Dynamic Behaviour of Materials
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Module No. #04
Lecture No. #13
Taylor's Experiment for Plastic Wave Propagation 1

Hello everyone, so we have discussed about, Plastic Wave of Uniaxial Stress, and Plastic Wave of Combined Stress, that is, shear and longitudinal Plastic Wave. So, we have seen, different experiments to produce, Plastic Shear Wave, with Plastic Longitudinal Wave. So now, in this lecture, we will discuss about the Taylor's experiment, which is also known as, bar impact experiment, bar of finite length. And, this is a classic experiment, to test the dynamic constitutive behavior of materials.

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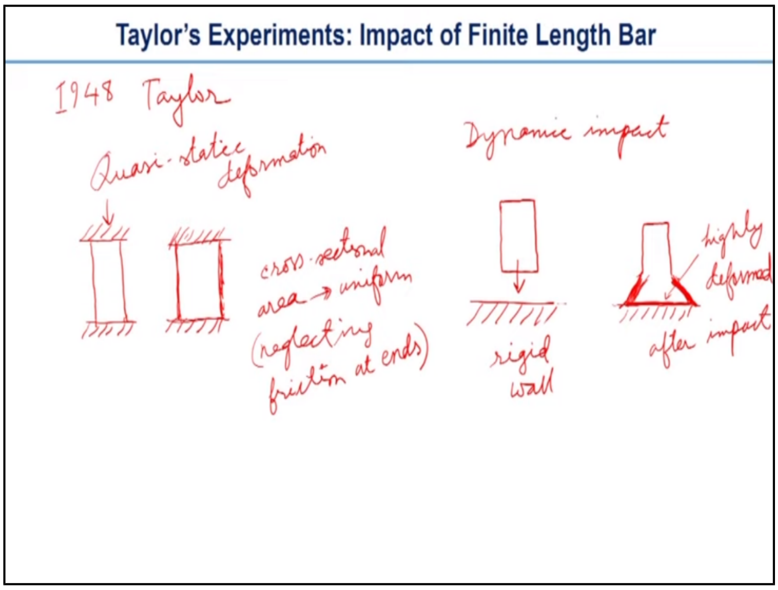

So, Taylor, in 1948, he developed first this experiment, during the World War II. So, Taylor described that, based on a sequence of events, that is, Elastic and Plastic Wave propagation. Before going details about that, we will try to understand, the difference between, Quasi static deformation, and Dynamic impact. So, suppose, we have a Quasi static compression here. This is a cylindrical bar, which is compressed.

Let us say, this bar is compressed, in the top side. So, after compression, what will happen is, the cross sectional area will be constant at any time, neglecting the friction at the ends. So, what will happen is, the cross section will be uniform, throughout the length. It got compressed, but cross sectional area will be uniform, throughout the length, during the test, and at any given time T, and we neglect the friction at these ends.

And so, now for Dynamic impact, let us assume that, there is a rigid wall. And, the cylindrical bar is impacting on this rigid wall. So, after the impact, the bar will take a shape like this. This is, after impact. So, the shape of the object, during the deformation, and after the deformation, they are not uniform. The part of the cylinder, which is close to the impact, that is, the part of the cylinder, that is undergoing the impact, with the rigid wall, so this will be highly deformed. So, if you see, this part, will be highly deformed, and other part, the deformation will be less.

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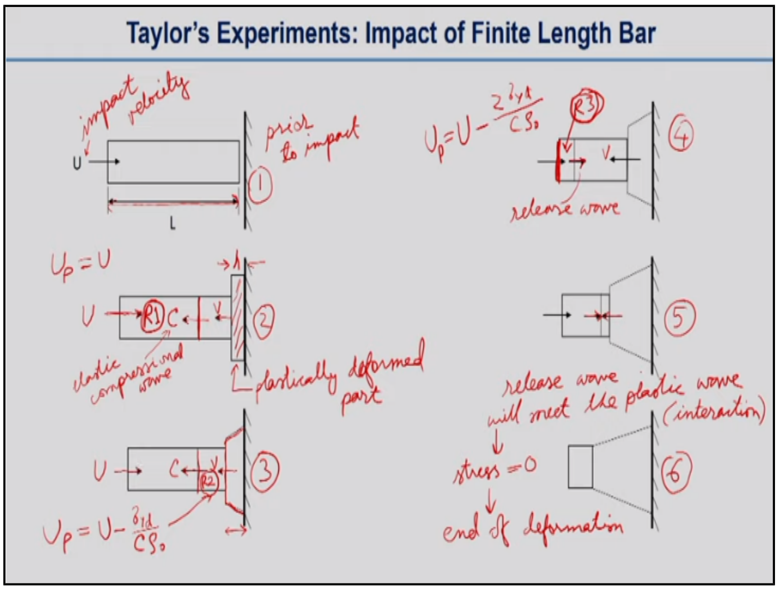

