Dynamic Behaviour of Materials

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Lecture-18
Shock Wave Rankine Hugonoit Treatment

Hello everyone, in the last lecture we discussed about analogous treatment that is analogous to Rankine Hugonoit treatment of shock waves that last was actually the treatment which was initially proposed by Davis and that is if you remember that is that a piston in a cylinder compressing a compressible fluid. So now here we will discuss the actual the shock wave treatment which is also known as the Rankine Hugonoit treatment, that is the conservation equation of mass momentum and energy.

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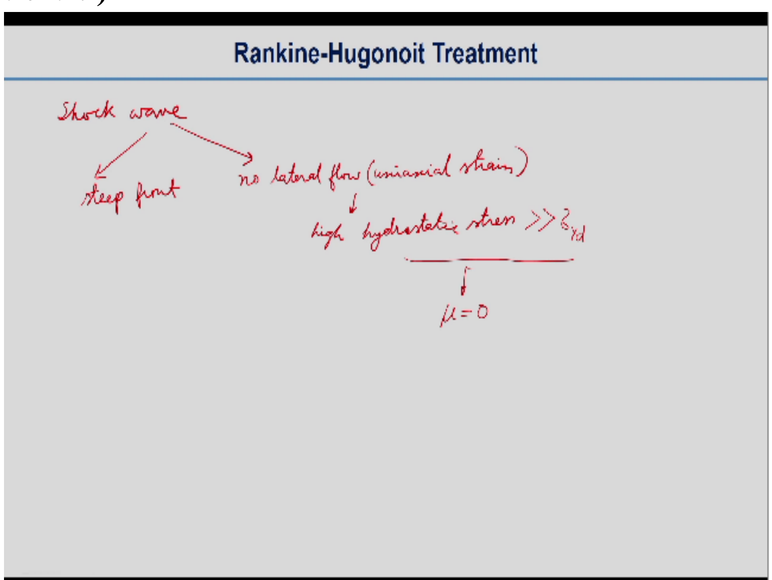

So as we know the shock wave has the characteristic of a steep wave front and also we know that no lateral flow that means uniaxial strain. So this allows a buildup of very high pressure or high hydrostatic component of stress**.** So if the hydrostatic stress is much higher than the dynamic yield strength that is yield strength at higher strain rate.

Then the shear stress resistance will be 0 and that will denote the shear modulus mu will be equal to 0. So this is, so if it is you need zeal strain that will develop very high hydrostatic stress and if is several order higher than the or you can call several factors higher than and sigma yield at dynamic that is sigma yd. So then the shear resistance will be 0, the resistance to shear that means shear modulus will be equal to 0 mean will be equal to 0.

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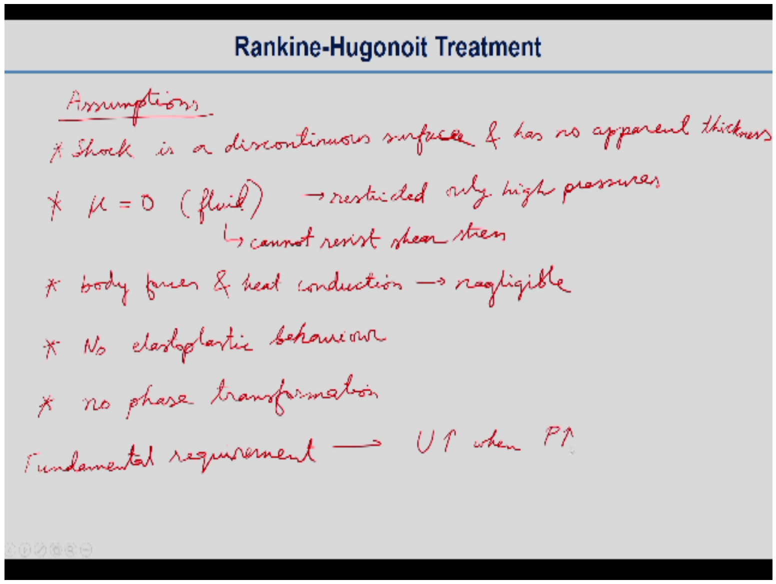

So if we note down this basic assumptions of this treatment whatever that is Rankine Hugonoit treatment, the assumptions for the shock wave as we know is first one is a shock is discontinuous surface and has no apparent thickness as so this is with very less thickness. So shear modulus mu = 0 that means it behaves as a fluid as we know that for fluid it cannot resistance resist shear stress.

