Dynamic Behaviour of Materials
Prasenjit Khanikar
Department of Mechanical Engineering
Indian Institute of Technology-Guwahati

Module No. #03
Lecture No. #10
Introduction to Plastic Waves

Hello everyone, so in the last lecture, we have discussed about, the Elastic Wave propagation. We discussed, Elastic Wave propagation, in a finite body, like cylindrical bar, when we impact a cylindrical bar, with a cylindrical projectile. Also, we discussed about, the Elastic Wave propagation, for an unbounded media.

We discussed about, Longitudinal Wave Velocity, Shear Wave Velocity, and Rayleigh Wave Velocity, which is, Surface Wave Velocity. We also discussed about, some additional considerations, we need to take care of, while analysing, the Split Hopkinson Pressure Bar experiment. So now, we will talk about, the Plastic Wave, in this lecture.

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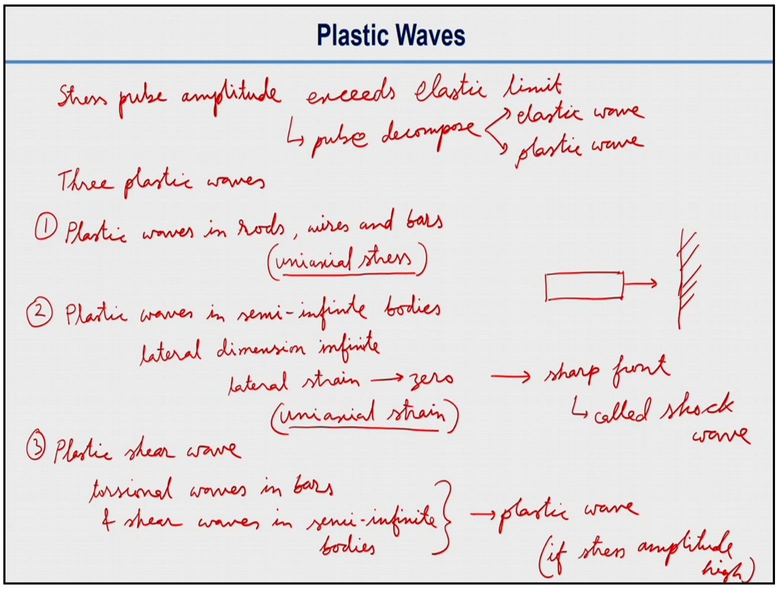

So, when the Stress pulse amplitude, exceeds the elastic limit, the pulse will decompose, to Elastic Wave, and Plastic Wave. So, we know that, when a material is stretched beyond a limit, that is, elastic limit, the plastic deformation happens to the material. So, similarly in this case, so when we have a dynamic loading, the Stress pulse amplitude, if it exceeds the elastic limit, then this wave will be decomposed into, elastic component, and the plastic component.

So here, we will discuss about, the three classes of Plastic Waves. Three Plastic Waves, that is, we can call, three classes of Plastic Waves. And, the first one is, Plastic Waves in rods, or wires, or bars. So, in this case, we have, let us say, one example is, we have a rigid target, and we have a cylindrical rod, or we can draw a little longer, let us say, the cylindrical rod, is hitting the rigid target. And, this is a case of, Uniaxial stress.

If, the Stress pulse amplitude is, sufficiently high, then plastic deformation will happen, and Plastic Wave will propagate through it. Then, the second one is, Plastic Waves in Semi-infinite bodies. So, this is the case, when the lateral dimension of that, suppose, we are talking about, this rod, are infinite. So, that will lead to, actually, lateral strains are zero, and which is a case of, Uniaxial Strain Uniaxial strain. So, the earlier case, we had Uniaxial stress, and here, it is Uniaxial strain, and this will lead to, a very sharp front wave.

And, that is called, a Shock Wave. So, when it is a Uniaxial Strain condition, and that is, the wave front will be very sharp, and that is called, a Shock Wave. And, the third case is, Plastic Shear Waves, Torsional Waves in bars, and Shear Waves in semi-infinite bodies. These can generate, plastic deformation or Plastic Waves. This can lead to, Plastic Waves, if that amplitude is sufficiently high, stress amplitude is high, that should be higher than the, elastic limit.

