductors at high removal rates compared to conventional machining methods
- Engraving on glass as well as hardened steel and sintered carbide
- Parting and machining of precious stones including diamond

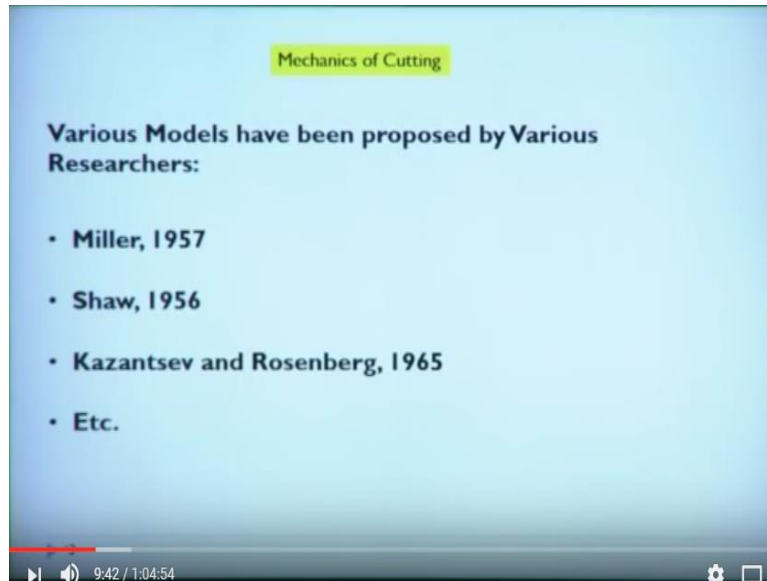
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So another application is the micro-ultrasonic machining okay. So in this case your tool is very small okay. So utilizes workpiece vibration, vibrating workpiece okay. So instead of giving vibration to this tool here vibration is given to the workpiece itself okay. So vibrating workpiece allows inferior tool system design without considering transducer, horn okay. So micro-holes of 5 micron diameter on quartz, glass, silicon made using tungsten carbide micro tools.

So sintered diamond tool having high wear resistance used to machine multiple holes using a single tool. So can be used for machining 3D shapes also okay. So other applications are cutting off parts made of semiconductor at high removal rates compared to the conventional machining methods, engraving on glass as well as hardened steel and sintered carbide, parting and machining of precious stones including diamond.

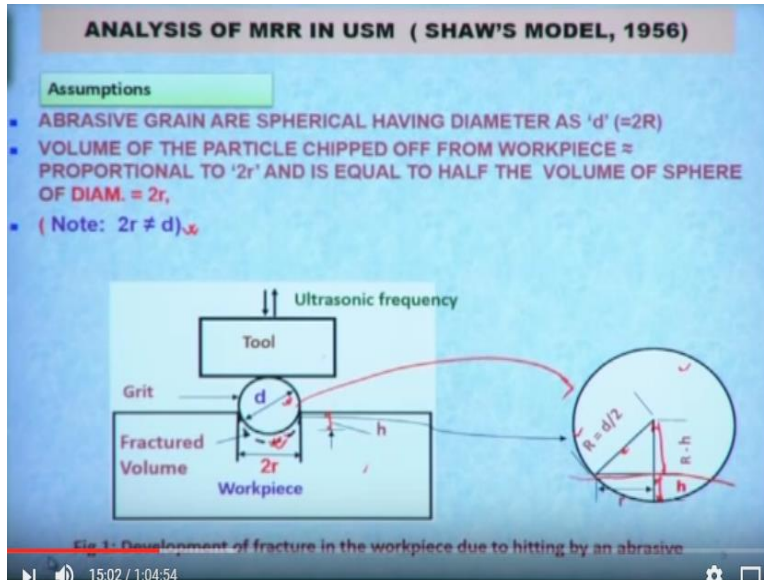
Now we shall discuss this material removal mechanism of ultrasonic machining process. How this material removal occurs and what is the rate of material removal during ultrasonic machining process.

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Okay, there are different kinds of models which are available in the literature so various models are proposed by various researchers and among this models okay so few very popular models which is which are Miller by Miller, he proposed in 1957; M. C. Shaw, he proposed in 1956; and also Kazantsev and Rosenberg, they proposed mechanical mechanism or cutting by ultrasonic machining process in 1965 and there are several other models are also available. So among these 3 models we shall discuss about this M.C. Shaw's model which is proposed in 1956 okay.

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So the generation of M.C. Shaw's model for material removal mechanism by ultrasonic machining process. Okay so there are few assumptions are there for this M.C. Shaw's model who proposed in 1956. Here he assumed that this abrasive particles are spherical shape having diameter d and radius capital R okay. So this abrasive particles have been spherical in shape having diameter small d or capital R radius capital R . So although these abrasive particles are not spherical in shape but for the simplicity of your of this M.C. Shaw's model he proposed that these abrasive particles are spherical in shape.

So volume of this particle chipped off from the workpiece surface is proportional to this $2r$. So as I discussed (()) (11:41) from this figure and this volume of metal removed or particle chipped from this workpiece is equal to the half of the volume of sphere of diameter equal to $2r$, here r is small r . So here you can see this is the tool here which is vibrating with a ultrasonic frequency with a frequency of f and with amplitude of $a/2$.

Now there are so many abrasive particles are there in between this tool and this workpiece there are so many abrasive particles are there. So moving some abrasive particle it is hit by by tool and it is impinging or penetrating into the workpiece surface here. So this is the diameter of this abrasive grain. So this is the diameter of this abrasive grain here and this is the small d or $2R$. Here r is the radius capital R is the radius of the abrasive grain okay.

So because of this impact force by this abrasive grain it will impact into the workpiece surface and it will penetrate with an amount of h , h is the small h is the depth of penetration. So because of this penetration it will make a spherical penetration into the hemispherical penetration into the

workpiece surface. So this is the hemispherical penetration we can assume that there is this is the hemispherical penetration with radius small r okay.

So volume of material removed is because of this penetration okay so volume of material removed is half of this considering this hemispherical penetration. So this is the fractured volume. So here this is the fractured volume and this this one is the depth of penetration how much this abrasive grain is penetrated into the workpiece surface.

Now this zone is now exaggerated here with a bigger picture okay. So this is the abrasive grain. So this is the abrasive grain here okay. So this abrasive grain now this is the radius of this abrasive grain is capital R, it is d by 2, small d by 2. Here h this h is the depth of indentation into the workpiece surface and this is the r here hemispherical this amount this much this much actually it is penetrated into the workpiece surface. So here this is the small r, small r which is the hemispherical zone which is removed or chipped off from the workpiece surface and so obviously this one will be this one will be capital R-h and this is R by that is d/2, capital R=d/2.

Now from this triangle you can consider this R-h whole square plus small r whole square equal to this d by 2 whole square okay. So here note that 2r small 2r equal to not equal to this small d okay. So small d equal to capital R.

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- MATERIAL REMOVAL BY 'THROWING' AND 'HAMMERING' ACTIONS ARE CONSIDERED.
- OTHER MODES OF MATERIAL REMOVAL (CAVITATION, CHEMICAL ACTION) ARE NEGLECTED.
- RADIUS OF THE SPHERICAL CRATER FORMED DUE TO FRACTURE = 'r'.
- RADIUS 'R' IN TERMS OF GRIT RADIUS (=R) AND DEPTH OF PENETRATION (=h) CAN BE EXPRESSED AS.....
 $r^2 = R^2 - (R-h)^2$ (Fig. 1)
 $= R^2 - R^2 + 2Rh - h^2$
 $\approx 2Rh$

NOW, VOL. OF MATERIAL REMOVED / GRIT / CYCLE) (V_g)

$$V_g = \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right) \dots\dots (1)$$

$$\approx \frac{2}{3} \pi (2Rh)^{3/2} \quad (r^2 \approx 2Rh \text{ FROM ABOVE})$$

$$= k_1 (hd)^{3/2} \text{ mm}^3/\text{grit-cycle} \quad (k_1 \rightarrow \text{Const., } h \text{ \& } d \rightarrow \text{mm})$$

Where, k_1 is a constant, and 'h' is depth of penetration.

