Dynamic Behaviour of Materials
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Lecture-19
Rankine Hugonoit Treatment and Shockwave Under Impact

Hello everyone, so in the last lecture we discussed about shockwaves that is a Rankine Hugonoit treatment of shockwaves and we have the Rankine Hugonoit equations. So today we will discuss a little bit more about Rankine Hugonoit equation or Hugonoit curve and then we will discuss about shockwave generation from impact.

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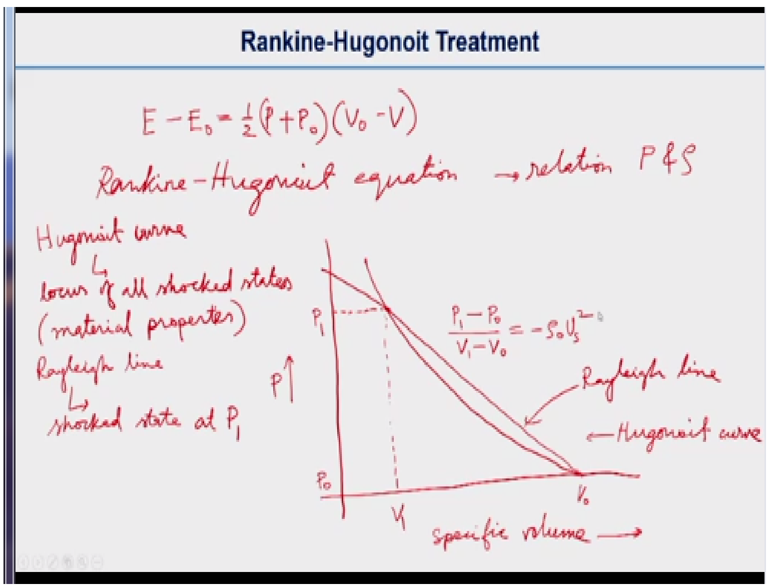

So what we discuss in the last class from the conservation of energy we had this equation E – E0 that is the internal energy half of P + P0 multiplied by V0 V P the pressure of initial and final configuration and V is the volume. So this is we talked about Rankine Hugonoit equation, so this is basically a relation say between P pressure and density.

So we will have a the graphical form of this equation or the Hugonoit curve, so here we have the pressure on the y-axis and specific volume, volume on the x axis which is 1 by rho 1 by mass density. So the Hugonoit curve will look something like this, this is the Hugonoit curve this is Hugonoit curve from the Rankine Hugonoit equation probably I will redraw it ok, so now this Hugonoit curve is if the locus of all shocked states of the material.

So this is essentially like describes the material properties, so relation between P and density, pressure and density. However this curve has some discontinuity as can be seen from this Rankine Hugonoit equation. So that is why it is difficult to find the pressure density relationship from the Hugonoit curve and then we use another line a Rayleigh line is a straight line.

This is called Rayleigh line and suppose at this point pressure is P1, so this Rayleigh line is intersecting the Hugonoit curve it at this point and that is corresponding to pressure P1. So Rayleigh line gives the shocked state of at pressure P at, so this is not P this is P1, so at pressure P1. So the Rayleigh line will give us the shock state, so that we can determine the other shock wave parameters.

So what we need to know from here is we want to have the slope of the curve, slope of the Rayleigh line. So what we can do is this is the pressure P1 and then pressure p0 is let us says the reference line and then we have the volume the specific volume V1 here and then V0. So the slope of this line will be P1 – P0 divided by V1 – V0, so we will try to find this slope, so that we can determined soft state at a different point at other points.