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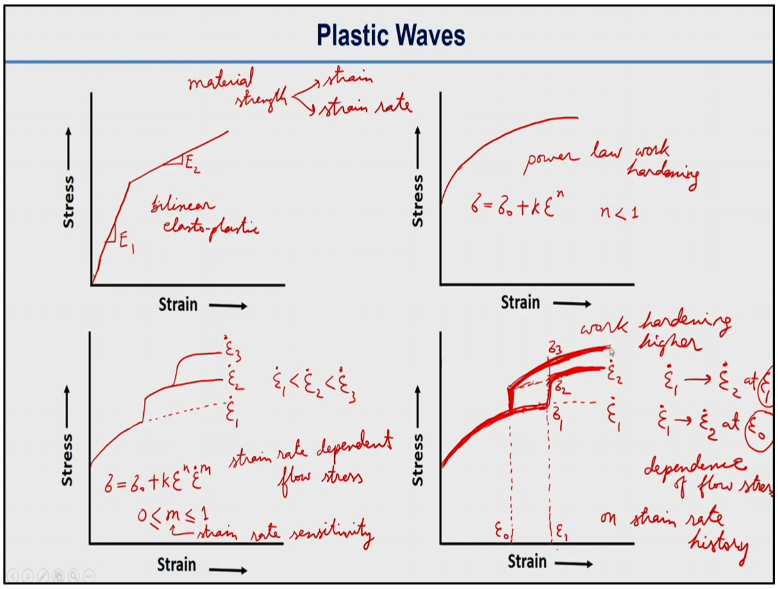

So, we will discuss, a few cases of, plastic deformation. As we know that, material strength is dependent on, Strain and Strain rate. So, we know, material strength depends on, Strain and

Strain rate. Now, in the simplest case, we call it the Bilinear Elastoplastic, where we have a straight line, in this equation, Elastic portion. And then, another straight line, with different slope, as the Plastic portion. E1, the Elastic stiffness. This is, the slope is the frontier, for the Plastic portion.

So, this is, we call as a, Bilinear Elastoplastic Behavior. These all are, stress strength curves. It will show, different stress strength. And, there can be another model, which is, we call, Power Law Work Hardening. So, the first one, is the very simplified case. But, in reality, we can get, these type of stress Strain relationship, which is, we call, Power Law Work Hardening.

Why we call, it as a Power Law Work Hardening? Because, this work hardening proportion, the rate portion, we can express the flow stress, Sigma equal to, Sigma-0 K Epsilon to the power N. So, the Sigma-0, is the ill strength, and K and N, are the two materials constant. N is, we know as a, work hardening or Strain hardening exponent. Some material can be, Strain rate dependent. So, in this case, third case, we will, Strain rate dependent Flow stress.

So, what happens here is, let us say, you have a flow stress behavior, like this. And, when you, this is, let us say at, Epsilon dot. And then, if we increase it, the Strain rate, it will look like this. And, if we increase the Strain rate more, so, let us say, this, the Stress strain curve will look like this. So here, Strain rates are in, increasing order. So, this is strain rate dependent, flow stress. So, the constitutive equations, we can write, in this form, K Epsilon to the power N, Epsilon dot to the power M, this strain rate more.

And then, this M will be, in between, 0 to 1. Sorry, other case, I did not mention here. So, N is, just smaller than 1. So, M is generally, referred as, the Strain rate sensitivity. So, in the fourth case, we will see that, the material behavior depends on the, Strain rate history. So, dependence of flow stress, on Strain rate history. So here, what we will show is, let us say, this is the material behavior, and that is, let us say, Strain rate, Epsilon dot.

And, if we, let us say, increase the Strain rate, at Strain Epsilon-1, then the material will increase, will so increase in the flow stress. And, what we did here is, at Strain Epsilon-1, we increase the Strain rate, from Epsilon Dot-1 to, Epsilon Dot-2. So, now the material behavior is, something like this. But, in case in case, if someone else, wants to increase the Strain rate, at a lower Strain, Epsilon-0, then what will happen is, so with the same Strain rate, actually, it should be like this, from the, our earlier predictions. But actually, it should, it goes like this.

