Dynamic Behaviour of Materials
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Lecture-38
Plastic Deformation at High Strain Rates 5

Hello everyone, so we have been discussing on plastic deformation at high strain rate, so we will continue this discussion. So today we will discuss on physically based constitutive models, so earlier we discussed empirical based constitutive models. So before starting this topic the physically based constitutive model, so we just want to a review a problem on that dislocation dynamics.

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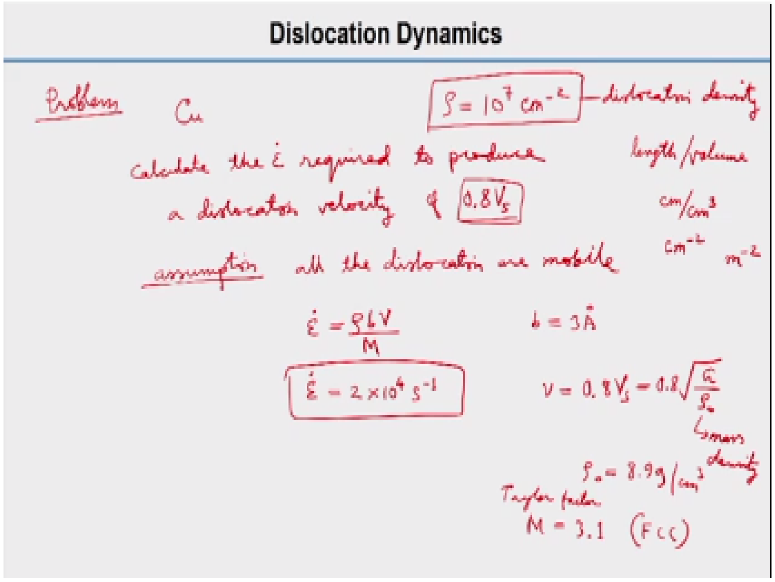

Dislocation Dynamics

Problem Cu $\boxed{\rho = 10^7\ cm^{-2}}$ — dislocation density

length/volume

cm/cm^3

cm^{-2} m^{-2}

Calculate the $\dot{\varepsilon}$ required to produce a dislocation velocity of $\boxed{0.8V_s}$

assumption all the dislocation are mobile

$$\dot{\varepsilon} = \frac{\rho b V}{M}$$

$b = 3\text{Å}$

$$\boxed{\dot{\varepsilon} = 2\times10^4\ s^{-1}}$$

$$V = 0.8V_s = 0.8\sqrt{\frac{G}{\rho_o}}$$

↳ mass density

$\rho_o = 8.9g/cm^3$

Taylor factor

$M = 3.1$ (FCC)

So this is simple problem and it is just a solve problem of the Mark Meyer's book. So the problem is so the material deformed is copper, this is under plastic deformation and so this has a dislocation density as 10 to the power 7 per centimeter square. So dislocation density as you know the length of the dislocation divided by volume centimeter per centimeter cube that means centimeter to the power - 2 or you can express it at meter to the power – 2.

So here the density is given, this is the density dislocation density, so we need to calculate the strain rate, I just write epsilon dot strain rate required to produce a dislocation velocity of 0.8

times of the shear wave velocity. So at that fortunately 0.8 V s probably the relativistic effects are also of important. But anyways what we will do for this problem is we will have the assumption that all the dislocations are mobile, there are no immobile dislocations all dislocations can move.

And there on slip planes, so they will move, so what we need to do here is, so as we know this formula so we will be using is rho b v, v is small v, the dislocation velocity divided by M. So here as you know rho is a dislocation density, b is the magnitude of the burgers vector, so that will be equal to let us say 3 angstrom. We need to get the dislocation velocity small v = 0.8 V s.

So V s will be nothing but capital G shear modulus by rho 0, so rho 0 is the mass density and the rho is the dislocation density ok, this is the dislocation density. So we can get the V from here, so rho 0 for copper, so it will be 8.9 gram per centimeter cube. So from here and again I forgot to mention that M, the orientation factor or it is also called Taylor factor orientation fraction probably I did not mention about Taylor factor.

