Dynamic Behaviour of Materials
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Module No. #03
Lecture No. #10
Introduction to Plastic Waves

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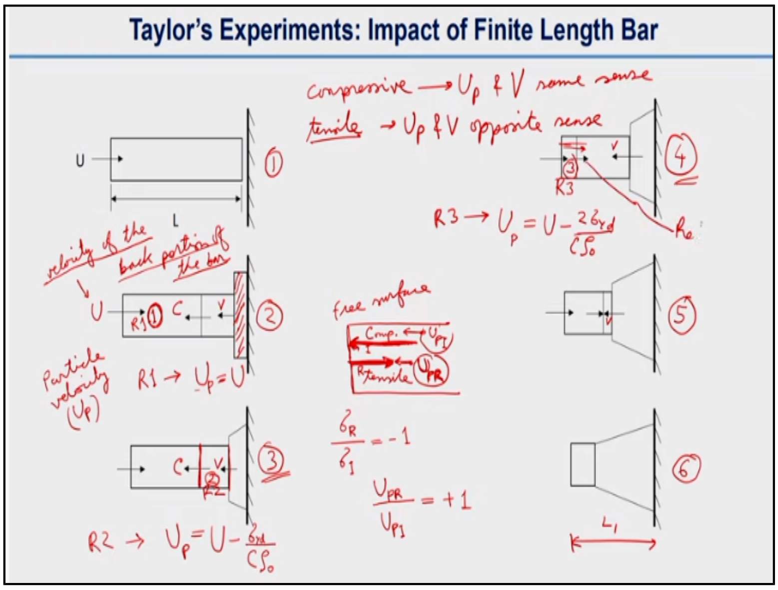

Hello everyone, so in the last lecture, so we have discussed about the Taylor's experiment, that is, impact of a bar of finite length. So, we will continue, this discussion. So, what we discussed in the last class is, the particle Velocity of that impacted bar, at different point of time. So, we will again go through that discussion, a little bit again. So basically, what we have here is, that there are different steps, or sequence, number 1, number 2, number 3, 4, 5 and 6.

So, we remember that, this C is the Elastic Wave Velocity, and the plastically deformed part, in step number two, that plastically deformed part will move at a Velocity V, which is the Plastic Wave Velocity. And, so what we did in the last class, so we found out that, for the Region 1, here I wrote, Region 1, or we can write it as, R1. So, I think in the last class we had, this was written as R1, this Region 1, and then R2, in this case, and then R3. So, we will write, R1, R2, R3.

So, the particle Velocity, for R1 case is, just U, which is U, is your Impact Velocity, and that is the Velocity of the back portion of the projectile, or that impact bar. So, this is actually you can see, that Velocity of the back portion of the bar. So, that is the Impact Velocity. So, for region R1, the particle Velocity is U. Or, we can write, U p equal to U, that means, U p is the particle Velocity, general symbol for other particle Velocity. For region 2, for R2, we can see that, the region is behind the Elastic Wave front, which is Elastic Wave front, is this one, and this is Plastic Wave front.

So, this, in between the Elastic and Plastic Wave front. So now, for particle Velocity U p here will be, U minus. So, the particle Velocity from the Elastic Wave, that we calculated as, Sigma C Rho0. So, Sigma is nothing but the, dynamic yield strength here, so this one. And then, if we talk about region R3, so then what is happening here is, so we have particle Velocity will be, that we talked that, U minus twice Sigma yd Rho0 C.

Why it is, twice Sigma yd, we did not discuss much about this. So, what happens, when the wave travels, you can see from sequence 3 to sequence 4, what happens the wave travels, and then come back from the free surface, so what happens, if we draw it here, this thing in a bigger way, so what happens is, this is the wave, which is compressive, I will write C, I will write compressive wave, and this is the free surface.

So, this free surface, will reflect the way back, and that will be a Tensile Wave. So, this will be a Tensile Elastic Wave. So, in the earlier discussion, in an earlier lecture, we show that, this reflected wave, divided by this Incident Wave, that the stress is minus 1, that means, for free surface, a compressive wave will reflect back, as a Tensile Wave. However, we also discussed that, the particle Velocity of the reflected wave, that is, relation with the particle Velocity of the Incident Wave is, plus 1.