So, what happen here is, let us say, this is our flow stress. If you see at a point, corresponding to Strain Epsilon-1, here the flow stress is Sigma 1, here the flow stress is Sigma 2, and here the flow stress is Sigma 3. So, what we did is, we have the stress-strain curve, and at Strain Epsilon-1, so we increase the Strain rate, from Epsilon Dot-1, to Epsilon Dot-2. This is at, Strain Epsilon 1. And then, what we got, this curve, we got this curve. And, in the second case, what we did is, we changed the Strain rate, Epsilon Dot-1, Epsilon Dot-2, at the lower Strain Epsilon-0.

So, what happens, in this case, the stress curve will go up, in this portion. And, it will not follow, the same slope, as this. So, it will have a, higher slope. So, that means, the work hardening is higher, so flow stress will depend on the, Strain rate history. That means, when we increase the Strain rate, whether we increase the Strain Epsilon-1, or whether we increase the Strain Epsilon 0, that will decide, what will be the work hardening. And we can see that, slopes of these curves, are different.

So, this slope, is a lower. This, another one, the slope is, will be higher. So, this would be flat, and this slopes would be higher. So, we will discuss about, the Plastic Waves, now. And, so here, we are discussing, actually the Plastic Waves, and the treatment of Plastic Waves, are complicated, complex, because of Strain rate effects, and Strain rate history effects.

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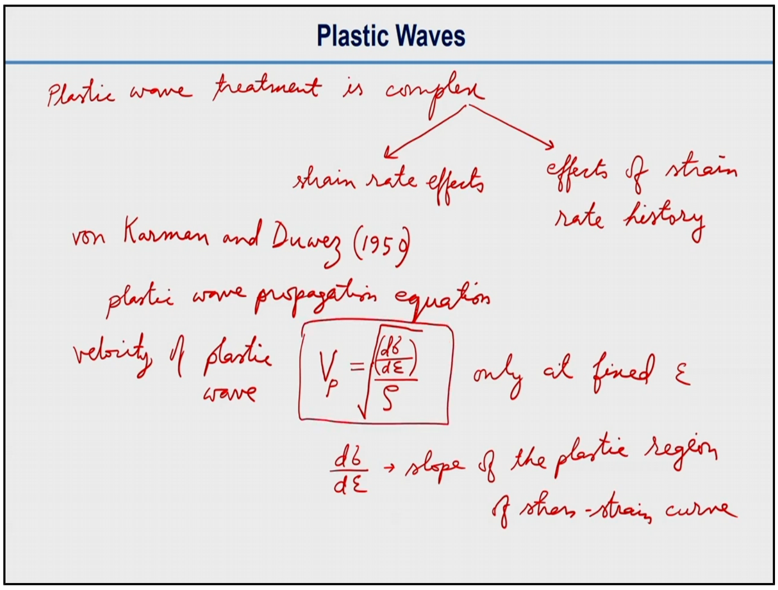

Plastic Wave treatment, is more complex, than the Elastic Wave, due to, first thing is, Strain rate effects, that we have just discussed. Strain rate effects, and effects of Strain rate history, what we have just discussed. So, in 1951, Karman and Duwez, proposed the Plastic Wave propagation equation, which is, the velocity can be given by, d Sigma d Epsilon, divided by mass density, square root of the whole thing.

So, this is the velocity of Plastic Wave, and it is only at, fixed string Epsilon. Because, the d Sigma by d Epsilon, which is, the slope of the plastic region, of the stress Strain curve. And, the slope, we define it, only at a fixed Strain, but that is why, the velocity of Plastic Wave, this is the point at fixed strain.

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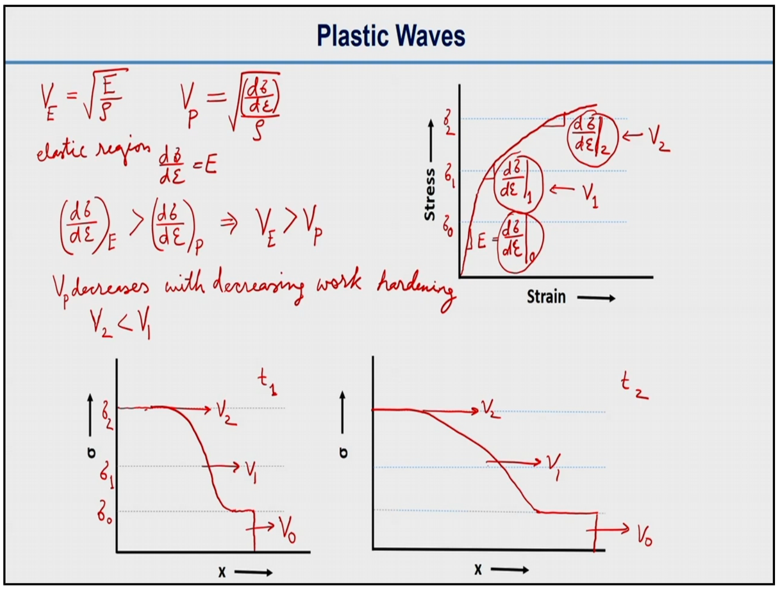