So we can assume it as a fluid and also we should understand that this is restricted to only higher pressures and body forces that includes gravitational forces as well and heat conduction negligible and there is no elasto-plastic behavior that means it will not show a elastic behavior a small load and then you go to a plastic behavior and material does not go does not undergo phase transformation, no phase transformation.

So the shock wave can produce phase transformation but in this treatment we will assume that there will be no phase transformation and no elasto-plastic behavior and also a fundamental requirement of the shock wave is fundamental the requirement is velocity of the pulse will increase when the pressure will increase that we already discussed the related to that. So velocity of the pulse U will increase when the pressure will increase, increase with increasing pressure.

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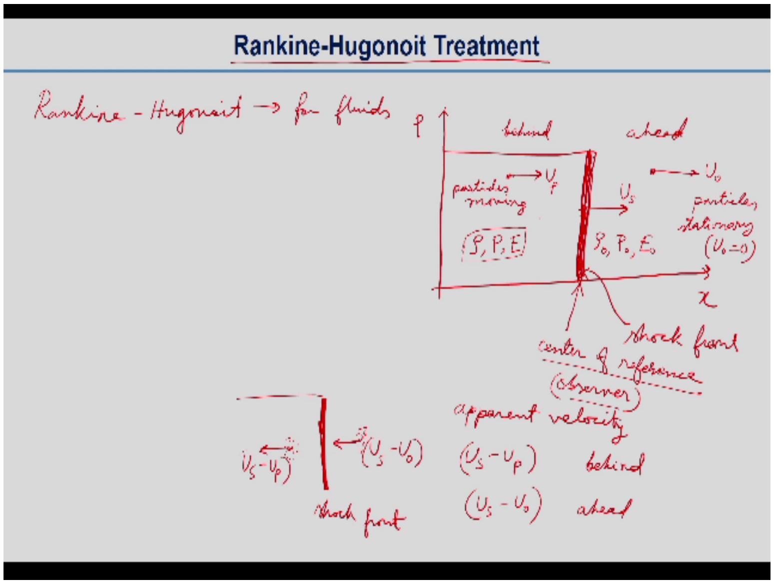

So let us assume a shock with front, let us say we will draw the shock wave front like this, let us assume this is X and P. So we can have a shock wave front we can draw a shock wave front like this, so this is the front we are all just I am drawing vertical line and just to show the importance of this boundary. So this is shock wave shock front we can write and that assume this front is moving forward with a velocity Us.

And the particle velocity behind the front when we called behind this means behind is this way and we will when we call ahead, ahead of the shock front, this will be this way because the shock wave is traveling from left to right and so particle velocity in this behind the front is Up and particle velocity ahead of the front is U0, originally will have the treatment for U0 = 0 that means a stationary ahead of the front.

So here we assume that these is particles moving and here we will assume that particles stationary but that means U 0 = 0. So we have here the behind the shock front we have the density Rho and then pressure P and then energy internal energy as E per unit mass and in this the other side we will have density Rho 0 V 0 and E 0. So because this is the initial one with the so that Rho 0 P 0 and E 0.

And after the shockwave travels through this region then this density and pressure will change. So the particles are moving actually at the behind the front and also at the at the shock front, so that is these values Rho P and E is defined at this behind and also at the shock front. We will assume that there is a center of reference at this shock front and we will assume that the observer is at the shock front.

So basically let us say it is moving with the front, this is basically as I told we have written here Rankine Hugonoit treatment, the Rankine Hugonoit treatment is for Hugonoit treatment is originally developed this for fluids as we know that for high pressure the we assume that even for that the materials we are assuming it has a fluid. So the displacement of the particle that means we have behind the front Up and ahead of the front U 0 that displacement is responsible for the pressure build-up later the shock.

