Dynamic Behaviour of Materials
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Module No. #04
Lecture No. #11
Plastic Waves of Uniaxial Stress

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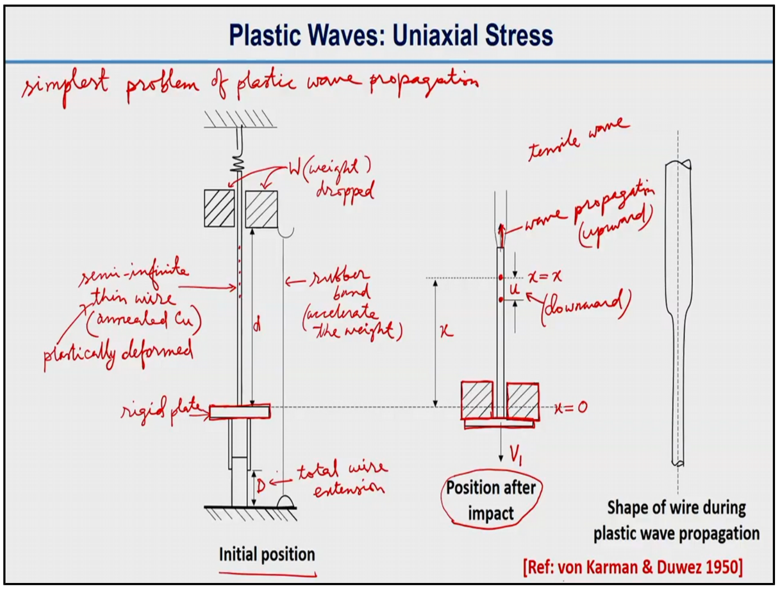

So, we are discussing about, the simplest problem of Plastic propagation, that is the experiment designed by, Von Karman and Duwez.

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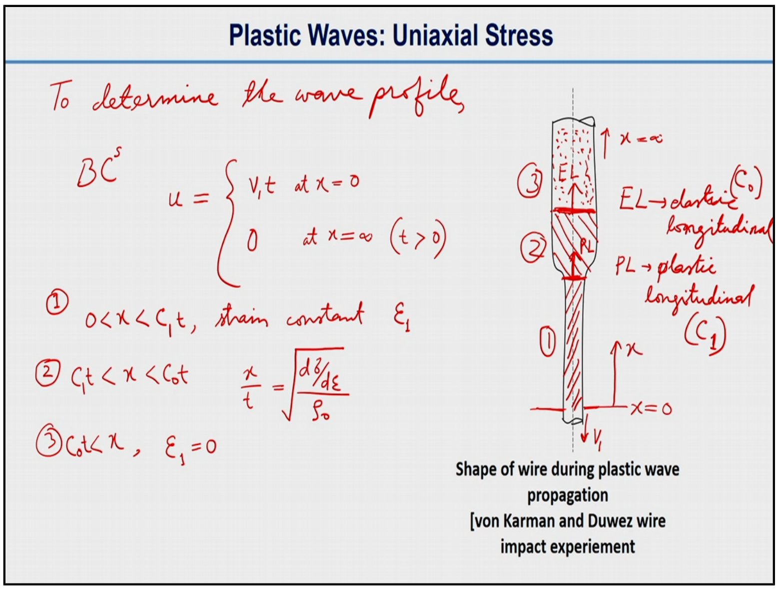

So, to get the Wave Profile, or to determine the Wave Profile, so we need to apply, the boundary conditions. I write BC, for boundary conditions. So, boundary conditions is equal to V1 T, which is, as we discussed earlier, that the V1, is that velocity of the plate, or when the weight will impact on the wire, impact the plate, so V1. And so, we have, suppose, this wire, this is X equal to 0.