Now, if we compare, this Plastic Wave velocity, with our Elastic Wave velocity, whatever we discussed earlier, in earlier lectures, so that we know, the Elastic Wave velocity, is square root of the ratio of, e by Rho. And so, even if we see this, both the equation, V-Elastic and V-Plastic, which is, the Sigma d Epsilon by Rho. So here, in the elastic region, so, d Sigma d Epsilon is e. So, if we draw the stress-strain curve, it will look like, this is the elastic portion.

And then, there will be yield. Strain hardening, it will go like this. So, first portion, in the elastic region, the slope equal to d Sigma d Epsilon, which is 0 at, Sigma-0 actually. So then, we will have, another slope, let us say, at Sigma equal to 1, which is in the plastic region, and d Sigma d Epsilon at 1. And then, Sigma 2, slope here is less. D Sigma d Epsilon at 2. So, we can see that, for elastic region, this is the slope.

And, for a plastic region, we got two slopes, for Sigma 1, and Sigma 2. The first slope is, higher than the, the other slope. So, and also, what we can see, from elastic and plastic is, d Sigma d Epsilon. For elastic, which is this one, is higher than, d Sigma d Epsilon plastic. So both of these, are smaller than the, elastic slope. So, that means, velocity of Elastic Wave, is higher than the, velocity of Plastic Wave.

So, that is an important conclusion from here. And also, V decreases, in the plastic region. Or, we can just write, VP, decreasing work hardening. V2, is smaller than, V1. So, V1 corresponds to, this. And, V2, corresponds to that. So, the slope of V2, is less. And, that means, V2 is smaller than V1. We will see that, how the velocity will look like, we will plot it with, Sigma versus X. So, the velocity, which is, we will start from the bottom. So, here the velocity is, high, and constant, and then, it will, look something like this.

So here, V2, as we can see, the V1. Sorry. We should write, this one also first, at Sigma-0, and Sigma 1, and Sigma 2. So, this is V1. And then, this is constant. This is V0, in the elastic region. And, this is, let us say, at Time equal to T1. And, at a later time, if we take this, with higher Strain, so what will happen here is, so this will be, more variation here. So, this is, V2, V1, and this is, V0. So, we can see that, this is, how the Plastic Wave travels, we can see that, different velocities.

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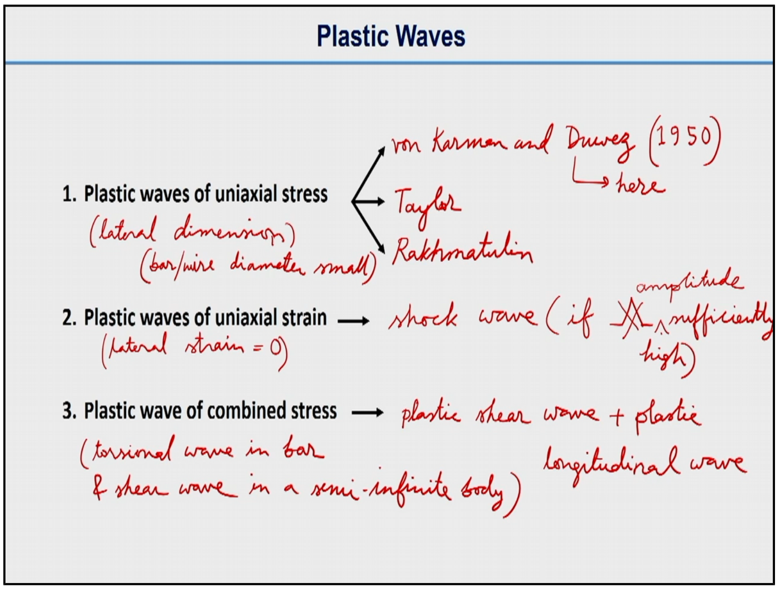