Okay so material removal by throwing. There are 2 types of models are there. One is the throwing model, another one is the hammering model okay. So other methods of other

mechanism material removal like cavitation, chemical action are neglected. So there is because these slurries are there suppose we are slurry it is a water based slurry. Suppose if you have to make you have to machine on a glass. So this slurry (()) (15:38) do the chemical reaction. So due to this chemical reaction maybe there are some material removal rate but here it is not considered. So material removal because of this chemical reaction and cavitation is not constant here. So only by throwing and hammering model it is considered.

So radius of this sphere or crater formed due to this fracture is small r . Radius capital R in terms of this grit radius and the depth of penetration small h can be expressed as okay. So from this triangle we can we can write this one small r this small r square equal to capital R square minus R minus h square okay. So after simplification we can write this one as capital R square minus R square plus $2Rh$ minus h square. So here we can see that this abrasive particle size is it is in the micron size. So this depth of indentation h which is penetrating this abrasive particle penetrating into the workpiece surface it is very less okay.

So now we can consider we can neglect this h square. So h is very small so you can neglect the h square so ultimately after simplification it will be $2Rh$ 2 capital R into h equal to R square. Now this volume of material removed per grit per cycle that is V_g here so V_g already we have assumed that this is equal to the half of volume of material removed by the brittle fracture okay considering small r as the radius. So this is half of this $4/3 \pi r^3$ the hemispherical zone fractured zone so this is the volume is $4/3 \pi r^3$.

Now we have considered this half so now we are putting this value of r so here small r r square equal $2R$ into h . So small r equal to root over $2Rh$ here. So now putting this value of R into this equation so we can calculate the volume of material removed per grit per cycle or per grit per cycle okay. So this is very simple here. So now here you can see $2/3 \pi$ and this this one we can consider as k_1 okay.

So this is the constant here and 2 into capital R we can consider this one as the d , d is the diameter of this grit and this is h as as it is we can put it here. So this is the volume of material removed so it is millimeter cube per grit per cycle okay. So here k_1 is the constant and h and d are in millimeter are expressed in millimeter. So h is the depth of penetration and d is the diameter of this abrasive grit.

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Let 'N' be No. of impacts / cycle
 INVERSELY PROPORTIONAL TO THE X-SECTIONAL AREA OF THE GRIT (d^2 OR d'^2)

$N = K_2/d^2$

- IS EVERY GRIT ACTIVE (THAT IS HITS THE WORKPIECE AND REMOVAL MATERIAL)?

LET ' K_3 ' → PROBABILITY THAT A GRIT UNDER THE TOOL IS ACTIVE

V_c (MRR / CYCLE) = $K_3 \times V_g \times$ No. OF IMPACTS

$$= K_3 K_1 (h d)^{3/2} (K_2 / d^2) = K_1 K_2 K_3 \sqrt{\frac{h^3}{d}}$$

$f \rightarrow$ FREQUENCY OF TOOL VIBR.

$$MRR_v = K_1 K_2 K_3 \sqrt{\frac{h^3}{d}} \cdot f \text{ mm}^3/\text{s} \quad (1)$$

• HOW TO KNOW DEPTH OF PENETRATION 'h' ?

THROWING MODEL HAMMERING MODEL

24:44 / 1:04:54

Now we have to consider how many impacts are there per cycle okay. So now if you consider this is the tool here so under this tool so many abrasive particles which is vibrating like this so under this tool so many abrasive particles are there okay. So how many impacts are there by this abrasive particle into the workpiece surface because of one movement or one cycle of the tool into the towards the workpiece surface.

So it is inversely proportional to the cross-sectional area of the grit. So now you can consider there are 2 examples are there here diameter of this abrasive particle is less it is small d and here diameter of this abrasive particle this is d' . Here this diameter of this abrasive particles are bigger size okay. So now this is the total this one is the diameter of the tool. So under this tool here as the abrasive particles grit size is diameter is small.

So we can accommodate more number of abrasive particles and here as the abrasive particle diameter is bigger in size so under the under this tool we can accommodate only very less number of abrasive particles. So this number of impacts is obviously it is proportional to the cross-sectional area of the abrasive grits okay. So cross-sectional area is πr^2 okay. So we can consider this one as d^2 okay. So it is proportion to the cross-sectional area proportional to the d^2 okay. So N number of impacts you can consider as the K_2 by d^2 considering K_2 as a constant.

So now the question is that is every grit is active. So active grit means just below the tool whatever abrasive particles are there whether all the abrasive particles are taking part into the material removal or not. It may happen that this or abrasive particle it is hit by the tool it again

hits another abrasive particles but it does not take part into the material removal. So that aggressive particle is not an active abrasive particles. Also it may happen that these one abrasive particle it is hit by the tool okay, so it may actually move towards the away from the workpiece okay. So it may move away from the workpiece okay. So in that case also it is not taking part into the material removal. So these active aggressive particles are those particles which are taking part actively taking part into the material removal.

So we can consider a constant of proportionality is K_3 its probability that what abrasive particle is taking part into the material removal that can be considered as K_3 okay. So K_3 is the probability that a grit under the tool is active and taking part into the material removal. So volume of material removed per cycle okay.

So it is K_3 into volume of material removed per cycle per grit and number of impacts okay. So how many impacts are there okay. So here number of impacts are N here. Now you can put this K_3 , V_g already we have calculated is $K_1 h d$ whole to the power $3/2$ and number of impacts N equal to K_2 by d square okay. So now $K_3 K_1 K_2$ and K_3 root over h cube by d okay. So this is the volume of material removed per cycle.

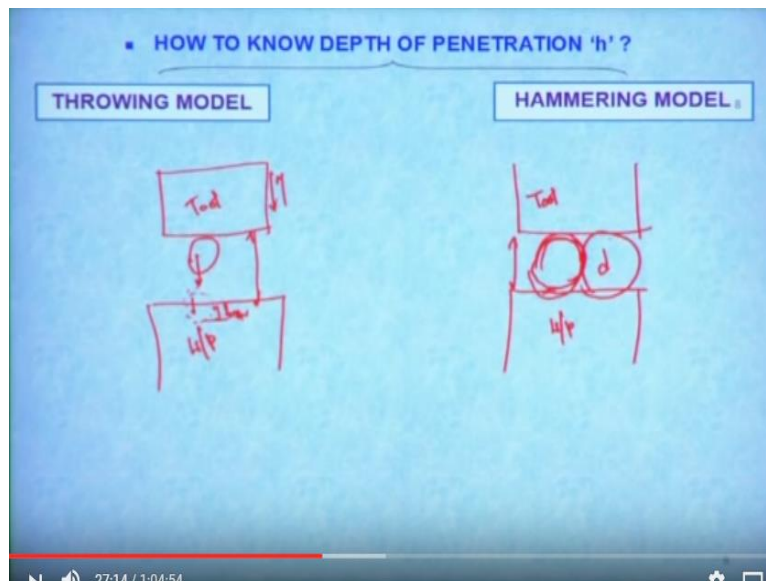
Now we have to calculate volume of material removed per or volume of material removal rate per any time okay. So if I have to calculate this per unit time now we have to multiply this V_c with the frequency. So frequency means how many times this tool is vibrating in one second. So if we multiply this V_c volume of material removed per cycle multiplied by how many cycles are there, how many times it is vibrating in one second then we can get the volume of material removal rate per unit time okay.

So this is can be taken as MRR_v . So here same as $K_1 K_2$ and K_3 root over h cube by d multiplied by f , f is the frequency of vibration of this tool. So generally it is in the more than 20 kilohertz. So here we can get this one volume of material removal rate equal to millimeter cube per second. So this is equation 1 here okay. So volume of material removal rate per second. So f is the frequency of tool vibration. So now this (()) (24:06) is that actually d we know here d is the diameter of this abrasive grits, f is the frequency of vibration that is machining parameter then h , h is the here h is the depth of penetration okay.