And so as we mentioned that the center of reference the observer is here, so that is why the apparent velocity behind the shock front behind is Us – U p and ahead of the shock front is U s - U 0. So that means this is the apparent velocities are actually for the fluid moving suppose this is the shock front**.** So the fluid moving inside the shock front will have a velocity U0 apparent velocity.

And because the center reference and we assuming at the shock front, so if you see from the shock front if you are moving with the shock front then the apparent velocity U s - U 0 of the fluid moving into the shock front. So this is the shock front and then the apparent velocity of the particles let us say here also we can draw the particles. So these particles go inside these the shock or front at the velocity U s - U 0.

And the particles inside the shock front I mean that means actually behind the shock front will go away from the shock front at a velocity U s - U p. So these are the apparent velocities whilst first start with conservation of mass and then conservation of momentum and conservation of energy like how we did for that the analogous treatment discussed in the last lecture.

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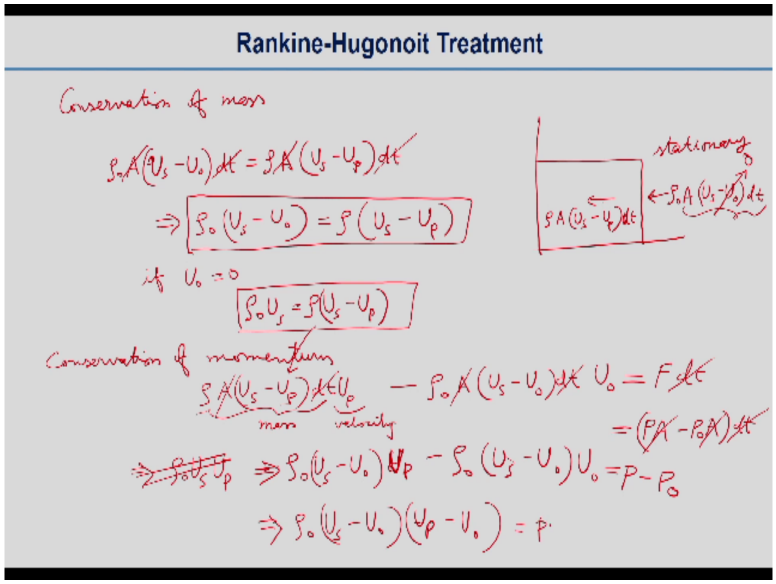

So conservation of mass again we are drawing the shock front, so mass moving towards the front this mass is A, A is the area of that cross-section A Rho 0 Us - U 0 dt we are assuming the time of travel is dt. So that is basically as you can understand the area or otherwise we can write it this way Rho 0 first and then area. So this will give us the volume multiplied by Rho 0. So which is the total is mass moving inside towards the front.

And mass going away from the front is we can write it as Rho density multiplied by area A Us U p dt, so that is as you know U s – U p and or in this side on the right hand side U s - U 0 due to the apparent velocity V. So earlier so now for conservation of mass we can write like Rho 0 A U s - U 0 dt = Rho A U s – U p dt. So we can cancel dt and A from both sides and that will give us Rho 0 U s U 0 = Rho U s - U p.

And if we assume that this U 0 = 0 that means stationary material actually stationary we call fluid head at the front shock front. So then if it is U 0 = 0, so this will give us Rho 0 Us = Rho Us – Up. So this relation will be important because we will use this relation in we will substitute this relations in other relations to get some interesting expressions. So after that we will try the conservation of momentum that is we know that momentum is mass multiplied by velocity.

And which will be equal to the impulse, so momentum change will be Rho A Us - Up dt, so this is the mass and then multiplied by U p the velocity mass and the velocity - Rho 0 A U s U 0 dt mass into the velocity U 0. So it is basically if you can see from the figure, so this is one is a head of the shock front and one is behind a shock front. So this difference in momentum will be equal to the impulse, that impulse will be F dt.

