Dynamic Behaviour of Materials
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Lecture-34
Plastic Deformation at High Strain Rates 1

Hello everyone, so we have discussed about the experimental techniques of producing dynamic deformation in earlier lectures. So in today's lectures we will discuss about the plastic deformation at high strain rate, so you will see some constitutive relations.

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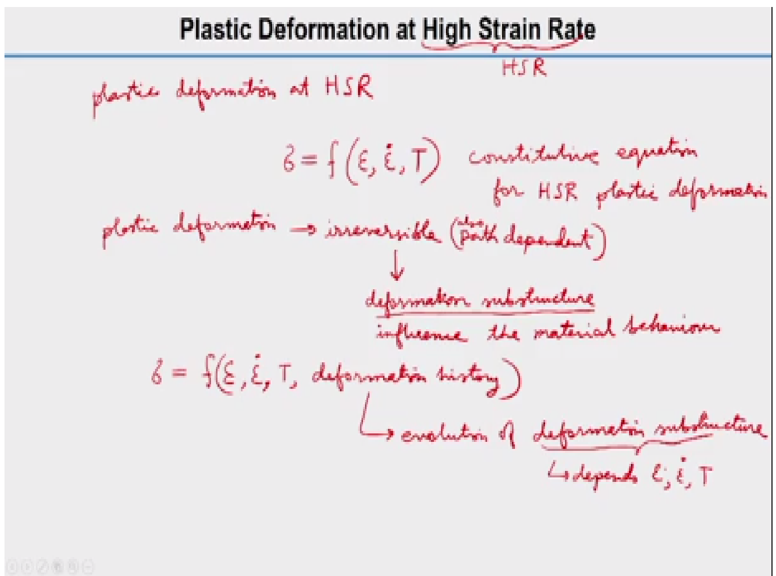

There is a relation between stress and strain or other quantities like strain rate or temperature and we will get these equations the mathematical relationships. And also we will see how the defect structures, the crystal defects influence these relations. So plastic deformation at high strain rate so I will write high strain rate as HSR so that will be easier for me.

So plastic deformation at high strain rate have a relation between stress and other parameters like strain rate and temperature. So as we know that in the elastic region we have only Hooke's law that is sigma = e epsilon. But here and if you are talking about the plastic deformation at high strain rate we need to consider the strain rate epsilon dot and we can also consider temperature here.

So this is the constitutive equation I am sure you know what is constitutive equations, you probably studied in your solid mechanics classes, constitutive equations for high strain rate plastic deformation. So as we know this plastic deformation it is irreversible and hence it is a also path dependent. So that is why the deformation substructures here in this book that using the phrase deformation substructures which means mostly the defect structures like dislocation structures in the material.

So this deformation structures influence the material behavior or the plastic deformation behavior. So what we need to do is because this deformation substructures influence plastic deformation. So we can right now rewrite the earlier expression like epsilon dot, T and deformation history. So why we were writing deformation history, so this will denote the evolution of deformation substructures.

So that means suppose in most cases the dislocations how the dislocations evolves, evolves means with time, with deformation how the changes happens in dislocation structure or the deformation substructure. And these deformation substructures also depends on the evolution depends on strain, strain rate and temperature, so that depends on that these.

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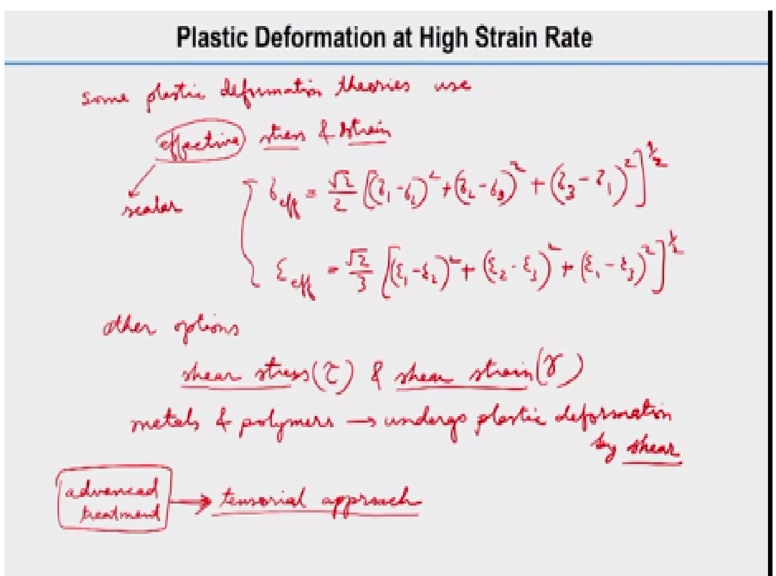

