Dynamic Behaviour of Materials
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Lecture-20
Shockwave Under Impact

Hello everyone in the last lecture we have discussed about shockwave produced under impact. So we discussed about a planar parallel and normal impact we derive the expressions for a pressure generated during this impacts. So in today's class we will continue that, so we will solve some numerical examples that is basically from the Mark Meyers dynamic behavior of materials book and will solve to a couple of problems and then we will talk speak a little bit more about this shockwave.

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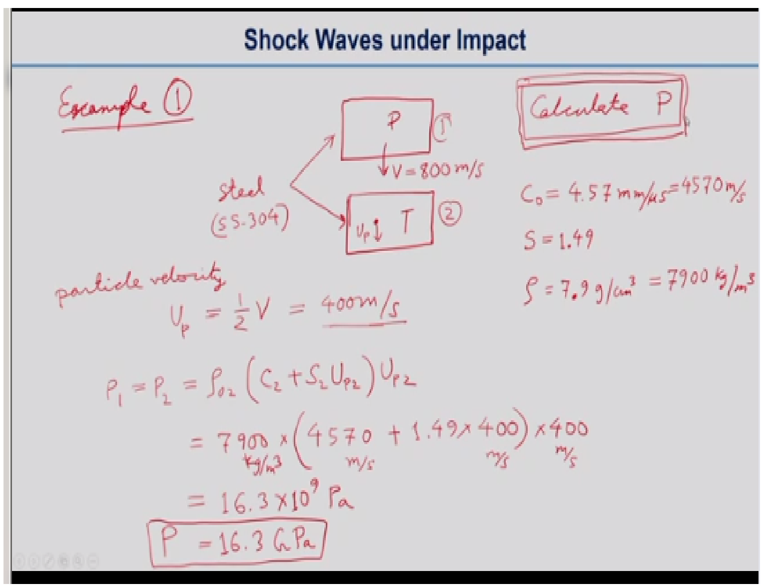

So, the first problem is let us say write example numerical example as a 1 it is so this is let us assume that we have a projectile. I will draw the projectile like what we did in our last lecture. So will draw it like this, so this is a projectile and this is the target and the projectile has a velocity impact velocity which is let us assume that 800 meter per second which is kind of close to the speed of ak47 bullet.

So now let us assume that, so we have the same material in both the projectile and this is let us say this steel and for example we have assess 304 this special kind of steel. So, **so** this is the steel we are using for both projectile and target, so here in this problem we need to calculate the pressure P generated by this impact. So this is our goal calculate the pressure P and we need the properties so we need the shock parameters needed for equation of state.

And those parameters are shock parameters c0 and s, so c0 is 4.57 millimeter per microsecond and s is 1.49, so 4.57 and also we need density this rho which is for still 7.9 gram per centimeter cube. So we can convert this 7.9 per gram per centimeter cube, **so** into kg per meter cube that will be 7900 kg per meter cube. So we will keep it into all in SI units and here for 4.57 millimeter that means again 4570 meter per second.

So we need to calculate the pressure and before that we need to have the particle velocity, in our last lecture we derive that the particle velocity for an impact which has both the projectile and the target as same material. That means if we have steel and both the materials then the particle velocity will be U p will be equal to half V the impact velocity. So this is particle velocity, particle velocity whatever particle velocity will be generated after the impact or so this U p = 1/2 V.

So which will be equal to 800 by 2 it is 400 meter per second and then from earlier equation where we had that pressure generated in the target and the projectile will be same and P1 = P2 which will be equal to rho02 where 1 is for projectile and 2 is for target. So rho02 that is the density of the target multiplied by the parameters for the equation of state thus s2 U p 2 this all for the target and multiplied by U p 2.

So in this case we can take the density as we know 7900 multiplied by C 2 which is nothing but the C 0 what we wrote it here there will be 4570 + s would be 1.49 without any dimension. And then U p 2 which we have found at 400 meter per second here 400 and then the whole thing multiplied by 400. So this will give us let us say it is a big number, so there will be 16.3 10 to the power 9 Pascal.

