Dynamic Behaviour of Materials
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Lecture-17
Introduction to Shock Waves- II

Hello everyone, so we have already discussed the basics or introduction of shockwaves, so in this lecture, so we will talk about more on shock waves.

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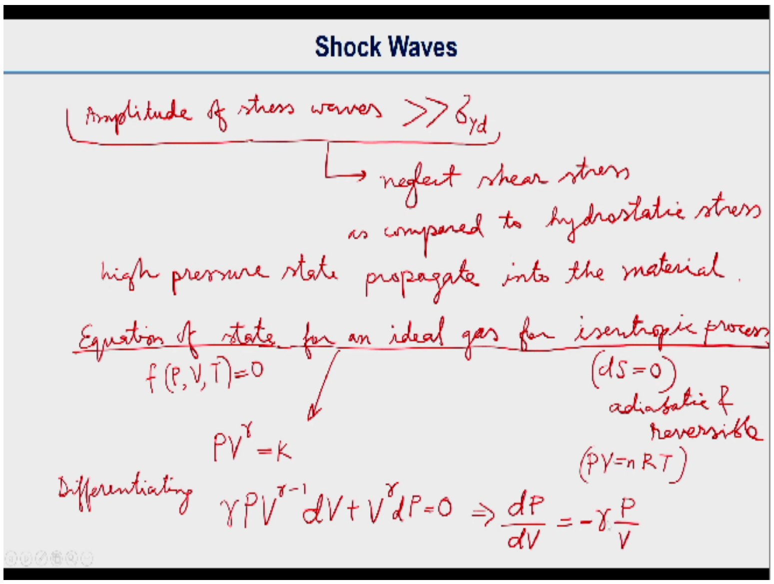

Shock waves are like produced when the amplitude of stress wave greatly exceeds the dynamic yield strength of the materials, so dynamic yield strength of low strength is that yield strength at that strain rate that is which is different than the quasi static or at slow strained, if it is the **if it is** greatly exceeds greater than the dynamic yield stress.

So we can neglect the shear stress as compared to hydrostatic component of stress**.** So that means a high pressure state will propagate through the material so into the material. The equation of state for an ideal gas, for isentropic process that means isentropic process means change in entropy is equal to 0 or isentropic means adiabatic and reversible. So we are neglecting the shear stress as compared to the hydrostatic stress.

So will now see the treatment for an ideal gas that is equation of state which is like a your Boyle's law, Charles law and like classical ideal gas laws like we got PV is equal to nRT. So similarly these are equation of state that means it is in thermodynamics, thermodynamic equation the relating the state variables which described the state of a matter. So that means in equation of state is actually we can in general form we can write it this way.

But for this isentropic equation of straight for an ideal gas or isentropic process, so this will be we write it as PV to the power gamma is equal to constant that is a K constant. So this is what we can do is we can differentiate we can get as gamma PV gamma to the power --1 dV + V to the power gamma dP equal to 0 which will give us dP dV = - gamma PV.

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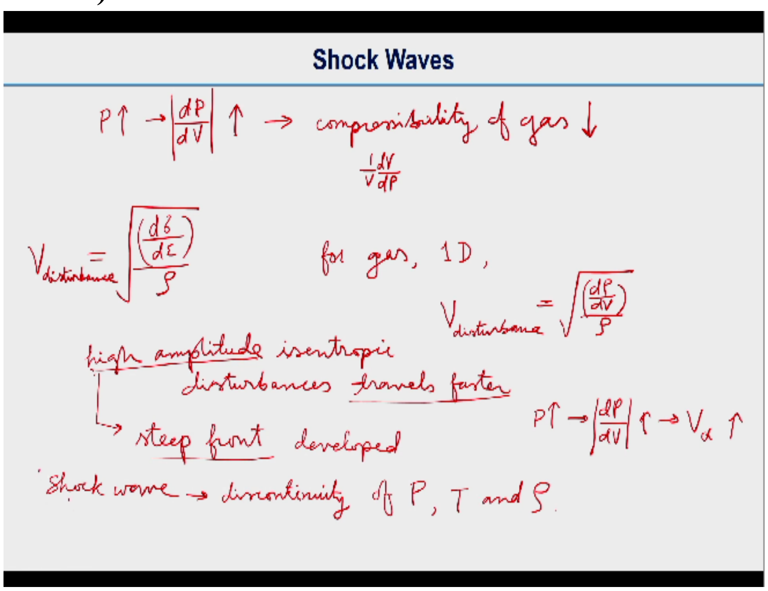