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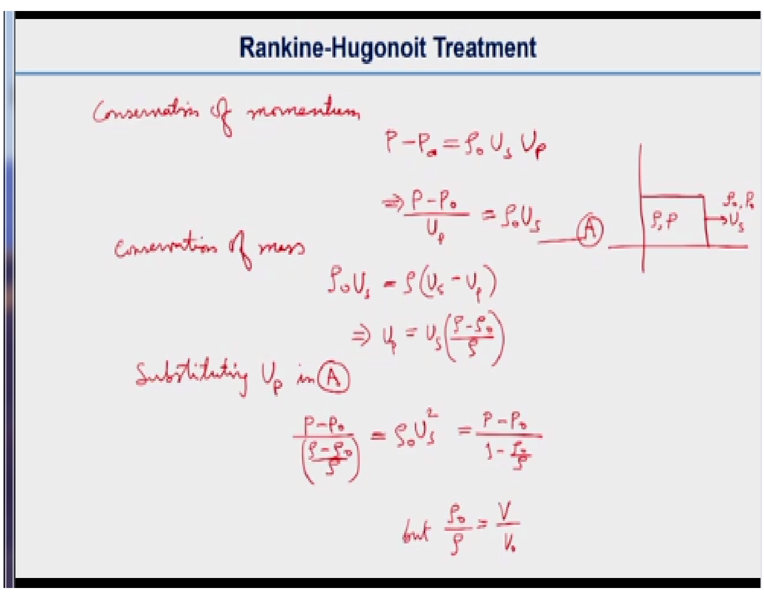

So how to do that from the conservation of momentum, so we have P – P0 = rho0 U s U p that is what we discussed earlier. So from here what we can get is P - P0 divided by U p is = rho0 U s again if we use conservation of mass rho0 U s, rho0 is the, so if I again draw this just if you do not remember. So this is our shockwave discuss in the last class shockwave with velocity U s, so behind a shock front.

So this density is rho and pressure is P and the head the shock front this is density is rho0 and pressure is P 0 and then U p is the particle velocity. So Rho0 U s = Rho 0 U s - U p and this will give us an expression of U p which is U s rho – rho 0 divided by rho. And if we replace substituting U p in A. We will found that P - P0 divided by rho - rho0 by rho and we have that U s, so that U s we can keep it and at the right hand side.

So that will give us U s square that is the velocity of the shockwave and which will be again we can write in this way P – P0 1 - rho0 by rho under left hand expression we can write it this way and but we know that this rho0 divided by rho is nothing but V by V0 the ratio the velocities volumes ratio the volumes.

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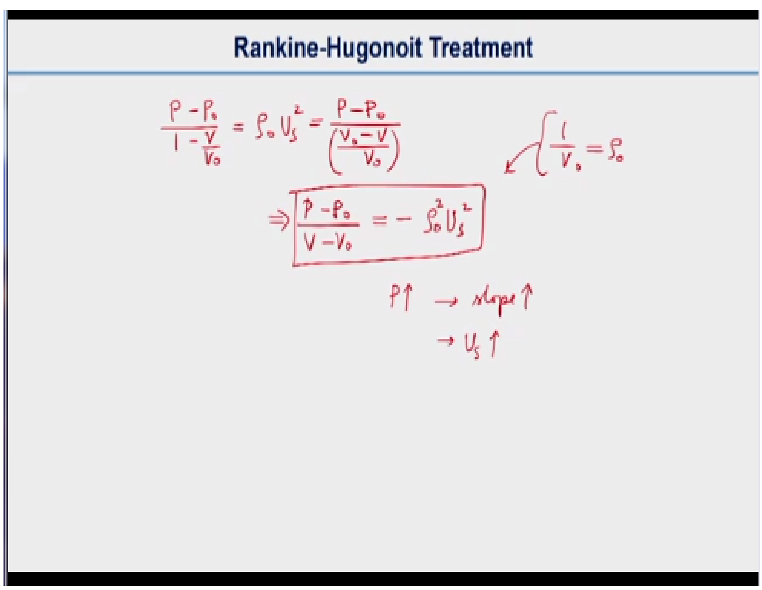

So using that relation if you we can write it like this and this will begin we can express it in this form which is equivalent to P – P0 divided by V - V0 and that is we are writing to oppose that is why the minus sign will come here rho0 U s square. But the another rho 0 will come from this V0 will be is equal to rho 0, so that will give us rho 0 square here, so this is the slope of that Rayleigh line.

So we will go back to that slide so what we got this the slope of the Rayleigh line which will actually can determine that this continuity we found in the uh Hugonoit curve. So that will be rho0 U s square, so from the Rayleigh line, so we can find out the shock state if we increase P the pressure that will increase the slope of Rayleigh line and also it will increase the velocity of shockwave.