So, the Taylor described the test, as a sequence of Elastic and Plastic Wave propagation. So, you will see, the sequence here. Suppose, this is the rigid wall, and this is first case, is the prior to impact. So, let us assume, this is case number 1, 2, 3, 4, 5 and 6. So, these are the sequences. So, prior to impact, this is U, if the impact velocity, you can see, this is not touching now. So, impact velocity is U, the length of the bar is L. So, if you see that, the length is decreasing here, so after the impact, this is up and the impact, the length is decreasing.

It is decreasing gradually, if you see, the step 2, 3, 4, 5, 6, it is decreasing gradually. So this part is the plastically deformed part. If you see, the Stress Wave, will start from the wall surface, and this is the Elastic Wave Velocity C, which will be ahead of the Plastic Wave. The Plastic Wave Velocity, we are adding A v. And this, let us say, this portion is H, which is the length of the h, which is the length of the plastically deformed part. So here, we have, U. So, this is actually U, is the same as the particle velocity here, we will discuss about that later.

This Elastic Wave is a Compressional Wave. And, even the Plastic Wave is following, the Elastic Wave. So, at the third step, so what will happen is, this plastically deformed zone will be, now longer. So, you can see that, plastically deformed zone is now, thickness is more. And then, as we told earlier, this is the Elastic Wave Velocity Wave front, this one, and this is the Plastic Wave front, and this is U. So now, at this position, at step number 4, so what is happening here is, the Elastic Wave hits the free surface here, this free surface, back surface, and it is going back now

Though, it is going back now, Elastic Wave, this is we call a Release Wave, that is reflected back. And here, the Plastic Wave is still advancing, and at some point, what happened is, this reflected Elastic Wave, will meet the Plastic Wave, and then, at that point, that means, we will call, Release Wave, the reflected wave, Elastic Wave. The Release Wave, will meet the Plastic Wave, at that point, the stress will become 0. And, that is the end of deformation.

This is basically, interaction of the Release Wave, with the Plastic Wave. So, Release Wave is the reflected Elastic Wave, back from the back surface, this back surface. So basically, this is the interaction. And, right here, interaction of Release Wave, when plastic wave will lead to, reduce the stress to 0. And, that is the end of the deformation. Now, as you can see this, now the length of the bar is, slowly reducing from 1 to step 6.

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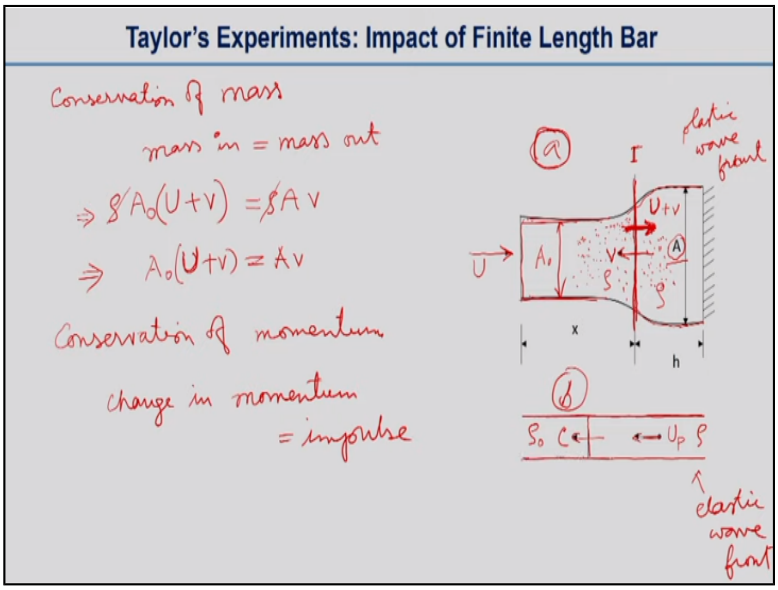

And, if we see in the details in this diagram, so we can see that, first one is the Plastic Wave front, and the second one is Elastic Wave front. Okay. So, this Plastic Wave front, you can see that, this is the plastically deformed area, this area is the plastically deformed area. Plastic Wave front is traveling at a velocity v. And, we know that, this is the area A, the cross sectional area. And, the area is A0, in this portion. This is the area, not the length.

