Dynamic Behaviour of Materials
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Lecture-15
Taylors Experiment Wilkins-Guinan Analysis

Hello everyone, in the last lecture we have discussed about the Taylor's impact test, an impact of finite bar. So today we will continue this discussion, so will discuss about some additional consideration and some additional analysis by Wilkins and Guinan on this Taylor's experiment.

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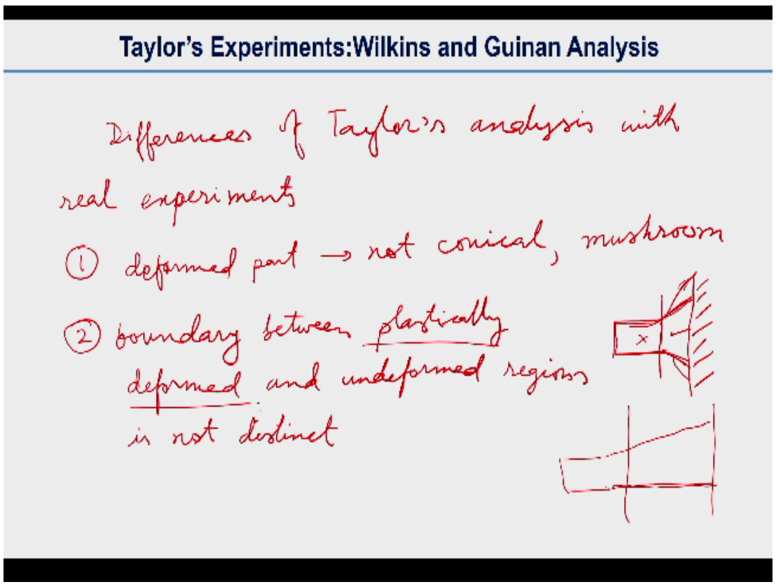

So what we have seen earlier is that the Taylor's experiment that will for the final shape of the deform bar may not be exactly what Taylor assumed. So the main differences may be with the real differences of Taylor's analysis with real experiments is number 1, the deform part may not be conical, but not conical we have seen that that may have mushrooming effect mushroom ship.

So basically when a body is impacted, so the Taylor analysis says that this is a residual, this will be conical, but then this may not be exactly that and this will may have some mushroom effect like something like this. So and the number 2 is the boundary between plastically deformed and elastically deformed we would say the plastically deformed and undeformed region is not distinct.

That means here the early analysis the Taylor analysis it says that there is a clear boundary between this plastically deformed and plastically undeformed part, but in real material so sometimes this can have some gradual deformation part, so we where we cannot actually exactly know the whether the boundary is that is the boundary between the plastically deforming and undeform part.

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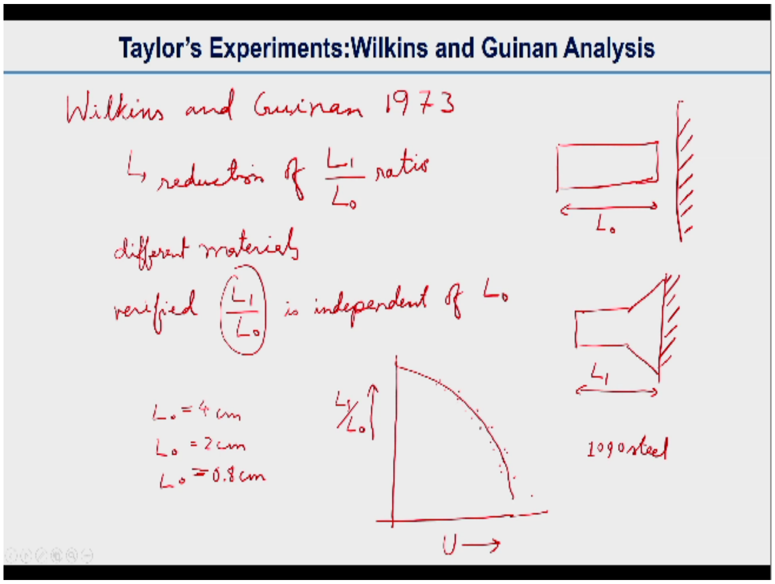

So Wilkins and Guinan in 1973 they have performed the simple mathematical analysis on the Taylor's test they studied the reduction of L 1 by L 0 ratio. So the initially the bar impacting the rigid wall is initial length is L 0 and the final length so if you are assuming this the conical shape the final length is L 1. So how this L 1 by L 0 ratio reduce that reduction ratio they did analysis.

And they did experiment on a number of materials, different materials we will derive their analysis, so first they verified that first they verified is L 1 by L 0 ratio is independent of L 0. So they have also performed one experiment on this they have one experiment on 1090 steel the material let us say is 1090 steel and this L by L 0 in the y axis and then velocity is on the x axis. So this is the plot looks something like this.