So, at X equal to 0, we have the V1 velocity, that Impact Velocity. So, u is equal to the displacement V1 T, at X equal to 0. Or, it will be equal to 0, so displacement will be equal to 0, at X equal to infinity. So, that means, up above, towards this direction, X is infinity, at a far away from, this extremity of the wire. So, at Time equal to, greater than zero, it is not at zero. It is, at Time t is equal to, greater than zero, so we will get, displacement will be zero.

So, displacement is very less, at the top, or it is zero, and it will increase, in this direction. So now, we will see, the different cases, as we know that, Elastic longitudinal Wave Velocity, and the Plastic Longitudinal Wave Velocity, are different. And, Elastic Longitudinal Wave Velocity, is higher than, Plastic Longitudinal Wave Velocity. So, as we know that, this Elastic Wave will propagate, after the impact, and then, this we write it as, Elastic Longitudinal Wave Velocity.

So, EL will be, Elastic Longitudinal Wave. And then, the Plastic Longitudinal Wave, will be somewhere here. So, this is Plastic Longitudinal wave. So now, the Plastic Wave, this is the wave front, and this is going, in that direction. And, we know that, this portion, so what will happen in the first case, this region, when X equal to 0, to X equal to C1 T. Or, we will write, this way, so X equal to 0, to X equal to C1 T.

So, the Wave Velocities are for, Plastic Longitudinal Wave Velocities, it is C1, C subscript 1. And, for Elastic Longitudinal Wave Velocity, this is C subscript A0, actually C0. So, coming to this again, so in the region, zero to C1 T, so this region, so our strain will be constant, and that is, we are denoted as, Epsilon subscript 1, that is equal to Epsilon-1. And similarly, in the region number 2, so let us say, this region, so this is, X is greater than C1 T, smaller than C0 T.

So, in this region, X by t will be, equal to, d Sigma d Epsilon and Rho-0. This will be, we will have an expression like this. And then, the other region, higher than X, higher than C0 T, so X greater than C0 T, the strain will be equal to 0. Because, the Elastic Wave, does not reach, at this portion, so that is why, the strain will be equal to 0. And, the Plastic Wave, as you can see, the Plastic Wave is, following the Elastic Wave.

And, that is why, these two regions like, region number 1, 2, and 3, so the three different region, so we can see it from here, and also we know, this X is increasing in this direction. So, here we can see, that the material flow, that particle velocity towards the, downward direction, and but, wave is in the, upward direction. This is because, this is a Tensile Wave.

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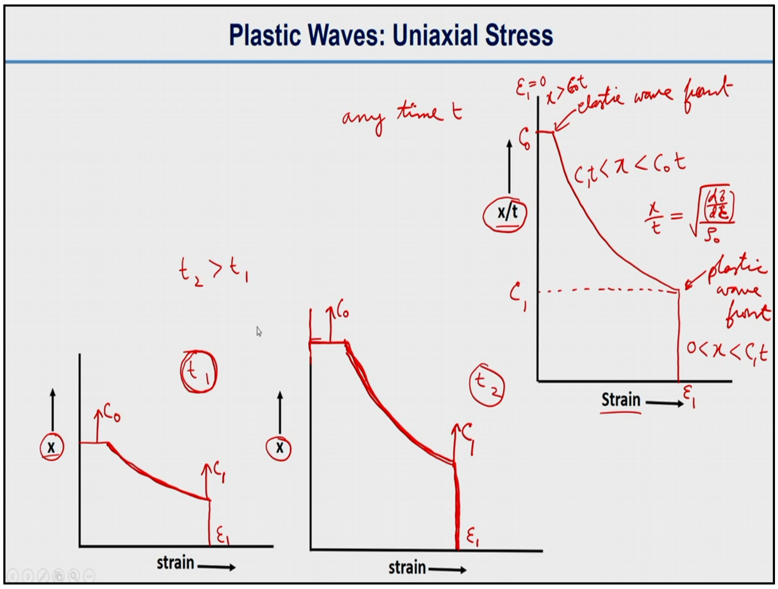

So, we will now discuss about the, Wave Profile. So, we will see the distribution, how the strain varies, with a function of, X by T. So, how strain will vary with, function of X by T. So, we know that, for a Plastic Wave, the strain will be, constant Epsilon-1. And then, so it will take a shape like this. And then, finally, this is C0 Elastic Longitudinal Wave Velocity, this is Plastic Longitudinal Wave Velocity. So, this is actually, Plastic Wave front, and this is, Elastic Wave front.