So h is the depth of penetration which is unknown okay. So now we have to calculate what is the depth of penetration at different measuring conditions. So that is why there are 2 types of models

are there okay. So Shah M.C. Shah he proposed 2 types of models, one is throwing model, another one is the hammering model.

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So now in next class I shall discuss what is throwing model and what is hammering model okay.

So there are 2 models are there, one is throwing model, another one is the hammering model.

Suppose this is the tool here okay. So this is the tool. So it is vibrating with an ultrasonic frequency. Now there is an abrasive particles which is which it hits now this is the workpiece here. So this is the workpiece okay. Now this tool it it hits the abrasive particles. Now this abrasive particle it move towards the workpiece surface and it will impinge or impact into the workpiece surface and it will generate this much of depth.

So this depth is called the depth due to the throwing model. Here this distance between this tool and workpiece okay it is higher than this distance is higher than the diameter of this abrasive particles okay. So this abrasive particle is hit by the tool. Now it will move through the slurry and it will penetrate into the workpiece surface. So this distance between tool and workpiece is more than the abrasive particles. So this is called the throwing model.

So another model is there is the tool here. So this is the abrasive particle here and this is the workpiece. So okay so this is the abrasive particle here okay. Now this is the workpiece here. So in this case this distance between this tool and this workpiece, distance between this tool and workpiece is less than the diameter of this abrasive particle. So it is less than this diameter of this abrasive particles. So here tool is in contact to the in contact with the abrasive particles and this

abrasive particle is in contact with the workpiece surface okay. So here this distance between this tool and the workpiece surface, bottom surface of this tool and the top surface of this workpiece okay it is less than the diameter of this abrasive particles. So in this case tool is every time it is in touch with the abrasive particles.

So there are 2 models, one is throwing model another one is the hammering model. So we shall discuss first throwing model and again after that we shall discuss the hammering model.

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GRAIN THROWING MODEL

- A GRAIN THROWN ONTO THE WORK BY THE TOOL
- TOOL DISPLACEMENT (OR VIBRATION (Y) FROM MEAN POSITION (t = 0) AT TIME t (Assuming sinusoidal vibration of the tool)

$Y = a/2 \sin(2\pi ft)$
VEL. OF TOOL; $V = \pi a f \cos(2\pi ft)$
MAX. VELOCITY; $V_{max} = \pi a f$

(ASSUMING THAT THE GRIT ALSO LEAVES THE TOOL AT THIS MAXIMUM VELOCITY v_{max})

Diagram: A tool is shown vibrating sinusoidally on a workpiece (W/P) surface. The displacement is labeled as $a/2$ and the time as t . The vibration is labeled "Tool vibration".

K.E. ($= \frac{1}{2}mv^2$) = $\frac{1}{2}(\pi/6 d^3 \rho_s) \pi^2 a^2 f^2$ (2)

So grain throwing model okay. So a grain is thrown on to the workpiece surface by the tool. A tool displacement or vibration from the mean position at t equal to 0 at time t assuming the sinusoidal vibration of this tool. So we are assuming this vibration, this vibration of this tool as the sinusoidal okay. So at time t equal to 0 from this mean position to at time t what is the tool displacement. So this displacement of this tool can be considered as the this tool movement can be considered as the sinusoidal. Now this displacement of this tool Y can be considered as a by 2 sin 2 pi f t. So a by 2 is the amplitude here and f is the frequency of vibration and t is the time period.

So velocity of this tool, so you can take the derivative d by d t. So it will be a by 2 into 2 pi f t okay into 2 pi f and then cos 2 pi f t. So after simplification it will be pi a f cos 2 pi f t. So what is the maximum velocity of this tool. So this maximum velocity where this cos 2 pi f t equal to 1 okay so then it that case this maximum velocity of this tool will be pi a f okay. So this is the tool.

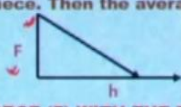
It is or abrasive particle it is hit by the tool here. It is it is impacted into the workpiece surface. So this distance here more than this diameter of this abrasive particles.

So equal to t is 0 and then after some time t it will hit the workpiece surface. So here one important assumption, assumption is that when this abrasive particle is hit by the tool okay it will gain the same velocity it will gain the same velocity as that of the tool okay. So this is the assumption we have considered here assuming that the grit also leaves the tool at this maximum velocity y okay. So now this sinusoidal will this sinusoidal profile or displacement is shown here. So this is the time period is t here, a by 2 is the amplitude of vibration okay, amplitude of tool vibration. Okay now what is the kinetic energy of this abrasive particle it gains when it is hit by the tool okay.

So this kinetic energy is we know that it is half mv square okay. So here m is the mass of this abrasive particles so this mass can be calculated as mass of this abrasive particle can be calculated as pi by 6 d cube okay so d is the diameter of this abrasive particle and rho a rho a is the density of this abrasive particle okay. So now v already we had calculated this v here. If we put this v here so it will be pi square a square and f square okay. So this is the kinetic energy of this abrasive particle after being hit by the tool. So this is mass here and this is the v square here.

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Full K.E. absorbed by the workpiece before the particle comes to rest. it will result in penetration ' h_{th} ' into the workpiece. Then the average work done,

$$W = \frac{1}{2} F h_{th}$$


(ASSUMING TRIANGULAR VARIATION OF FORCE (F) WITH THE DEPTH OF PENETRATION (h))

Average contact stress acting on the workpiece surface (σ) = Brittle fracture hardness (H)

$$\sigma = F / \pi d h_{th} \quad (\pi r^2 = \pi 2 R h_{th} = \pi h_{th} d)$$

$$F = \pi h_{th} d \sigma$$

IT IS KNOWN THAT K.E. OF A PARTICLE = WORK DONE

$$K.E. (= \frac{1}{2} m v^2) = \frac{1}{2} (\pi/6 d^3 \rho_a) \pi^2 a^2 f^2 = \frac{1}{2} \pi H h_{th} d h_{th}$$

$\underbrace{\pi/6 d^3 \rho_a}_m \quad \underbrace{\pi^2 a^2 f^2}_{v^2} \quad \underbrace{\frac{1}{2} \pi H h_{th} d h_{th}}_{\text{Work done}} \quad \sigma = H$

$$h_{th} = \pi a f d \sqrt{\frac{\rho_a}{6H}} \quad (2)$$

$$MRR_v = K_1 K_2 K_3 \left[\frac{\pi^2 a^2 \rho_a}{6\sigma} \right]^{3/4} d f^{5/2}$$

35:50 / 1:04:54

Now one more assumption is that full kinetic energy of this absorbed by this workpiece before this particle comes to the rest. So when this abrasive particle is impinging or impacting into the workpiece okay into the workpiece so this full kinetic energy of this kinetic energy gained by

this abrasive particle it is absorbed by the workpiece and this abrasive particle starting with a very high velocity it will come to a rest position okay. So it will result in the penetration of h into the workpiece surface. So this is the depth of penetration because of this throwing model okay. So it will penetrate equal amount of h so then the average work done.

So if we consider as a triangular so this one if we consider the triangular profile okay with F_{max} equal to here okay so triangular variation of this force with the depth of indentation if we consider okay. So what is the work done by this abrasive particle before before coming to the rest condition okay. So it will be half of h , F half of F multiplied by this h , h is the depth of indentation, depth of indentation.

So as if in the triangular variation of the force F with the depth of penetration. So average contact stress acting on the workpiece surface so it is equal to the brittle fracture of the hardness or brittle brittle fracture hardness of the workpiece surface okay. So average contact stress so how much force is covering because of this penetration of this abrasive particle divided by that cross-sectional area.