And they use different L 0 values let us say L 0 3 different value like 4 centimeter L 0 = 2 centimeter and L 0 equal to let us say something below 1 centimeter or 0.8 centimeter. So they have got all the points for all the 3 samples, so that means that that L 1 by L 0 that ratio is independent of L. So they have these points follow this curve.

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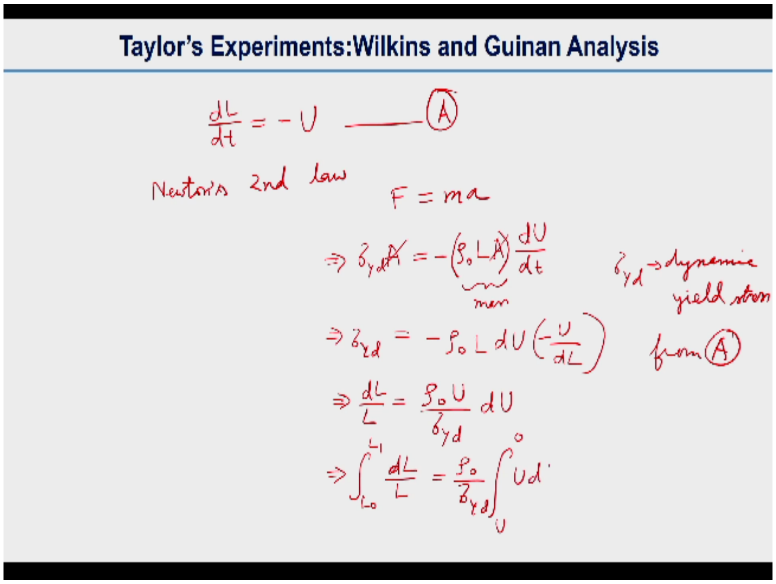

And so what the derive first we will start with the change in length with time is the instantaneous velocity U, so the - sign because this decreasing length and then they applied the Newton's second law. So Newton's second law that force equal to mass into acceleration, the force on that impacted bar is nothing but the sigma yd the stress multiplied by the area. So area the cross sectional area of the bar and sigma yd is the as we know dynamic yield stress.

And we know that the stress in that plastically deform area will be sigma L that will be constant throughout and so sigma yd x multiplied by will give us the force mass will be Rho 0 we have the density of the material that projectile a bar multiplied by L into cross sectional area. So this will give us LA multiplied by the volume and which density it will give the mass. So this is mass and acceleration is dU by dt.

And this is actually the acceleration wherever negative sign here, so because it will the velocity is reducing, so from here we can cancel out this area and then sigma yd will be equal to - Rho 0 L dU. So from this expression we can if we combine it here so 1 by dt, so we can write as U by dl, so from A, so this we can rearranged as dL.L Rho 0 sigma yd dU. So if we take the now integration from L 0 to L 1 dL by L.

So this is from initial length to the final length and here we have Rho 0 sigma yd which are constant out of the integration sign it will be the velocity will change from U to 0 Udu.

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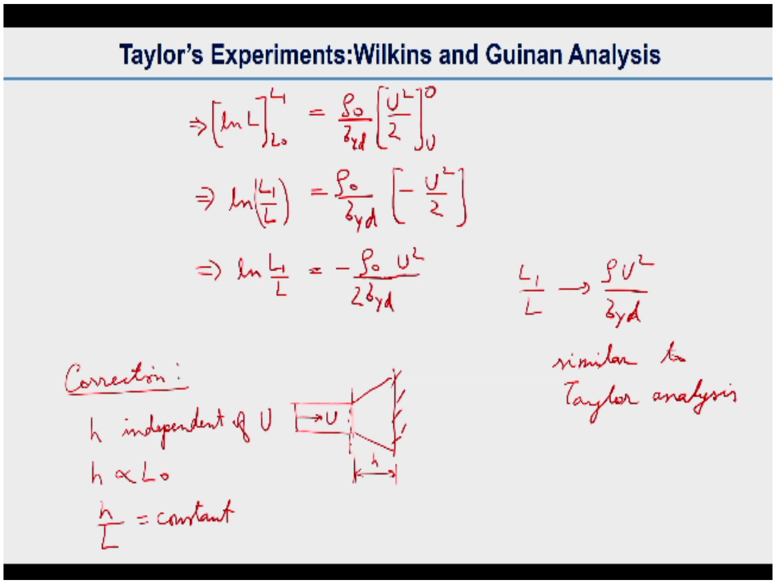

This will look like Ln L, this is limit L 0 to L1 Rho 0 sigma yd U square by 2 U sorry so 0 so then what we will get is Ln L 1 by L and equal to Rho 0 sigma yd we have a - here if we this will be -, so this will give us finally Ln L 1 by L = - Rho 0 sigma yd U square by we can write a 2 here. So this is the expression and it shows the dependence of L 1 L 2 with Rho U square sigma yd which is also similarly we got it for the Taylors analysis.