So, as we discuss, this is the Number-1 region, where 0 XC is X, take the value from 0 to C1 T. And, this region, where X is greater than C1 T, and smaller than C0 T, where we discuss that, X by T, will look like, this. We will take the expression, d Sigma by d Epsilon, divided by Rho-0 N, square root of the whole thing. And then, here, X greater than C0 T, strain will be equal to 0. So, it is at any Time T, and for different times, after the impact, we can get different Wave Profiles.

So, suppose at Time equal to T1, and Time T2, so we draw this. So, this will look like, this is at Epsilon-1, so this profile will, look like this. This is your, C1, and this is your, Elastic Wave Velocity, C0. So, this is the distance, that you can see, from the previous slide, so X, in this direction. And, here at Time T2, this is at, Time T1. So, at Time T2, so it will look like this. So, we will extend this part, little above , so this is C0, this is C1.

So, as we can see that, at Time T2, this has the same maximum strain. But, we can see that, this portion of the curve, it is like, spreading out. So, you can see the, difference of the Wave Profile in, for Time t equal to T1, and then, Time t equal to T2, where, your T2 is, greater than T1. So here, just important to mention that, this is what, we plotted here is, X Pi T, it can be for any Time T. And, then here, we are keeping on the X and the Y axis, in both the cases, and there are two times T1 and T2, where T2 is higher than T1.

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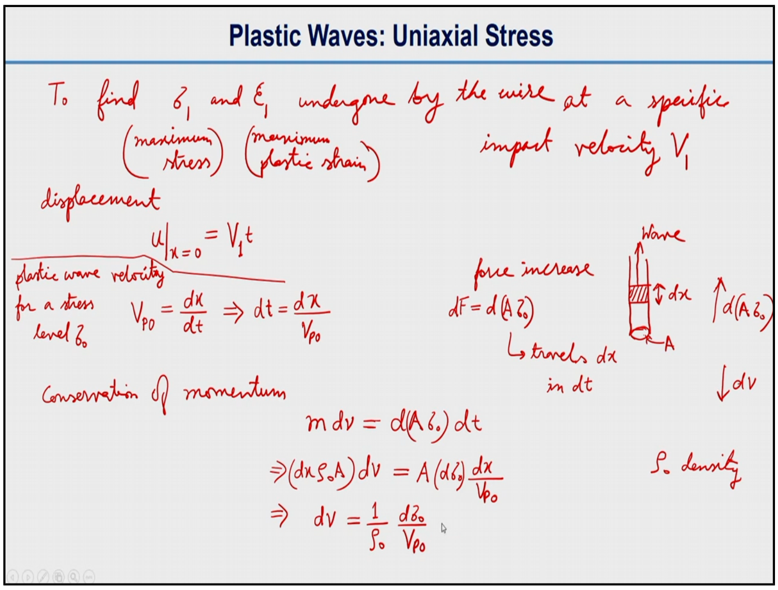

So, now, we want to find, the maximum stress and strain. Strain means, it is Plastic strain, undergone by the wire. So, this is the maximum stress, and this is maximum Plastic strain. So, we want to find, the maximum stress, and maximum Plastic strain, undergone by the wire, for a particular Impact Velocity, V1. And, we will now derive the expression, of the strain, in terms of, the Impact Velocity V1, V subscript 1.

And, we will again find the, the maximum permissible Impact Velocity, and that means, which value of the Impact Velocity, can produce the necking in the wire, so that means, making instability in the wire, so that is the maximum permissible Impact Velocity, that will get the derivation of that, very soon. But, before that, what we want to do is, we want to get the strain, in terms of the, Impact Velocity.