So cross-sectional area is the πd into h , h is the here h is the indentation depth okay. So now we know this πr^2 equal to $\pi 2r$ into h equal to we can consider this one as the πh into d . So like this this πh has come. So it is a cross-sectional area. $2R$ square equal to $2Rh$ we already we have calculated this one okay. So this one is πh into d okay. So this $2R$ equal to d and this is h here okay.

So now this force after simplification we shall get force equal to σ multiplication of ndh okay. So it is known that this kinetic energy of particle is equal to the work done by this particle into the workpiece surface. So if we equate these 2 equations okay so this kinetic energy already we have calculated. This is the kinetic energy and this is the work done by this particle here okay. So here work done is half F , F is already calculated from here so F equal to $\pi h d \sigma$ equal to here brittle hard brittle fracture hardness of this workpiece material and then multiplied by this h okay so this h here.

So half is this same half is here. So now here we have considered σ equal to this stays on this on this workpiece surface is equal to the brittle fracture hardness of this workpiece okay. So now after simplification we shall get the get the penetration by the throwing model. It is equal to $\pi a f$, f is the frequency of vibration, d is the diameter of this grit. Then ρ_a , ρ_a is the density

of this abrasive particle divided by 6H, H capital H is the hardness of this workpiece material and it is root over okay.

So now if we note this hardness here so we can put this hardness value into the earlier equation where we have calculated the volume of material removed. So this volume of material removed is $K_1 K_2 K_3$ root over h cube by d okay. So this h cube already we have calculated h cube here so if we put this value into this equation 1 for calculation volumetric material removal rate okay so now we shall get in this form this is the volumetric material removal rate in case of throwing model.

(Refer Slide Time: 35:54)

This value of ' h_{th} ' from Eq. (2) can be substituted in Eq. (1) derived for evaluation of MRR_{vth} by the throwing model.

$$MRR_v = K_1 K_2 K_3 \left[\frac{\pi^2 a^2 \rho_a}{6\sigma} \right]^{3/4} d f^{5/2}$$

In the same way, h can be derived if the material removal takes place due to hammering action of the abrasive, as follows.

35:54 / 1:04:54

So this value of hth from equation 2 can be substituted in equation 1 for deriving the for evaluation of the volumetric material removal rate by the throwing model. In the same way h can be derived if the material removal takes place due to the hammering action of this abrasive okay. So already we have calculated this throwing model the depth of indentation in the same way we can calculate the depth of indentation by this hammering model okay.

(Refer Slide Time: 36:26)

Grain hammering model

- Gap between tool and W/P < diameter of abrasive
- Abrasive will partially penetrate in the tool (h_{th}) and partially in the W/P (h_{wh})
- Total indentation (h_h) on the tool (h_{th}) and workpiece (h_{wh}) is given by

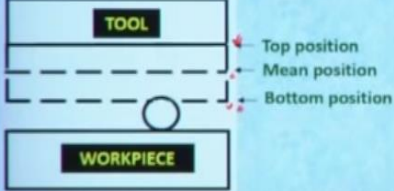
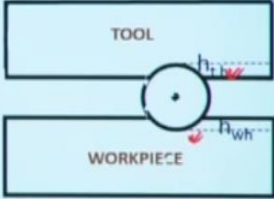
$$h_h = h_{th} + h_{wh}$$



Fig. 14: (a) Schematic diagram of a grain hammering workpiece

Fig. Partial penetration of a grit in the tool and workpiece

- It is assumed that the grain is hammered into the workpiece (Gap between the bottom face of the tool and top face of the workpiece < Grit size)
- Mean speed of the tool is low
- Mean static feed applied to the tool (F) = mean force of tool on the grit

38:18 / 1:04:54

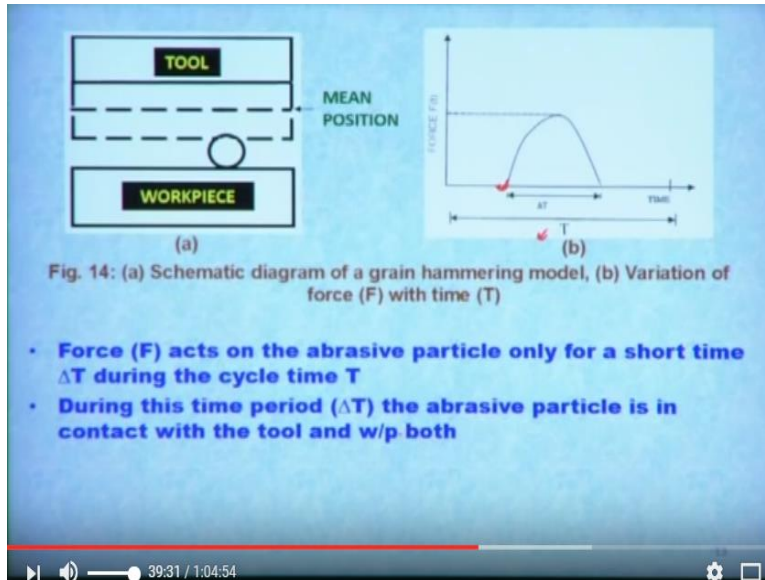
So this is the okay. So here this is the tool here tool at the top position here. Now it will come to the mean position. After that it will come to the bottom position and this bottom position it will hit the abrasive particles because here gap is less than the gap between this tool and the workpiece top surface of the tool and the workpiece surface okay.

So here from this top position it will come this tool will come to the mean position then it will come to the bottom position. When coming to this bottom position it will hit the abrasive grains okay. So gap between this tool and workpiece is less than diameter of this abrasive in case of throwing model hammering model so abrasive will partially penetrate into the tool. So here penetrating partially into the tool and also it will partially penetrate into the workpiece here.

So total depth of indentation here in case of hammering model it is h_{th} okay plus and depth of indentation into the workpiece which is h_{wh} . So it is a submission of this h_{th} because of this tool indentation because of this indentation into the workpiece.

So there are some assumptions are there in case of hammering model. So it is assumed that the grain is hammered into the workpiece. So gap between this bottom face of this tool and the top face of the workpiece is less than grit size of this abrasive particle. Mean speed of the tool is low in this case and main static feed applied to the tool which is F equal to the main force of tool on the grit okay.

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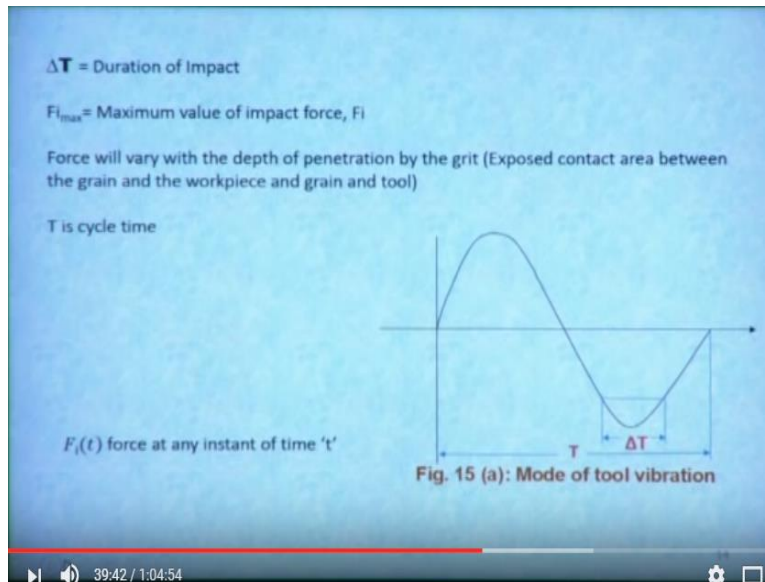


So force acting on the abrasive particle only for a short time ΔT okay. Now you can see so from this top position it will come to the mean position after that it will come to the bottom position. So while coming to this bottom position it will touch at this point to the, this tool will touch in contact with the abrasive and then abrasive is in contact with the with the workpiece okay. So at this point only there is a contact between this tool, abrasive, and workpiece starts.