There were some relation for Taylor analysis also , we worked on that, so this is similar, so now that Wilkins and Guinan they have a correction on this, so correction is they observed that this the thickness of the plastically deform zone, so plastically deform zone means we have this residual, so we have that assuming it is as a conical. So plastically deform zone the thickness is actually independent of velocity.

So what happens when it hits first and then that H will increase and finally there it is got a fixed value of H and this H is independent of velocity U that is this impact velocity U and proportional to L 0 the initial length or we can write H by L = constant, they defined in new boundary conditions here , that means this boundary between the these 2 and that is as you know that it will move to a fixed position the H will move to a fixed position after the deformation.

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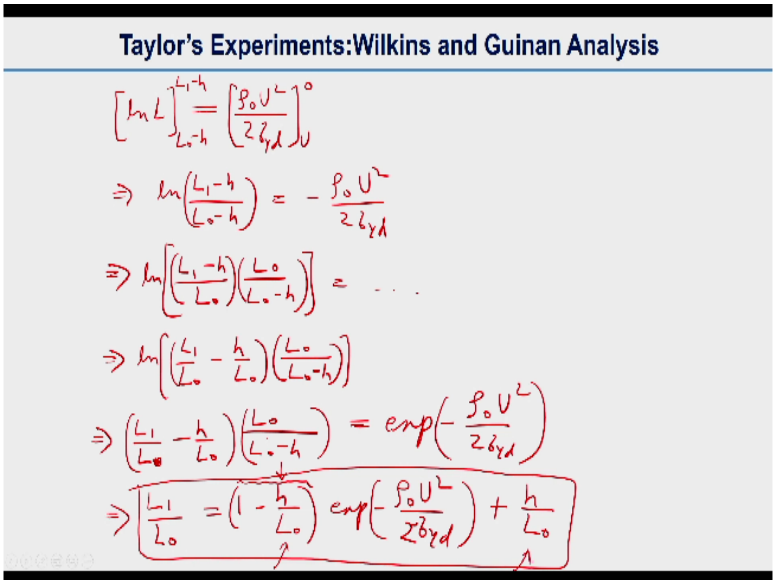

From the earlier relation so this Ln L, so if we want to get this H 0 by L out because H 0 does not depend on U and then the our earlier relation what we showed is that that is a function of You, U square at L1 by L U square. So what they did is they have that H 0 part of H 0 by L = constant that part out of this relation. So for that they have the limit L 0 - H 2 L 1 H = Rho 0 U square twice sigma yd U to 0.

So this will give us Ln L 1 - H L 0 - H this is Rho 0 U square twice sigma yd, so what we can do in the left hand side little rearrangement we can do it like L 0 L 0 - H, this and then this will be let us say the right hand side will remain same and then this will be L n L 1 - L 0 in this ratio and H - L 0 and then this will be same L 0 L 0 – H. So finally what we can do is we can keep this out L 0 - L 0 H - L 0 will give us this exponential - Rho 0 U square twice sigma yd.

And then finally we want to get a relation L 1 by L 0 which is 1 - H by L 0 I am not writing the intermediate steps here, so directly you can find it out twice sigma yd + H by L 0 that H by L 0 will come here and this if we invert it L 0 by L 0 - H this will give this term and so this is the expression where the H by L 0 term is you can see H by L0 term this one term here and then one term here.

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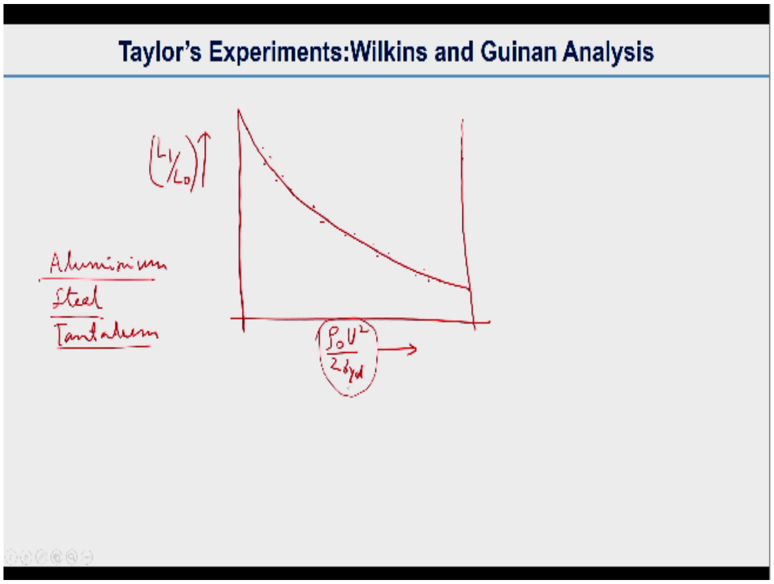