We are writing this way. So, area A0, which is the smaller, than the area A, is in the plastically deformed part. So, as you know, this impact velocity was U. And, we will say that, this is the interface I, that is the interface between, the plastically deformed part, and the other part. Similarly, for Elastic Wave, we know that, when the Elastic Wave will go through this area, let us say, this area.

So, we are showing it in the second diagram, let us say, the first diagram is in diagram A, and the second one is diagram B. So here, what is happening is, the Elastic Wave front, with the velocity C, it is advancing towards the left. And, this is, let us say, the particle velocity U p. And, we can say that, the density is different here. This is Rho, and this is Rho0. So, it is the initial density, and Rho0, and Rho, because the compressive Elastic Wave, is changing the density.

But, in the other case, in the diagram A, we have the Rho, you know, on both the side of the interface are same. So, this Rho, and this Rho, they are the same, they are both equal. But, in the Elastic Wave case, we are assuming that, the compressive Elastic Wave, we reduce the density. So basically, in the right hand side, the Rho0 is the initial mass density, and the right hand side, because of the compressive wave travels through that part, and that is why, it compresses the material, making actually the density higher.

So now, we will try to derive some relations between, strain and stress, and impact velocity. So we will see, how to go forward. So, in the first case, this is diagram A, so we can have conservation of mass. So, the virgin material, that means, the material on the left side of the interface, so this material, moves into the interface, with a velocity U plus V.

So, that means, I will tell you, this is, mass in, will be equal to, mass out, for a conservation of mass. This velocity, so we twitch this, the thicker line, you know, so this v1 is the for Plastic Wave front. But, the mass, the virgin material, will enter into the plastically deformed part, that means, towards the right side of the interface, and the velocity will be, U plus v. Because, as you know, this impact velocity is U, and the interface is coming towards left side, with a velocity V, that is the Plastic Wave front velocity.

Or, what we can do is, we can write the V, somewhere here, to make it even clearer. So, now this part of the material, on the left side of the interface, will enter inside the interface, that is, towards the right side, with a velocity U plus V. Because, U is the velocity of the impact, and V is the velocity at which, the interface is coming towards the left side, so that means, on the relative velocity, so we have EU plus v, and multiplied by your area A0.

And, we have Rho. So, this is mass, Rho into, area into that velocity, and the mass in, and which will be equal to, on the right hand side, area will be A, Rho is same, and then this v, we call the material that is on the right hand side, that is going away from the interface, at a speed of v. Because, at the speed of v, the interface is going towards the left side, that means, the material on the right side, is going at a speed of v, towards the right side is going away from the interface.

So, this is the conservation of mass equation. So, as we know, both the density are same, in this case. And, so basically, we will end up with, A0 U plus v A v. So, after that, we will use the conservation of momentum, which means that, the change in momentum, is equal the impulse so, I have noticed that, these lines were not shown earlier, so you can draw this line.

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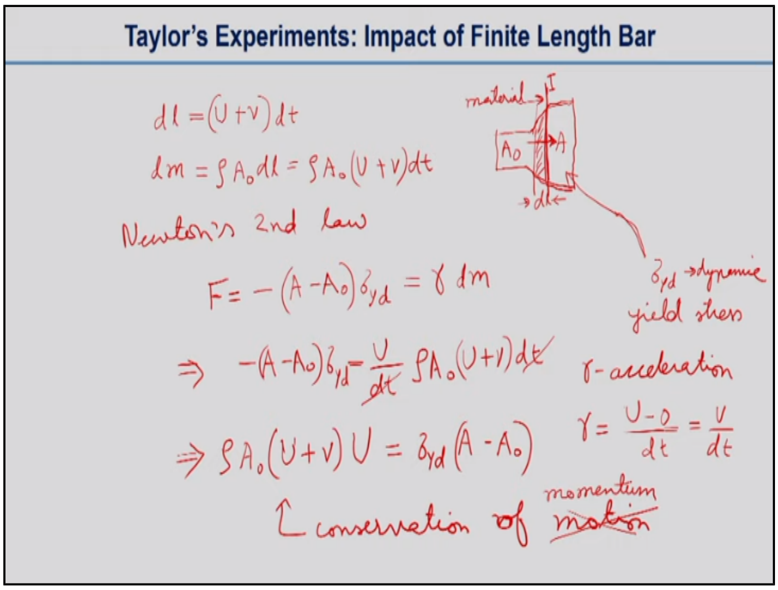