So at this point only actually this force will start force will increase okay from its 0 value. For a time period of T capital T only this ΔT it is in touch with the abrasive particle. This tool is in touch with the abrasive particle and abrasive particle in touch with the workpiece and as well as both to as well as with tool also.

Now force will increase then after that it will reduce to 0 for a delta time okay. So force F acts on the abrasive particle only for a short time ΔT during the cycle time T. During this time period ΔT the abrasive particles is in contact with the tool and workpiece both okay.

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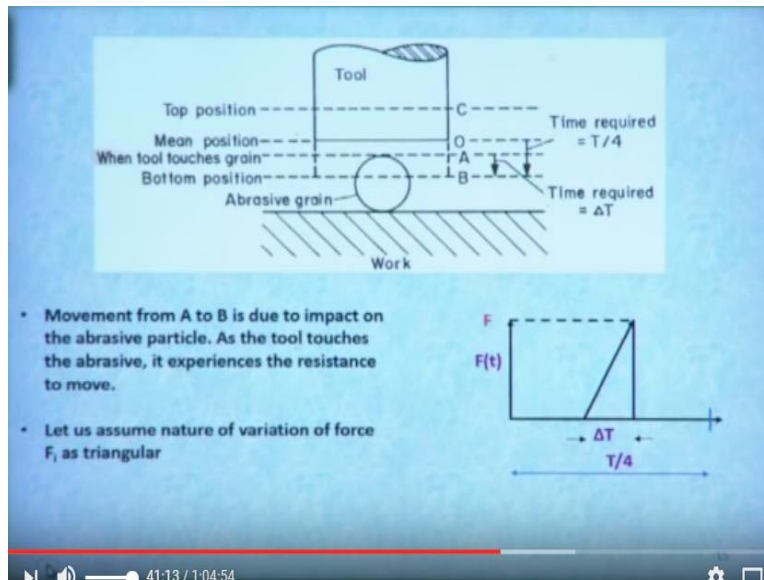


So ΔT , here ΔT is the duration of impact. So $F_{i_{max}}$ is the maximum value of this impact. So here also we are considering the sinusoidal vibration of this tool. So at this time for this ΔT time it is this tool is in contact with the both this abrasive particle is in contact with the both the tool and the workpiece. So $F_{i_{max}}$ is the maximum level of the impact force F_i . Force will vary with the depth of penetration by the grit exposed contact area between the grain and the workpiece of the grain and the tool and T capital T is the cycle time, cycle time okay.

So $F_i(t)$ is the force at any time instantaneous time t here.

So what will be the average force okay because this force is varying at time for a ΔT time okay so you can take the average value of this force by taking the integration from T capital T equal to 0 to T total time period okay. In this time period it varies from $F_i(t)$ to okay so $F_i(t)$ is the force at any instantaneous time t okay so multiplied by dt . So it is averaged over this total time period 1 by T . So this will give the average force okay on the abrasive particle okay.

(Refer Slide Time: 41:12)



So here you can see this is the tool here. So it is the C is the top position of this tool and then O it will come to the mean position of this tool and after that after that it will come to this bottom position. So at this means it will touch the abrasive particle at point A and after that this tool will come to this bottom position B. So this tool is in contact with the abrasive particle from A to B. So this from here to here it is time period $T/4$ because this tool is coming, it is going to the mean position, then it will go to the bottom position, and then it will go to the mean position, it will go to the top position.

So total this is the T, cycle cycle T capital T. Here from O to A B it is time period, it is $T/4$ and in between this A and B it is in touch with the workpiece. This abrasive particle is in touch with the workpiece as well as with the tool okay. So this is the time period ΔT . This is the top position, mean position, this is the tool touches the grain at this point and this is the bottom position okay.

So for movement from A to B here is due to the impact on the abrasive particle and the tool touches the abrasive it experiences the resistance to move. So let us assume nature of variation of this force F_i as a triangular. So here also like in throwing model also we are considering this nature of vibration on this tool. We are considering as a triangular although it is not a triangular but for the simplicity of this calculation we are considering this one as a triangular okay.

So this is the total here to here it is $T/4$ okay cycle sum of the one fourth of the cycle. At one fourth of the cycle only ΔT time and so it is in touch. So here from 0 to it will go to the maximum level this force okay for this delta time, time ΔT okay.

(Refer Slide Time: 43:20)

Grain hammering model

- Abrasive will partially penetrate in the tool (h_{th}) and partially in the W/P (h_{wh})
- Total indentation (h_h) on the tool (h_t) and workpiece (h_w) is given by

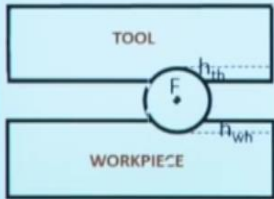
$$h_h = h_{th} + h_{wh}$$


Fig. Partial penetration of a grit in the tool and workpiece

So again so this abrasive will partially penetrate in the tool as well as into the workpiece so total depth of penetration is equal to the hammering model indentation if the indentation due to the into the tool and indentation into the workpiece.

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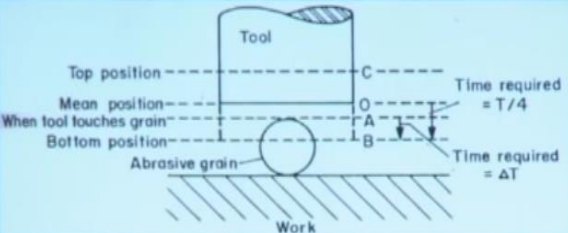


Fig. 15 (b): Various positions of the tool while hitting workpiece via a grit

Mean vel. during quarter cycle ($T/4$) from O to B is $\frac{(a/2)}{(T/4)} = 2a/T$

Time (ΔT) required to travel from A to B (h_h)

$$\Delta T = \frac{h_h}{(2a/T)} = \frac{h_h}{a} \left(\frac{T}{2} \right)$$

$$h_h = h_{th} + h_{wh}$$

So what is the mean velocity between the quarter cycle $T/4$? So from here to here it is the $T/4$. What is the velocity here? At this point this tool is actually going half of means it is going to the amplitude amount okay so this is $a/2$ amount, it is moving. So amplitude of vibration is $a/2$ here okay. So what is the velocity here? Velocity we can calculate this $a/2$ divided by $T/4$ okay. So this with this time $T/4$ it is moving $a/2$ amount of depth means $a/2$ amount of depth okay. So so

this is the velocity. After simplification will give this $2a/T$. So this is the mean velocity between the quarter cycle of the tool, mean velocity of the tool during the quarter cycle.

So what is the time required to travel from A to B here. So time required, so from A to B actually it is it is moving this hh amount, hh is the total indentation depth due to the hammering model okay. So this hh by means total indentation depth divided by it will give the we if we divide with the mean velocity of this tool then we will get the ΔT amount what is the time required for for a penetration of this full penetration okay into the tool as well as into the workpiece okay. So from here we will get this ΔT this value of ΔT here okay.

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- Force (F) acts on the grit by the tool starts increasing as soon as the grit gets in contact with the tool and W/p at the same time
- It attains maximum value and then starts decreasing until attains zero value
- Movement from A → B is due to impact on the abrasive particle as the tool touches the abrasive, it experiences more resistance

Assuming nature of variation of force $F(t)$ with time t as triangular

$$\int_0^T F_i(t) dt = \left(\frac{F_{i,max}}{2} \right) \Delta T$$

So this force act on the grit by the tool starts increasing as soon as the grit gets in contact with the tool and workpiece at the same time. It attains maximum value then starts decreasing until it attains the zero value. So it from here to here its maximum value and then it will go to the zero value here. So movement of A to B is due to the impact on the abrasive particle as the tool touches the abrasive and it experiences more resistances here.

So assuming the nature of vibration of the force F t with time t as a triangular variation of this variation of this force F t with the time t okay. So this integration of 0 to T F_i t dt it will be it will it will be expressed as F_i max is the maximum force by 2 into ΔT . So this is the F_i max here, so this is the F_i max here okay by 2 (()) (46:35) this triangular area so area of this triangle is F_i max by 2 into this ΔT here okay. So this is the integration will give (()) (46:50).