So they plotted that L 1 by L 0 the plot I did with Rho 0 U square twice sigma yd by the this term and then found an interesting result for different materials. So as we have an intercept here and as we know this in the early expression so we have H by L 0 term on this side, they verified it for 3 materials aluminum, steel and tantalum. So they have did all the experiments and they got that for all the 3 materials they have these experiment results lie on very close to this line.

So that is an important contribution from Wilkins and Guinan because first they verified that that L 1 by L 0 ratio is independent of L 0 and the secondly they showed that that H the thickness of the plastic zone is independent of the velocity of impact and that H 0 is depends only on the initial length L 0 H by L 0 = is a constant and by doing that they showed the interesting result here that aluminum, steel and tantalum show the same trend that is if we plot it L 1 by L 0 versus this term that is U squared term with the constant Rho 0 divided by twice sigma yd.

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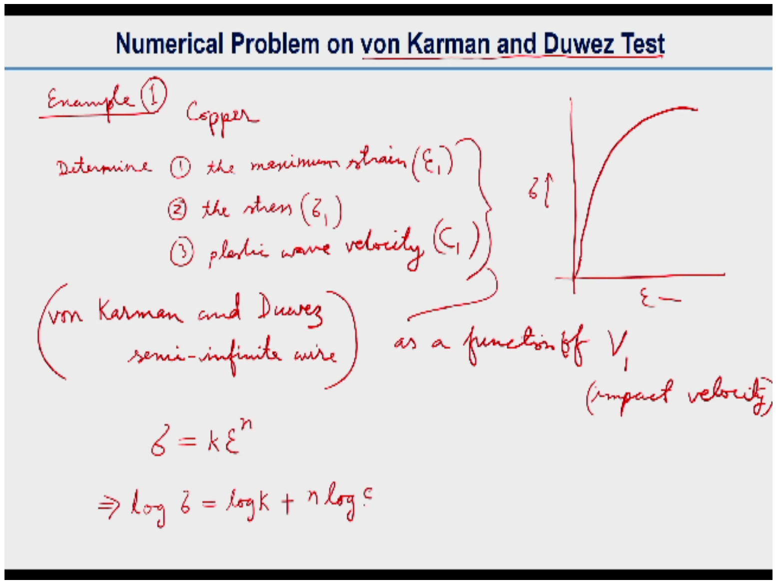

So with that so we will we are finishing this analysis part and then we will have some problems on these on plastic wave propagation. So first problem we will work on is on von Karman and Duwez test you know if you remember that the wire the plastic deformation of the semi infinite wire. So in this case lets example these examples you can find it in the Mayor's Mark Mayor's book.

So this is this solved examples we will work on here, this is let us say we have a material copper, copper wire and we are given a stress-strain curve up copper, the common stress-strain curve for this copper. So now to find from here is determine so determine the maximum strain and the stress. So stress at the maximum strain will as we you know we write it as epsilon 1 here that is a sigma 1.

And we need to get plastic wave velocity which we denote as C 1, C 0 is for elastic wave velocity. So these 3 we need to find out as a function of V 0 the impact velocity V 0 so that is a sorry V 1 the impact velocity of the drop weight, so we need to find in front in terms of that and that is one common do is all right small von Karman and Duwez wire experiment, that is semi-infinite where a plastic deformation.

So we have the stress-strain curve, so from there that we need to know that that as we know that the plastically deform part we have a relationship K epsilon to the power n. So what we can do is in the logarithmic scale we can plot that and that will be like log the natural logarithm of sigma that will be again log k + n log epsilon.