So, basically, we will discuss here, three types of Plastic Waves. The Plastic Waves of Uniaxial Stress, and Plastic Waves of Uniaxial Strain, and Plastic Waves of Combined Stress. So, that we have, even discussed in the, few slides earlier. So, for Plastic Wave of Uniaxial Strain, which that means, lateral Strain is zero, or like it is, infinite lateral dimension. So, this is called, Shock Wave, if amplitude is sufficiently high.

And, for Plastic Wave of combined stress, so we have like, Plastic Shear Wave plus Plastic Longitudinal Wave. Basically, in the upper two cases, Case number 1 and 2, we mostly focused on, Longitudinal Wave. But, in this case, we have, Shear Wave, as well. And, that means, let us say, Torsional Waves in a bar, and the Shear Wave in a semi-infinite body, can also generate, Plastic Waves. So, first, we will talk about, Uniaxial stress, where the lateral dimension is very small.

That means, suppose if we are, talking about a bar, the bar radius is very small. And, there are, you can write that, the Bar or wire diameter, small. And so, there are Three Theories of Plastic Wave generally, we follow. And, so this three theory, the Plastic Wave, these are initially proposed by, first one is Von Karman and Duwez, and then, Taylor.

And, the third one is, Rakhmatulin. So, these are independently developed theories. And then, we will mostly discuss on, the first one, which is published in the year of 1950. So, we will discuss, here. Sorry, this, the capital lambda is not correct, because that is, the stress amplitude. So, the capital M, that we use for, wavelength. So, basically the Shock Wave is for, the Plastic Wave. That means, if the amplitude is, sufficiently high.

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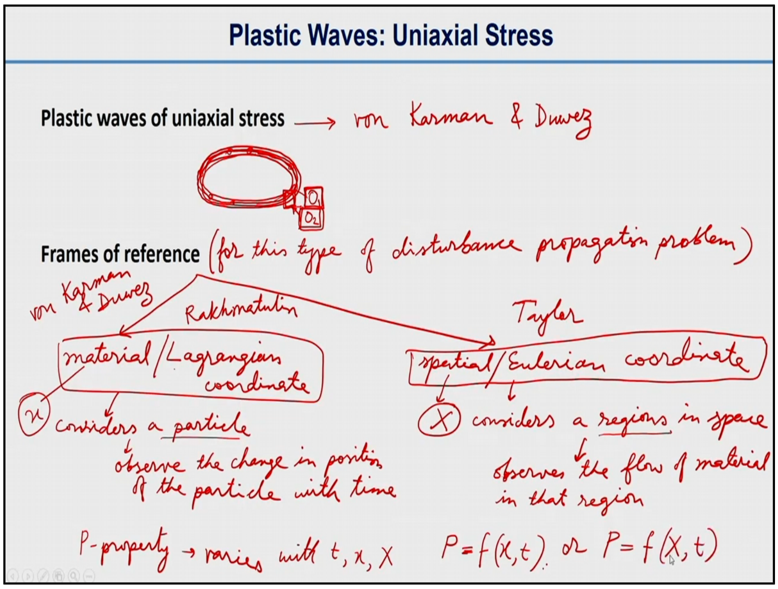

So, Plastic Waves of Uniaxial stress, so what we will discuss here is, Von Karman and Duwez Theory. And then, we know, two frames of reference, which probably, you have studied in, your under graduate courses, probably in Fluid Mechanics. So, we have two frames of references, for these type of disturbance propagation problem, even in fluid mechanics. For fluid displacement problems, you have encountered, these two frame of reference, at least, you read about these differences between, these two frames of references.

So, this type of disturbance propagation problem, like our stress wave propagation. So, for this type of propagation problem, you know, the first one is, material or, Lagrangian coordinate, and the second one is, spatial or Eulerian coordinate. So, what happens in the first case, one considers a particle of the material, and then observe the change in position of the particle, with time.