(Refer Slide Time: 46:51)

F_{avg} be the average force acting in one cycle, then

$$F_{avg} = \frac{1}{T} \int_0^T F_i(t) dt = \frac{1}{T} \left(\frac{F_{i,max}}{2} \right) \Delta T = \frac{1}{T} \left(\frac{F_{i,max}}{2} \right) h_t \left(\frac{T}{2} \right)$$

OR, $F_{i,max} = \left(\frac{4F_{avg} a}{h_t} \right) = \left(\frac{4F_{avg} a}{h_t + h_w} \right)$ **FORCE/GRIT**

Maximum force / grit = $F_{i,max}/N$

47:38 / 1:04:54

So so average force is 1 by T 0 to T F t dt okay. So Ft dt we already got this Fi max by 2 into del t here okay. So it is 1 by T Fi max by 2 and del T already we have calculated in the earlier slide. It is hh by a into T by 2 here. So from here after simplification will get this Fi max. So this is the Fi max here. It can be represented with respect to the average force. So final expression of this Fi max is 4 into average force multiplied by a and then divided by ht plus hw okay. So it is hh, ht plus hw is hh, total indentation depth okay. So maximum force per grit is given by Fi max divided by N here okay.

(Refer Slide Time: 47:56)

MAX STRESS DEVELOPED IN THE W/P $(\sigma_w) = \frac{F_{i,max}}{N} \left(\frac{1}{\pi d h_{wh}} \right)$

MAX STRESS DEVELOPED IN THE TOOL $(\sigma_t) = \frac{F_{i,max}}{N} \left(\frac{1}{\pi d h_t} \right) = \sigma_w \frac{h_{wh}}{h_t}$
 $\Rightarrow \frac{h_t}{h_{wh}} = \frac{\sigma_w}{\sigma_t} = 1$

$$\sigma_w = \frac{F_{i,max}}{N} \left(\frac{1}{\pi d h_{wh}} \right) = \frac{4F_{avg} a}{(h_t + h_w)} \left(\frac{1}{N \pi d h_{wh}} \right) = \frac{4F_{avg} a d}{\pi K_2 h_w^2 \left(\frac{h_t}{h_w} + 1 \right)}$$

N be the no. of grains under the tool

$$N = \frac{K_2}{d^2}$$

Stress σ_w can be replaced by Brinell hardness number (H). Both are the same ($\sigma_w = H$)
 Above equation can now be simplified as

51:18 / 1:04:54

So maximum stress developed on the workpiece can be calculated as $F_i \max$ so divided by N here. So this is the maximum force per grit and divided by the cross-sectional area of the grit. So cross-sectional area can be given as $\pi d h$ wh okay.

So maximum stress developed in the tool also we can calculate. So this is the $F_i \max$ again same $F_i \max$ divided by number of grits under the tool and divided by 1 by $\pi d h$. So h is the depth of indentation into the tool here. So $\pi d h$ is the cross-sectional area of the tool, cross-sectional area of this abrasive into the tool here okay. So $F_i \max$ by N by πd can be represented by can be substituted by σ_w so you can put it here as σ_w then it will be h by h okay.

So here one more modification is there. So this h by h by h okay or h by h this ratio of this ratio of this so this ratio of this throwing model divided by this hammering model can be represented as this σ_w by σ_t equal to you can consider this one as a, j . So this h by throwing model indentation by throwing model by indentation by by indentation sorry indentation by into the tool divided by indentation into the workpiece is required to stress into the workpiece and stress maximum stress developed into the tool equal to j here okay.

So now we have to calculate the maximum stress developed into the workpiece equal to $F_i \max$ by N divided by $\pi d h$ wh okay cross-sectional area into the workpiece okay. So this $F_i \max$ by N again we can put it here 4 average a by h plus h total indentation depth. Then again this is the N again see this N is here and this cross-sectional area into the workpiece okay.

So now if we take this value this common to this h as (j) (50:35) here so it will be h by h plus 1 here. So h by h already we know this is the j so this will be j plus 1 so that is why it will become so this one will become j plus 1 here okay. So K_2 is the constant of proportionality. It has come because of this N . N equal to K_2 by d square okay. After simplification will get this σ_w here. So stress σ_w can be replaced by Brinell hardness number H .

So both are the same σ_w equal to H here. So now this above equation can be simplified.

So N here considered here number of impacts. It is inversely proportional to the diameter of this d square d square of this abrasive particle okay. So it is the cross-sectional area of this abrasive particle.

(Refer Slide Time: 51:38)

$$h_{wh} = \sqrt{\frac{4F_{avg} ad}{H_w \pi K_2 (j+1)}}$$

SUBSTITUTE FOR h_w TO GET

$$\rightarrow MRR_v = K_1 K_2 K_3 \left[\frac{4F_{avg} a}{H_w \pi K_2 (j+1)} \right]^{\frac{3}{4}} d^{\frac{1}{4}} f$$

MRR_v FROM THROWING model << MRR_v FROM HAMMERING model

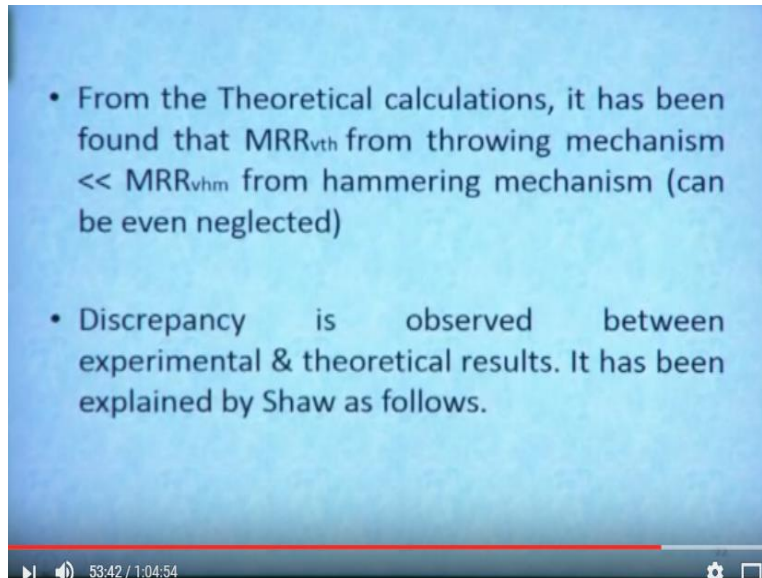
Okay, so after simplification this h_{wh} will come in this form root over 4 average a amplitude of vibration, d is the diameter of this abrasive particle by h_w is the hardness of this workpiece material then K_2 is the constant of proportionality and then j plus 1 j equal to h_{th} by h_{wh} so h_{th} is indentation into the tool and h_{wh} is the indentation into the workpiece okay.

So now if I substitute this value of this h indentation into the equation 1 for calculating material removal rate will get this final equation this form K_1 plus K_2 plus K_3 this $(\frac{4F_{avg} a}{H_w \pi K_2 (j+1)})^{\frac{3}{4}}$ $d^{\frac{1}{4}}$ f average into a by hardness of this workpiece material K_2 into π into j plus 1 whole to the power 3 by 4 and d to the power 1 by 4 into f , f is the frequency of vibration, d is the diameter of this abrasive grit here. So other things everything is known here okay.

So one thing we want I want to tell here so although it is shown here this material removal rate is proportional to $d^{\frac{1}{4}}$ so theoretically whatever we get material removal rate by using hammering model okay. We do not get the same thing same amount experimentally. So whatever this experimental years and this theoretical years it does not matches well. So there are some simplification or modification Mr. Shaw M.C. Shaw who has who has made some simplification there into his model into his hammering model okay.