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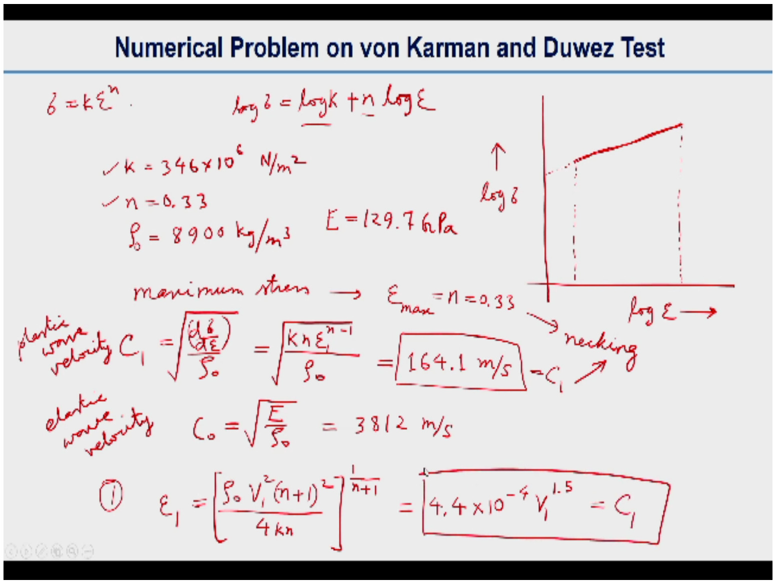

And this we can plot it, we will plot it here so log natural logarithm of strain epsilon and log of sigma. So this will look something like this, so here will be yeah this line is nothing but what we showed earlier this log of sigma = log k + n log epsilon. So n is the slope and this part is our the intercept here, we need to get the properties also although I did not mention that in the earlier slide.

So these properties will be because we know the stress-strain curve then we can get the properties, so these properties are the K value will be 346 10 to the power 6 this will be in a Newton per meter square and then we have the N the hardening exponent is 0.33 and we have our density it is copper 8900 kg per meter cube and then young's modulus of copper is 1 29.7 gigapascal.

So similarly so what we wanted to know is the maximum strain, so maximum strain and we know that the maximum stress will stress is related to it is a little to the necking instability. So for necking instability we found out that the maximum strain is equal to N and that is 0.33. So we will use this and so what we can have is the C 1, C 1 is the plastic wave velocity. So we need to find C 1 and C 0.

So plastic wave velocity is d sigma d epsilon divided by Rho 0 ok, we will write Rho 0 as a density, so which will be from this expression only x from sigma = K epsilon to the power n. So this will be K n epsilon to the power n - 1 the derivative of sigma which respect to epsilon and this will be divided by Rho 0. So if you calculate, so we have K here, this we have K, we have n and we have epsilon actually epsilon 1 the maximum we can take that equal to n hardening coefficient.

Because this is at necking maximum strain corresponds to the necking instability. So this will if we do these all calculations this will give us 164.1 meter per second, so that is the plastic wave velocity and what about elastic wave velocity. Elastic wave velocity is C 0 and as you know this is simple E by Rho 0 which will give us 3812 meter per second as we know it is much faster than the plastic wave.

So this is actually for as you know these are true for slender bar or cylinder not for unbounded medium. So this is the maximum like plastic wave velocity so 164.1 meter per second which is corresponds to making, so if it is the plastic wave velocity above that, so the wire will break wire will not know which tend more deformation more plastic deformation. So now our first part is the maximum strain we need to find maximum strain.

And then sigma 1 and then C 1 all in terms of function of V 1 which is the impact velocity. So first part so what we need to do is epsilon 1 which we have from our earlier equation that we already derived. So that I am not going to write differently, so this directly we are putting this Rho 0 V square V 1 square, this is n + 1 square 4 by K n to the power 1 + 1 1 by n + 1. So this here we know the density, we know the n which is 0.33, we know K**.**

And so we need to get the maximum strain in terms of the V 1 the impact velocity, so if we calculate I am not writing to all the steps, but it will be 4.4 into 10 to the power - 4 V 1 square, so that is what sorry this is 1.5 actually, so how it get 1.5 to the power and that you can understand this is square and this will give us 1 by 1.33. So that will eventually will give you 1.5 because 1 by 1.33 is like 3 by 4 and so this will be 1.5. So this is the first relation, so we now check the second and third.

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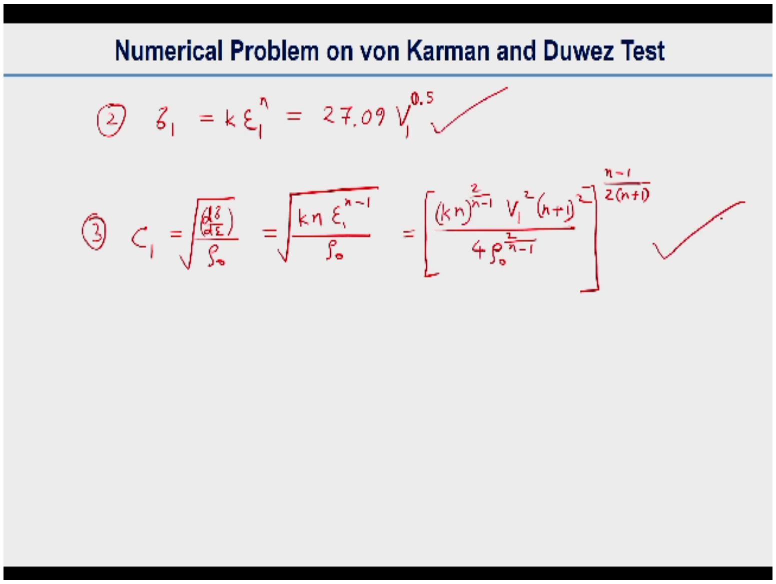

