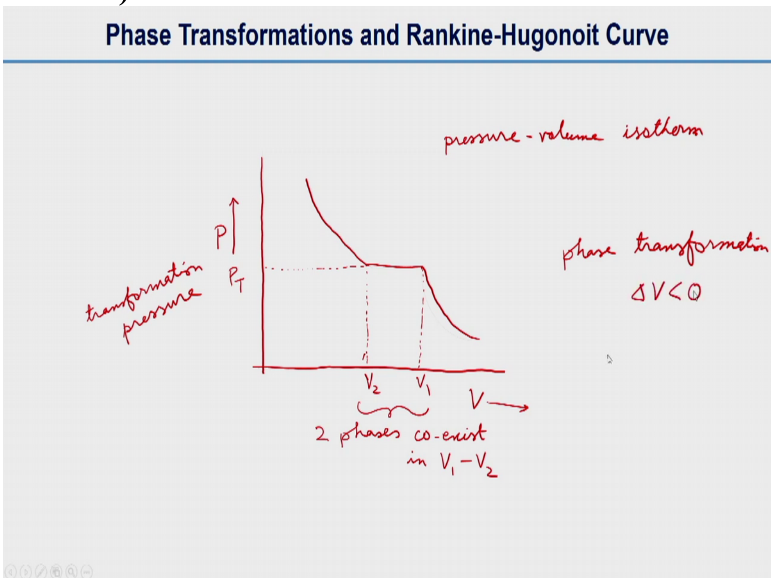
Dynamic Behaviour of Materials
Prof. Prasenjit Khanikar
Department of Mechanical Engineering
Indian Institute of Technology – Guwahati

Lecture-29
Shock Wave Induced Phase Transformations 2

Hello everyone, so, in the last lecture we have discussed about different phase transformation s, we talked about diffusion and diffusion less transformations and we also talked about another classification, first order and second order phase transformation. So, in today is class we will continue these discussions and we will see the relation of this phase transformation we to Rankine-Hugonoit Curve-

So, as we know that the Shock Wave loading is a very limited time. So that is why this thermodynamic equilibrium is not possible because of the short time. So, non equilibrium processes are important here however, we will assume we will create the equilibrium processes in this discussion.

(Refer Slide Time: 01:32)



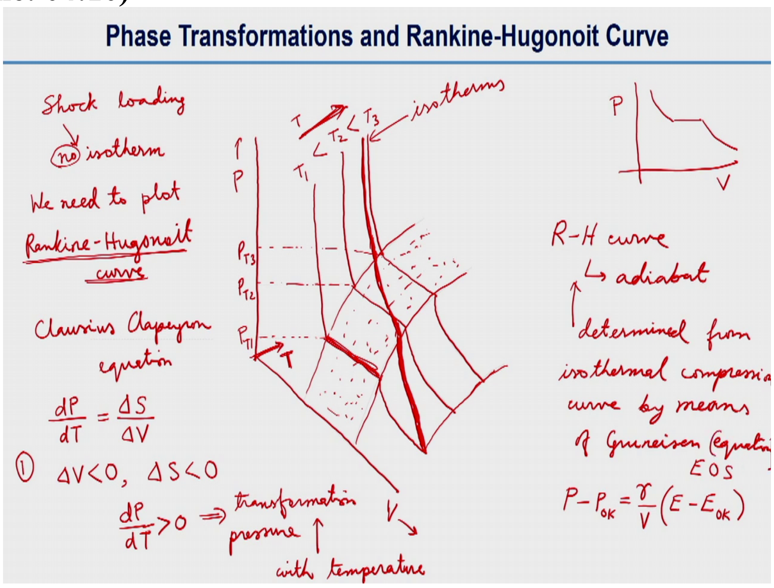
So, we will see first the transformation and with a PV Isotherm is a volume Isotherm that means at constant temperature. So, we have V volume in the x axis and P pressure in the y axis so, for we are drawing a phase transformation that Isotherm on a PV plane. So, the Isotherm we look like something like this. So here, these constant pressures are drawn V 1 V 2. So, these are the volumes had. So, this pressure is corresponding to P T we can write this is transformation pressure.