So all the you needs if you see that this is in all are in SI units this is kg per meter cube and here also these are meter per second meter per second meter per second. So this will be the final result will be 16.3 into 10 to power 9 Pascal, so which is nothing but we can write as Giga Pascal 16.3 Giga Pascal it is a very high pressure 16.3. So pressure is 16.3 pressure generated due to the impact, so that is what we wanted to calculate.

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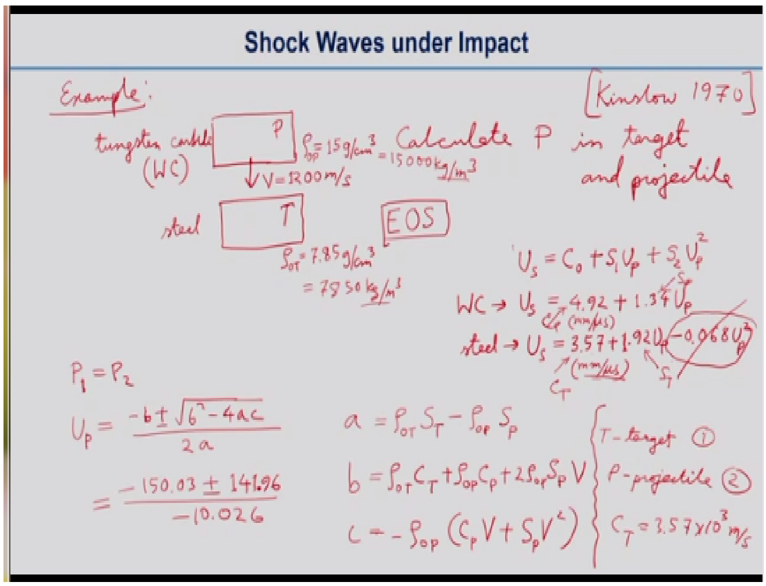

So this is our final answer, so we will discuss about one more problem now, this problem is from Mark Meyer’s that our textbook what we are following Mark Meyer’s book but again originally it is solution the analytical solution is from another literature that is Kinslow 1970. So in this case we will take 2 different materials for projectile and target, so let us this is a projectile and this is the target.

So our projectile is tungsten carbide or you can write WC is the symbol we use for tungsten carbide. And so here this target is still let us assume that the velocity will be 1200 this is meter per second and that is the speed at which the projectile will impact on the target. So what we need to calculate, so we need to calculate the pressure again in the target and the projectile, pressure generated in the target and projectile.

So for that we need the equation of state for these 2 materials equation of state for these 2 materials and also we need that density of these 2 materials. The density of steel this rho I will write rho0 T, T for target and this will be let 7.85 gram per meter centimeter cube little this is a little bit different than the earlier one. Because that was ss 304 which 7.9 and this is 7.85 and here tungsten carbide is very heavy material it has very high density that is 15 gram per centimeter cube.

Now for equation of state for these 2 materials are for let us say for tungsten carbide the equation of state is U s the velocity of the shockwave is equal to ok I will write it is it should look like c0 + s1 or s U p. So now in this case this is for tungsten carbide or better I will write this is a common equation and then we will say that tungsten carbide what is the equation and for steel what is the equation.

So for tungsten carbide this equation is looks like 4.92 1.34 U p, so U p is not known now the particle velocity and then again for still and the equation of state is 3.57 1.92 U p – 0.068 U p square. So your concern about these last term, so as you know actually for the equation of state is this is only the linear relationship. But we can extend you may have the more terms here, here if you write s1, s2 that can be U p square or there can be even higher terms that we already discussed about that.

So for still due to some complex behavior not maybe all still but for some material then that may be arising due to some complicated behavior like phase transformation also. So we have one more term which is a quadratic term U p square unit of these 3.57 or 4.92 we should understand that this is in millimeter per microsecond this is again millimeter per microsecond and this is s1 and s2 dimensionless.