So for number 2 so we need to find out sigma 1, so sigma 1 as we know is nothing but this is the relation K epsilon 1 to the power n. So that is how it is now simple, so we know K, we know n and we have a expression for epsilon 1 in terms of as a function of V 1. So that will be equal to 27. 09 V 1 0.5 this one and the third one V 1 to get the plastic wave velocity. So plastic wave velocity which is derivative of sigma with respect to epsilon divided by Rho 0 whole thing square root take n and K n epsilon 1 to the power n - 1 after differentiation.

This is Rho 0 this and then we have the expression for epsilon 1 in terms of V 1 and then I am writing the final expression, so K n to the power 2 - n - 1 V 1 square n + 1 square for Rho 0 2 n - 1 the whole thing we will use the square bracket here, instead of this circular bracket, so square bracket n - 1 twice n + 1. So yeah so we got the second relation in terms of sigma, the sigma 1 in terms of V 1. And then we got the third relation the plastic wave velocity in terms of impact velocity. So now we will move forward and we will discuss another problem which is a little bit same the example we have shown here.

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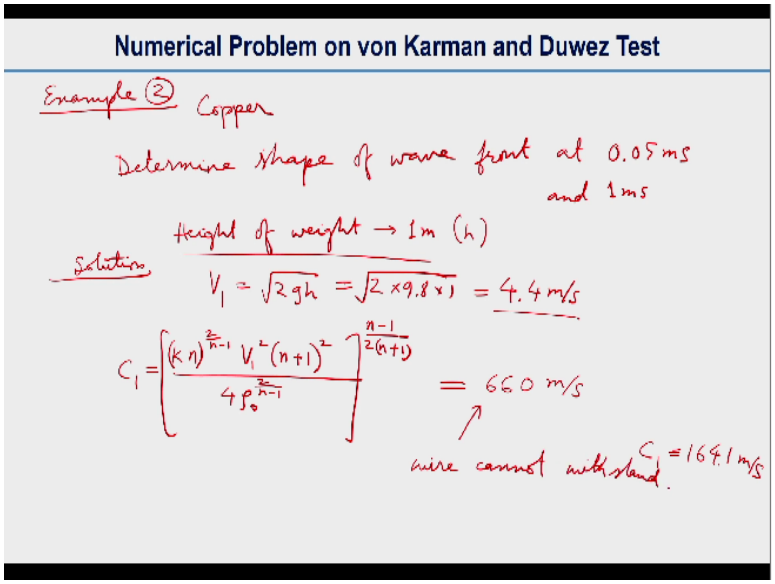

This is example number 2, so for the given material above like for copper we need to find the shape of the wave front, so we need to determine the shape of the wave front shape of the wave front at first 0.05 millisecond and 1 millisecond. So after the fall of the weight, so let us say we have the von Karman and Duwez experiment the weight the height of the weight was set at height of weight that means the initial height of the weight is at 1 meter from the extreme end of the wire.

So that means the weight will fall 1 meter and then it will hit the wire and the lowermost end of the wire. So we need to first calculate the impact velocity that we can easily find out from this height of the weight, weight where we kept first, so that is nothing but that V1, so this is the solution we are starting from here. So V1 is nothing but 2 gh, h is this one 1 meter. So this will give us 2 into 9.8 into 1.

So this will give us 4.4 meter per second, so that is the impact velocity, that weight hitting the wire a lower end of the wire and now what we need to find is the velocities plastic wave velocities and elastic wave velocities, so that we can draw the wave profile. So we will now find a plastic wave velocity from the above whatever we discuss in the earlier problem the plastic wave velocity expression from here.

So this will be like kn, kn to the power 2 – n V 1 square n + 1 square divided by 4 Rho 0 2 divided by n - 1 and to the power n - 1 to n +1. So here if you see we know K from the earlier problem we mentioned about K, so K we have the value of K, we have the value of n and then we have the density so and we know the impact velocity now. So earlier we did not know that but we know for this problem we have the impact velocity with us.

So this will give us C 1 = 660 meter per second, so 660 meter per second is a very high velocity that we know that wire will not withstand why because our C 1 the maximum C 1 that is corresponds to the nicking we found in earlier problem as C 1 = 164.1 meter per second. So now this problem and the wire will break or wire cannot withstand such a high plastic wave velocity.