So this F dt we can write it as pressure into area PA - the P 0 A dt, so that is the force P A by P 0 multiplied by dt. So now if we try to simplify it with mass conservation, so what we can do is you can see here Rho U s - U p if we use this relation here so that will give us A we can already cancel it out so what you can do is all As we can cancel it out and also dt we can cancel it out, dt, so now then we will use the mass conservation the relation that will give us Rho 0 Us U p. So we need to use this relation for conservation of mass.

So that will give us Rho Us - U p = Rho 0 Us - U 0. So there will be this will be Rho 0 Us - U 0 multiplied by U p - Rho 0 Us - U 0 U 0 = P - P 0. So this will give us Rho 0 Us - U 0 U p - U 0 = P = P 0.

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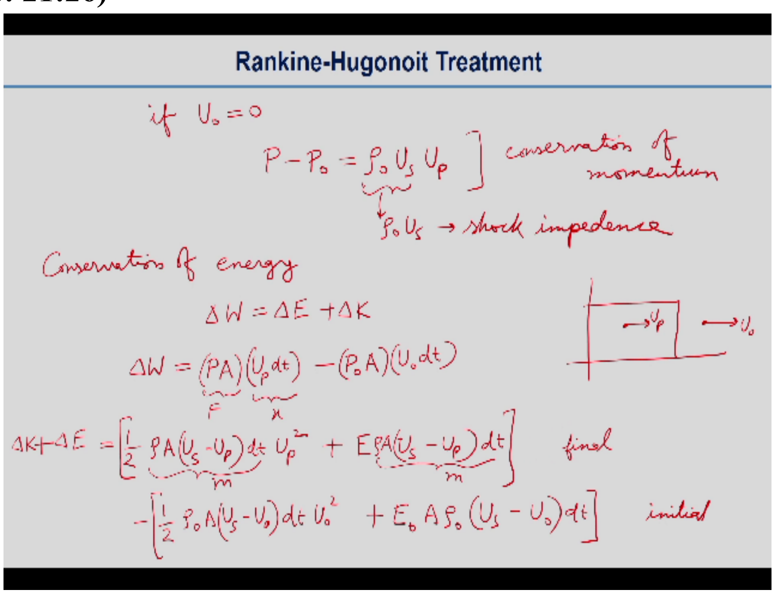

So if U 0 = 0 the conservation of momentum relation will be equal to P - P 0 will be equal to Rho 0 Us Up. So this term Rho 0 Us is known as shock impedance like we got earlier shown impedance. So and this is the relation of conservation of momentum for U 0 = 0 and then we will come to the third conservation that is conservation of energy, so conservation of energy will give us the work done is equal to our total energy difference. So that means let us say internal energy and kinetic energy.

Now the work done by pressure P minus the work done by pressure P 0 which we can write it as PA the force into the distance of the travel by the particles that is written as U p dt - P 0, the area will be same the particle velocity will be U 0 multiplied by time will give us the distance. So basically we can write F into X, so this is considering the original reference system so we are not taking like apparent velocity or something.

So then about the difference in total energy which probably we can write it as delta K kinetic energy and delta E internal energy. So which will look like the half of Rho A Us - U p the apparent velocity of the particle moving away from the shock front that is to go into behind going towards the left side of the stock front if you can draw again this front. So the this way we are showing the kinetic energy this is the m and V square is U p square.

So sorry what we wrote here is should be in this direction but the m is this correct so m is this expression and then the velocity of particle ahead of the sub front is U 0, so that we will see in later so this is the final kinetic energy plus our final internal energy which will be internal energy per unit mass multiplied by the mass so that will be like density Rho A Us E - U p dt.

So this is also mass and this is for final configuration and then minus the initial one which will be equal to half of Rho 0 A Us - U 0 dt U 0 square + E 0 E Rho 0 Us - U 0 dt. So this is initial that means before the shock front travels to the material so this is the initial one and then when the shock front travels enters that area and then that is the final one.