So what we will do now is from our earlier derivations earlier relations we discuss in the previous class, so we need to set that the p1 is equal to p2 and that will give us something that the expression for U p. And that will be like little complex expression is b square - 4ac the root of the quadratic equation divided by twice a. So we will write what is a, b, c here but again I want to tell you so for in this case we will ignore this quadratic term.

So we will not consider this quadratic term because that will make our calculations simple, so this U p that is what discuss we discuss in the last class. So what is a, b and c, that is rho0 T, T for target I will write T for target in the subscript and P for projectile or we can even use number 1 and number 2 as well 1 or 2 whatever we discussed in the earlier derivation we keep it as 1 and 2 but today we will keep it as T and P.

So this is rho, rho0 T sT – rho0 P, P for projectile and then sp then b is rho0 T, c T for target rho0 o p for projectile cp. And then twice rho0 p there is the density of the projectile material and sp for projectile and V the impact velocity and then c will be equal to – rho0 p for projectile c this shock parameter for the projectile material multiplied by V the impact velocity then rho p V square this would be s1 sp V square this will be Sp V square.

So this is the expression and then we know that rho0 T s you can see 7.85 gram per centimeter cube or we can write it as 7850 kg per meter cube here actually we did not write the subscript. But this is rho0 p and that will be equal to 15,000 kg per meter cube and then we have this c0 is cT or cp these are nothing but the this, so this is c T for the target and this is c for projectile, similarly this is s target and this is s projectile.

So now using this what we can get is the expression we are not going to calculate in each and every step. So I will write the final expression this will be 150.03 + - 141.96, so divided by – 10.026, so if a + - sign here, so we will get 2 roots out of here.

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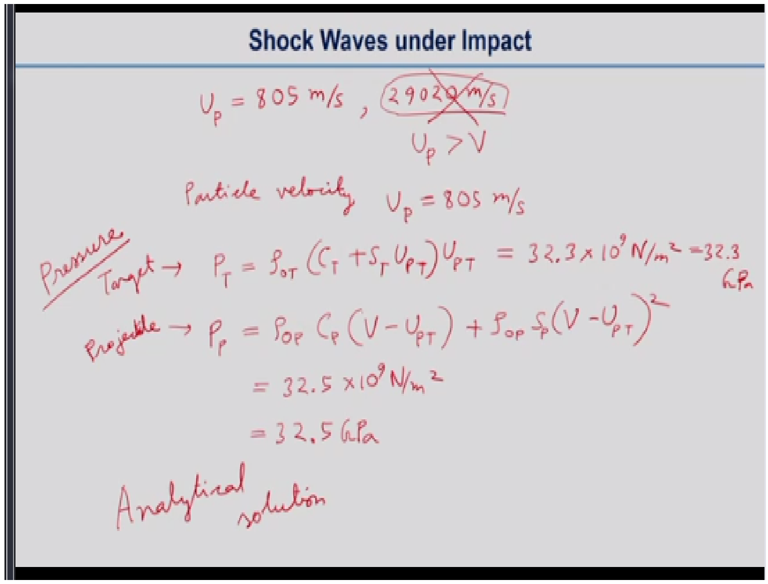

And then these 2 roots are fortunately the both are we have positive roots, actually this is we can write in kilometer per second or this is nothing but actually 805 meter per second which is 805 per meter per second. So actually what we can do is that in these calculations you need to be careful now, so because we can have this densities in kg per meter cube. Then we can have these C0 values which is let us say CT, CT will have 3.57 into 10 to the power 3 meter per second here it is millimeter per microsecond.

And that way if you convert everything into meter per second or kg, so what will happen is this I think we can directly write it here so U p the one root is U p 805 meter per second and another one is 29020 meter per second. So but this is a very high value we can see that this U p is 29 kilometer per second that is quite high and anyways both the roots are positive we can take only the positive roots not negative.

And here we can see that there is a very high value which is even higher than this U p is higher than our impact velocity V. So our particle velocity cannot exceed the impact velocity U p, so that is why we will neglect this one and we will only go with U p you can do 805. So that means our impact particle velocity in this impact will be 805 meter per second that is the particle velocity because of the impact.