So some plastic deformation theories use some effective stress and strain to express these you know plastic deformation like sigma effective = root 2 by 2, sigma 1 - sigma 2 square + sigma 2 - sigma 3 square + sigma 3 - sigma 1 square to the power half square root of the whole thing. Or maybe effective strain which will look like this epsilon 1, epsilon 2 square + epsilon 2 - epsilon 3 square + 1 - epsilon 3 square half.

So these are these effective stress and strain they are taken as a scalar quantity although these are tensors . We are not going to use this, so there are some other options like shear stress and that is tau and shear strain that is gamma can be used to express these plastic deformation theories. So because matters and also polymers they undergo plastic deformation by shear.

So that is why we can use shear stress and shear strain to plastic deformation and then to come up with some theories. So also more advanced analytical treatment and computational schemes they use tensorial approach which we are not probably we are not using that tensorial approach. Because it will be little difficult for us because you have different backgrounds, so we probably did not study about the tensorial approaches earlier.

So that is we are talking about the advance theories or you know analytical treatment and computational methods, so I would just write analytical treatment. So these theories probably will require tensorial approach with fully calculations which tensors.

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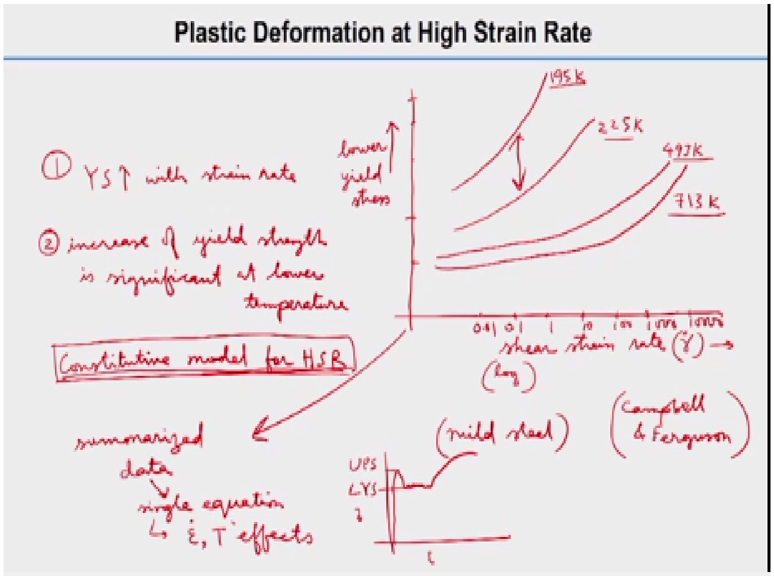

So if we plot the yield stress with shear strain rate, so that is we can call let us say gamma dot. So this plot whatever we will be showing is from Campbell and Ferguson, so the yield stress as it is for mild steel. So here they are using the lower yield stress as you know there will be the yield point phenomena, so this stress strain curve will look like this, so this is the lower yield stress LYS the upper yield stress is this one.

So this is the stress strain diagram we are seeing but here what we are showing is the yield stress. So how it look like is suppose we can see that the yield stress will increase with strain rate. And so what is happening here is this is in logarithmic scale, log scale, so let us say you have 1 here 10, 100, 1000 like this 10000. So here let us say 0.1, 0.01 something like that.

So here we are not plotting what about mega Pascal values here, but this is the first curve is for very high temperatures 713 Kelvin. And then second one is 493 Kelvin probably these are very this would be close to each other. And then the third one let us say 225 Kelvin and the last one is 195 Kelvin. So just to show you the variation here, so there are 2 observations they made that is yield strength increases with the strain rate**.**

And the number 2 is that increase of yield strength is very prominent or significant at lower temperature**.** Lower means as compared to d 713 or 493 Kelvin these are lower, so the increase the difference is higher in that at lower temperature range. So we will what we will discuss next is a the constitutive model for high strain rate plastic deformation behavior.

So what we need to do is, so whatever we have seen in this plot, so that can be you know the summarized if you can summarize the data you know data means that from the plot into a equation the mathematical equation we if we can plot into a single equation. That means that should include that strain rate variation and temperature these effects, so we will try to find.