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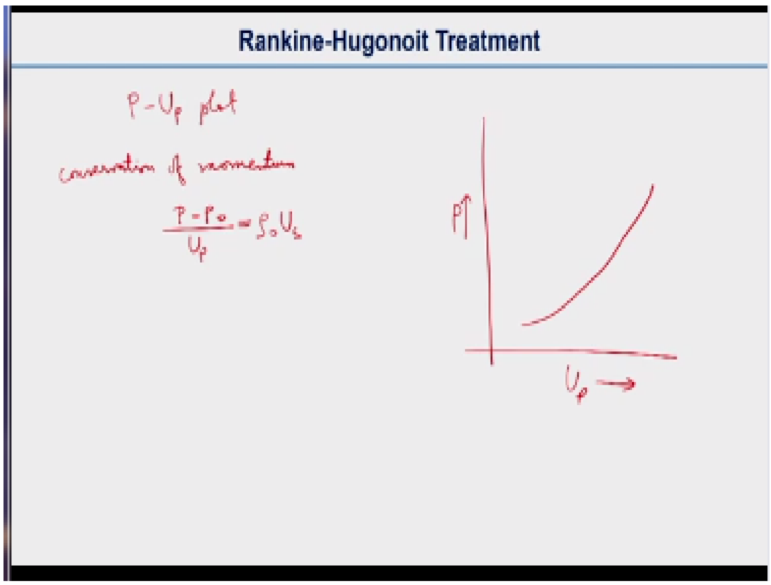

And again we can have another plot P U p, plot so this plot is pressure verses particle velocity, so pressure verses particle velocity from conservation of momentum P - P0 divided by U p will give us the rho0 U s. So this will give us a this type of curve, so we have seen that pressure versus specific volume and pressure versus U p that is the particle velocity. So these are the different curves or relationship between the shockwave parameters.

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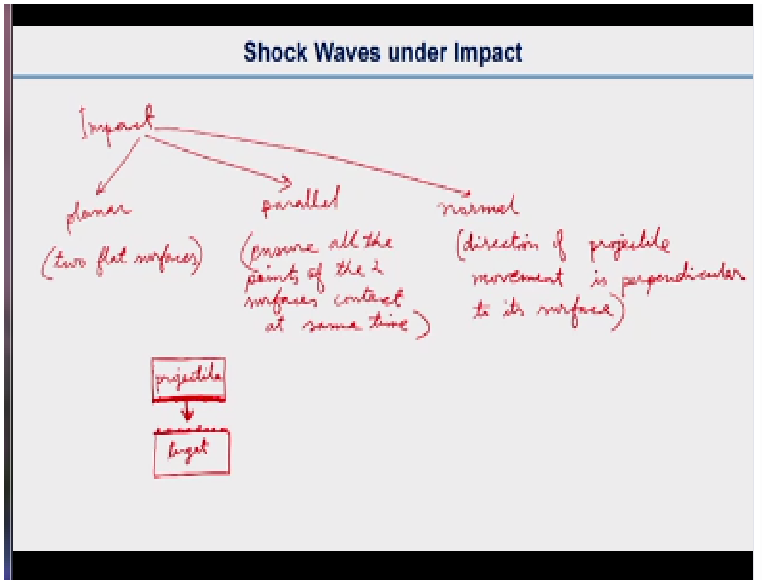

So next we will discussed about producing shockwaves generating shockwaves with impact, so we will consider the simplest case of impact very simple case of shockwave generation. So this impact will be have 3 characteristics first one is this should be planar and the second it should be a parallel impact and third it should be normal. So these are the conditions for the simple shockwave generation.

So planar means it is impact of 2 flat surfaces we will draw the diagram here, so first maybe so we have this projectile and this is the target and target. So this is the projectile has hitting the target, so we have 2 flat surfaces, so this is a planar I impact and it is parallel plate will ensure that all the points of the 2 surfaces contact at the same time, so we have all the points so here we have in the projectile we have all the points and here we have in a target.

So all the points in the 2 surfaces will at the same time, so it should be exactly parallel to each other and the normal means the direction of projectile movement is perpendicular to it is surface. So the projectile surface is this one so it direction of movement is perpendicular this it is the surface of itself I mean that is not the target surface but this is should be projectile surface.