So, to find this, at a specific Impact Velocity V1, so that, we know that, earlier, we showed this V1, when the weight impacted the plate. This is the V1. So, what we will do is, we will try to get the displacement, of the bottom part of the wire. So, bottom part of the wire, that displacement, in terms of, as a displacement, as a function of Time, that means, in terms of, Impact Velocity.

So, displacement at X equal to 0, that is the bottom part of the wire, will be equal to, V subscript 1, that is the Impact Velocity, multiplied by Time. So, why we are doing this? Because, we want to get, the expression of maximum stress or the maximum strain. So, for that, we are starting from, displacement. So, displacement, we know that, the bottom part of the wires, displacement, we can find it out, from the Impact Velocity.

So, we need to consider here, the force increase. Suppose, we have the wire. We take a small element, here. So, we know that, there will be a force increase, that means, the force increase, which we will write as, DA Sigma-0, we will call it as, the force increase. Because, as the Plastic Wave or the Elastic Wave, is moving upward, that means, this is, let us say, the wave direction, so what will happen is, the force will increase, from the bottom part to, the top part.

And, we are taking a small element, DX here. And, we assume that, this force increase, travels the distance DX, in Time DT. So, we know that, the Plastic Wave Velocity, for a particular stress level, Sigma-0. So, let us say, we will write here, the Plastic Wave Velocity, for a particular stress level, Sigma-0, it can be written as, DX by DT. So, at Time DT, I mean that, wave crossed this DX distance, and during that, the force increase.

That is, we can write, DF, force increase equal to, DA Sigma-0, that is, area multiplied by the Sigma-0. Area is the, cross sectional area. So, this will be equal to, DX by DT. Now, V P0 is equal to, DX by DT. So, what we eventually will do is, so here, we will write, the displacement. So, what we eventually will do is, we will apply the conservation of momentum, for this small element.

So, this, we can relate this force increase, which is traveling upward. So, force increase, A Sigma-0 is traveling upward. And, we discussed that, it crosses the distance DX, in Time DT. So, if we apply the conservation of momentum, so then, it will be equal to, M DV DA Sigma-0 DT. So, this is, M is the mass of that element, and DV is the differential of the velocity, and so, which will be, equal to the force increase into, the Time required to, cover that distance, DX.

So, as we know that, this velocity, and DV, we have in the downward direction. So now, the mass of this segment, if we consider the, Rho-0 as the density. And, we know, the cross sectional area is, A. So, the mass will be written as, DX Rho-0 A, into V, the small V. We can write, this is small V. It looks like capital V, here. So, we can write it as, small V, which will be equal to, DA Sigma-0 DT. So, we actually can keep out this, the outside, is a constant, for that plastically deformed portion.

So, we can write it, this way, this Sigma-0. So then, we can, cancel out the, area term. So, what we can find from the above, that relation of Plastic Wave, V P0, so we can find that, DT equal to, DX by V P0 V P0. So, we are in this, we can use that, or rather, we will use it, here itself. So, this is DX V P0. Okay. So now, what we can do is, DV, we can cancel out, DX NA. And, that will give us, 1 by Rho-0 d Sigma-0, divided by, the Plastic Wave Velocity, at a stress level, the Sigma-0.