So it means we can write when pressure will increase dP dV modulus of dP dV will increase and that means the compressibility of the gas will decrease, the compressibility generally we have V 1 by V dV dP. So this compressibility of gas will decrease, so earlier we found that the disturbance velocity d sigma d epsilon by Rho to the power this disturbance that we can call the velocity of disturbance**.**

So now for a gas if we assume like one-dimensional conditions the velocity of disturbance will the disturbance means let us say stress wave in this case. So or it is we will call it is let us say the pressure discontinuity as it is little to high pressures of pressure discontinuity. So this disturbance velocity will be covalent to are dP dV vt Rho, so this is for gas and one-dimensional case.

So now that says that high-amplitude isentropic disturbances travels faster that means high amplitude means you have a high pressure and that as you can see from here, if pressure increases dP by dV increases or velocity will be higher or it will travel faster. So this is I will write it here P dP by dV and your velocity of disturbance as a slight VD will be higher. So what happened the high amplitude part will have steep front.

So steep front will be developed because the high amplitude disturbances will travel faster than the low amplitude disturbance. So this shock wave is also defined as a discontinuity pressure, temperature and density, Rho, so it is also defined this way.

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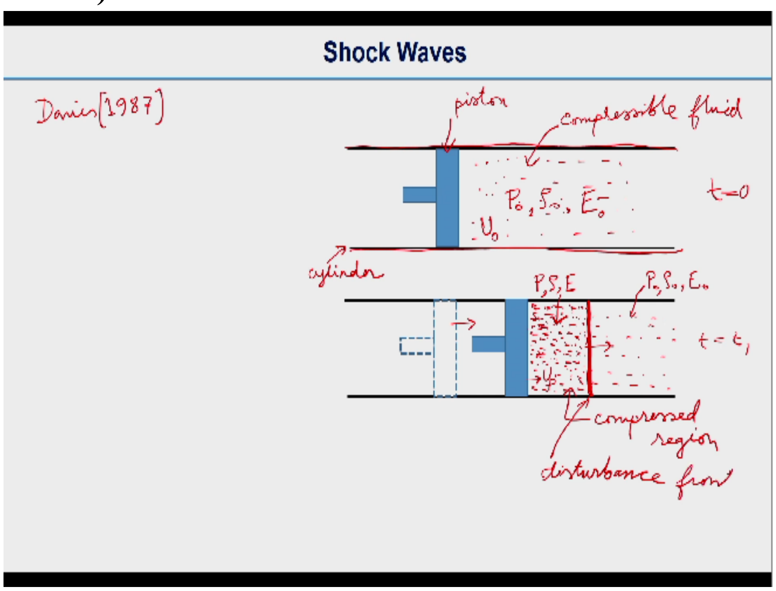

So this concept of shockwaves and then it is strickman's especially what we learn a little later about Rankine Hugonoit treatment that is that represent Rankine Hugonoit conservation equations for shock waves. So that these equations can be easily understood if we look at the another treatment of which is analogous to this Davis 1987 which is analogous to that not exactly what we will be doing is not exactly the shock wave treatment.

But it is something analogous to it or it is a the concept is same. So what it shows is a cylinder, this is a cylinder with a piston inside, so this one is the cylinder and the piston will move towards right. So this is a piston now he does rest let us say this is idealized piston and this is this part we have this has compressible material or in this case as we know we can assume it as a fluid compressible material.

Then let us say this is at time t = 0 and the pressure here is P 0, density here is Rho 0 and internal energy per unit mass is let us say E 0 and the velocity of these particles which is U 0 and that can be if it is stationary it can be equal to 0. So now at time t = to t1 let us say the piston moves forward and so this velocity of the piston is will write as a Ut, so what will happen is **so** so we have this compressible fluid.