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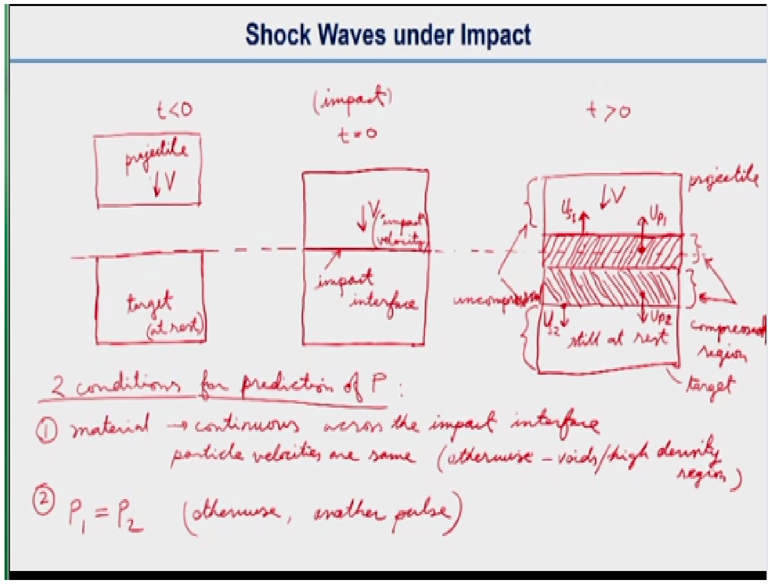

We will again discuss this problem in details, so let us assume that we will draw a line first, so first case is, so we have the target here need to make this some that is a rectangular block in 2D we have seen. So this is the target and our projector is something like this, so projectile is coming at a velocity V, so this is at rest it is velocity is 0 this is at rest. So now it will have a impact at the time of impact, so this is the impact surface.

So we can call this is T = 0 and T is actually before impacted we can call that and this is we can write that the point of impact this is at the just the impact point the second case and the third case is this is during impact which is T is get in 0. So impact is still happening and the projectile will go little deeper from that reference line and this is our target we can see here the impact velocity is the same as VA what we see in the left hand side figure.

So this is impact velocity V the impact velocity and this is the impact interface we call is a impact plane or impact interface, interface we are saying because it is the interface between the target and the projectile. And now in this case but during the deformation process, so we will see that there are 2 waves shockwave generated on the both side projectile and both projectile and the target side.

So these are the compressed region, so this one and this one they are compressed region of target and a projectile the velocity of the wave shockwave. This is actually that compressed region that means the shockwave front is this boundary, So U s1 is the velocity of the shockwave and as we can understand this portion and this portion are uncompressed and this is this portion is still at rest this portion and this portion is still moving at velocity V.

But if you take a particle velocity will call this as U p1 of the compressed region and similarly ok I did not the other one. So this shockwave the one is as I as we discuss that this is going upward which is shockwave of the projectile. So I will write it here this is the projectile and this is the target, so the one is going upward to the shockwave generated in the projectile and one is going downward that is shockwave produced in the target. We will write the velocity as U subscript s2, so 2 for target 1 for projectile.

Similarly for particle velocity U p 1 for projectile and U p 2 for target material we need to have 2 conditions here, so 2 conditions of this impact . So that for the prediction of pressure P and then subsequent we can even predict the particle velocity. So these 2 conditions are first one is the material has to be continuous that means continuous across the impact interface.

So this side and this side, so this should be continuous, continuous means the impact it means particle velocities are same and otherwise what happens otherwise we may get some voids or high density region. So that I think we can understand that if the particle are not traveling in the same velocity there will be some voids or there will be some very high density regions that means higher than the density of the original materials , so that can generate.

So this is condition number one and the condition number one the 2 is pressure has to be same in the both of them P1 = P2 in the compressed region. So P1 and the projectile should be equal to P2 in the target, so other ways there can be we can imagine that there can be another pulse another wave. So if the pressures are not same we cannot assume it to be the same wave.

So that can the pressure difference can create another pulse, so these are the conditions we will follow when we will derive the relations. I mean we want to determine the pressure of this shockwaves of the simple case of shockwave generation on the impact and also we will get the particle velocities. So now in this case the interesting point here is to.