So, that is the pressure at twist the transformation will occur and this range of volume V 1 to V 2, 2 phases coexist sorry 2 phases will coexist in this range of volume that means in V 1 to

V 2 volume. So, this is I am writing this is a pressure volume Isotherm this curve is. So, this is what we are talking is not shock loading but any general phase transformation on this PV diagram.

Whatever we saw here is the phase transformation process, phase transformation process it that for that is for Delta V smaller than 0 that means in this in this phase transformation process the volume will decrease of the transformation.

(Refer Slide Time: 04:10)



So, now in case of shock loading, so, we do not have the Isotherm. Isotherm shock loading we do not have the Isotherm. So, I will write no Isotherm because the temperature will change and so, we need to develop we need to plot the Rankine-Hugoniot Curve for this shock loading Rankine-Hugoniot Curve. So, we will see how to draw that. So, what we will do is we will try with the earlier whatever the previous slide.

We saw the pressure volume diagram. So, this is a 3 dimensional plot pressure on this axis volume on in this axis and then we have temperature in another axis that is T temperature in another axis. So, if you draw these Isotherms, so, the Isotherm, there are several Isotherm we can draw. So, this is one Isotherm make it can understand this is the same as what we did in the earlier curve plots.

Suppose, this is I am drawing here again. So, we have done that this is when Isotherm for a phase transformation. So, in this case, we are drawing this at let us say temperature T 1. So, this is an Isotherm because at constant that means at constant temperature. Then at T 2, we can draw another one so, we will draw another one T 2 and so similarly at T 3 so, this T 3 we will draw. The T 3 is higher than T 2 we can draw this Isotherm.

So here T 2 is higher than T 1, sorry T 3 is higher than T 2 and T 2 is higher than T 1. So, that means our T is increasing this way. And that is obvious you can see from already shown this T is a T axis is in this direction and now this 2 phase region is this one. So, this is our 2 phase region so, the 2 phases coexist in that region. So, these are all Isotherms. These are Isotherms. If you remember the earlier lecture.

So we have shown that the Clausius Clapeyron equation, our relation we have seen that dP by dT is equal to delta S by change into P divided by change into volume. So, here in this case as we in the earlier slide we talked about this has a negative volume change volume will decrease and this is the first case we are talking about. So, we will show another case. So, in this case.

So, what we are assuming that the volume will decrease anyway the Phase Transmission volume will decrease and that entropy will also decrease. So, if volume also increase, decrease and entropy or decreases then dP by dT term will be will increase. That means the transformation pressure will increase with temperature. So, this means the transformation pressure transformation pressure will increase I write this arrow with increasing temperature we simply right with temperature.

So, here in this case you can see that because temperature is increasing in this direction. And you can see that the transformation pressure which is in this diagram, it may be a little complicated for you. But if you see this, the pressure corresponds to these this part is for added for Isotherm temperature T 1, so, this is the pressure. So, this is P 1 and then if you see the second Isotherm the pressure is P 2 and even in the third Isotherm.

You can see that the pressure P 3. So, what we found you that transformation pressure increases, that is all we can write it as PT 1 similarly we cannot because the transformation process of that. We also we can write P T1, P T2 and P T3 probably that will be you know make it clear so whatever we because that is a transformation pressure not any other pressure so now we wanted to talk about the Rankine-Hugonoit Curve.