So, that means, whatever particle Velocity here, that is U pi, and then U pr, the reflected wave, so this is the Incident Wave, and this one is the reflected wave, so here also, the particle Velocity will be, in this direction, and this will be U pr. So, we can see that, both the sense of the reflected and the incident particle Velocity, is the same. Or otherwise, so we discussed that, for compressive wave, U p and wave Velocity.

Let us say, we write as, wave Velocity as V, they have same sense, and for Tensile Wave U p and V, has opposite sense, that we discussed earlier. So, here you can see that, for the compressive wave, that particle Velocity, and the compressive wave Velocity. But, this is the propagation of wave direction, for compressive wave, so that, U pi and this Incident Wave, is in the same direction. Similarly, for the reflected wave, the Tensile Wave propagation direction, is opposite to the U pr direction. So, that is what, the above statement says that, for Tensile Wave, the U p and VR, opposite ends.

So, it is because, both U pi and U pr in the same direction, so we need to subtract that from the U, which is the Impact Velocity, or as we discussed, that is the Velocity of the back portion of the bar. So, ultimately, if we want to know the particle Velocity, inside the Region 3, R3, which is actually left of that reflected or Release Wave front, this is the reflected wave front is, so-called release wave, then it will be, U minus twice of the term Sigma yd C by Rho0.

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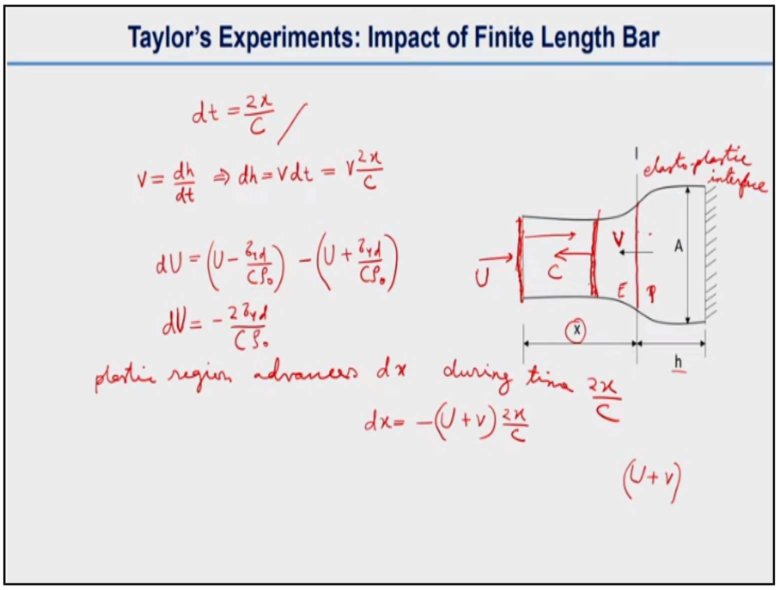

So, now we will go ahead, and we will discuss that, something else, I mean, we are continuing it. So, we want to see, how much time it takes, for this Elastic Wave, this is the Elastic Wave front, how much time it takes, from this plastic boundary, that is, that plastic boundary means, we can call elastoplastic interface, or you can simply call, plastic boundary interface or boundary, so because, this side is plastic, and this side is elastic, so how much time it takes, for the Elastic Wave front, to go forward, and reflect back at the free surface, reflect back and to reach again, in the elastoplastic boundary.

And, that is, we can call the time taken is, DT twice X by C. So, this is X. So, that is, time taken to, by the Elastic Wave front, to travel from elastoplastic interface, to the free surface, and reflecting again to the plastic boundary, or elastoplastic interface. So, this is twice X by C. And also, for Velocity of Plastic Wave propagation, that is V, we know that can be written as, DH by DT, because h is that thickness of the plastic zone. So, we are talking of the same time interval DT.

So, suppose, we have DH equal to v DT, at the same time interval, so we will see that, DH is the plastically deformed region thickness, at time DT, so which we can write is, from the above relation, we can write it as, V twice X divided by C. And also, if you want to know that, du, that is the change in Impact Velocity, or change in the Velocity of this back portion of the impact on the bar, so that will be equal to something like, U minus Sigma yd C Rho0, minus which is, before reflection, and then, this will be equal to Sigma yd C Rho0, so which is after reflection.