And then what we can do is then we can interpolate or extrapolate to predict the material behavior at different strain rate or a different strains or temperatures. So that is what we will do that to find a constitutive model for a suitable or appropriate constitutive model for high strain rate.

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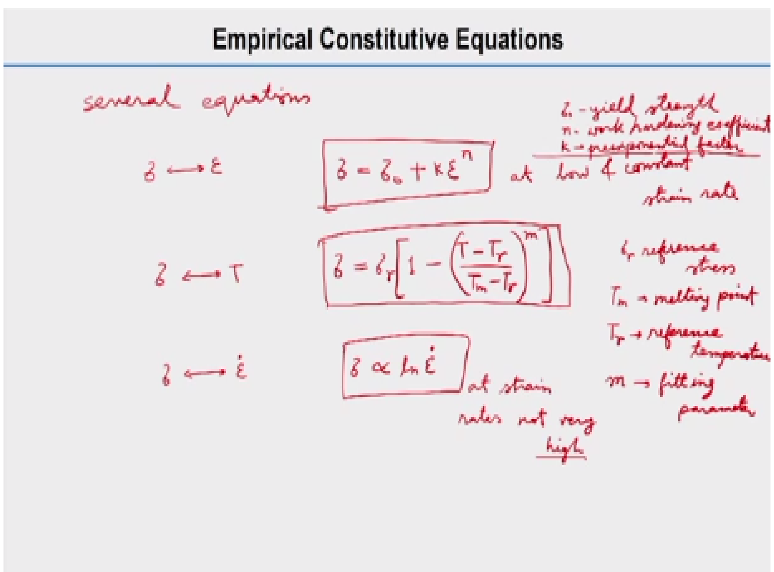

There are several equations were proposed and even an equations means constitutive equations they were proposed and even successfully have been used. So that is objective of this equation is to summarize the yield stress behavior the variation of the yield stress with strain rate and temperature. So suppose if we try to have these relationship between stress and strain in the plastic region.

And that there is a relation which is a good approximation, so this is at low strain rate low and constant strain rate. So and then if you want to talk about the flow stress dependent on temperature. So there is another model that is put forward is like this 1 - T – T r T m – T r so we will see what these means. Similarly if we want to see the variation of flow stress with strain rate or effect of strain rate.

So that stress is proportional to nature logarithm of the strain rate, so here in the first equation. So this sigma 0 is the yield strength and n scholars work hardening coefficients and k is the pre exponential factor. That I think all of you know that this is work hardening or strain hardening exponent or we can call coefficient. And then k is also a constant, this is we generally write as pre exponential factor.

So similarly for the flow stress temperature dependent relation that is sigma R is the reference stress T m here is the melting point melting temperature, T r is the reference temperature**.** So m is a experimentally determined this is the fitting parameter that you know when you want to fit your experimental curve. So with this equation, so we may need this fitting parameter.

And then the third relation is the sigma that is directly proportional to natural logarithm of epsilon dot, that is the strain rate. So that is also not very high strain rate at strain rate it is not very high probably this relation yeah it is not for very very high strain rate but we can go to moderate high strain rate.

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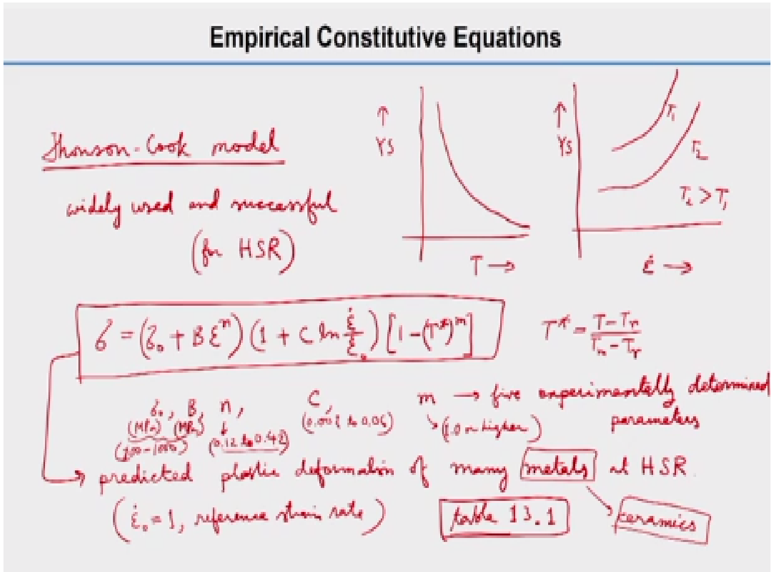

So if we want to plot these variations, so we can get like this yield strength and the temperature curve will look something like this. So that with temperature increase so yield strength will decrease and the curve looks like this. And again if we draw the yield strength variation with strain rate epsilon dot, so this will vary like this. And if you are talking about a lower temperature, so it will look like this, so this is T1 and T2 here T2 is higher than T1.