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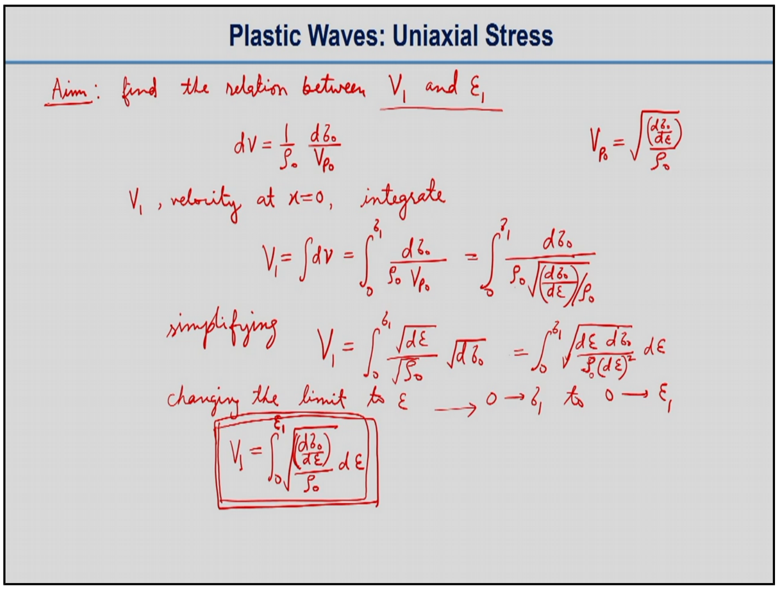

So, now our aim is to, find us a relation between, the Impact Velocity, and stress or strain. So, then we will, find the maximum permissible Impact Velocity, that means, which corresponds, relates to the necking of the wire. So, our aim is to, find the relation, between V1, and stress, Sigma or Epsilon. So, where we will write, let us say, V1 and Epsilon-1, we will get the relation for this, for Impact Velocity, with the strain. So now, from the earlier expression, so the earlier expression was, this expression.

The differential of the velocity, that is, DV, small V,1 by Rho-0, d Sigma-0 V P0. So, if you want to find the velocity, at X equal to 0, that is, V1. So, V1 is the velocity, at X equal to 0. So now, what we need to do is, we need to integrate the Wave equation. Why we are integrating? Because, the velocity is 0, at the top, at the maximum, at the bottom. So, if we integrate this differential of V, so what we can get is, the V1 is, small V, and that will be, with a limit of, 0 to Sigma-1.

So, this will be, d Sigma-0, divided by V P0, which will be again, 0 to Sigma-0. We know that, the Plastic Wave Velocity, V P0, can be written as, the square root of d Sigma-0 d Epsilon, divided by Rho-0, the Mass Density. So, then, what we can write is here, we have earlier expression, we have a Rho-0 here. So, this is Rho-0 d Sigma-0 d Epsilon, divided by Rho-0. And, this is, d Sigma-0.

So, if we simplify it, simplifying V1 is equal to, integration from 0 to Sigma-1, d Epsilon, divided by Rho-0 d Sigma-0. So, this can be, we can further, what we can do is, we can change the limit. Changing the limit, to Epsilon strain, that is, from 0 to Sigma-1 to, 0 to Epsilon-1. Because, we want to, find the relationship of V1, with respect to Epsilon-1. So, we want to change the limit to strain, instead of this stress. So, what we can find is, okay, we will do the calculations, here itself.

So, this is basically, 0 to Sigma-1, which will be like, d Epsilon and d Sigma-0, divided by Rho. So, what we can do is, we can get here, d Epsilon square. And, outside, we can get, one more d Epsilon, so that, in here, we can write it, in terms of, 0 to Epsilon-1, d Sigma-0, d Epsilon, Rho-0, the square root of the whole thing, d Epsilon. So, this is now, we have changed the limit. So, this is the relation, we got for V1, with a limit 0 to Epsilon-1. Sorry, I left its square root, over Rho-0, and please note that. And also, this Rho, so there have been, Rho-0.