So, this will give us as, D U equal to twice Sigma yd C Rho0. So, this is one relation. And then, the plastic region advances a distance DX, during time DT, that is, let us say, twice X by C, here from the above. So, that we can write as, DX minus U plus V multiplied by the time. How we got this? That is, actually we earlier discussed, the mass going into, from the left side of the boundary, to the right side.

The virgin material goes inside the plastically deformed part, that is from conservation of mass, we showed that, the mass in, is U plus V, which is because, the back surface is going at a Velocity U, and this interface is coming at a Velocity v, so if you want to know that mass going inside this plastic region, is actually the U plus V, because the elastoplastic interface is going towards left, and then this back portion of this bar is going towards right, at a Velocity U.

So, this will be, U plus V, we are writing minus because, we are writing that plastic region advances, that is right to left, so it is minus U plus V, which is opposite to the Impact Velocity, so multiplied by the time twice X by C.

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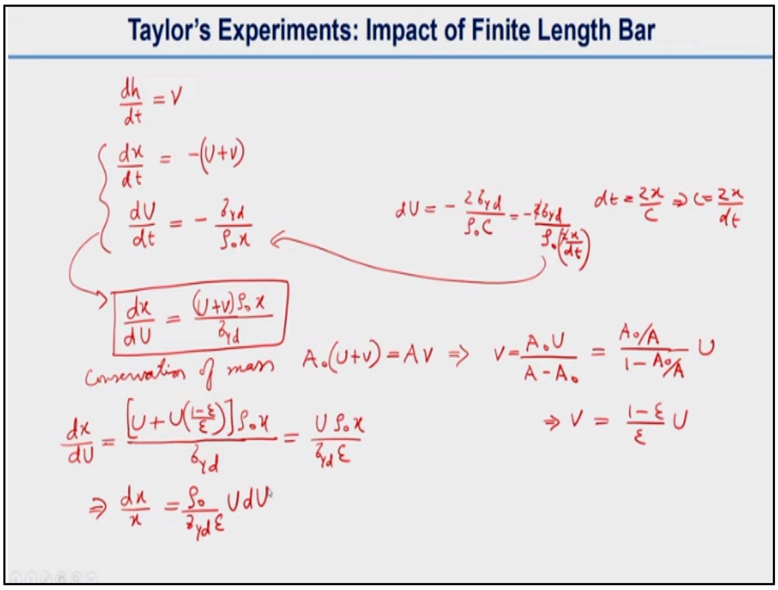

So now, from this discussions, what we can know is, DH by DT. So, DH by DT equal to V, which is a Plastic Wave Propagation Velocity, the definition. And then, DX by DT, which we just found out, U plus V. This is nothing, but this relation. And then, we have du, DT, which will be equal to, Sigma yd Rho0 X.

How we got this relation? And, it is actually, we have du equal to twice Sigma yd divided by Rho0 C. And, if we replace the C, because C is nothing but, we got that, DT equal to twice by C, or we can write that, C equal to twice X by DT. So, if we replace this, let us say, C minus twice Sigma yd, divided by Rho0 twice X DT, and what we will get is, here from we can cancel out the two, and then finally we can end up with, this relation.

So, from these two relations, what we can get is, DX du is equal to, U plus v Rho0 X, divided by Sigma yd. So, from conservation of mass, we know that, what we discussed in the earlier classes, plus A0 v Av, so that, why the area is different, because the plastically deformed part, and the other part, has a different area. So, this can be, we can found out that, v is equal to A0 U, divided by A minus A0.

And, in terms of strain, so what we can do is, A0 by A, 1 minus A0 A, multiplied by U, and this will give you, 1 minus Epsilon into Uv. So, what we got is, Plastic Wave Propagation Velocity v, in terms of strain, and the Impact Velocity U. So, what we will do is, we will substitute this v, in this above equation, so that will give us, DX du will be equal to, U plus U 1 minus Epsilon divided by Epsilon, this Rho0 X divided by Sigma yd.