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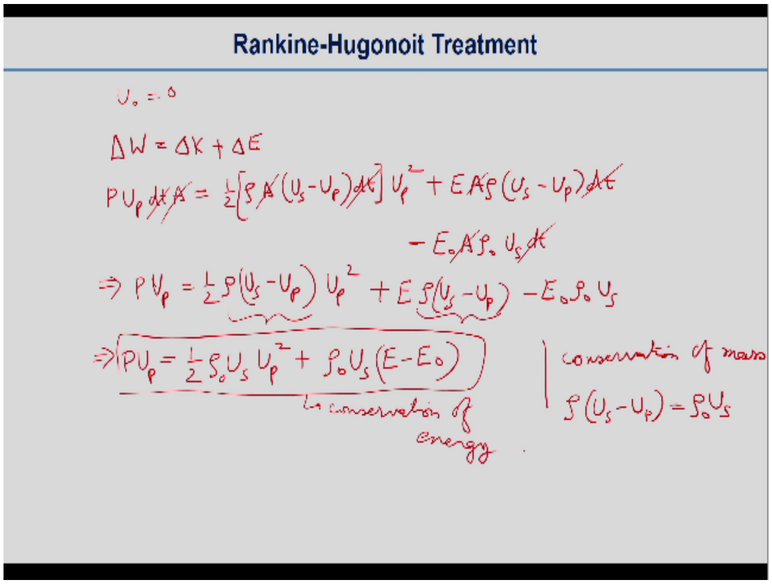

So now if we can simplify it so what we can do is we can take U 0 = 0 that we know that particular stationary ahead of the shock front and that will give us if we simplify it basically, so if we have this delta W, delta K +, delta E and that will give us P U p dt multiplied by A is equal to half of Rho A Us – E P U p ft the whole thing to the power U p square related like this + E a Rho Us - U p dt minus the third term will be 0 because that way that has an U 0 square we subscript You 0 square. So and the fourth term will be E 0 a Rho 0 Us - unit will be that unit part will be 0 and dt.

So now we can cancel out the dt and every term and also A the area in every term. So finally this will look like this half of Rho Us - U p U p square + E Rho Us - U p - U 0 Rho 0 Us. So from conservation of mass**,** so we got that Rho Us - U p = Rho 0 Us. So if we substitute that that will give us P U p this part if we substitute half Rho so Rho 0 Us that is the sonic shock impedance and then U p square plus we can simplify it more Rho 0 Us E – E0. So that is again we are simplifying this portion with the conservation of mass. So this is basically what we found is conservation of energy**.**

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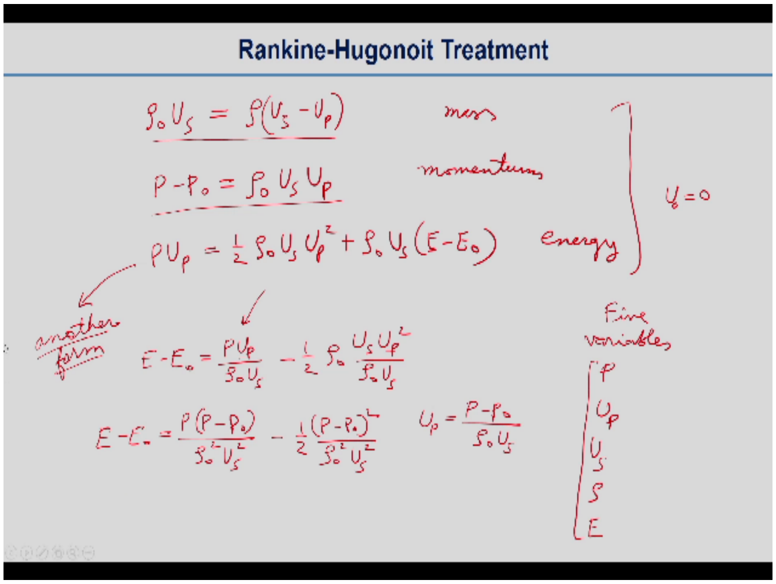

So what we found in our discussion here is that the first one is conservation of mass that is if we summarize it the 3 conservation equation, so this is conservation of mass and then second one is P - P 0 Rho 0 U s U p and that is conservation of momentum and third one is the conservation of energy half Rho 0 Us U p square + Rho 0 Us E - E 0 that is conservation of energy and all of these are for as we know the head of the shark from the practical velocity particles are stationary A 0 = 0.