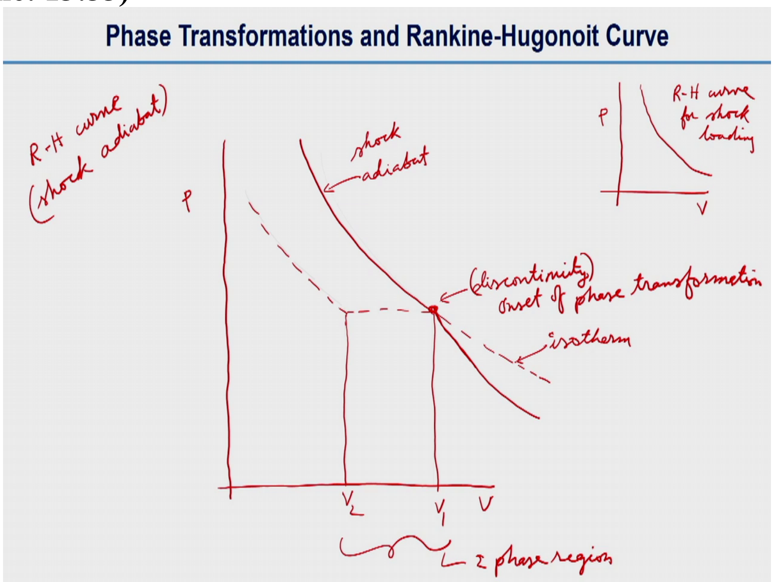
So, as we already discussed that for shock loading, we do not need the Isotherm so, no Isotherm that means, we do not need Isotherm we need Rankine-Hugonoit Curve. So, how to draw the Rankine-Hugonoit Curve? So, it is a little different than this Isotherm this point is actually did we can draw a line here for the reference line whether this Isotherm starts from so now that is a different line but now we want to draw this Rankine-Hugonoit Curve which will go something like this.

This Rankine-Hugonoit Curve it is difficult to understand from this but anyways we will in the next slide I will show you just need to understand that this is not on a single PV plane like our earlier the Isotherm. So, it will go something like this. So, that means the temperature will change. So, it may be a little difficult to understand in this case. So, in the next slide we will have a simple curve and this Rankine-Hugonoit Curve.

I will write RH curve is adiabat not an Isotherm adiabat represents a derivative process and this can be this curve can be determined from Isothermal compression curve compression curve from by means of Gruneisen equation that we have already found that in an earlier lecture, so by means of Gruneisen equation. So Gruneisen equation is something like, if you remember P minus pressure at 0 Kelvin, absolute 0 equal to Gruneisen constant divided by V with internal E minus internal energy that absolute 0.

So, this is the Gruneisen equation or actually this is the equation of state we can call this is Gruneisen equation of state from there, we can determine the Rankine-Hugonoit Curve.

(Refer Slide Time: 13:33)



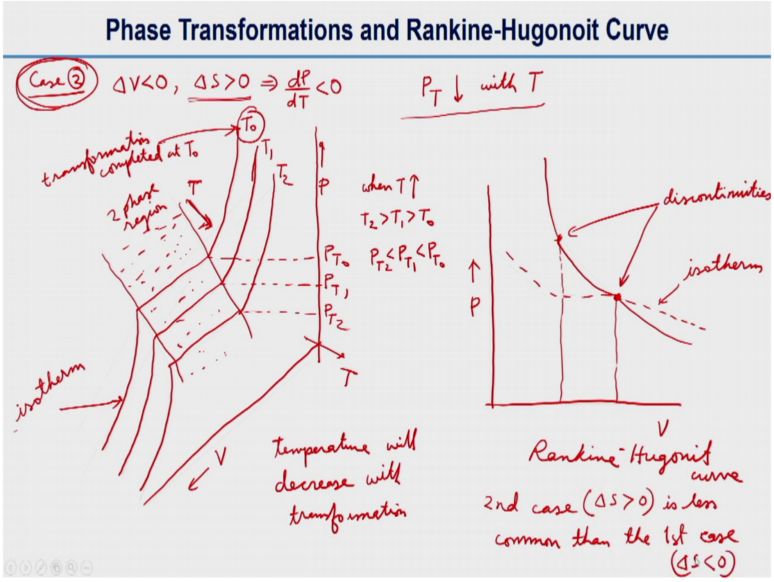
So, we will see it in 2D plot which will be easier for us to visualize. So, the early plot was complicated because we have even temperature axes. And so, now here we have in a simple PV plot, the same case, same idea bit here. We can call Rankine-Hugonoit Curve or even it is known as shock adiabat because it is adiabat process shock adiabat on a PV plane, so, first we will draw this on the Isotherm what we got in the earlier curve.