So now, it will be more clear to you, somehow due to some technical problem, these lines were not shown, and these arrows. So, we are discussing about, the conservation of momentum. So, we have the bar, with the plastic deformation, that means, bar impacted on a rigid wall, and this part is the plastically deformed part. And, let us assume that, as we know, we have this area A here, and area A0 here. And, let us assume that, we have the interface here.

Let us assume that, the material is entering, from left side to the right side, material flow. So, this small length d l, is the material entering into the plastically deformed area. So, d l will be equal to, as we discussed, this is U plus v, that is the relative velocity of the material, going inside the interface, inside means, from left to right of the interface. And, let us assume that, the time taken is d t, and also the mass of this small element, let us say, Rho A0 dl.

So, this is equal to, Rho A0 U plus v d t. So, yeah, let us assume that, this is the interface. So now, from the Newton's second law, so we have force equal to, acceleration into the mass. So, force here is A minus A0, into the stress, and that means, Sigma y d is nothing but, the dynamic yield stress. So, in the plastically deformed zone, so the stress within that region is constant, and that is Sigma y d. Because, the plastically deformed zone stress, does not exceed that stress limit.

So, this is dynamic yield stress, it is not static yield stress. So, yield stress at the highest strain rate. And, that is why, so across the interface I, so this force will be A minus A0 into Sigma y d, which is compressive, so it is actually minus sign. So, this is the force, which is equal to minus of A minus A0 Sigma y d, that will be equal to acceleration into mass.

So, acceleration, we will write Gamma, and the mass is d m. Gamma is the acceleration of that element. So now, we have this acceleration. So basically, the material is going inside, this plastically deformed part, and the initial velocity is U, and the final velocity is 0, that is, that happens in time d t. So, this will be equal to, U d t. Now, if you have this as Gamma U d t, and d m from the above expression, Rho A0 U v d t, and on the left hand side is, minus A minus A0 Sigma y d.

So, d t will get cancelled, and we can write this, by rearranging, U plus v U is equal to, Sigma y d A minus A0. So basically, this is the conservation of momentum. Sorry, this should be conservation of momentum, not motion. So now, we have two relations with us, one is the conservation of mass, initially we got that expression, and then another relation is from, conservation of momentum.

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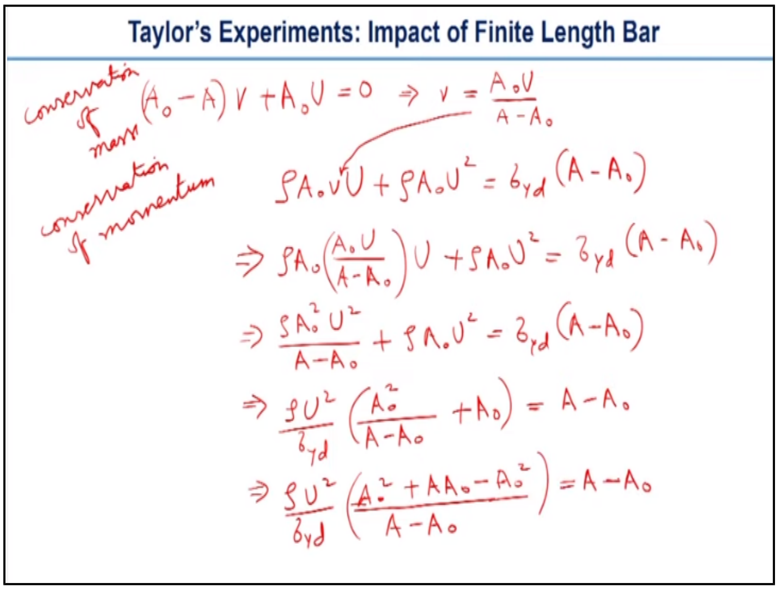

So, we will combine these two, to get a final expression. So, what we will do is, from conservation of mass, we found that, we will try to eliminate the v, from this two, A minus A0 v plus A0 U is equal to zero. And, if we have the v, del v equal to A0 U, A minus A0, this is conservation of mass. From conservation of momentum, what we can get is, Rho A0 v U plus Rho A0 U square, equal to Sigma y d A minus A0. So, if we substitute this v, with this, so we will find Rho A0, A0 U A minus A0, U plus Rho A0 U square, is equal to, Sigma y d A minus A0.