So although this is we have written this conservation of energy as in this form but there is another form of conservation of energy is more popular. So we will work on that will derive that as well and here we can see that we have 5 variables in these 3 equations we have 5 variables. So those are P U p Us P is the pressure U p is the particle velocity, Us is the velocity of the shock wave front and then another one is density of the fluid.

And then E the internal energy per unit mass, so let us see that, so we will require one more relation that you can understand form here. So we will discuss about that later because we have 3 equations and we have 5 variables. So if we get one more relations then what we can do is we can determine these parameters of shock wave parameters in terms of another one, so that we will discuss and so before that we want to see what is the other form.

Other form of these energy equation which is more common, so we will go little quickly, **so** so if you see this equation so if you want to simplify E – E 0 = P U p Rho 0 Us - half of Rho 0 U s U p square divided by Rho 0 Us from the conservation of momentum what we got is conservation of momentum we got U p to this conservation of momentum equation we got that U p = P - P 0 E - P 0 divided by Rho 0 Us.

So if we substitute U P and that will give us E – E 0 P - P 0 Rho 0 square U s square - half of E - E 0 square divided by Rho 0 square U s square, again from the conservation of mass we can substitute this. So that will give us will it in the next.

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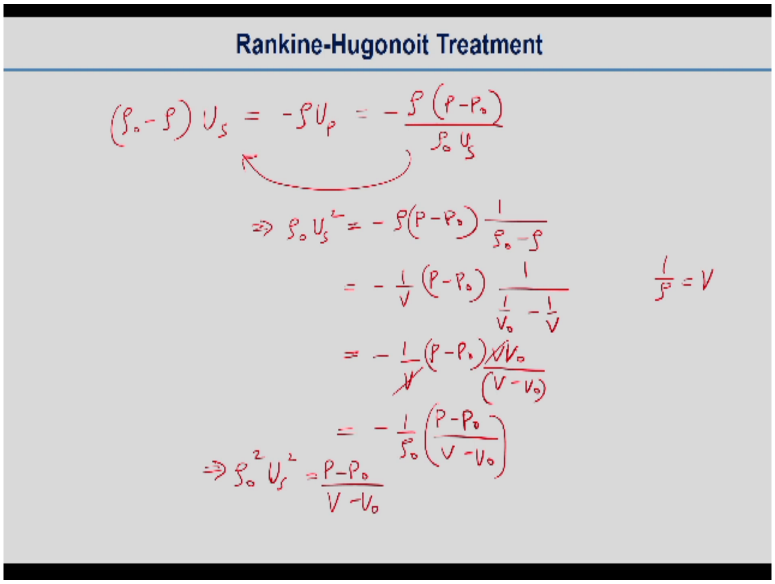

So from the conservation of mass which we use it will get expression like this which will be equal to minus Rho U p and again if we use the conservation of momentum that will in this relation that will give something like this. So now if we combine this we get Rho 0 U s square - Rho P - P 0 1 divided by Rho 0 – Rho. So if we know that 1 by Rho = V volume V. So in here if we do this substitution so this is V only.

So P – P 0 1 by V 0 1 by V and 1 that will give us 1 by V P - P 0 will have V multiplied by V 0 V - V 0, we can cancel V and V here. So this we can write there will be one remaining V 0 so that will be can be written as 1 by Rho 0 P - P 0 V - V 0. So from here nothing more you can get is Rho 0 square U s square = P 0 V 0 V - V 0.