So, this is Isotherm whatever we found in the earlier slides. So, this is the Isotherm as we know the mix phase joined is, V 2 and V 1 that is the both the phases will be will exists. Now, if you want to draw the adiabat will look something like this. So, this dash line is Isotherm that means at it is a constant temperature process. And this is the adiabat or we can call shock adiabat. That is the Rankine-Hugonoit Curve.

And now, we can see, it may not be very clear from this diagram, but they were discontinued in this point. You can see that this curve is not continuous like our earlier curve. So, what in the earlier lectures when we did study about Shock Wave, we saw that the pressure versus specific volume or volume, we got a curve like this and that is we call the Rankine-Hugonoit Curve for a for shock loading.

So, now in this curve, whatever we are drawing here is the same one, but we waited discontinuity. So, this discontinuity exists because of the phase transformation. So, this discontinuity will represent or will this indicate onset of phase transformation I will write within bracket discontinue onset of phase transformation. So, the phase transformation will start from here and till the volume V 2.
So, after that this ah the region and in between V 1 and V 2 will be 2 phase region 2 phase will coexist. So, and probably higher pressure we will get an again one single phase. And both right hand side of this 2 phase region and the left hand side of the 2 phase region and we will have a single phase but in this we want to V 2 will get a mix phase. So, this is you should understand that this is on simpler PV plane but the earlier one was with PV plane in the third axis is the temperature axis.

So, that is where we found some difficulty to show you the this tic RH curve. So, that was we found it difficult, but in this case here it is look very simple and the so the same case, we have that was as we discuss that the case 1, where the volume will decrease and also the entropy will decrease and that gives us the pressure will the transformation pressure will increase with temperature.
(Refer Slide Time: 18:20)



So, we will try to see at the second case there another cases case 2 where again we will have volume negative volume changes because for this phase transformation , but the entropy changes positive here and that gives us dP by dT is smaller than 0 form Clausius Clapeyron equation. So, this dP by dT will be smaller than 0. So, very quickly I will draw even if it is not very clear, you can follow the textbook for this diagram.

So, and this is pressure volume in this direction and the third direction is temperature. So, we will draw the Isotherms into look something like this. And then the second one second Isotherm will look something like this. And the third one again, not different, they all have the same and we have this mix phase or 2 phase regions here. So, this is 2 phase, region. And so, and if you can see that in this case that you know transformation pressure decreases or I will just write it PT transformation pressure will decrease with temperature.

That means, if we increase the temperature the transmission pressure will decrease here, we can see that let us say T 2 T 1 T 0 this direction. So, in increasing T is in this direction and when the temperature is increased, if you see the transformation pressure PT 2 PT 1 and then PT 0. So, when you know T 0 T 1 T 2 increases that means here in this case actually we cannot try to clear.

So, when T increases that means T in this case T 2 T 1 T 0. So, and then the pressure will decrease, pressure will decrease. So, PT 2 PT 1 PT 0 so, actually with transformation, the temperature will decrease, so the transformation is happening when no temperature will decrease, I will write it here. Temperature will decrease in this case in the second case, decrease, we transformation and probably in this case we the completion will happen T 0 temperature.