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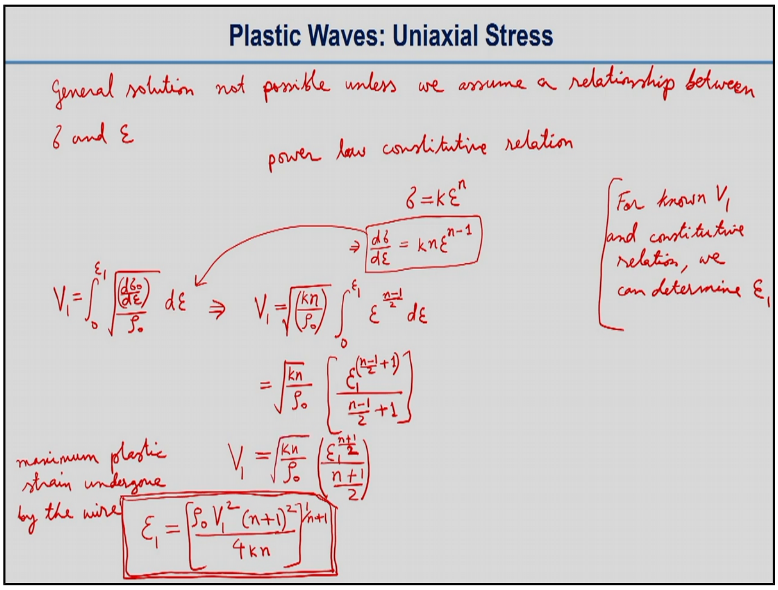

So, to get a general solution, we need to have the, stress and strain relationship, here. Because, general solution, is not possible, for the above equation, unless we assume a relationship between, stress and strain. So, we know that, the common power law constitutive relation, we will be using, that is, all of us know that this, K Epsilon to the power N.

So, Sigma equal to, K Epsilon to the power N, where earlier, we have discussed about this. So, this K and N are, two material constant. So, this is the relation between, the Sigma and Epsilon, in the Plastic region of the stress-strain curve, that we have already discussed, in a previous lecture. So basically, what we want to do it, from here is, for a known V1, and a constitutive relation, which we have just, we have written here.

So, what we can do is, we can determine the strain, that is, Epsilon-1 is the maximum Plastic strain, that undergone by the wire, so that expression will find it, in terms of the V1, that is the Impact Velocity. Now, let us see, how to get that. Because, we have this constitutive relation, which is Sigma equal to, K Epsilon to the power N. Now, we have the earlier relation, which we got this one.

So, we will try to use our constitutive relation, in this expression, and to find out, the Impact Velocity. So, from here, d Sigma by, d Epsilon by, we are doing this, because we need to know that, d Sigma by d Epsilon expression. So, this is, look like this. And then, from the expression in the previous page, so what we can do is, V1. Okay, I will write the expression, or what we have found, in the previous phase.

So, we found in the previous phase, that Impact Velocity expression is, 0 to Epsilon-1, write it like this, d Sigma-0, d Epsilon, divided by Rho-0, whole thing square root, d Epsilon. So, now from this, what we can do is, we can use these expressions, in these, you can use that, for the expression of V1. So, what we will get is, the K N and Rho-0, will come out of this, integral, so this will be, square root of this, and then integration, 0 to Epsilon-1, Epsilon to the power N – 1, divided by 2, d Epsilon.

So here, we know that, this expression will be, KN Rho-0. This, will be, Epsilon to the power N – 1, divided by 2, plus 1. So, better, we will make it look, more clear. So, divided by, N – 1, divided by 2 plus 1. So, we have a 1/2 limit from, 0 to Epsilon-1. So, directly, what you can do is, you can write this as, Epsilon-1. Because, the 0 will give a, 0 value here. So now, we can write, V1 KN Rho-0 N plus 1 divided by 2, and then, Epsilon-1 to the power N plus 1, divided by 2. Okay.

So, ultimately, we will express, Epsilon-1, in terms of V1, so the opposite one, why we are writing, V1 square. Because, we had a, 2 here. This is true. So, that will go, this side. So, this will be, V1 square, N plus 1 whole square, 4KN, the entire thing, to the power, 1 by N plus 1. So, this is the expression for, Maximum strain, or we will write, Maximum Plastic strain, undergone by the wire. So, this is the expression. Sorry, I have left, the square root, over KN divided by Rho-0.