Now when you were compressing with this piston so some part of the fluid like this part will be compressed now. So density will be higher here, so this is the compressed region and we can write the pressure, the density and as the chronic mass for this region like this and the other region it will remain the same P 0 Rho 0 and E 0 and then this boundary is called the front of the disturbance. So this boundary is we will call I will write it here disturbance front**.**

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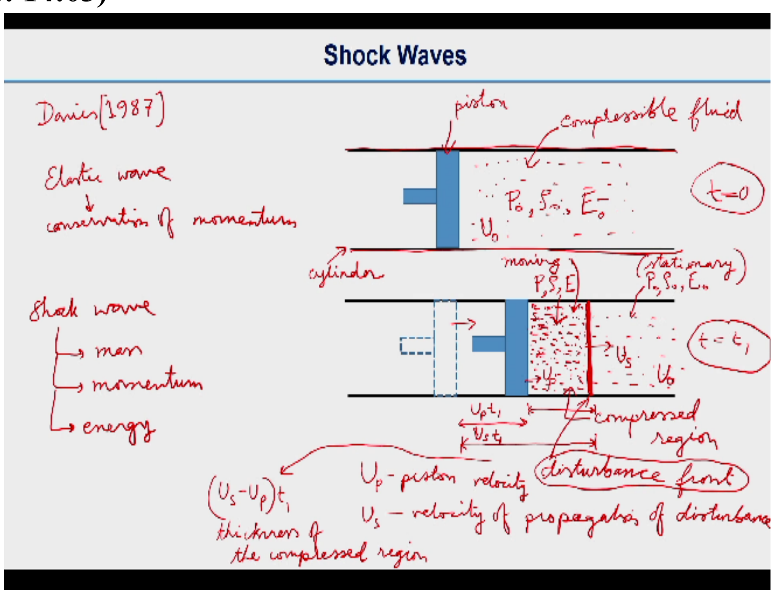

So this disturbance is let us say we are here we are not calling it as a shock wave front or something. So here we will just say that this is pressure discontinuity, the disturbance and this is the disturbance front. So this disturbance front will move forward with a velocity Us and as we know this velocity of these particles are U 0 in this side and here we have the piston velocity as Up.

So I will write it here U p is piston velocity and Us is velocity of propagation of the disturbance, we can assume that this part which is not compressed that is a stationary part of the fluid and this part is moving part, that stationary part may not be always stationing but for now we can assume it is stationary and the part that is compressed is we can call as a moving part moving the material or fluid.

And the boundary between the 2 is the disturbance front there is a disturbance front, when the piston will move it travels a distance Up t1 and at the same time the disturbance will move a distance Us t and so it is too messy here, but so what we can do is the other part that this part I will write it somewhere else. So this part is Us - Up multiplied by t1. So that is the thickness of the compressed region.

So sorry this is Us t1 that is because time t = t1 and the earlier one was when piston is at rest and it is t = 0. So now here what we will do is we will apply the conservation of mass, conservation of energy and conservation of momentum. So earlier also we discussed in the for elastic wave propagation, so what we did is for elastic wave we had conservation of momentum.

So we used conservation of momentum, so here for shock waves we will use conservation of mass, conservation of momentum and conservation of energy as well, so but this is exactly not the shock wave treatment, but this is analogous to it that is piston in a cylinder which was initially proposed by Davis in 1987.

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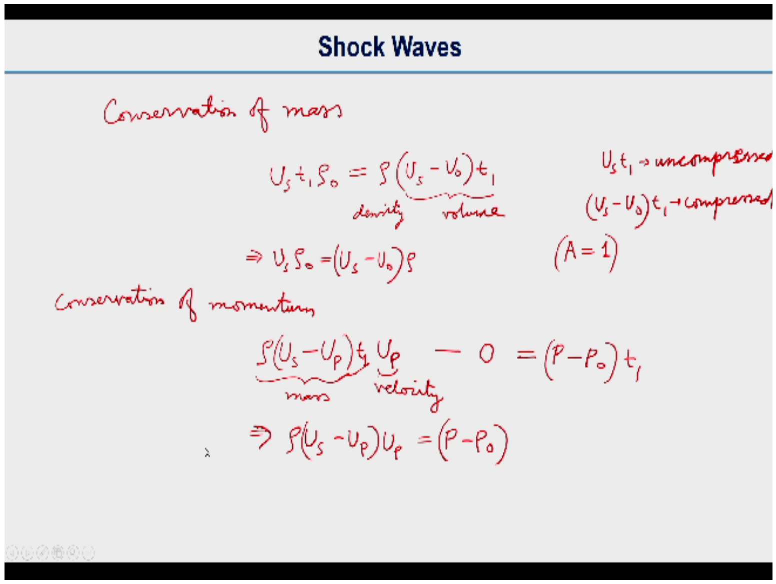

So first we will go for conservation of mass, conservation of mass the initial uncompressed material of fluid the this is uncompressed this part, uncompressed material, so here the mass of this is we can have it as Us t1 Rho 0. So this is the initial mass uncompressed material which is after compression actually it will look like Rho Us - U 0 t 1 ok. So initially the thickness was Us t1 uncompressed and Us – U0 t1 is a compressed mass thickness.