In the other case, one considers a certain region or space, actually region in space, and observes the flow of material, in that point of space or in that region. So, it is basically, Von Karman and Duwez. For, Material and Lagrangian coordinate they use, Von Karman and Duwez. And, also Rakhmatulin, he used the Material, and Lagrangian coordinate. And, on the other hand, the Spatial or Eulerian coordinate, Taylor used it for his, Theory of Plastic Wave.

Now, we can try to understand, these two coordinates, like this. I am sure that, you have already studied in your, you know, undergraduate, about these two frame of reference. So, a simple example is, also here. Let us assume, a stadium of 400 Meter track. And, so there are, let us say, a 5 Kilometre or 10 Kilometre race, is happening, in that track, and you are the observer. This is, you.

So, this is the observer, O, let us say. And, let us say, out of those, the runners, let us say, 20 runners are running here, and one of them is your friend. So, what will happen, in that case. You will see, your friend only, where he is, whether he is in this position, or this position, or this portion. So, you will focus, on your friend, because you know, only one person, so that is why, it is like, you consider the particle. This is actually, Lagrangian coordinate system, and let us say, you were the Observer-1.

And then, another person, who is the Observer-2, in the same location, let us say. Observer-2, who does not know, any of the runners or those athletes. So, what will happen is, he will focus only on the small region, let us say, this region. He will only focus, on this region, and he will see, all the people, who are crossing, by that area, so that is like, consider a region in the space. And so, Observer-2, if we consider that say, Eulerian Frame of reference. And, Observer-1, who is like, you observing only your friend, not the other runners.

So, that is Lagrangian coordinates, Lagrangian frame of references. So, any property, let us say, P, the property. And, p is a property, that varies with, Time T, and then spatial coordinate X, so that, these coordinates, we will write it as X, and the material coordinate, we will write it as small X. The small X, and this capital X, so it varies with, small X and capital X. So, p can be, either expressed as a, function of small XT, or p can be expressed as, function of capital X and T, so that is the two frames of references.

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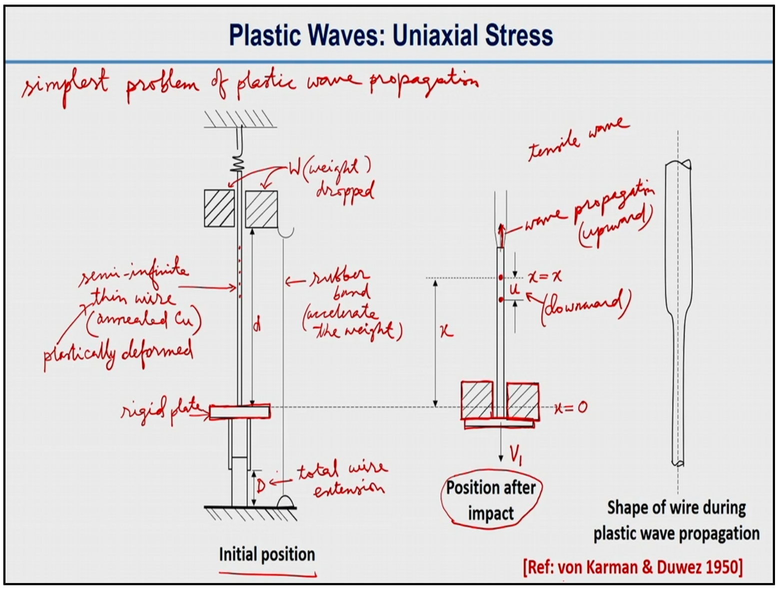

So, we will talk about, the experiment, that problem, that Von Karman and Duwez has, when they used in 1950, so that is probably, the simplest problem of Plastic Wave propagation. So, this is, from the reference, Von Karman, if you can see, the lower bottom corner, from Von Karman and Duwez, in 1950. So, this is, the simplest problem, of Plastic Wave propagation. So, this is the initial position. We will focus on the, initial position.

So, this is a, semi-infinite Tin wire. This is, let us say, made of annealed copper. It is, very ductile material. So, we have some weight here, W. This is, weight. And, so this one rigid plate, attached to the wire. And then, this weight will be dropped. And, this rubber band, will accelerate the weight, the falling of the weight, accelerate the weight. And then, the weight will impact, on the rigid plate, thereby, deforming the wire.