So, this part, U plus U 1 minus Epsilon divided by Epsilon, will give us, U by Epsilon. And then, finally the expression will be, Sigma yd, Epsilon Rho0 X. So, we can write as, DX by X Rho0 Sigma yd Epsilon U du

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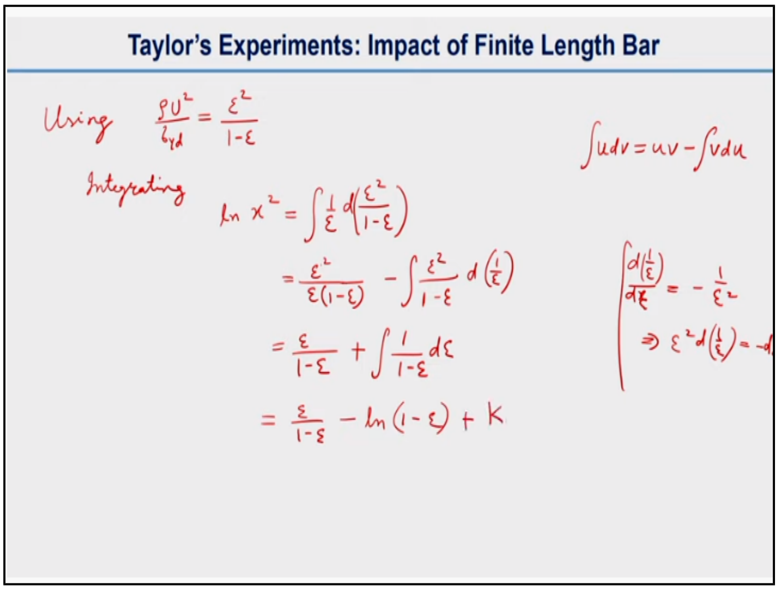

So, this expression, if we can use one earlier expression, which is Rho U square, Sigma yd, is equal to, Epsilon square 1 minus Epsilon, and if we can replace U, from this relation to the other relation, and then if we integrate, what we can do is, using this relation, in the earlier relation, what we got in the earlier slide, and then if we integrate it, so I am writing the final expression, I am not going into the detailed derivation of this, just left few steps.

And, then finally, what we can get is, LN of X square integration, 1 by Epsilon, D Epsilon square, 1 minus Epsilon, which will give us, integration by parts, that will give us as, which is just integration, that is, U dv is equal to Uv minus v du, so applying that, what we will get is, Epsilon square, Epsilon into 1 minus Epsilon, minus Epsilon square minus 1 minus Epsilon, and D 1 by Epsilon.

So, actually D Epsilon will be, minus 1 by Epsilon square, that means, what we can get is, Epsilon square, D 1 by Epsilon, differential of this, and is equal to, minus differential of Epsilon, so that way, we can write it as, 1 minus Epsilon plus, 1, 1 minus Epsilon, D Epsilon. And, that can be written as again, Epsilon, when we do the integration, with a constant of integration K.

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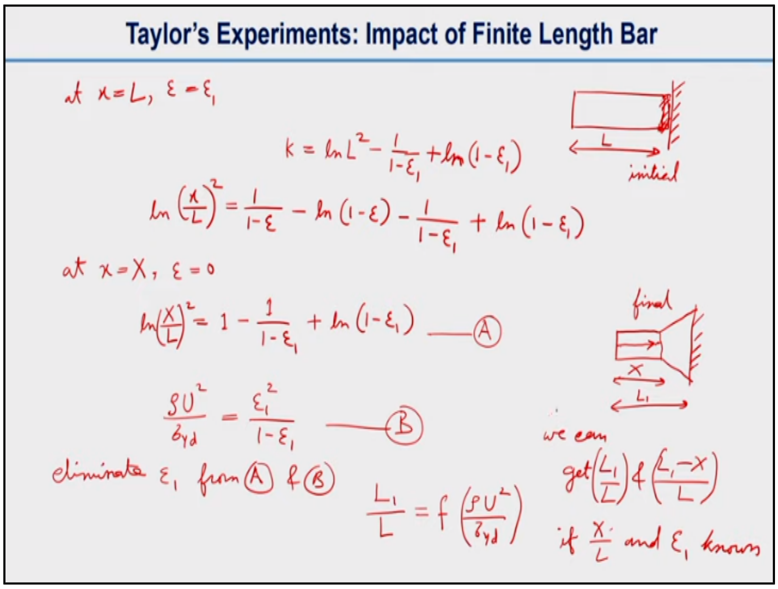

And, we will use some boundary conditions here, like, at X equal to L. L means, first the initial length of the bar, when impacting the rigid wall. So, this length is L. So, when it impacts the rigid wall, so let us say, the strain is, Epsilon equal to Epsilon 1. And, let us assume that, Epsilon is equal to Epsilon 1. And then, if we use that relation, so which will give us as, you know, the constant of integration K, which will be equal to, Ln L square, minus 1 minus 1 by Epsilon 1 plus Ln 1 minus Epsilon 1.