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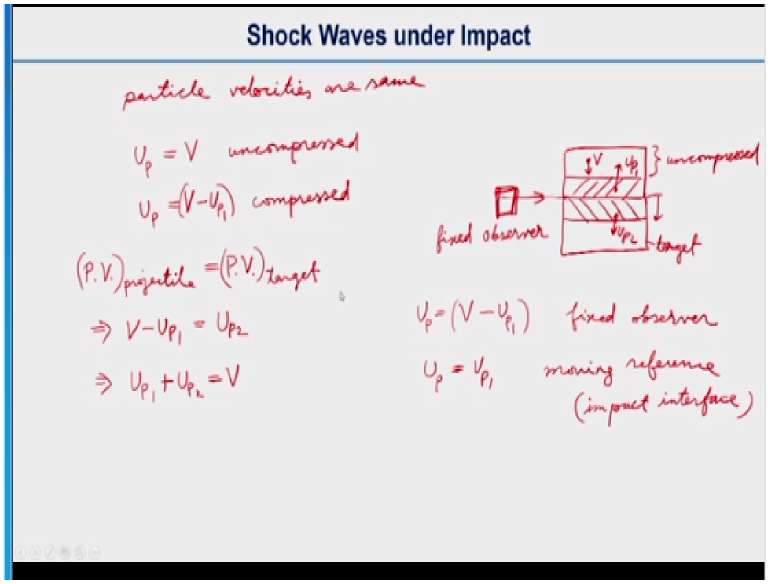

So the to establish the particle velocities the condition one we told that the particle velocities are same, so here we show that we have a particle velocity U p 1 and we have a particle velocity here which is U p 2. So the particle velocities are same to show that we need to take a fixed reference frame suppose this fixed reference frame means this is our observer.

So if the fixed observer outside of that, that can see that the particle velocity in the original material before the impact or the part of the material which is uncompressed is still V1 that means the particle velocity U p is V for uncompressed region. And then particle velocity for the compressed region if you think this will be reduced V- U p 1, so U p 1, so actually if you see V- U p 1 is the particle velocity.

When we have a fixed observer and this particle velocity is actually U p 1 for the projectile if that is a moving reference. So moving a reference let us say this moving reference if we take it as impact interface so if is the reference is the impact interface that is moving in this direction. So then the particle velocity is U p 1 and if we take a fixed observer outside the system that is projectile target system.

So in this case if you know that this target is the lower one target system the target projectile target system. So then our particle velocity will be V - U p1, so and this will give us our condition the particle velocities are same that condition means that this in the projectile that means particle velocity I will write PV of projectile is equal to particle velocity PV of in the target or you can simply like even U subscript P for P V.

So this will give us V - U p 1 = U p 2 which is the particle velocity in the target, so this is will give us U p 1 + U p 2 = V which is the impact velocity, V is the impact velocity, so to determine the pressure.

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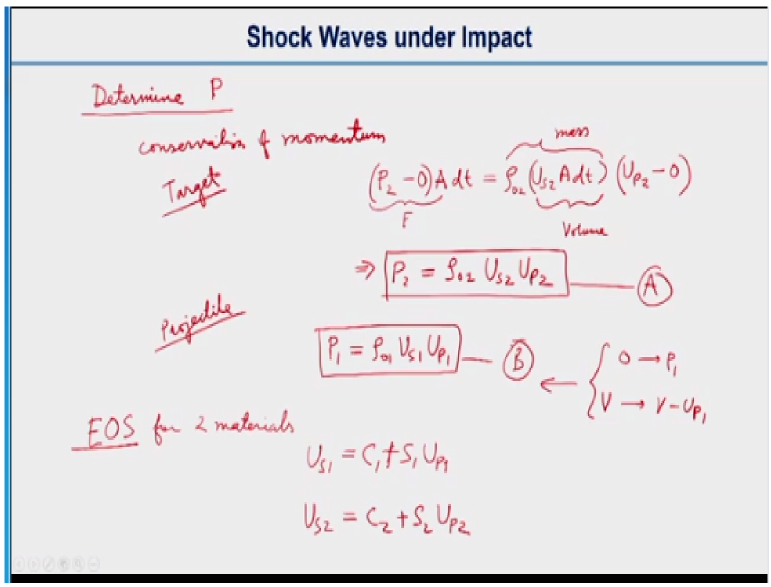

So basically our goal is to determine the pressure and also the particle velocity subsequently, so to determine the pressure, so we will first let us say we will use conservation of momentum that is nothing but the final that the impulse is P2 - 0 A dt, P2 is the pressure which increases from 0 to P2 multiplied by area that will give us the force multiplied by dt that is the impulse and for the change in momentum is rho0 2 let us say U s 2 A dt.