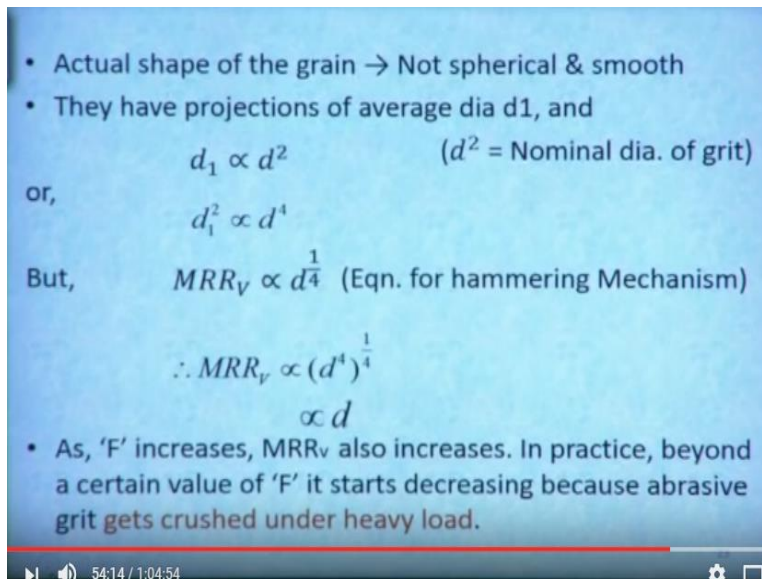
Here this material removal rate is proportional to the d to the power $\frac{1}{4}$ or d to the power 0.25 okay. So that modification we shall discuss in the next slide okay.

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So from the theoretical calculation it has been found that MRR by throwing model from the throwing mechanism is very very less than this hammering mechanism okay. So material removal rate from this throwing mechanism is very less than the hammering model okay. So we can neglect that material removal rate from the throwing model. So discrepancy is observed between the experimental and theoretical results and it has been explained by Shah as follows okay.

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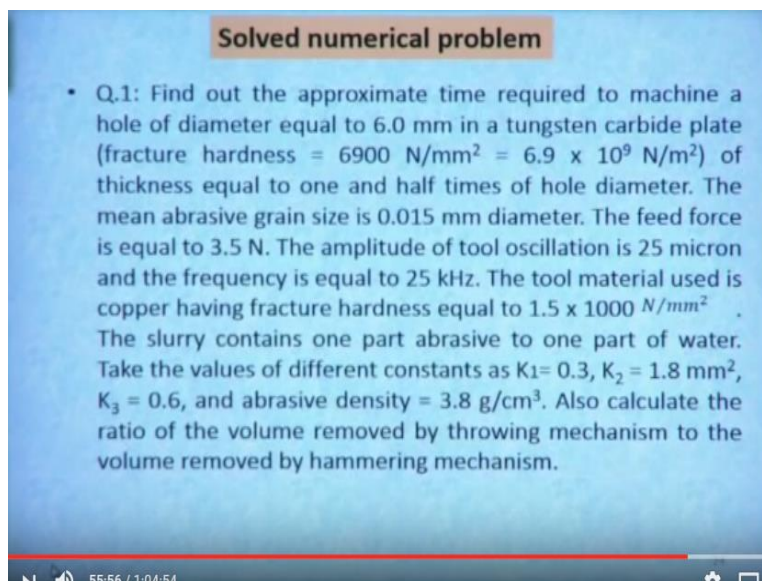
So actual shape of this grain it is not a spherical, it is not a spherical it is random in size. The abrasive particles are random in size but we have considered this one as spherical and smooth. So it is not at all spherical and smooth. So they have a projections of average diameter of this d_1

okay so they have a projections average diameter of this d_1 and this d_1 is proportional to the d square. So d square so this projection diameter is d_1 . It is proportional to the d square d square is d is the diameter of this abrasive grain.

Now if we take the square of this d_1 square is proportional to the d to the power 4 here okay. So now this material removal rate is proportional to the d to the power 0.25. So from this hammering model here okay. So now this here so now if we put this value MRR_v volumetric material removal rate is equal to so d_1 so if we put it here d to the power 4 to the power 1 by 4 equal to this will become proportional to this d okay.

So now if we increase the force by the tool so MRR_v material removal rate will also increase but in practice beyond a certain value of this F it starts decreasing okay. So if we increase the force beyond an optimum value so this material removal rate again it will decrease because this abrasive grit will get crushed under the heavy load. So this abrasive grit will it will be crushed under the heavy load okay.

(Refer Slide Time: 55:56)



Solved numerical problem

- Q.1: Find out the approximate time required to machine a hole of diameter equal to 6.0 mm in a tungsten carbide plate (fracture hardness = $6900 \text{ N/mm}^2 = 6.9 \times 10^9 \text{ N/m}^2$) of thickness equal to one and half times of hole diameter. The mean abrasive grain size is 0.015 mm diameter. The feed force is equal to 3.5 N. The amplitude of tool oscillation is 25 micron and the frequency is equal to 25 kHz. The tool material used is copper having fracture hardness equal to $1.5 \times 1000 \text{ N/mm}^2$. The slurry contains one part abrasive to one part of water. Take the values of different constants as $K_1 = 0.3$, $K_2 = 1.8 \text{ mm}^2$, $K_3 = 0.6$, and abrasive density = 3.8 g/cm^3 . Also calculate the ratio of the volume removed by throwing mechanism to the volume removed by hammering mechanism.

So now we shall discuss some problem okay. So this from this hammering and throwing model. So this from this material removal mechanism we shall discuss one problem here okay. So now I am reading that problem. Find out the approximate time required to machine a hole of diameter equal to 6 mm in a tungsten carbide plate of thickness equal to one and half time tungsten carbide plate which has a fracture hardness of 6900 Newton per meter square or it is equal to 6.9

into 10 to the power 9 Newton per meter square and of thickness equal to one and half times of the hole of the diameter okay.

This thickness of this hole or thickness of this sheet is 1.5 times of the diameter of the hole okay. So now if it is diameter of this hole is 6 so now this thickness of this sheet will be 1.5 into 6 means 9 mm. The mean abrasive grain diameter is 0.015 mm. The feed force is equal to the 3.5 Newton. So this average force to the tool is 3.5 Newton.

The amplitude of the oscillation is 25 micron here and frequency of frequency is equal to the 25 kHz and tool material used in copper having the fracture hardness equal to the 1.5 into this 1000 Newton per meter square. The slurry contains one part of abrasive to one part of water. So mean this C value of C is equal to 1 and take the values of different constants this constant this t constant constants are there K1 K2 and K3, these values are given here. So K1 equal to 0.3, K2 equal to 1.8 and K3 equal to 0.6 okay and now this abrasive grain density is 3.8 gram per centimeter cube.

Also calculate the ratio of the volume removed by the throwing mechanism to the volume removed by the hammering mechanism or V volume of material removed by throwing mechanism by volume of material removed by the hammering mechanism and we have to calculate what is the time for machining of 6 mm diameter hole and 9 millimeter diameter or a 9 millimeter thick tungsten carbide plate okay.

(Refer Slide Time: 58:26)

• **Solution:**

Following data are given:

- Hole dia = $6 \times 10^{-3} \text{m}$,
- plate thickness = $1.5 \times \text{hole dia} = 9 \times 10^{-3} \text{m}$,
- mean abrasive grain size (d) = $1.5 \times 10^{-5} \text{m}$,
- feed force (F) = 3.5 N,
- amplitude of tool oscillation (a/2) = $25 \times 10^{-6} \text{m}$,
- frequency of oscillation (f) = 25000 cps,
- fracture hardness of workpiece material,
 $\sigma_w = H_w = 6.9 \times 10^9 \text{ N/m}^2$, fracture hardness of tool material
(Ht) = $1.5 \times 10^9 \text{ N/m}^2$,
- abrasive grain density (pa) = $3.8 \times 10^3 \text{ kg/m}^3$,
- $\lambda = H_w/Ht = 4.6$,
- $K_1 = 0.3$, $K_2 = 1.8 \text{ mm}^2 = 1.8 \times 10^{-6} \text{ m}^2$, $K_3 = 0.6$, $C = 1$.