And, that will give us, if we replace that in the earlier expressions, so we will end up with, Ln x L, whole thing to the square, 1, 1 minus Epsilon minus Ln, 1 minus Epsilon minus, 1 minus Epsilon 1. There is a constant of integration. So, 1 minus Epsilon 1. So, again, at x equal to X, so Epsilon equal to 0, then this actually the end of the deformation, when the Elastic Wave, that is released from the free surface, that is back surface of the impacted projectile, the bar.

So, that okay, I will draw it here. Suppose, when this is the rigid wall, so the final shape of the projectile will be, like this. So, what happens, when the Release Wave, goes back from the free surface, this is the Release Wave, and then meet the Plastic Wave here, so then the deformation will end, and the strain will be 0, at that point. So, this distance, we can call it as X, and then, total distance is L1. This is the initial configuration, and this is the final configuration.

So now, if x equal to X, and Epsilon equal to 0, so then what we can get, this relation can be written like, 1 minus, because Epsilon equal to 0, in the above expression, 1 minus 1 divided by the 1 minus Epsilon 1 plus Ln of 1 minus Epsilon 1. From an earlier expression, what we know is, Rho U Square Sigma yd, which is dynamical strength, is equal to, Sigma 1 square divided by 1 minus Epsilon 1.

So now, what we can do is, if we eliminate Epsilon 1, from these relations, this A and B, if we try to eliminate Epsilon1 from A and B, so we will end up with an expression of L1 L2, we can find out alone at that final length, and divide by L, is a function of Rho U Square Sigma yd is a function of this. So, Taylor actually plotted this, and prove this model with experiment, so we can get both L1 by L and L1 minus X divided by L. So, we can get these, if X by L and Epsilon 1, known.

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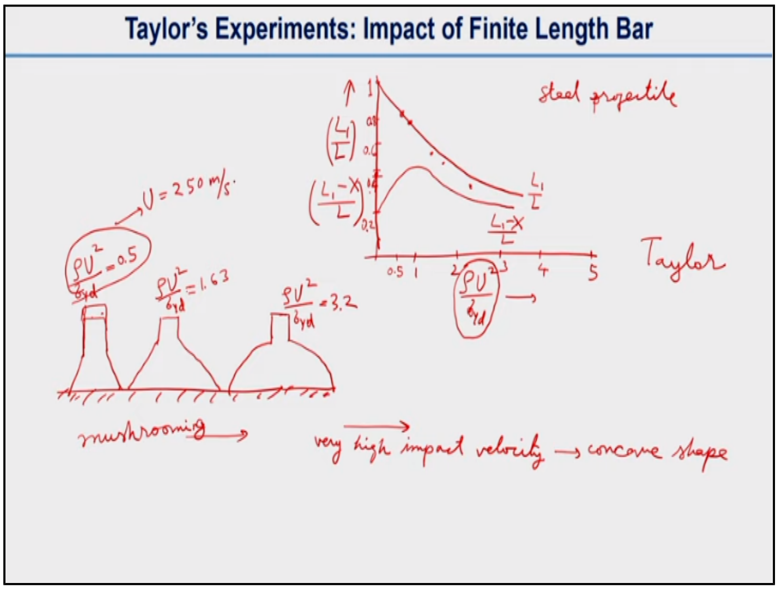

So, what we know from the Taylor's plot is, Rho U Square Sigma yd, which is a 1, 2, 3, 4, 5, and in this direction, L1L, or maybe, we can plot even, L 1 minus X by L, both, we can plot in the Y direction. So, for L1L plot, this looks, something like this. So, this is L 1 by L, and this is 1, let us say 0.2, 0.4, 0.6, 0.8. And, also for L 1 minus X 2, this will look like, something like this. So, this is L 1 minus X by L.

So basically, whatever the model, the theory, Taylor proposed, and the experiments observed results, very close to the prediction is very good, when this value is Rho U square by Sigma yd is around 0.5. But, if you go away, and so this will show some deviation, so the prediction is not so good. It is something like this. These are all, from the Taylor's original experiments, and then theoretical predictions.