This one is the volume and the entire including rho0 this is the mass multiplied by change in the velocities of the particles. So this is the mass of that compressed region, so ultimately we will get a relationship like P2 equal to this is for the target we are talking about this is for a target until so P2 = rho0 subscript rho0 2 actually and then U s 2 U p 2. Similarly for the projectile, for the projectile we can have P1 = rho0 1, U s 1, U p 1.

So this is again from 0 to P1 pressure and then the particle velocity are from initial V to V - U p 1. So this will give us this, so pressure in the projectile, pressure in the target, so these are from conservation of momentum and also the equation of state that we discussed in the last lecture the equation of state for the 2 materials means the materials of the target and the materials of the projectile.

So we have 2 relationship here U s 1 C1 + s1, so as you know this is equation of state is the relationship between the shockwave velocity and the particle velocity and C1 and s1 are 2 empirical parameters. So similarly for target this is C2 + s2 these are material parameters has to multiplied by U p2. So if you substitute these equations substitute these EOS into these 2 equations let us say I will write A and B.

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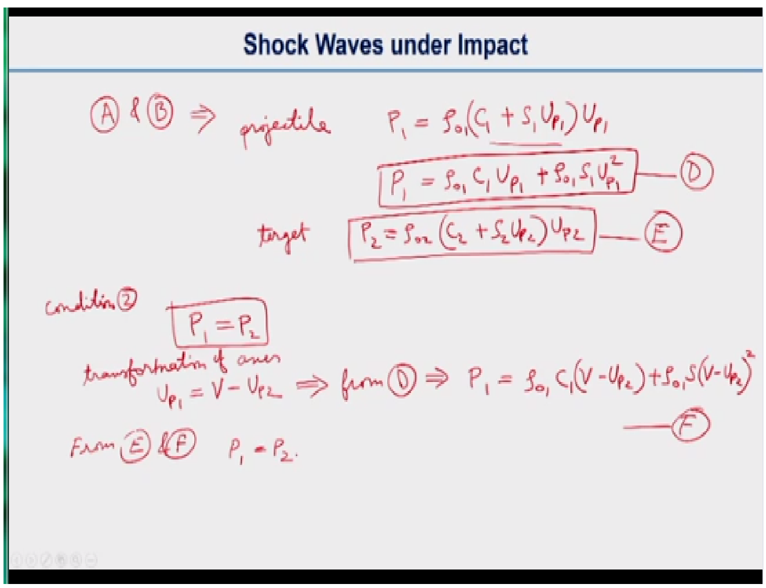

If we substitute the EOS in A and B, so from A and B, so what we will have is for the projectile P1 = rho 0 1 C1 + s1 U p 1 multiplied by U p 1 that you can understand this one is we get it from we got it from the EOS of the projectile material. And then this will give you rho0 1 C1 U p 1 + rho0 1 s1 U p 1 square similarly for the target P2 = rho0 2 C2 s2 U p 2 U p 2. So now we will set the condition P1 = P2 which is our condition number 2.

So what we will do is we will replace U p 1 in this equations so that will give actually that we know from earlier this U p 1 is V - U p 2. So that means here we have the transformation of axes is that means our reference frame is now as we know that that is the fixed observer outside of the system. So if we substitute this relationship in let us say we have D and E.

So in D let us say from D so what we will get is P1 rho0 C1 V - U p2 + rho 0 1 s V - U p2 holding square then we have this condition P1 P2. So what we will do is from E and F ok, so from E and F and if we are using P1 = P2.

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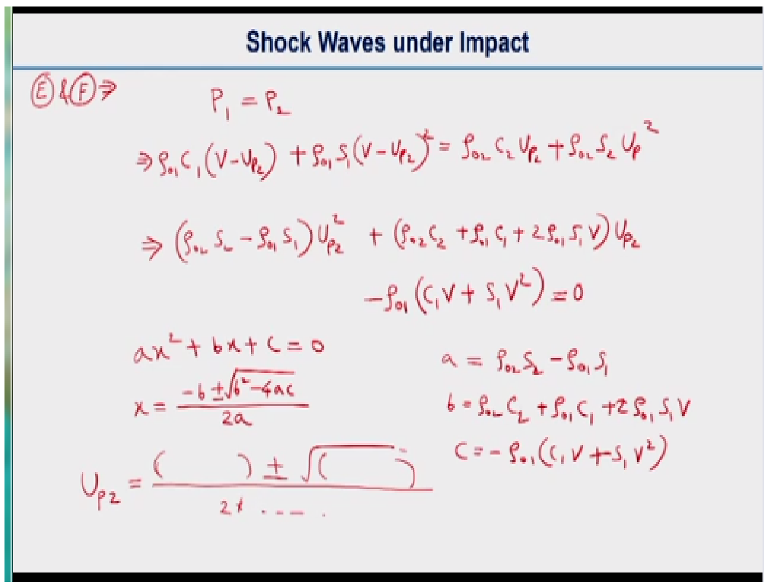