And then, little simplifying, A0 square, U square, A minus A0 plus Rho A0 U square sigma y d A minus A0. We will try to simplify it more, taking that term common, A0 square, A minus A0 plus, A0 A minus A0. So, we will take the term common, and then, we will again find this term, in terms of A and A0. So, how it will look like is, okay, we will keep it here itself, on the left hand side, A minus A0, A0 square, plus A A0 minus A0 square, okay, is equal to, A minus A0.

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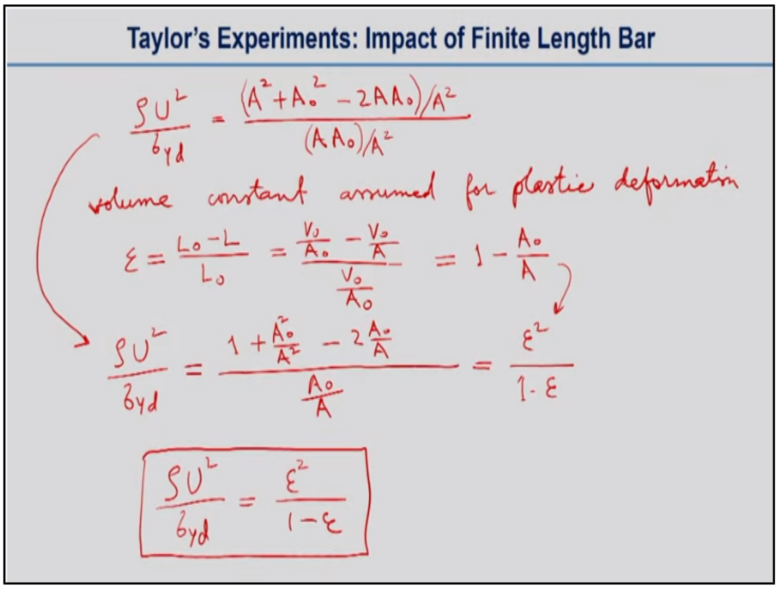

So, further rearranging, Sigma y d equal to, A square plus A0 square minus twice A, A A0. So, what we will do is, we know that, the volume is constant, to get the expression in terms of strain, so what we do is that, we know the volume is constrained during plastic deformation. So, we assume, it actually like that. So, assume for plastic deformation, which is not the case in Elastic deformation.

Plastic deformation, we can assume it as constant, strain is equal to L0 minus L. We know, this length of the bar, initial length, and the final length, divided by L0, which will be given as, V0 A0 minus V0 A, because volume is constant, divided by V0 A0, and that will give us, 1 minus A0 by A is the string. So now, in the earlier expression, this expression, what we can do now is, on the top, so what we can do is, we can divide, both numerator and denominator, by A square.

And then, we will see, if we can reduce that, to an expression of string. So, this above expression will give us, okay, 1 plus A0 A, this is square minus twice A0 A divided by A0 by

A, which will be equal to, and the above, it is actually from this, we can write as Epsilon square, and the below it is, one minus Epsilon. So now, we have arrived at the final expression.

So, we have an expression of the mass density, with the impact velocity, with a dynamic illustrious, which will be equal to, as a function of string. So, Epsilon divided by 1 minus Epsilon. Sorry, so we left the square here, that should be Epsilon square. Okay. So now, we will talk about the, Elastic region. But, already we have seen that, as we discuss in the figure B, in the bottom figure, so we have the Elastic Wave Velocity C, and particle velocity U P, and the densities are different. So, we can even draw that, here.