So these are the plots the shape of the plot it does not look very good, so these are the shape of the plots. We will talk about now Johnson Cook model a plasticity, so which is a very widely used and successful for specially for high strain rate, I will write HSR the high strain rate for high standard plastic deformation behavior. Johnson and Cook basically summarize and then use the basic ingredients like the stress dependence on strain, a temperature and strain rate.

So using these basic ingredients and proposed a relationship say which is equal to sigma 0 + B epsilon to the power n 1 + c natural logarithm of epsilon dot divided by epsilon dot, epsilon dot, epsilon 0 dot. And then 1 - T star to the power m, so the power for the T star only. So this equation have 5 experimentally determined parameters, so here you got sigma 0 B and n, here you got c and here you got m.

So these are the 5 experimentally determined parameters, so this equation this model it is was very successful. And so has been predicted and the plastic response plastic deformation of many specially metallic materials, this is for metallic materials. Many metals at high strain rate or under high strain rate loading, so this is predictions was actually predictions are very successful for many metallic materials.

And these parameters are already available, so you can see in the let us say table 13.1 in the Mark Meyer's book. So you can see all these parameters in a tabular form, and then for many materials these are already available. Suppose for n is whatever you can see from this table is the range is 0.12, 0.42 generally this varies from 0.12 to 0.6 but whatever I mean I am focusing is that these only this table.

And this table has I will just speak I will not write all those, so OFHC corporate cut is brass, nickel 200 Armco iron and then carpenter electrical iron 1006 steel 2024 Aluminum, 7039 aluminum and 4340 steel. And there are 3 more materials, so these materials have a you can see the range of n is like that and then c will be it is a smaller value 0.0072, 0.06.

And then m which is the thermal actually exponential thermal softening exponent which is mostly it is equal to 1.0 or even higher or higher, even there are some lower values are also available. So and sigma 0 and B are which are we express in terms of mega Pascal. So they are also kind of it can vary from 100 to 1000 these range are let us say 100 or even less than 100 to 1000 in this range these parameters.

But these are experimentally determined and so for many materials these are already available. And we will we did not discuss about this T star, so T star = T – T r which is the reference temperature and then T m melting temperature – T r. So this we already got this in the earlier slide, so this is we already wrote, epsilon dot in this equation epsilon dot 0 as a reference strain rate.

It is taken generally as 1 for convenience this is actually reference strain rate and then sigma 0 is measured at reference temperature T r. So T r the reference temperature, sigma 0 is measured at that temperature. Also as I told her this is used for mostly for metals but sometimes it is also modified for ceramics as well. This Johnson Cook model has been modified for many materials.

This is some of the researchers they modified the equation a little bit and so that it can fit to I mean different kind of materials. And the so some researchers even they tried it for ceramics and they were able to get reasonable prediction from this Johnson Cook model for ceramic as well.

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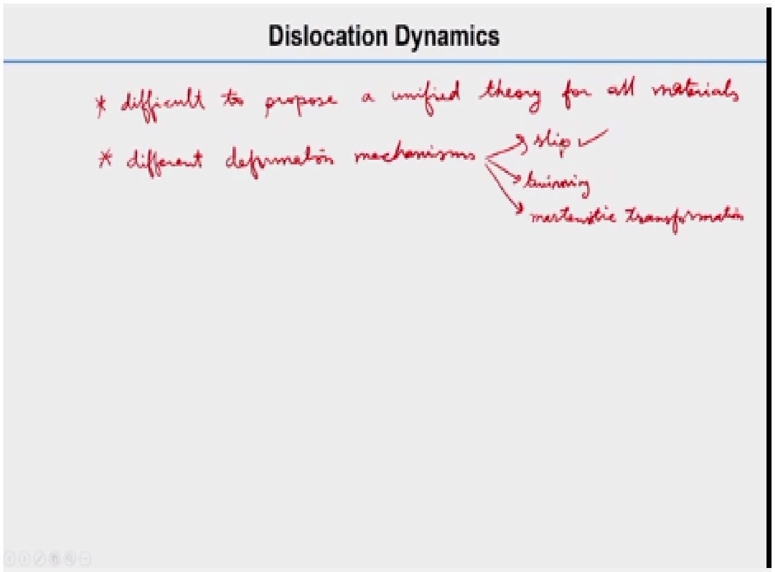