So, this will be, plastic deformation of the wire. Actually, we are focussed on the, wire, now. So, this string wire, will be plastically deformed, when the weight will impact, on the plate. And, the allowable deformation here is, D, you can see that, this is one end will, and then, you can vary this distance. So, what will happen, when the weight will impact here, that say, this is small D, that the portion, the wire, from the weight, initial position of the weight, to the rigid plate. So now, the weight will travel, a distance, D, small D.

And now, the capital d is the, the allowable deformation. And, you can say, the total wire extension, this is total wire extension. And then, Weight W is sufficient enough, not to be deaccelerated, significantly, when the plastic deformation is happening. So, when the plastic deformation is happening, if the weight is low, then there may be some de-acceleration, but that is why, the weight is intelligently chosen, so that, it will not have much, de-acceleration.

So, now we will see, the position, after impact. This is the position, after impact. Now, if we see that, this plate is impacted, by these weights. And then, by these weights, in generating a velocity downward, Z1. So, if we consider this, this dotted line, this initial position of the plate, as X equal to 0. And then, this is a point on the wire it as, X equal to X. That means, this distance is X. So, this displacement of this point, into downward direction is, U.

We should focus on that, this wave will travel, on the upward direction. This is the, wave propagation. So, that we should understand, this is a tensile wave. So, wave propagation is upward, but material displacement is downward. So, the senses, opposite here. So, this is, wave propagation is upward, and material displacement is downward.

So here, we should mention that, in these experiment, there some equidistance mark, is kept on this wire, if you see, on this. So, to get the displacement here, so here we have to mark, so we have this point, is displaced by a distance, U. And, the current position, it is, at this location. So, we are, considering a particle, at X, at a Time T, the particle is, let us say, displaced by U.

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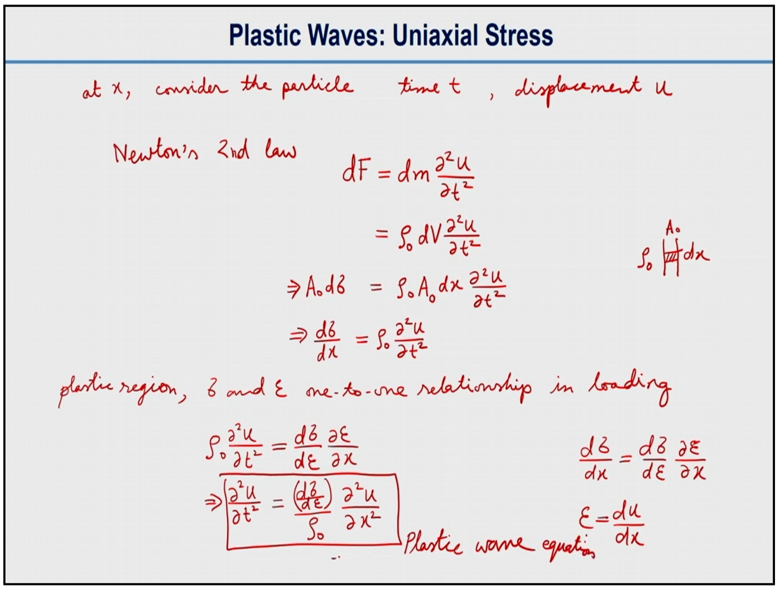

So, at the position X, that consider the particle, and which is at X, and Time T, and displacement is U. So, we will apply, the Newton's second law here, for that particle. So, DF equal to DM, second partial of with U, with respect to T, which is the acceleration term. And then, DM is the, mass of that small element, a particle, which can be written as, the initial density Rho0, into the volume of this, the small element, and with again, second partial of U, with respect to T.

And, then again, we can write, initial mass density into, initial area, the cross-sectional area. And then, DX is the, thickness of the small element. So, this is the DX, and cross-sectional area. This is, initially the, A0. And, we have, Rho0 is the initial density. And, this will be equal to, as we know, A0 d Sigma, area multiplied by the, stress acting on it. So, that will give us, d Sigma by DX, is equal to, Rho0, this.