And we are assuming A equal to that cross sectional area A = 1 unity and that means density mass into density. So this is our mass into density sorry volume. So we have this volume into density. So this is for uncompressed the left hand side, in the right hand side is the compressed. So when you cancel out the time t1, so the final expression we can write it as Us – U0 multiplied by Rho.

So this is the relation we get from conservation of mass and then conservation of momentum we will use conservation of momentum which will give us the final momentum which will be Rho Us - Up, So mass and this masses at this point and this is the velocity, so this is the final momentum and the initial momentum is 0 we do not the piston is at rest. So it is 0 and then is equal to the impulse that we know that equal to the impulse which will be the final pressure minus the initial pressure multiplied by t will give us the impulse.

So now if you cancel out the t from both we can write here t1 for both, so Us - Up multiplied by Up will give us P - P 0. So this is the equation of conservation of momentum.

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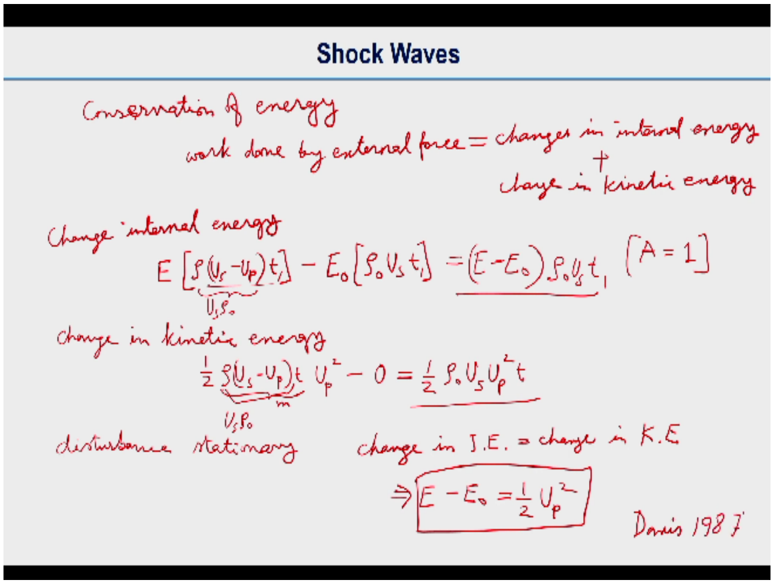

So we have talked about conservation of mass and conservation of momentum. Now we will see the conservation of energy, we know that the conservation of energy is like work done by external force is equal to change in internal energy plus change in kinetic energy. So change in internal energy in this case is the E the internal energy per unit mass of the changes in the final internal energy which will give it as Rho Us – Up t which is the mass of that and then Rho 0 Us t.

So this is we will see from this figure, so Us t1 is the thickness of the compressed, so uncompressed region earlier and then the thickness of the compressed region is Us - Up t1. So that is the mass if we talk about mass, then and we are assuming area equal to unity here everywhere we are assuming area equal to unity. So this is for the mass of the uncompressed region. so and this will give us actually E – E0 = Rho 0 Us t1.

So this is t1 is actually this part from the conservation of mass we got that Us Rho 0 = Us - U 0 multiplied by Rho. So that will give us Us Rho 0, so now finally expressional with this and change in kinetic energy kinetic energy will be the final kinetic energy 1/2 Rho Us U 0 sorry Us by Up t that is the mass and velocity square minus the initial kinetic energy that is the piston will be at rest.

So that will give us 1/2 Rho 0 Us Up squares the same this from the conservation of mass this part will be Us Rho 0, so this will give us this expression. Now for a stationary disturbance front is stationary, if actually disturbance is stationary, so then that work term will not influence here. So we will have only change in internal energy equal to change in kinetic energy.

So that will give us these 2 term if we equate it so we will get it as E - E 0 = 1/2 of Up square. So this is basically from actually initially a proposed by Davis 1987 and which is for a compressible fluid in a cylinder and pushed forward by idealized piston and then we applied the conservation of mass momentum and energy and we can see the final expressions from these equations.

So in the next lecture we will be talking about Rankine Hugonoit treatment of shock waves which will be very similar analogous to this treatment, that is why we have gone through this treatment in this lecture. So the rest of the shock wave discussion we are keeping it for the next lectures, thank you.