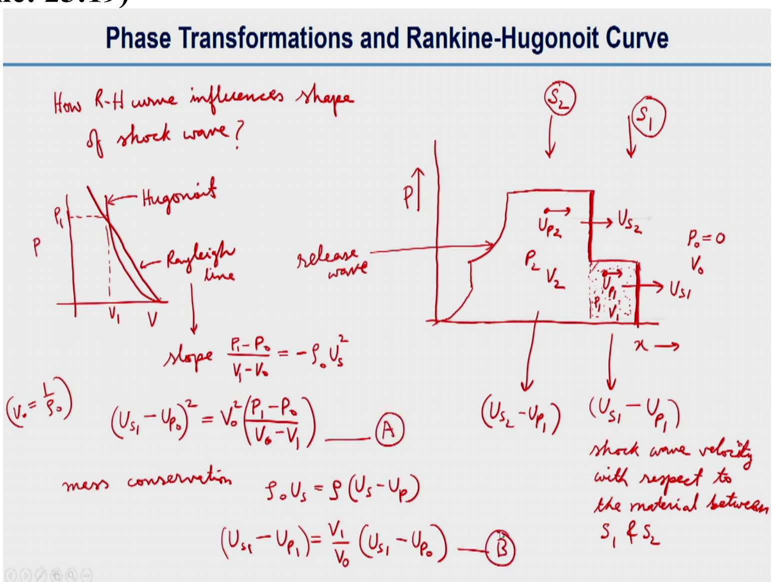
The phase transformation will be completed transformation completed at T 0 temperature. So, you can see that the T is increasing this way, but the transformation happening in the opposite way and the notice the lowest temperature and the transformation will complete sorry, I am not write completed, transformation completed at T 0 this case can be again in a simpler PV plane we can draw like our previous case.

So, in this case what will happen first will draw the Isotherm dash line Isotherm this is our Isotherm and we will have the adiabat that Rankine-Hugonoit Curve adiabat is look something like this. So, in this case there will be 2 discontinuities one second discontinuity is there this is as you can understand that we are drawing the Rankine-Hugonoit Curve. And in the first case we are showing the Isotherms here mostly these are Isotherm in the Rankine-Hugonoit Curve.

Probably it is difficult for us to draw in the first diagram then we are showing it here and we can see that there are 2 discontinuities that is the difference with the first case where our entropy change is negative in this case the entropy change is positive and that gives us the transformation pressure will decrease with temperature. And this is important to mention that this second case is less common.

Second case is less common than the first one second case means delta S is greater than 0 is less common than the first case. Then the first case, which is as you can understand delta S is smaller than 0.

(Refer Slide Time: 25:19)



So, now you will check how this Rankine-Hugonoit Curve influences the shape of this Shock Wave how R-H curve influences shape of Shock Wave so, for that we need to draw a draw shock front shock front of 2 waves. So, what we will do is will draw a shock front of 2 waves the wave one we will write this wave as S 1 or you can just say this wave is S 1, Shock Wave 1 and this wave as S 2 the Shock Wave 2.

And if you see that Shock Wave velocity is U S1 and in the second case Shock Wave velocity is U S2 and if you know you know that this is basically a x versus distance versus pressure plot these are the fronts the first font and there is another Shock Wave which is following the first one the high pressure wave that is U S2 and we know that the material ahead of this will have pressure that is would be 0 and also volume V 0.

And then V 0 will be particle velocity associated with this particular velocity in the first behind the first Shock Wave this is the shock front and the behind the first shock front is U P1 and similarly in the behind the second wave the wave velocity sorry particular velocity is U P2 and then we can see that in between these Shock Wave S 1 and S 2 this area will have pressure P 1, volume V 1 and in in this area pressure P 2 volume V 2.

And also, we can see that the release wave release wave can be drawn like this. So, we will probably discuss later about these releases. So, this part I will draw an arrow from here and we will write, release wave we have, we know what is released, we discussed earlier. And so, basically, we are dealing with now 2 Shock Waves S 1 and S 2 and we should understand that the material in this region material this region with has a velocity U P1 that is the U P1 is the velocity particular velocity.

So, for this for first Shock Wave for first time with the, the relative velocity which respect to the material of this part is relative velocity will be U S1 – U P1. So, this is we can write, Shock Wave velocity with respect to, through the material between shock front S 1 and S 2. So, and similarly, what we can do is the velocity again with respect to this material of the second Shock Wave will be U S2 – U P1.

So, these are the relative velocities, what we can do is now is we can remember these early equations or earlier from earlier classes. So, what we had is and PV plot or pressure versus specific volume plot. So, what we discussed is this Rankine-Hugonoit Curve and then we found that particular pressure so, if you take the slope this is, we call it the Rayleigh line. So, this Rayleigh line and this one is the Rankine-Hugonoit Curve.