So now we will see we will do the calculations let us say the pressure generated in the target that will be rho0 T CT all this subscript T for target s T, U pT, U p T. So this will give us 32.3 10 to the power 9 Newton per meter square we can write 32.3 Giga Pascal. And then pressure generated in the projectile or we can write it here target and projectile, so these are both pressures.

So rho0 p, p for projectile c p V - U p T this will be for target U p T not for projectile, so if you remember our earlier derivation, so this s p V - U p T square this is also T not p. So from this we will get the value as 32.5 multiplied by 10 to the power 9 Newton per meter square that is 32.5 Giga Pascal. So we have the pressures are there they equal actually almost equal we actually our condition was the pressures are equal.

So anyways we got this pressure generated in the target and projectile which has a very high value 32 Giga Pascal 32.5.

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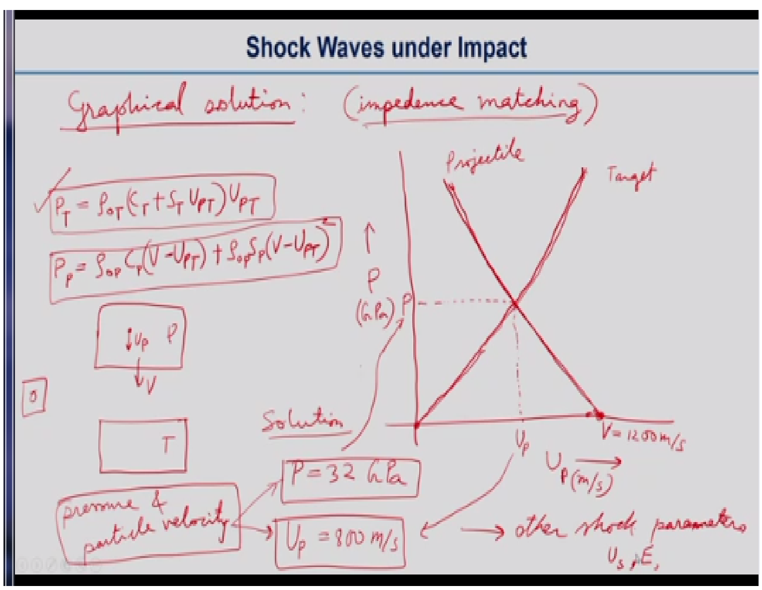

But there is another way to the solution, so what we did is the analytical solution we will go back and will write here, so this was the analytical solution. So we will go for one more solution which is graphical solution this you may find it to a faster solution. So graphical solution is this is actually also called a impedance matching technique. So basically we need to plot this here on the x-axis the particle velocity and the y-axis the pressure it is a pressure we are writing in Giga Pascal in particle velocity as we know this will be meter per second.

So now if we draw this, so these 2 equations what we derived earlier in earlier classes, so p2 that is for target rho0 2 C2 + s2 U p 2 and then for pressure p1 is rho0 1 C1 V - U p 2 + rho0 1 s1 V - U p 2 whole square so this is for projectile. I do not want to confuse you again, so probably we can write all this as T so this all 2 will be replaced by T sorry rub that is just to make you not much confuse.

So these all one we can replace by p for projectile this is p and this will be p, so pp means oh sorry this is T this one is T for target. So only these 2 are target by other way it is expression will have all p subscript, so now so these 2 equation that we derived in the earlier classes we will plot it here. So this would look like I will do very rough sketching here then you can check these all these diagrams in the book, so it is directly from the book.

So this will look something like this and that is for the target, so how the pressure will vary with respect to the particle velocity and similarly for that means what we are doing is we are plotting these equations for the target equation. And then we will do the other one, so this one will look like something like this and then this is actually for the projectile and as you know this projectile this one should be a impact velocity that is 1200 meter per second, so this is 1200 meter per second.