So we will do in the next slide, so this will give us from E and F, I will write it again from E and F and as we know P1 = P2. So probably we can here write it again P1 = P2, so that will give us rho0 1 C1 V - U p 2 + rho0 1 s1 V - U p 2 square = rho0 2 C2 U p 2 + rho0 2 s2 U p square. So this will we can rearrange I am just omitting few of the steps and we can rearrange and can get rho0 2 s2 – rho 0 1 s1 U p 2 square + rho0 2 C2 + rho0 1 C1 plus twice rho0 1 s1 V U p - rho0 1 C1 V + s 1 V square = 0.

So this is as you can understand we are having a quadratic expression in terms of U p 2 and this is nothing but your like ax square + bx + c = 0 and we know that the root of this equation will be equal to x = - b + - b square - 4ac square root of the whole thing and divided by twice a. So here in this case as we know a = rho0 2 s2 – rho0 1 s1 b = rho0 2 C2 + rho0 1 C1 plus twice rho0 1 s1 V and c = - rho0 1 C1 V + s1 V square.

So if we want to get the this expression, so U p 2 will be a somewhat a long expression, so probably I do not write it here. So we can see that is why what will happen is this you know you can replace this a, b, c in this equation and so this will give you twice a 2 into a from U p.

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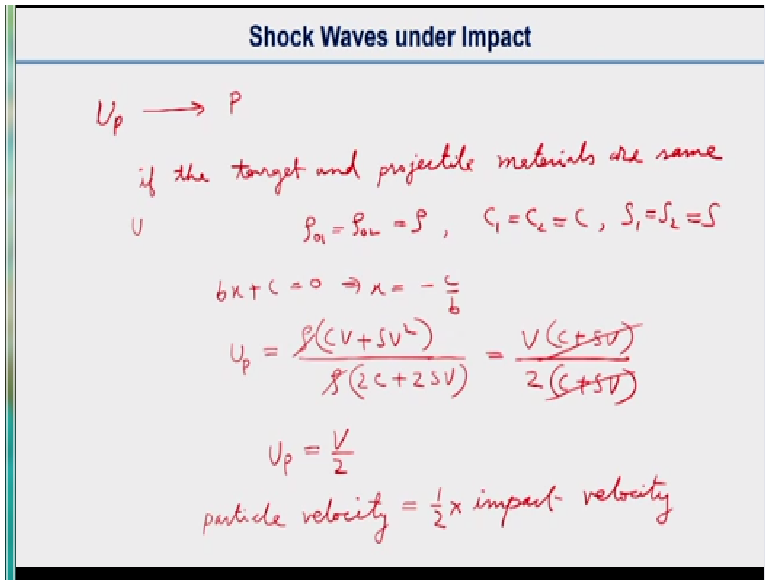

We can even get the expression for pressure, so if the target and projectile material are same. So what will happen then rho0 1 rho0 2 = rho, similarly C1, C2 = C and s1, s2 = s what will happen in this case. If you see the other expression though we have the a, will be equal to 0 for same material. So if a = 0 then we are left with only bx + c = 0 and this will give you x = - c by b right.

So **-** u and then which will be nothing but U p equal to actually rho C V + S V square divided by rho twice C + twice S V, so this will be rho will be cancelled out here V C + S V divided by twice of C + S V. So this will cancel out and finally this U p we get it as V by 2, so basically the particle velocity will be V by 2, so that means keep the material are same, particle velocity will be half of impact velocity, so this will be half of impact velocity yeah.

So that is all for today, so we will be discussing little bit more about it that graphical solution of this whatever we discuss today. And then we will discuss about the methods of getting the equation of state of shock waves, so this is all for today, thank you.