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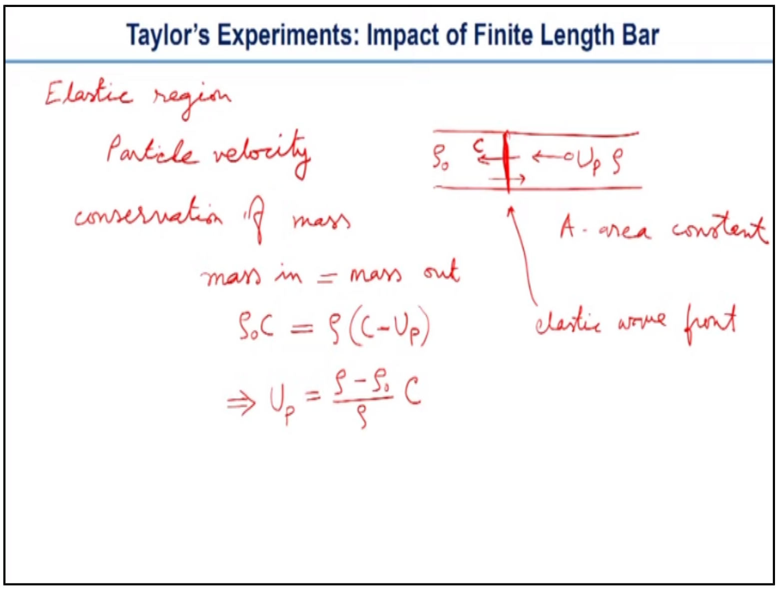

We can draw it here, again. So, the Elastic Wave front, which is moving from right to left, it is C is the Elastic Wave Velocity, and the particle velocity is, we are referring as U p. So now, initially, the mass density is Rho0. But, when the wave travels from right to left, so the right side of this wave front, where the compressed Elastic Wave already passed through, that the density will be little higher. This is the Rho, the other one is Rho0.

Now, we are talking about this Elastic region. So, we will talk about the particle velocity, in the Elastic region. So, conservation of mass here, in this case, you can see that, the area is constant, let us say, it is only A, so then the density are different. So, what will happen is, mass in, okay, we will write it like this. So, conservation of mass in, is equal to, mass out. So, mass going from the left side of this interface, to right is, Rho0 C.

Because, the interface is moving towards left, at a velocity C. And then, mass out is, that is the density Rho, multiplied by C minus U p, that is the relative velocity. Because, C is the velocity, that interface is moving towards, left from the right side. And, U p is also the velocity of the particles or materials, towards the left side. So, we will end up with this expression, for conservation of mass, for this Elastic Wave front, which you can see here. Okay.

So, from here actually, we can get the particle velocity, in terms of density and Wave Velocity, which will be, Rho minus Rho0 divided by Rho, into C, that Elastic Longitudinal Wave Velocity. So, this is equal to, we will see, what is the relation of this, with the strain.

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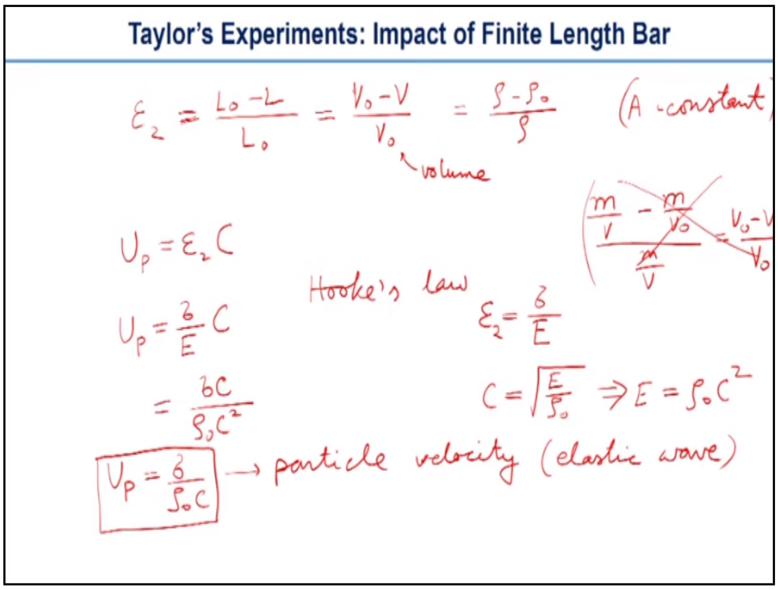

So, now we will try to have the strain expression, and relate with this particle velocity. So, the strain, we write it as Epsilon 2, which is because, the earlier we wrote Epsilon 1, and this is for the Plastic Wave front, we are trying to get the expression, so that is why, we are writing as Epsilon 2, and that will be equal to, L0 minus L divided by L0, that is the length of the bar. So, and this will be equal to, V0 minus V divided by V0. This is the volume actually, not any velocity.