Because the projectile we are plotting U p that U p cannot be higher than the impact velocity the it will start from this and then the particles in the projectile. So if you draw this, this is the projectile p it is coming at a velocity V and it is hitting the target T. So the particle velocity in the projectile before heating will be 1200 meter per second if you have an observer outside this is observer now after heating it will change.

And then we can have this plot p p, p versus U p like that and similarly for target the initial velocity was 0 the particle velocity is 0 and it will increase and it will go to other extent. So this is these 2 equations we plotted and that means for projectile when U p will increase this pressure will decrease and for the target which U p pressure will increase.

So now the solution, the solution of this is the intersection of this curve, so this p that means this is almost what we got is if you plot it accurately so what we will get is it will be close to 32 Giga Pascal. So this point will be the solution is p will be equal to 32 around 32 Giga Pascal or what we got in the earlier analytical solution. So what we did it here is we plotted these 2 equations we know the relation between the pressure and the particle velocity.

Because we have the other parameters with s and density and the parameter c and s and that is for the target, so what we did is we have which start from the this point and we got this for this curve for target. And similarly we have another curve which starts from that V = 1200 meter per second that is the highest maximum particle velocity and that curve looks like this and if these 2 curve when it will mean that means as we have already in earlier derivation we set the condition that the pressure should be equal.

And our even the particle velocity should be equal, so these 2 from these 2 conditions what we can get is p is 32 Giga Pascal our U p whatever we found U p will be around 800 meter per second that was we got like 805 meter per second. So this is because our condition to earlier that the pressure and the particle velocity will be same in the projectile and target. So this is the graphical solution or impedance matching technique.

So this is as you can see that this is a faster technique than the analytical one, so also if we get these parameters like p or U p or U p 1 or U p 2 that means for target or projectile this basically will say that pressure and particle velocity. If we know these 2 parameters we can obtain the other shockwave parameters and using a Rankine Hugonoit equation.

So if we get the pressure and particle velocity then what we can do is from here other shockwave parameters can be or shock parameters we can what are the other parameters as we know that we got total 5 parameters and the other parameters are will be shockwave velocity U s or the internal energy like E, so these we can find it out.

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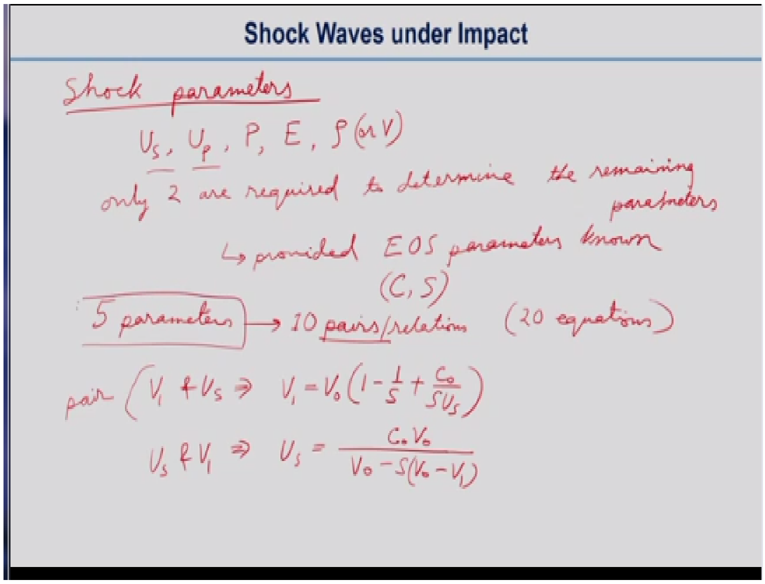

So we know the shock parameters are that earlier we discuss that there are 5 parameters U s shockwave velocity, U p particle velocity then we have the pressure and then internal energy and then density or we can write it in terms of V as well the volume. So we have these 5 parameters and then if we know 2 of these parameters only 2 are required to determine find the remaining ones remaining parameters provided we have EOS parameters known EOS parameters means c and s.