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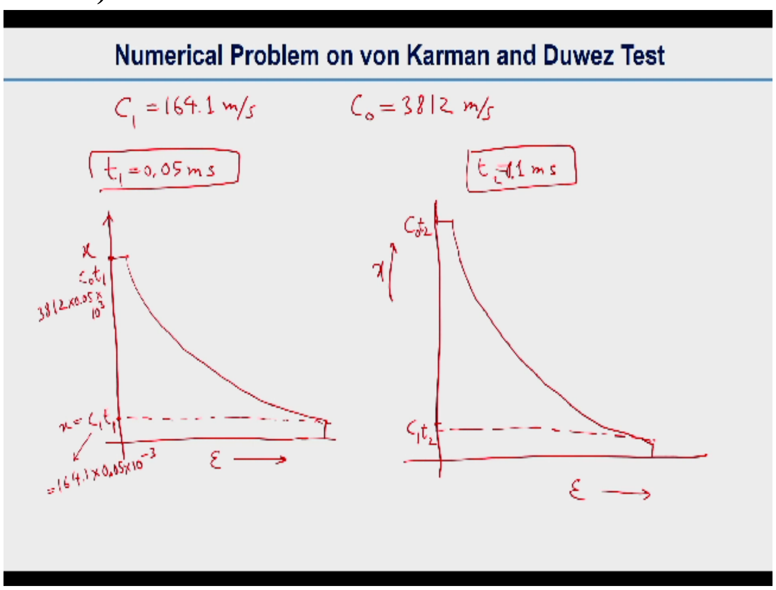

We need to plot the wave profile, so what we will do is we will assume then C 1 as 164.1 meter per second, so what we got the maximum one and then we know that C 0 = 3812 meter per second that we calculated in the last problem. So now we need to have the wave profile for 2 time, first one is 4.05 millisecond and the second one is time = 1 millisecond, so what we can do is for the wave profile as we know the x-axis will be epsilon.

And then y-axis will be X, so the wave profile from earlier if you want to draw is sorry this is 0.05 millisecond one and so this will be like first what we need to do is we will have a C 1 t, so C 1 t time is t = 0.5 millisecond, so this will give as X = C 1 t which will be equal to this C 1 multiplied by t and that means 164.1 0.1 into 10 to the power - 3 sorry I made a mistake here 0.1 millisecond not 1 millisecond.

So this is 0.1 and this is 0.05 millisecond ok, that is correct, now so this is point what we need to do is here is 0.05 millisecond and then similarly here we need to get the X = C 0 t, C 0 t will give us 3812 multiplied by 0.05 10 to the power 3. So now we can plot that, so this will something like this and as we know this will correspond to this. So similarly for this what will be the difference is the distance X how much it will cover.

So the difference will be it is at higher time, so it will be the higher C 0 t, so we have C 0 t, so let us assume this is t1 and this is t2. So here is C 1 t 2, so if you write it clearly so this will be C 0 t 2 here and this is X and this one will be C 1 t 2 here and let us C 1 t 1, so this will be they were front look taller as you know the C 0 t2 will be twice as high as this one, so that is the difference here, we have one more problem on Taylor's analysis.

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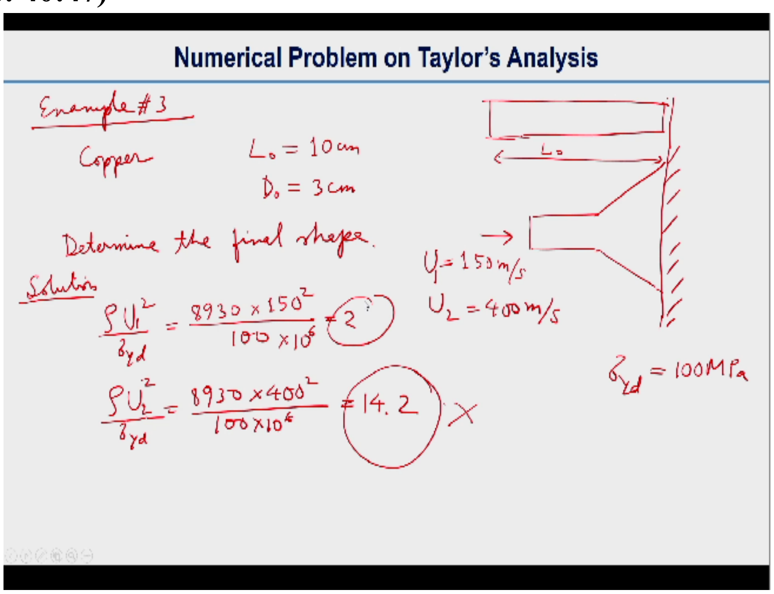

So let us say we have the example number 3, so this is on Taylor's analysis. The Taylor's analysis is as we know the Taylor's analysis assuming a conical shape of the deform bar, so we have a copper material and then initial length of the bar when it hits the target that bar was the initial length L 0, L 0 is 10 centimeter which D 0 the diameter of the bar is 3 centimeter and we have a velocity of this bar velocity when heating is 2 velocities let us say we are assume 150 meter per second.