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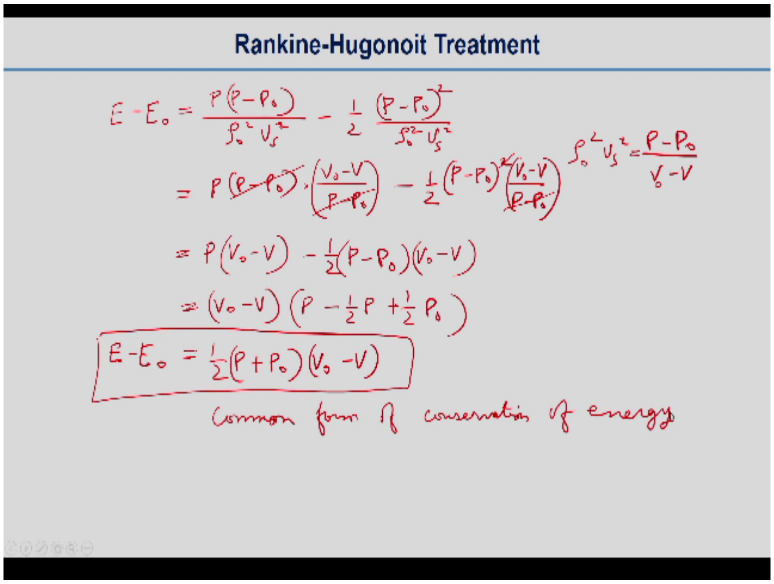

And if we substitute this relation in an earlier relation that is earlier relation of conservation of energy is equal to P P - P 0 divided by we do not use s square minus half of P - P 0 whole square Rho 0 square U s Rho 0 square Us square, so the previous expression if we substitute here that means the previous expression what we got is Rho 0 U s square = P - P 0 V - V 0 sorry this is P - P 0 V 0 – V. If we substitute this what we will get is P P - P 0 multiplied by V V 0 – V P - P 0 and then minus half of P - P 0 square.

And then V 0 – V P – P 0, so this will be V so this actual cancel out, so that will get P V 0 - V + - half of P - V 0 V 0 - V V 0 – V, if we take it out as a common factor so P - half of P + half of P 0 which will be equal to half of so we will try it first P + P 0 V 0 - V E – E 0. So this is the more common form of energy.

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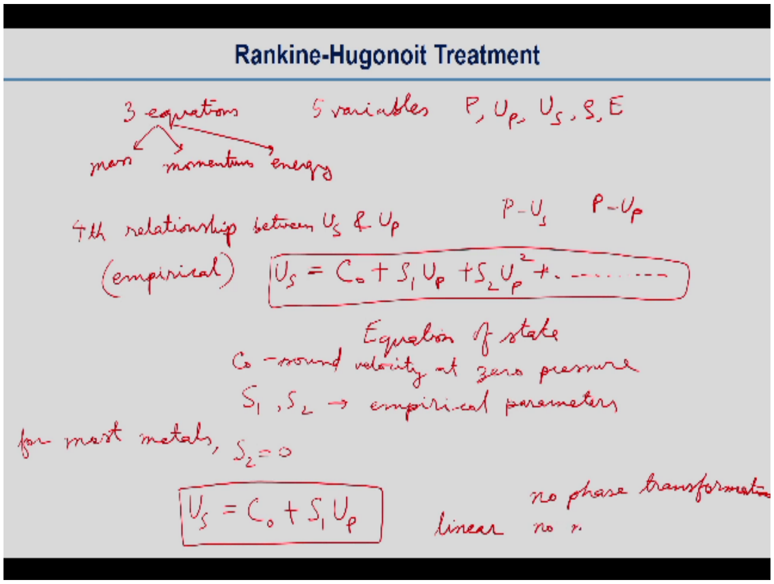

So as you have discussed we had 3 equations that is conservation of mass, conservation of momentum and conservation of energy and we had 5 variables that we found as P U p You s density and then E and these 3 equations are mass momentum and energy conservation equation. So if you want to know the relationship let us say of this shock wave parameters let us say we want to need a relation between U and Us or P and let us say U p, so what we need to have one more equation.

So that we can express these parameters in terms of other parameter, so there is another the fourth the relationship that is relationship we required is an empirical one empirical that is experimentally determined empirical one that is we can have the relations between **U s** U s and U p shock wave velocity and particle velocity. So U s will be equal to C 0 + S1 U p + S 2 U p square + dot dot dot I mean that means we can have more terms.