As you can see that there are different models, so these models will have exceptions so it is difficult to you know propose 1 theory proposed a unified theory for all materials. So that is not possible because so materials are all different they follow different rules, so it is not possible to get a unified theory. And also this is the first point and then the second point is there are different deformation mechanisms.

So different deformation mechanisms that means plastic deformation mechanism, that we already discussed in our materials basic lecture. So as we know we have dislocation slip, twinning and phase transformations or mostly we discussed about martensitic transformation. So here in this section we will only talk about slip, that is dislocation slip not the other 2.

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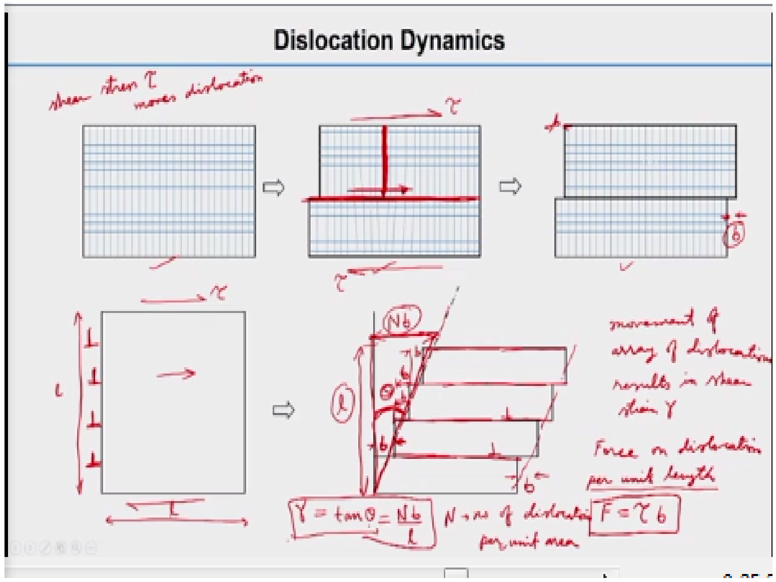

So here we will talk about this 1edge dislocations, here we have edge dislocation I hope this figure is clear to you. So this is an extra half plane, so this extra half plane these lines vertical lines or an horizontal line, so the atomic arrangements we are not drawing the atoms but showing only the lines. So these whatever I am highlighting with red color, so that is the extra half plane and if you say where is the dislocation line that is perpendicular to the plane of this diagram.

So and then this is the slip plane, the slip plane is you know we were seeing in 2-dimensional but if you extend it to the third dimension you will get the slip plane. So now this dislocation is moving towards right and then it is now after this you know first it was let us say we are talking about a perfect crystal. Here we have a dislocation and then if the dislocation moves, so there is 1 step created that is equal to burgers vector B the magnitude of this.

And here also maybe this is same as the burgers vector B one step, 1 atomic distance, here we are doing is we are applying some shear stresses tau which is in this direction also you can show. This is we are applying some shear stresses, so shear stress tau moves dislocation. Here in this case we are showing only one edge dislocation and here again in the lower figure what we will see is that let us say we have a square domain a material, that is say L and L length.

And then let us say we are applying some shear stress tau and there are some dislocations are can be shown with this symbol, that is a symbol for dislocation. So we are drawing let us say 4 dislocations here, so these dislocations when they will move this direction they can have, so this is 1 slip plane let us say number 1 dislocation move and can create a step here.

Similarly the other dislocation in a parallel slip plane will move and create a step, step means the making a step here. Similarly another dislocation can move and make a step here and then another parallel plane here. So the finally the shape will be look like this, so this final shape will be look like this. So this angle let us say assume this angle is theta and , so now what we will do is, so this second figure that means in the lower figures of the above talked figures.

So below 2 figures what it says is that moment of array of dislocation movement of array of dislocations results in shear strain gamma. So air array means as you can see we have drawn 4 dislocations here, and also on the same slip plane just I want to clarify this under a same slip system there will be more dislocations.