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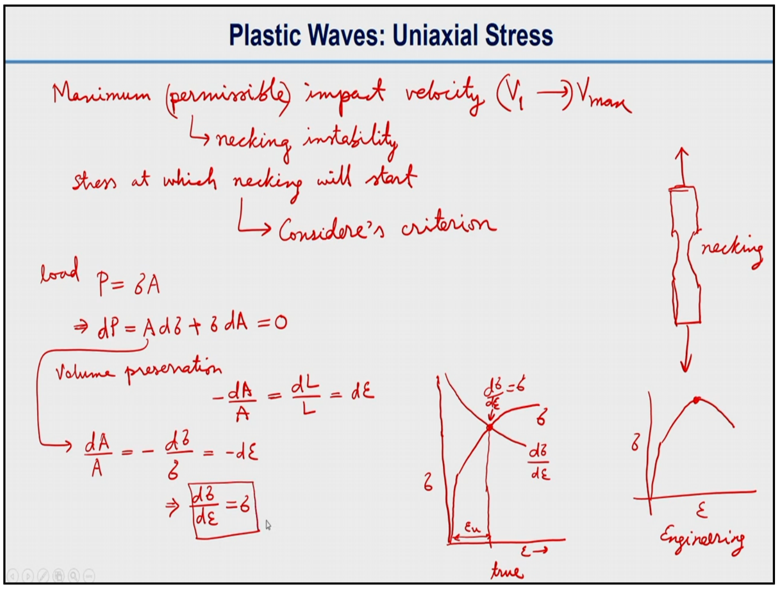

We want to find, the Maximum Permissible Impact Velocity, that is related to the, necking of the wire, that means, if we exceed that velocity, the necking instability will happen, to the wire. So, we do not want, necking in the wire. Because, that will eventually fail the material, so that is why, we want to calculate now, what is the maximum permissible velocity, that will not allow, any necking in the wire. So, Maximum Permissible Impact Velocity.

So, instead of V1, what we will write, V maximum. So, this is, nothing but the V1, we will write, V maximum, here. So, this is permissible, the word, this means, that corresponds to, necking instability. I hope you, all of you know, what necking. So, suppose, in your tensile test, with a universal testing machine, of a ductile material, when you are doing this tensile test, the ductile material will form a neck, in this region, so neck formation.

So, this is called, necking. And, in the engineering stress-strain curve, you know, for ductile material, so this ultimate tensile strength point will be, the point for the necking. So, we want to calculate the, V-max. For that, so we need to know, find a stress or strain, at which, the specimen will start to neck. Stress, at which, necking will start, can be found out, with something called, Considere's criterion.

Considere's criterion, is used, to get the stress at which, the necking will start. So, as we know that, the necking starts, at the maximum load. So now, as we know, the load P, is can be written as, Sigma-A, Sigma, multiplied by the cross sectional area. So, if we write differential of the load, which we can write as, AD Sigma plus Sigma DA, and DP will be equal to 0. Because, that happens, in the maximum load, the necking.

So, again from volume preservation, we know that, minus DA by A, the cross sectional area, equal to DL by L, is equal to, Epsilon, the string. Sorry. Yeah. This actually, d Epsilon, will give us the, d Epsilon. So, from this relation, so what we can get is, DA by A, equal to, minus d Sigma by Sigma, will be equal to, minus d Epsilon, from this relation, about volume preservation. So, now, from this, we can have a relation, d Sigma by d Epsilon, is equal to Sigma.

So, if we have it, this is what we have drawn is, engineering stress-strain curve. And, let us say, if we draw a true stress-strain curve, true Sigma, so the true stress-strain curve, let us say, will look like this. And so, this is the variation of Sigma. And then, the d Sigma by d Epsilon, which is also known as, the work hardening. So, this will look, something like this. This is, d Sigma by d Epsilon.