So basically, if Rho U square divided by Sigma yd is equal to 0.5, the shape will look like this, and the prediction is good. But then, when it has higher value, so Rho U Square Sigma yd is equal to, let us say, this is the wall, let us say, so if it is equal to 1.63, so it will be look like this. And, if it is even higher value, so it will take a, maybe this kind of convex shape, and this is Rho U Square Sigma yd is 3.2.

So now, this way, we call this as Mushrooming effect. The shape of the bar, will look like this. And, although it is little not exactly correct, so it may be the first one, we can draw a little longer. So, if we go this way, and if the Impact Velocity is very high, so this is increasing Impact Velocity, if it is very high Impact Velocity, the shape of the deformed projectile or bar will, can be very different.

And, it can even take a concave shape. So, for this Taylor's experiment is for steel projectile, or steel bar. And, for a steel bar, this value 0.5 is can be, the U the Impact Velocity is like, 250 meter per second. So, you can imagine, how fast it will be, for the other cases. One important result of the Taylor, this analysis or the Taylor's experiment is, the Velocity of Plastic Wave propagation.

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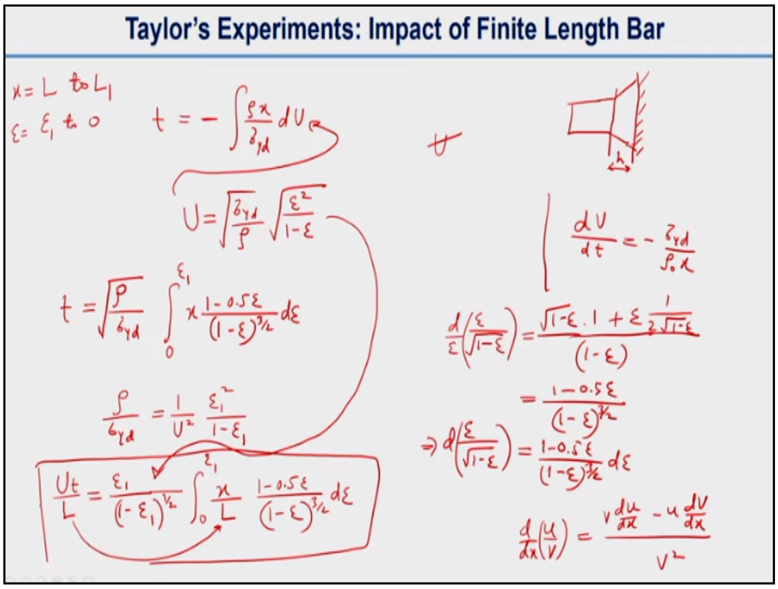

So, Velocity of Plastic Wave propagation, we wrote it as, DH by DT. So, that is nothing but, the change in the total distance, of the elastoplastic interface, from the impact plane, that means, whenever you have some deformation like this, so this elastoplastic interface, so distance here is H, so this distance, how fast it is moving away from the impact plane, and it will give us the Plastic Wave Velocity. So now, the total time from the, initial length L, to finite length L1, so the total time is like, what we can found out, from this is, Rho X Sigma yd du.

So, how you got is, earlier one expression, we know that, du by DT is like, Sigma yd Rho0 X. So, we assuming Rho here, not Rho0, let us say. So, that is the expression, we got. So, this is the total time, and from another equation, we have the U, in terms of strain, what we found is, Sigma yd by Rho and Epsilon square 1 minus Epsilon, entire thing square root. So, what we can find is, if you replace this one, U, in this du, so what will happen is, D Epsilon, Epsilon 1 minus Epsilon.

So, this will take a shape like, it is like D Uv DX, which will be like, V square V du DX minus U dv DX, so that way, this expression will be like, 1 minus Epsilon into 1 plus 1 minus Epsilon, which is square of the square root, and then, E1 by twice 1 minus Epsilon. So, this will give us something like, 1 minus 0.5 Epsilon, divided by 1 minus Epsilon to the power 3 by 2. And, so basically, D E1 minus Epsilon square, will be equal to 1 minus 0.5 Epsilon, divided by 1 Epsilon to the power 3 by 2 D Epsilon.

So, finally if we want to get an expression of the T, which will look like, Rho by Sigma yd, square root of the whole thing, into multiplied by the integration of X, will be here, and 1 minus 0.5 Epsilon 1 minus Epsilon 3 by 2 D Epsilon, and the entire thing will take a limit from 0 to Epsilon 1, because at X equal to L, the strain is Epsilon, and then X equal to Ln, and the final strain will be equal to 0. This will be, 0. And, we have a minus sign, here.