If they are known and then only we need to know only 2 of these parameters, so that we can find the others. And so we can have from 5 parameters we can have 10 pairs of these 5 parameters pairs or you can say that 10 relations that can be we can have on the both way then that can be been 20 equations that means suppose we have a relationship between U s let us say V1 and U s. So that can be written as this relationship is written as V0 1 - 1 by s + C0 s U s.

And otherwise if you write their relation like U s and V0 there will be like U s = V0 - s V0 - V1 and on the numerator it will be C0, V0. So just to show you that this is now one pair this is the pair which we talked about this pairs, so and then there can be 2 equations from these pairs and you can reverse it and that is how you can get 20 equations out of these 5 parameters.

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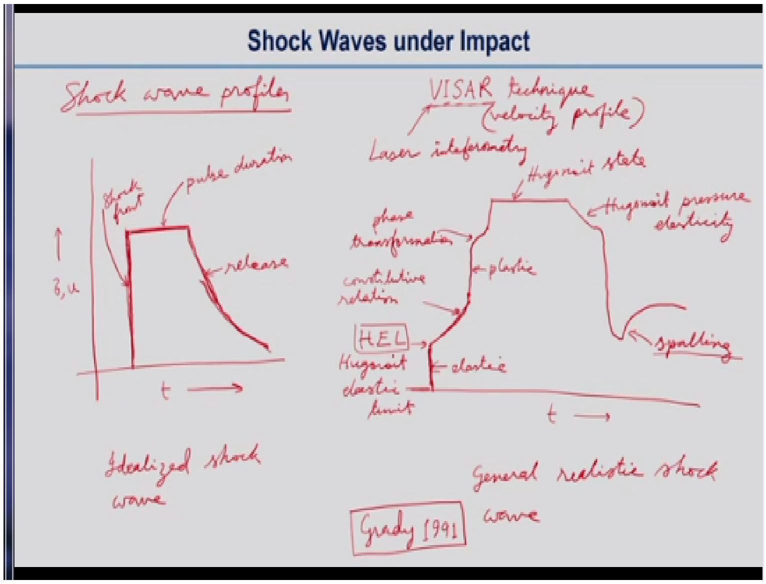

Then we will talk about this shockwave profiles will show some plots shockwave profiles, so first we will see a real idealized shockwave. So there is one idealize shockwave which the plot will be different than the real ones look something like this, this one is idealize shockwave profile. So this has time in and x direction and then stress sigma or displacement will write small u for that in this y direction.

So this will look like something like this, so stress will increase with time sharp increase that is actually on this continuity, this is this continuity it is going to like a sharp increase this is we call shock front. Here that is actually pulse duration and this is we call the release, here you can see that first part you can see this continuity as a vertical it is going vertically up and we plate to here at the top that is well continue for the pulse duration.

And then release that will be the gradually decrease gradual reduction of the stresses, however in the real case it is a general realistic shockwave profile. So what will happen here is with time we will show this actually a velocity profile this one is a velocity profile we will discuss about this VISAR this is call this is VISAR technique, we can discuss about that later in later chapters.

So this is actually nothing but velocity inter interferometer system for any reflector, so that is a technique to know the velocity profile. So here what will happen is first there will be a steep rise of pressure or particle velocity, so will be steep rise. And then there will be will have some slope in that part it will go again steep up and this will be look like this and then the play to comes something like this it will go down and go up something like that.

So this is a elastic part elastic, it is elastic modulus this limit is called HEL which we will discuss again in later. So this is Hugonoit elastic limit, so after this limit the special or the velocity let us say particle velocity will have a increase like this and this portion will depend on the constitutive relation of the material constitutive behavior of the material this part.

And then there is actually the plastic part this will give us the plastic modulus here will this will be the elastic modulus in this part in this range. And then this can happen due to phase transformations as we will discuss later that due to high energy of the shock the material can change the phases I mean phase transformation can happen. And then this that pulse duration plate will get and that is it is colours Hugonoit state.