And, this point, where d Sigma by d Epsilon, is equal to Sigma, this point, corresponds to the ultimate strain, that is actually, the strain at necking. So now, we got this relation, d Sigma by d Epsilon, is equal to Sigma, at the point of necking, finally, to get our, maximum Impact Velocity. The maximum, we can call, permissible Impact Velocity, we want to do, something more.

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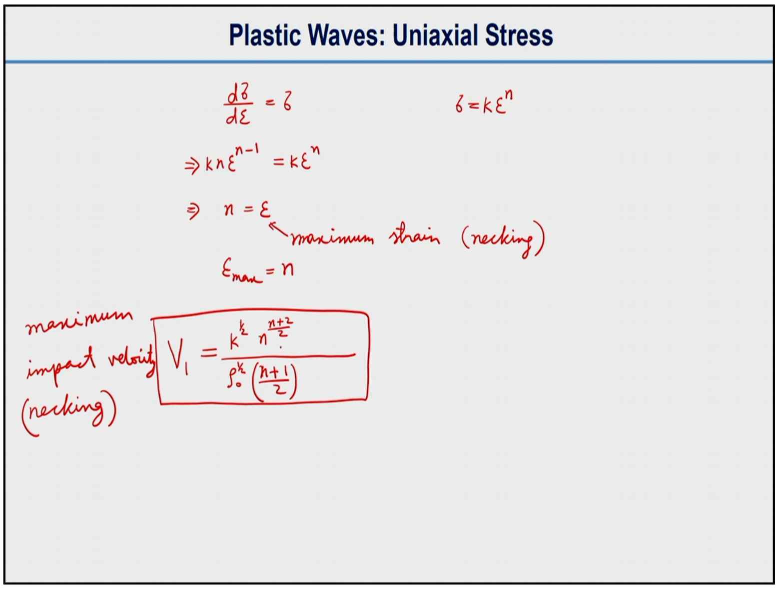

So, what we will do is, d Sigma by d Epsilon, is equal to Sigma. And, we know that, the Sigma, the constitutive relation is, Sigma equal to, K Epsilon to the power N. So here, if we use this, KN Epsilon, to the power – N, and then here, K Epsilon, to the power N, which will give us, K will be cancelled out, which will give us, N equal to Epsilon. So, basically, that this Epsilon is, the maximum strain at, that is, related to, necking.

Or, otherwise, we can write is, Epsilon maximum is equal to, and the maximum, which is related to necking. Okay. Before, going to that, I just wanted to tell you this, for engineering stress strain, so d Sigma by d Epsilon, will be different. This will be, zero. For, ultimate tensile strength, this will be equal to zero. So, what we had in the earlier expression is, we had the maximum Plastic strain, in terms of V1, that is, Impact Velocity.

We will use this expression, however, we need the other way around. So, we need the maximum permissible V1, that is the Impact Velocity, in terms of Epsilon-1. And then, we know that, the Epsilon can be replaced by M, that work hardening exponent. So, then the final V1, we can write it as, K to the power 1/2, N to the power N plus 2 by 2, Rho-0 1/2, to the power 1/2, and then, N plus 1 by 2. So, this is the expression, for V1.

So, this is the, maximum Impact Velocity, that means, that lead to, necking instability. Or, you can say that, this expression is, without any stress and strain term like, without any Sigma or Epsilon term, so we have only these constants, KN and Mass Density Rho-0. So, what we have discussed in this lecture is, Plastic Wave of Uniaxial Stress. So, we discussed about, the simplest experiment, designed by, Von Karman and Duwez.

So, we have derived the expression, for the maximum Plastic strain, experienced by the wire, in terms of the, Impact Velocity, V1. And also, we discussed, the maximum Impact Velocity, that can produce necking. And, this is all, for today's lecture. So, we will discuss about, Combined Plastic Shear Wave and Longitudinal Wave, in the next lecture. And also, we will discuss about, the Taylor's experiment. Thank you.