So, this will be, now we can take, from 0 to Epsilon 1, we reversed it with the minus sign, so this is a total time, and we have a relationship with Rho Sigma yd, we have a relation like this. And then, what we can have from these two, Ut L Epsilon 1, 1 minus Epsilon 1, square root 0 to Epsilon 1, x, L is coming for, this L on the left hand side, and then also the U. The U is nothing but, I will show on the top. So, okay, we already have that, sorry, so we do not need to write this U here. So, this U will be, that way, and this L is from here, so 1 minus 0.5 Epsilon, divided by 1 minus Epsilon 3 by 2 D Epsilon.

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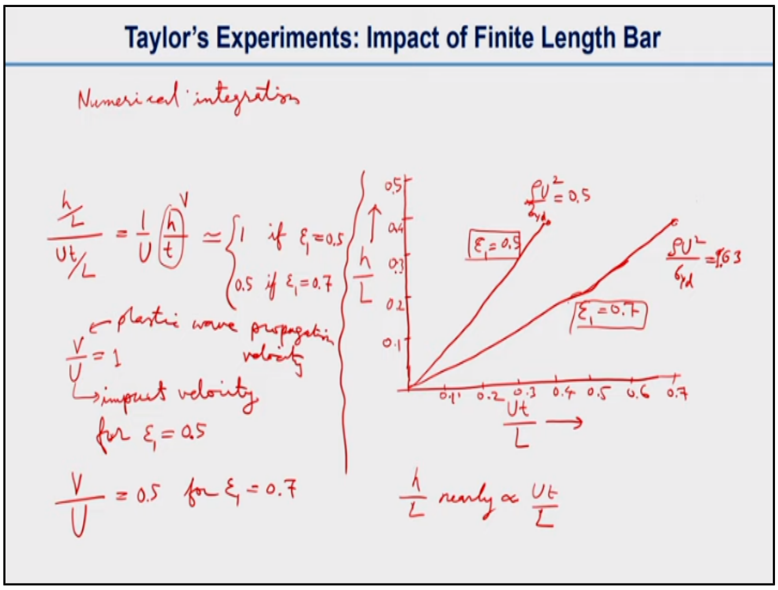

So, if this relation, if you want to do numerical integration, of the earlier expression, will give me some curve, so that can be in terms of Ut by L, and this way you can have, hl is the thickness of the plastically deformed region. So, that is like, if you have this as, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6 and 0.7. So here, 1, 2, 3, 4, 5, 0.1 0.2, 0.3, 0.4, 0.5. So, what will happen is, this will look, something like this.

It is kind of a straight line, close to a straight line. So, this will be, for this one is for, Epsilon 1 equal to 0.5, and this one is equal to, Epsilon 1 equal to 0.7. So, the value Rho U Square Sigma yd is 0.5, as we discussed. And, this is the actually, end state here, so it will end here. So, Rho U Square Sigma yd, it is 1.63, that we have earlier, even shown that, one of our, this 1.6 to the same ratio here, we are taking.

So now, actually we can write as this, h by L, Ut by L, is nothing but that, 1 by U h by t, is equal to 1. This will be, equal to, close to, 1, or we can write, strain Epsilon 1 is equal to 0.5. And then, and if L equal to 0.5, if Epsilon 1 equal to 0.7, so here, we can see these two plots, same in to same, plotted in two curve. So, the basically, this H by T is nothing but, the Plastic Wave Propagation Velocity, so that means, your V by U is equal to 1, so which is the Impact Velocity.

Impact Velocity U, and this is Plastic Wave Propagation Velocity, they are equal for our strain, that Epsilon 1 is equal to 0.5, the maximum strain, or it can be equal to the v, actually. V and U is equal to 0.5, for Epsilon 1 equal to 0.7. So, yeah, this is mostly, it is almost proportional. So, what we got hit here is, h L by L, nearly proportional to, Ut by L.

So, this is what, from the Taylor's experiment. So, we will continue this discussion, we have some more discussions on it, for Taylor experiment. And then, we will start, the another chapter of Shock Wave. So, before going to Shock Wave, we will have a little discussion about, these Taylor's experiment, so we will try to solve, some numerical problems. Thank you.