And this equation is known as equation of state for a material and the shockwave, so what is C 0, C 0 is sound velocity at zero pressure and S1 S2 they are empirical parameters. Now for most metals we can assume S 2 = 0 and the higher this parameter will be 0 and that will give us a linear relationship between U s and U p which will be equal to S 1 U p.

So this is the equation of state, this has a linear relation and it is valid for material with if it does not undergo phase transformation. So no phase transformation this is valid for no phase transformation or no material porosity**.** So if the material undergoes phase transformation under shockwave or if the material has porosity, this linear relationship of Us and that in stock wave velocity and the particle velocity will not hold and that will have we need to consider other relationship.

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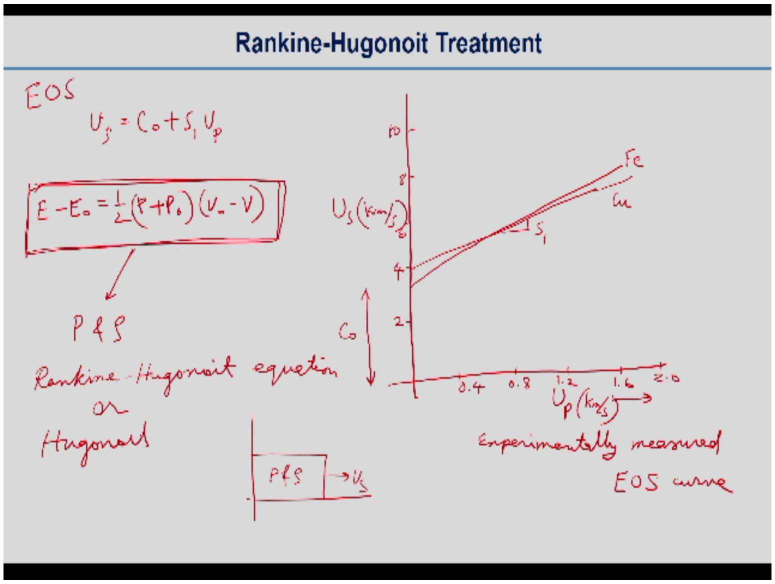

So this relationship of U s and U p which is C 0 + S1 U p and can be plotted, so we have experimentally measured equation of state will write U s curved, this is U s equation of state curve. So this is x axis will be U p which is let us say in kilo meter per second because this waves travel very fast and then us which is also in kilometer per second. So if you keep like this and then so this is 0.4 0.8 1.2, 1.6, 2.0.

And here if we have 2 4 6 8 10. So our curves will look something like this let us say for iron or some other material . So let us say this is for iron or if you have some other material let us say for copper, this can be little different maybe this is iron the earlier one and this may be copper . So this will be straight lines as you can see this intercept you can see that C 0 and then slope is S1. so these are the relations we can have from this equation of state as we discuss this is valid for no phase transformation and no material porosity.

And also the equation the earlier equation E - E 0 = half of P + P 0 V 0 – V, so this equation actually established as a relation between pressure and density and that is immediately behind the shock wave that means this is the shockwave so it is traveling with U s. So in the behind the shock wave what is the pressure and what is the density. So this equation the earlier we derived from conservation of all the conservation equations.

So this will is the relationship is known as this relationship between P and Rho is called Rankine Hugonoit relationship and also or it is called only Hugonoit. So we will discuss the rest of part of it in the next lecture. So what we discussed today is the Rankine Hugonoit treatment for shock waves and it consists of the 3 conservation equations, conservation of mass, conservation of momentum and conservation of energy.

And we also discussed about another relationship which has the relationship between U s and U p the shock wave velocity and particle velocity which is an empirical relationship that can be linear straight line when there is no phase transformation under shock wave or the material does not have any porosity. Otherwise they can this relation can be nonlinear and also we discussed about the Rankine Hugonoit equation. So with that so we will finish today's lecture and so we will discuss the rest part of it in the next lecture, thank you.