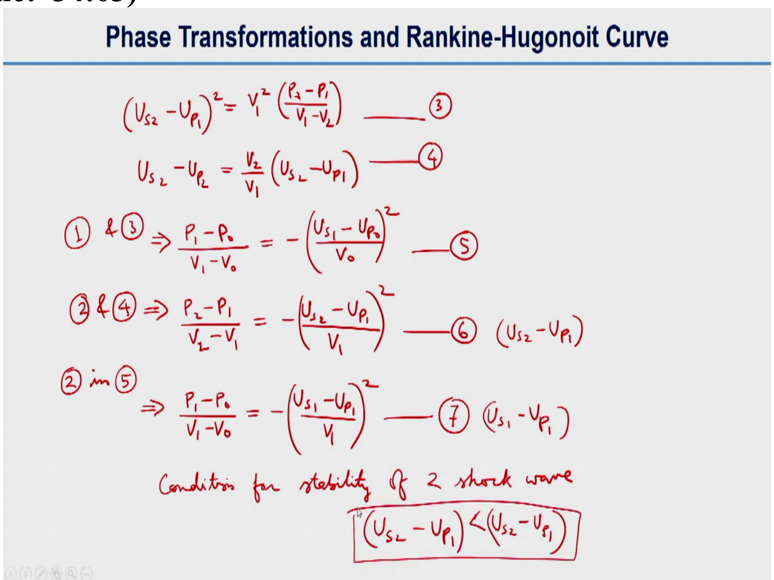
So, the slope of this Rayleigh line is important to us, we earlier we saw this. So, suppose P – P 0 as you know the P is the pressure at the behind the Shock Wave and P 0 is the head of the Shock Wave. So, similarly V by V 0 so, let us in this case you can take pressure this pressure

P or we can write P 1 or V 1 here. So, this means P 1 by V1 P 1 – P 0 by V 1 – V 0 that will give it as a slope and which is equal to Rho 0 initial density multiplied by Shock Wave velocity square US square.

And from this low bend we have even we can say that US 1 UP 0 square is equal to what we can get that for this case what we are doing is U S1 – U P0 that is the ahead of the Shock Wave U P0 and then V 0 square is nothing but the V 0 is the one by Rho 0 the volume and density relation. So, this will be P 1 – P 0 divided by V 0 – V 1 if you can understand from here that this minus sign comes because we are writing here, V 0 – V 1 instead of V 1 – V 0.

So, again from mass conservation, this is also we discussed in earlier lectures. So, mass conservation, which the equation was Rho 0 U S is equal to Rho U S U P please revise these from earlier lectures. So, from this mass conservation also we can have a for our problem today's problem. So, we can write this expression, V 1 – V 0, U S1 – U P0. So, you can understand the V 1 V 0 we are writing in place of Rho 0 and Rho which is does just inverse of it. So, from these 2 relations, I will see you will right this is a and this one is B.

(Refer Slide Time: 34:03)



So, similarly in case this is we are writing in terms of the points one in not like U S1 and minus U P0 similarly, we can write other equations, which has spread to U S2 – U P1 similar to A. V 1 square P 2 – P 1 the slope of this curve V 1, V 2, this we will and then another equation, which is similar to B the earlier and earlier slide U S2 U P2 which comes from the mass conservation V 2 / V 1 into U S2 – U P1.

So, let us assume this is as. Okay better we can write this All in 1 2 3 4 or maybe all right 1 2 3 4 that will be better give us better make it the picture more clear. So, this will be 3 this will be 4 then from 1 and 2 from sorry form 1 and 3, what we can find is please go through this, if you if it does not look very clear to you the simple calculations it seems in order was referred the textbook.