58:26 / 1:04:54

So now we shall see what are the informations are given here for the solution of this problem. So hole dia is given 6 into 10 the power 3 meter so everything here now is converted into (()) (58:35) okay so meter so we have converted this length into meter. So this plate thickness is 1.5 times of hole diameter so it is 9 into 10 to the power of 3 meter here. Mean abrasive grain size is 1.5 into 10 to the power minus 5 abrasive grain size and feed force is 3.5 Newton and amplitude of tool oscillation a by 2 is 25 into 10 to the power minus 6 meter.

Frequency of oscillation is 25000 cps cycles per second okay. Fracture hardness of this workpiece material is sigma w or you can consider as Hw equal to 6.9 into 10 power 9 Newton per meter square. Then fracture hardness of this tool material is 1.5 into 10 to the power 9 Newton per meter square. Abrasive grain density is 3.8 into 10 the power 3 kg per meter cube and then this here lamda is written or we can put this one as j so j equal to this Hw by Ht equal to 4.6, 4.6 here and K1 K2 and K3 this value and C equal to 1, these values are known here.

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The following procedure should be followed to solve the given problem:

What is the time required to machine the hole?

We know that the volume (V) of material removed during USM can be calculated using the following relationship:

$$V = K_1 K_2 K_3 \sqrt{\frac{h^3}{d}} \cdot f \text{ mm}^3/\text{s}$$

59:55 / 1:04:54

So there are different steps. The following procedure should be followed to solve the given problem. So what is the time equal for machining the hole of 6 millimeter diameter okay. So now volume of material removed we know this equation 1. So this is equal to K1 K2 and K3 root over H cube by d. So here unknown is the H. So H we have to calculate first by hammering model as well as by throwing model okay.

(Refer Slide Time: 1:00:25)

Step 1.
Calculate the value of "h" which is different for throwing model (h_{th}) and for hammering model (h_{whm}).

Step 2.
After knowing the values of h_{th} and h_{whm} calculate V_{th} and V_{whm} by substituting these values in above Eqn. Find total volume of material removed per unit time (V_s) by adding V_{th} and V_{whm} .

Step 3.
Calculate the total amount of material to be removed to make the required hole. Divide it by V_s to find the total time required to make the hole.

Step 4.
Find the ratio of h_{th}/h_{whm}

This step 1 this calculate the value of h which is different for throwing model and hammering model. So after knowing the values of this h for throwing and hammering model calculate the volume of material removed by throwing model by this earlier equation and volume of material removed by the hammering model by substituting these values of this h into the equation last in the last equation okay. So now we can find out the total volume.

So total volume is the V_{th} volume of material removed by throwing model plus volume of material removed by hammering model. So this total volume can be per unit time is V_s equal to V_{th} plus V_{whm} . Now calculate the total amount of material to be removed to make the required hole okay. So divide this divide it total amount of material removed required to be removed by this V_s will give the time for making the hole okay. So this ratio is h_{th} by h_{whm} .

(Refer Slide Time: 1:01:32)

- Following the above steps, all calculations are made as follows.
- **Step 1.**
- In Eq. (3.5), except "h" all other parameters are known. Let us calculate h_{th} as

$$h_{th} = \pi a f d \sqrt{\frac{\rho_a}{6\sigma_w}}$$

$$= \pi \times (50 \times 10^{-6}) \times (2.5 \times 10^4) \times (1.5 \times 10^{-5}) \sqrt{\frac{3.8 \times 10^3}{6 \times (6.5 \times 10^9)}}$$

$$h_{th} = 1.78 \times 10^{-5} \text{ mm}$$

Penetration (h_{whm}) in the workpiece due to hammering is given as

So after that we have to calculate the ratio the indentation depth by this throwing model by indentation depth by the hammering model. So following the above steps for calculations okay h_{th} equal to $\pi a f d$ then root over ρ_a by $6 \sigma_w$ ρ_a is the density of this abrasive particle, σ_w is the hardness of this workpiece material, a is the amplitude of vibration, f is the frequency of vibration and d is the diameter of this abrasive particle. So all these values are known here. After putting all these values into this equation into this equation we can get this h_{th} equal to 1.78×10^{-5} mm. So now we have to calculate the penetration into the workpiece by the hammering model.

(Refer Slide Time: 1:02:23)

$$h_{whm} = \sqrt{\frac{4F_{avg}ad}{\sigma_w \pi K_2 (\lambda + 1)}}$$

$$= \sqrt{\frac{4 \times 3.5 \times (2 \times 25 \times 10^{-6}) \times (1.5 \times 10^{-5})}{\pi \times (1.8 \times 10^{-6}) \times (6.9 \times 10^9) (1 + 4.6)}}$$

$$h_{whm} = 2.192 \times 10^{-4} \text{ mm}$$

Step 2

Now, volume removed by throwing V_{th} is given as

$$V_{th} = K_1 K_2 K_3 \sqrt{\frac{h_{th}^3}{d}} \cdot f$$

So by hammering model it is 4 average a average to the vibration is the diameter of this abrasive particle, sigma w is the hardness of this workpiece material then K2 is the constant lamda is the ratio by throwing model by indentation by throwing model by hammering model okay. So here we known all these values. After that we shall get this we can get this on this indentation by the hammering model here.

So now calculate this if we put this values into this equation we can calculate the volume of material removed by this both the models here okay.

(Refer Slide Time: 1:03:03)

$$= 0.3 \times 1.8 \times 0.6 \sqrt{\frac{(1.78 \times 10^{-5})^3}{1.5 \times 10^{-2}}} \times 2.5 \times 10^4$$

$$V_{th} = 4.97 \times 10^{-3} \text{ mm / s}$$

Volume removed by hammering is given by

$$V_{whm} = K_1 K_2 K_3 \sqrt{\frac{h_{th}^3}{d}} \cdot f$$

$$= 0.3 \times 1.8 \times 0.6 \sqrt{\frac{(2.192 \times 10^{-4})^3}{1.5 \times 10^{-2}}} \times 2.5 \times 10^4$$

$$V_{whm} = 0.2146 \text{ m}^3/\text{s}$$

So here this is the volume of material removed by this throwing model and here volume of material removed by the hammering model. So by putting all these value h we know both the cases okay. So it will be w so this is w h wh okay by this hammering model.

(Refer Slide Time: 1:03:31)

Step 3.

Time required to drill a hole =
 Volume of the hole to be drilled/ Volumetric MRR ($= V_{whm} + V_{th}$)

$$= \frac{(\pi/4) \times 6^2 \times 9}{0.21987}$$

$$= 19.289 \text{ min}$$

Step 4. Ratio,

$$\frac{V_{th}}{V_{whm}} = \frac{0.00497}{0.2146} = 0.023$$

Thus, it is evident that the material removed by hammering is much more than by throwing (approximately 43 times). Hence, for approximate calculations, V_{th} can be ignored as compared to V_{whm} .

So now step 3 is the time required to drill the hole equal to this volume of hole to be drilled so volume of hole to be drilled is $\pi/4 \times d^2 \times h$ is the diameter of this hole 9 is the depth of this hole so total volume of material removed it is equal to $\pi/4 \times d^2 \times h$ divided by the volumetric material removal rate total volumetric removal rate by hammering model as well as plus throwing model okay.

So it is coming as the time required is 19.289 minute okay. So now the second answer is the answer to the second question is that volume of material removed by throwing model by volume of material removed by hammering model okay so here both the volumes so it is coming as 0.023.

So thus it is evident that the metal removed by this hammering model is much more than the material removed by the throwing model. So approximately here 43 times. Hence for the approximate calculation V_{th} volume of material removed by different model can be ignored as compared to the volume of material removed by the hammering model okay.

Thank you.