So after 1 dislocation here for a simplicity we are showing that just 1 dislocation is going out of the crystal. But there can be more dislocation on the same slip system there can be 100s or 1000 of them. So the movement to the area of dislocation will result in a shear strain that is gamma, so we will see how to calculate these gamma. So this shear stress the force on the dislocation per unit length in this case is force on dislocation.

So we will write per unit length because the dislocation is a line defect, so it looks like a line, I have not shown you any figure. But if you see in transmission electron microscope TEM the dislocation will be seen in that transmission electron microscope and this dislocation will look like a noodles like Magi noodles. So these are lines basically though that may not be a straight line what we are seeing is some of our diagrams even in materials basics lectures.

We saw a straight line is a dislocation but that may not be the case that can be a curve, so and that may look like some noodles. So what happen is, so when you are calculating the force on the dislocation, so you need to calculate the force per unit length not the entire length of the dislocation, entire length can be very high. We even do not know and we it is difficult to discretize and say that ok this is 1 dislocation or something like that, it may be you know continuous curve line.

So that force will be equal to tau that is the shear stress multiplied by the magnitude the burgers vector so burgers vector. So that means that is the dislocation burgers vector you can write or that offset that caused by the dislocation. Here you can see this is the burgers vector magnitude, so if 1 dislocation goes out of this surface that is the distances B. So this is the force per unit length acting on the dislocation.

Now we will see another thing that the gamma I will write it here only although it does not look good. But this gamma that shear strain will be equal to tan theta in this case from your definition of shear strain. We know that let us say this portion is L and we need to know this distance, so how to get these distance is we know we are assuming this 1 step is B or in this side also you can see that this is B or even this portion is also let us say B.

And again this portion is B, so here we are not assuming let us say array of dislocation. But let us say we are talking about only one dislocation now, so then only one step and this step length is we can say that equal to B. So now if you have 4 dislocations we have here also B, so if you have 4 dislocations, so then it will be the total distance will be 4B.

Now instead of writing 4B we can write N b, so n is number of dislocations per unit area. So you can get that gamma = tan theta which will be equal to now N b by L you can see this is if you see this triangle and this angle is theta, triangle means I want to tell you about this triangle. And this is the perpendicular and the base so N b by L will be equal to gamma ok.

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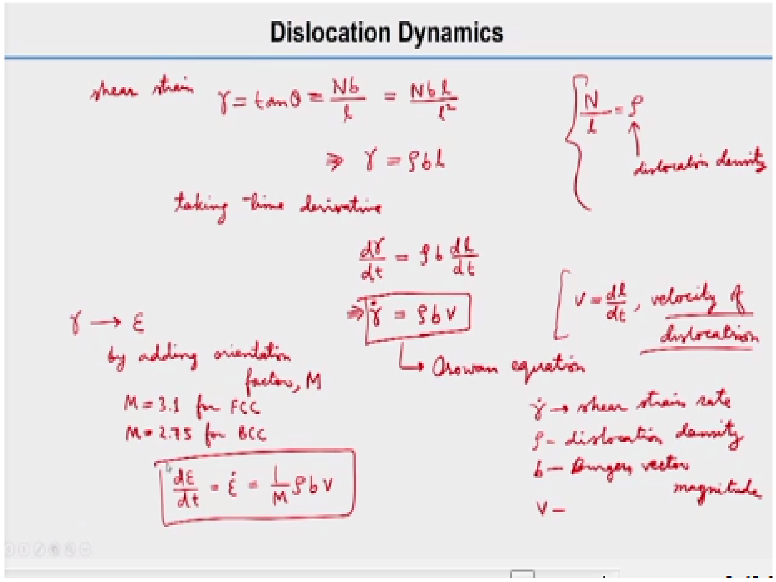

So we will write again here, so gamma will be tan theta which will be equal to N b, N is the number of dislocations per unit area, b is the magnitude of the dislocations burgers vector. And then divided by L, L is the our domain or size of the materials sample and then what we can do is we can write it like this N b l divided by l square. So we are multiplying l both above and below.

And so we should know that the number of dislocations per unit area and divided by l square will give us the dislocation density rho, that is the dislocation density, so this is dislocation density. So here what we can do is we can write now N by l as rho N b l, so that is equal to gamma the shear strain. So here I will write shear strain, shear strain produced by the dislocation movement.