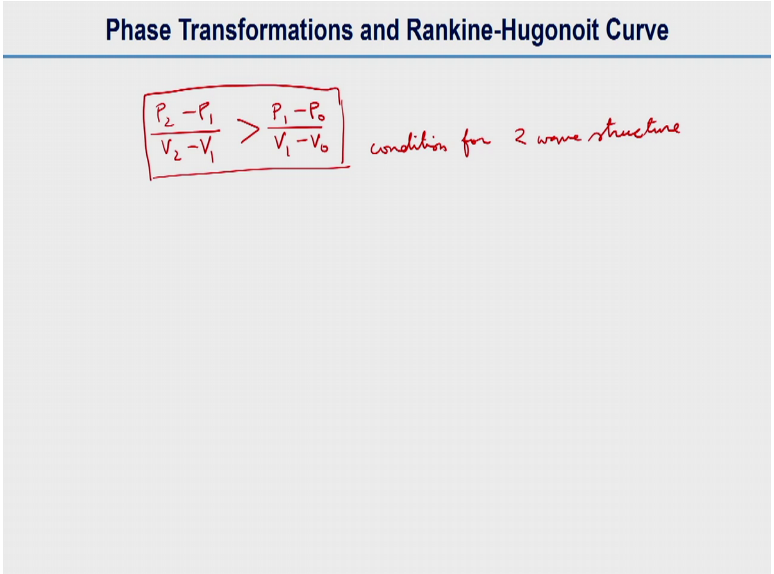
So, U S1 U P0 divided by V 0 square. And then if you see number equation the expression number 2 and 4. From there, we can have P 2 – P 1. V 2 – V 1 is equal to minus U S2 U P1 divided V 1 whole square. So, this is number 5, and this is number 6. if we substitute 2 in 5, if you substitute 2 in 5, that will give us P 1 – P 0 divided by V 1 – V 0 is equal to minus U S1 U P1 / V 1 so why we are doing that, because we want to have this in terms of U S – U P1.

So, our relations would be U S – U P1. So, equation number 7 is we have in terms of U S – U P1, the separation and even equation number 6 we have in in terms of U S2 – U P1 which we needed for this problem. So, we will see now this problem in the previous slide. So, what is happening here is the first wave first Shock Wave is traveling at the speed of U S1 but we as we told that Shock Wave velocity.

Which has let us say the material is U S1 – U P1 and in the second also second wave relative velocity is used U S2 – U P1. So, in the 2 phases to be stable, that condition for stability the stability of the 2 Shock Waves or to shock fronts is that U S2 – U P1 should be greater than sorry smaller than up one, so, why so? Because if that is the case, the first wave travels always ahead of the second wave but, so what I wrote in this slide is this would be, this would be U S1 U P1 should be greater than U S2 – U P1 okay.

So, if it is not better than the other one, then what will happen the second way will overtake the first wave, second wave overtake the first wave that means there will be only one single wave 2 waves will not be will not exist that means, 2 waves stability will not be there only single wave will exist if this condition does not satisfy. So, this is the condition for stimulated or to shock waves. So, what we can do is we can give you the number 8. So, now, we will see from number 6, number 7 and number 8.

(Refer Slide Time: 39:45)



The condition will be P 2 – P 1 / V 2 – V 1 > P 1 – P0 / V 1 – V 0. So, that is the condition that is the condition for 2 waves 2 Shock Wave structure. If that does not satisfy that means the second shock wave will overtake the first or that means, now, it will be only 1 Shock Wave instead of 2. So, with that, so, we are closing for today. So, we have discussed about pressure volume, Isotherm and Rankine-Hugonoit Curve.

For corresponding to phase transformation that is shocking news phase transformation. So, what I discussed about the 2 Shock Wave structures so, you can see here we have shown 2 Shock Wave structures and although I wrote it here, so, what is the how the R-H curve influences the shape of the shockwave sorry, but we did not discuss much about it I somehow actually missed that point.

And but anyways, you do not need to worry about it because this will be covered in in the next class. So, what you understand now is the first is the 2 Shock Wave structures and then we will discuss in the next lecture. So, how these 2 Shock Wave structure is related to the phase transformation that means shocking news phase transformation. So, that is all for today. Thank you.