And, we know that, the volume is not constant, as we have seen that, we already discussed that, the density will be different, and volume will be different. So, this will give us actually, Rho minus Rho0 as Rho, because area is constant. If you are wondering, how we get this from volume to density, so it is like, if you start from here, Rho minus Rho0, that will be your mass into V minus your mass, the same mass, by V0, and divided by, that is M by V, so which will be equal to, V0 minus V divided by V0.

Yes, that will be, will lead to this, if we do that. So, this is not needed. So, just to tell you that, if it is V0 minus V, it will be opposite Rho minus Rho0 divided by Rho. So, now we understood that, the earlier expression, what we got is, Rho minus Rho0 divided by Rho, which is actually related to strain, so that is Epsilon 2. So then, what we can get that, U p is equal to Epsilon 2 into C, and the material is Elastic, we can use Hooke's law.

We are talking about, Elastic Wave propagation. So, we can use, Hooke's law, which will give you, we will write Epsilon 2 here, so Epsilon Sigma by E, that is the Young's modulus. So, that is, from here now, you can write, U p is equal to Sigma by E and C. So again, we know that, the Elastic Wave Velocity C is equal to, Young's square root of the ratio of Young's modulus by density, which is Rho0 the initial density in this case, and that is why, or we can write even, E as Rho0 C square.

So, in this case, so if we write Rho0 C square here, and then Sigma C, so U p will be Sigma Rho0 C. So, this is the particle velocity, due to the Elastic Wave. But, if you consider the impact velocity U, then the particle velocity will be different. And then, we can find out the particle velocity, for three different regions in the, if we see the sequence of steps, earlier.

So, what will happen, when this is in a region number 1, let us say. Or, do not be confused with the other numbers, 1,2,3,4. So, this is region one, or better I would write R1, Region 1 here. This case is, when the Elastic Wave is, has not crossed that region yet. And, this is the Region number 1. And, the Region number 2 is, somewhere here, R2, that is inside the Elastic Wave region, that means, that is on the right hand side of the Elastic Wave front.

And then, third region is, let us assume that, after the Release Wave reflects back from the back surface, let us assume, this is R3, Region 3, so that is towards the back surface. This is the region, near the back surface, when the Elastic Wave is already reflected back, from the back surface. So, now these three regions, if we see the particle velocity, in the Region 1, R1, the particle velocity is nothing but the impact velocity.

Because, the Elastic Wave has not reached that part, and only the impact velocity, will be the particle velocity here. And, what happens for the Region 2, so the particle velocity U p will be equal to, U minus sigma dynamic yield strength actually, divided by C Rho0, whatever that expression we have just now derived, so that is the expression of particle velocity, due to the Elastic Wave propagation. But, in this case, because not only the Elastic Wave propagation, this term, but also the impact velocity will have a role.

So, that is why, the resultant will be, U minus Sigma, this is actually Sigma subscript y d. This N in the Region 3, so what will happen is, when the particle at the free surface, will have actually twice the velocity, that we have derive for Elastic Wave profile, that means, because that wave reflects back, from the back surface, that is why, for the particle velocity, for the Elastic Wave, that will be twice of Sigma y d divided by C Rho0.

So, this is the particle velocity, due to the Elastic Wave. And, that is because of the wave comes towards left, and reflects back from the surface. And, that is why, this will be twice of that. And, with that, the impact velocity will be that, the resultant will be the particle velocity is equal to, the impact velocity U minus, Sigma y d by C Rho0. So, there are what we learned here is, so we have the Region 1, and Region 2, and the Region 3 here, so we have different particle velocity.

So, in the Region 1, it is only the impact velocity of the bar, and in Region 2 and 3, we have both, the impact velocity, and the particle velocity, contribution from the Elastic Wave is contributing to it. Sorry, some of the lines here, in the earlier slides, specially the arrows were not shown properly, that is probably, some technical difficulty with the machine, so I am inserting the slide again. So, I have noticed that, especially these arrows, were not shown, in that earlier in the recording.

So, I have to insert it again, to show these arrows. So, I hope, that will clarify your confusion. I think, sorry about that, that is due to, some technical difficulty of the machine. So, that is all for this lecture. So, we will continue this discussion, that is, this Taylor's experiment, which is actually the experiment, for the constitutive behaviour of material, at high strain rate, and that is nothing but the impact of bars of finite length. So, we will continue this discussion, to the next lecture. Thank you.