So now we can take the time derivative, if we take the time derivative then we get that d gamma by dt rho b derivative of l with respect to t. So d gamma by dt is nothing but the shear strain rate with a dot on top of it and then rho b and we can write v small v. So v is the equal to dl by dt. So this dl is you can see we have l here this l so the dislocation moves let us say in time T you know if it moves in the x direction.

And then if we take the derivative of l with respect to t that will give us the velocity of dislocation. So this equation is very important and this is called as Orowan equation. So that equation shows the relation between shear strain rate, so here if you want to know this gamma dot is shear strain rate, rho is the dislocation density. We have already written there dislocation density and the v is the burgers vector magnitude of burgers vector actually burgers vector magnitude.

And v is the as you know velocity of dislocation we have wrote here, the shear strain can be converted to longitudinal strain epsilon by adding orientation factor. So here in this book it is not discussed in details but other books probably form on the mechanical behavior of materials Meyer's and Chawla. So they are referring it to it, and then this is orientation factor that is m, so m is can be different for different crystal structures.

So from this book from Meyer's and Chawla, so they are saying that m they are taking m 3.1 for FCC material and m = 2.75 for BCC body centered cubic metal. So what the expression I did not write yet, so d epsilon by dt that is epsilon dot will be equal to 1 by rho bv, so the m factor is different for different materials.

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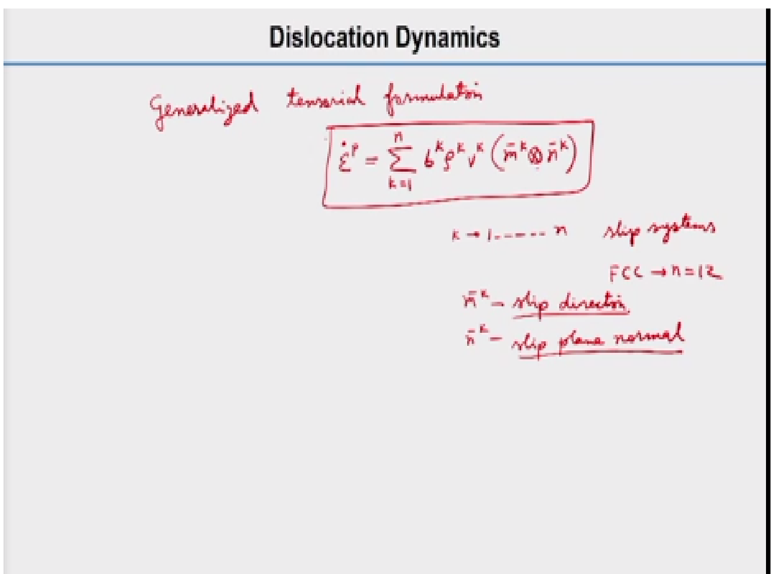

In a generalize tensorial form just I want to write that but we are not going to discuss at all, this will be little advanced for you. Generalize tensorial formulation is, so plastic strain rate is k = 1 n b I will let you know what is k and then rho V k. So you know what is b rho V and then m k and this n k, so these are the vectors I will tell you what is this.

And so basically what happens here is this k 1 to n is the number of slip systems, so these are the slip systems. So as you know there are different number of slip systems, so in FCC materials this n can be 12, so for each list slip system of the contribution should be calculated. And here m and n, so this is we writing the product of tensor product of m, m is for each slip systems this slip direction.

As I told you in earlier class of materials basic this is the slip direction and n is the slip plane normal direction, these are the vector slip plane normal direction. So just to let you know that the slip direction in the slip plane normal direction can you know influence this. So that is why although whatever we got early as a simple expression but then this slip systems can influence this the strain rate.

Here the strain rate in a longitudinal strain right, so this is we do not need to discuss more on it but this is a important result. So with that so we are actually closing for this lecture. So what we discussed here is we discussed about the constitutive relations empirical constitutive relations which is a very successful one equation is Johnson Cook model. And we also discussed about the basic high standard plastic deformation relations between stress function of strain, strain rate and temperature.

And that relation can be is summarized in with the help of this Johnson Cook model which is the most widely used. But there are other models also available in it, and then we also found about this R1 equation, the R1 equation gives the relationship between shear strain rate with the density of dislocation. And this the velocity of dislocation, so we will continue this discussion, so we have more to discuss about the plastic deformation behavior at high strain rate, so thank you.