So now, we want to have a, wave equation, which is, this is the relation, we got. Now, what we are doing is, we want to derive the wave equation, from here, for the Plastic Wave propagation equation. For that, we want to get the wave equation, in a form, which is similar to the, Elastic Wave propagation equation. So, for that, we know, that Sigma and Epsilon, in the plastic deformation region, has one-to-one relationship, in loading.

So, in unloading, it can be different. In unloading, this is irreversible. That, we are not going to discuss it, now. But, that may not be, a one-to-one relationship. And, this is in the, for plastic deformation, plastic range. I will write, plastic region. So, this has, one-to-one relationship. So, for one-to-one relationship, what we can write, d Sigma DX is equal to, d Sigma by d Epsilon. So, we are writing D, not partial derivative, because, it is one-to-one relationship, and partial of Strain, with respect to X, so that, strain is a multivariable function.

And then, we can write the above equation as, Rho0, second partial of u with respect to T, equal to, d Sigma d Epsilon, partial of Strain, with respect to X. And then, we know that, Epsilon is equal to, DU by DX. So, from that, what we can do is, d Sigma by d Epsilon, divided by Rho0. So, the left hand side will be, it is something like this. And, this will end up with a, like an equation, like this. So, this is, Plastic Wave equation.

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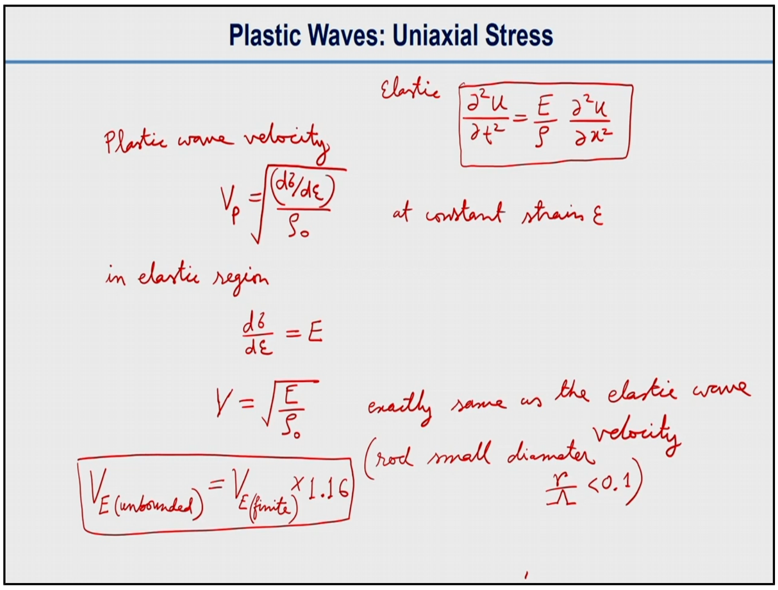

So, if we compare this equation, in the previous slide, with the Elastic Wave equation, for a finite body, so we had it like this. So, this is, Elastic Wave equation. So, if we compare with this, so our Plastic Wave velocity, is VP, which is d Sigma d Epsilon by Rho0, then the whole thing is square root, and that is, at constant string, or fixed Strain Epsilon.

Because, this Sigma by d Epsilon, we define at a constant Strain. And, if we see again, in elastic region, so as we know, the slope of the curve is e, so what we can do is for, if we substitute that, so our V will be equal to, e by Rho0. So, which is exactly, the same as the, Elastic Wave velocity. So, what we are, showing it here is, if you take the Plastic Wave velocity, and then, substitute this d Sigma by d Epsilon, which is the slope in the plastic region, of the stress-strain curve.

If we know that, in elastic region, the slope of the curve is, the e, and that is, Young's modulus. And, then after substituting, we get, e by Rho0, which is exactly the same, what we got earlier for, Elastic Wave propagation. By the way, this Elastic Wave propagation is, for a finite body. And, for basically, when the rod or bar of small diameter, these equations are valid, let us say, for R divided by wavelength is, 0.1.

And also, this is different than, the wave velocity, in unbounded media, which is lich for, V-Elastic for unbounded, is equal to, V-Elastic of the finite body, or this, whatever we got for, let us say, for rod, that will be, 1.16 times of this. So, 1.16 times of, for the Elastic Wave propagation velocity, in a rod, will be equal to, Elastic Wave velocity, in unbounded medium. So, that is all, for today. So, we will continue, in the next lecture. Thank you.