Let us say this U is 150 meter per second and we write U 1 and U 2 will be 400 meter per second. So there are 2 velocities and then assume that yield stress is equal to 100 megapascal. So determine the final shape of the bar, so what we can do first is we need to get this term out, so this sigma yd, so that will be we know the density is for copper 89 will write 30 kg per meter cube.

And then U square first we will take this one U 1 and then let us say we will write U 1 here and then sigma yd is 100 megapascal 100 into 10 to the power 6 we are writing all in SI unit. So this will give you a factor 2 and then for the other velocity, so what we can do this is square sorry so 8930 right, this will be 400 square 100 into 10 to the power 6 and this will give us a 14.2.

So now we know this 14.2 is a very high value and if you remember the earlier discussion in earlier classes so we cannot go with this because Taylor's analysis will not reveal it for this that will not we cannot predict the shape up from this shape from Taylor's analysis. So we will go with only this value for 2.

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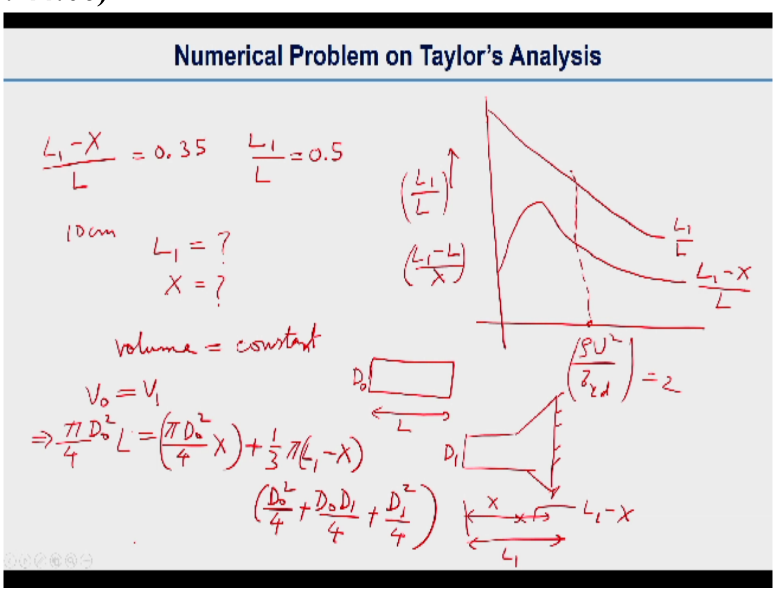

And then we if you remember the 1 plot between this x-axis Rho U square sigma yd and then in the y-axis we have we may have L 1 by L, the L 1 is the as you know L 1 is the final shape final length and the L 0 is the initial one. So L 1 by L and then and L1 by L and then another issue L 1 by L divided by X. So both of this we can plot with this Rho U square by sigma yd. So that we found that this curves that we already did it in the earlier lectures.

So this curve is L 1 - X divided by L will look like this and then L 1 by L will be something like this. So if you see that figure from the book so what you can get is if the value of this term is equal to 2, so you can find the values of this and this so this is found to be L 1 -, so L 1 - X divided by L is found to be 0.35 and then that is from the plot now you can find it in the book and that is found as 0.5.

So we know the initial L, so that is 10 centimeter, so from there you can find the L 1, you can find a logical L 1 and you can find X, then we have to find the diameter as well because so what you can do is you can assume that the volume will be constant, so the volume will be constant and then we have as you know this after count after the impact this portion we keep this portion as X and this will be L 1 - X and total distance is L 1.

So this is this part is L 1 – X, so now if the volume is constant suppose the initial volume is the and the final volume is constant, so what we can do is the initial as you know this is only the initial will be as L and d D 0 will write D 0 and this is let us say the initial is I will write L and d 0 writing square L is = pi D 0 square by 4 X, so you will you can do the calculations by yourselves as I am this that is even given in the book.

So this will be L - X multiplied by D 0 square by 4 + D 0 D 1 by 4 + D 1 square which is D 1 is the final one, D 1 square by 4, so with this we can find a diameter, so and then we can find get the final shape L L1 X and the diameter, so final diameter. So that is all for today, so that is actually end of the plastic wave chapter and then we will discuss on the shock waves , so in the next lecture thank you.