And then the unloading curve that is it is actually elasticity first and then first elasticity it is been written as Hugonoit pressure elasticity. And then first it will be elasticity and then it will show plasticity in the later portion and this is actually related to spalling that we will discuss in the next lectures. The spalling is nothing but the wave when it reflects back from the free surface and that can actually generate fractures.

So this will recall as this is call as spalling and we will be discussing that later and so this is basically this curve is from the reference Grady 1991 although we have taken it from Mark Meyer's book. But this is actually the originally from the Grady’s 1991 and just to let you know that what is Weiser I already told you that what is this velocity interferometer system for any reflector and this based on laser interferometry.

So this based on laser interferometry, so that means this is the based on interference fringes that appear when different laser beam interacts. So that we can discuss later about these this one.

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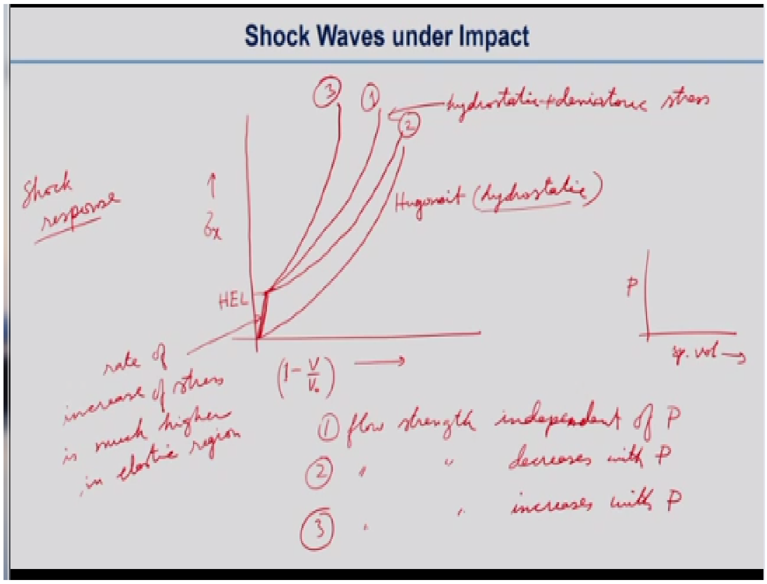

And so at last in this lecture we will talk about this shock response in a for 3 different cases, so this will be this curve is a sigma x variation with 1 - V by V0. So this is little bit different than what we got the Hugonoit curve earlier because this is if you see the x axis we have V by V0 the earlier we plotted P pressure with specific volume. So but this curve the Hugonoit curve will take a shape like this and however the pressure volume curve of a material is not exactly the like a Hugonoit curve.

Because it looks something like this and this is because the Hugonoit curve has only hydrostatic component a part and this may have both the hydrostatic and deviatoric. Because in Hugonoit we have ignored the deviatoric stresses, so and so this is the Hugonoit elastic limit HEL this point. So this rate of increase of stress is much higher in this rate of increase of stress is much higher in elastic region that you can see from here.

And then there can be 3 cases here, the first case is flow strength, flow strength of a material is independent of pressure P. So this is let us say this curve and then second case is the flow strength of material decreases with pressure decreases with P. So this curve can be look like this, so if this is the number 1 and this is number 2 and the number 3 is the flow strength of material increases with pressure.

So this can be something like this, so number 3 these 3 different other cases we can draw it like this. But the basically the Hugonoit curve is different because we have we do not have the deviatoric component of the stress, we considered only hydrostatic part. So that is why it has a difference with the other you know curves of different materials, so yeah that is all for today.

So what we discussed here if you try to remember and in the previous lecture also we discussed about the impact of projectile which a target that is a planar impact planar parallel and normal impact. So today's lectures we discussed a couple of numerical examples and to how to determine the pressure and even we discussed about the graphical method which is called impedance matching technique.

And also we saw some shockwave profiles and shock response of some materials. So in the next lectures we will discuss about equation of state for shockwaves and also we will have some discussions on the shockwave interaction or reflections, so that is all for today thank you.