But yeah that is for FCC materials that is 3.1 that is for FCC because copper is FCC, BCC the value will be different. So from there what we can if you substitute all this value we can directly I am not going to calculate, so the calculation steps, so this is 20,000 per second. So this is the strain rate that is the answer, this is very high standard so 20,000 per second so if you want to get a dislocation velocity which is a very high dislocation velocity that probably the relativistic effects will also be dominant in that case.

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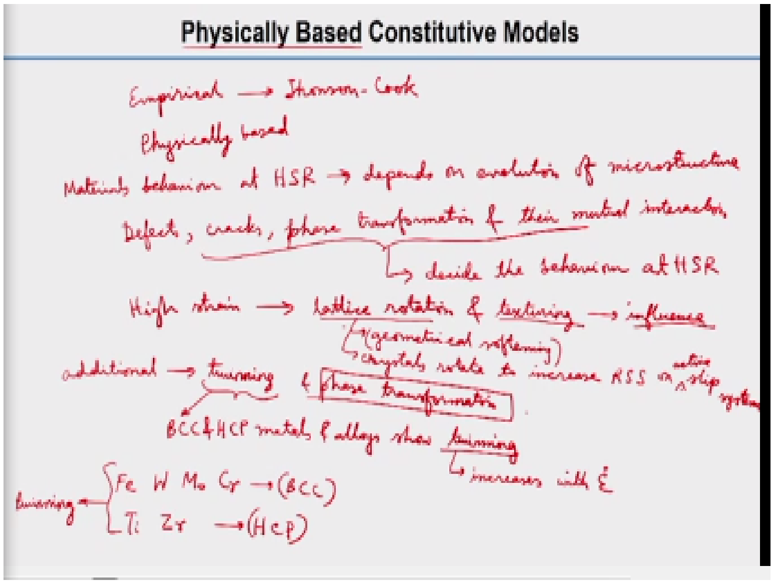

Physically Based Constitutive Models

Empirical → Johnson-Cook

Physically based

Materials behavior at HSR → depends on evolution of microstructure

Defect, cracks, phase transformation & their mutual interaction

↳ decide the behavior at HSR

High strain → lattice rotation & texturing → influence

(geometrical softening)

crystals rotate to increase RSS on active slip system

additional → twinning & phase transformation

BCC & HCP metals & alloys show twinning

↳ increases with $\dot{\varepsilon}$

twinning ← Fe W Mo Cr → (BCC)

Ti Zr → (HCP)

So now we will discuss about physically based constitutive models, so what is physically based constitutive model these type of models are based on physical processes. So earlier what we discussed is empirical constitutive model which is like we got Johnson Cook constitutive model. And now we will discussed on physically based constitutive model that based on some physical processes.

So we will talk about that in later I mean will be talking and today's class only but before going to some constitutive model, so we want to we would like to know that materials behavior at high strain rate I like HSR for high stand rate. Depends on evolution of microstructures, so these microstructures can include defects these are like cracks and phase transformation will influence and actually and their mutual interaction.

This will decide the mechanical behavior at high strain rate, so in addition high strain not high strain rate at high strain lattice rotation and texturing also influence. So what is lattice rotation, lattice rotation is if we check the that in some literature, so that can we call the produce geometrical softening, what is geometrical softening and that we probably can even discuss later and an another chapter.

But this the basic idea of lattice rotation is that the crystal rotates crystals we would say that grains will rotate to increase the resolve shear stress increase RSS the resolve shear stress on slip systems. Basically on active slip system we should call**,** active slip systems are those slip systems where the dislocation activity happens. Because not all slip systems will be active because the dislocation movement will happen with the dislocation movement will be restricted to some the slip systems only.

So these rotation is called lattice rotation and also texturing is I am not going to write here but you please study about this. Texture means the preferred orientation of some grains that is like some grains orient would like to orient in the you know same direction. So some mini grains are in the same direction the orientation suppose one on direction of many grains are you know directed towards the same you know global direction.

So these capturing and lattice rotation influence the behavior, this is influence means the influence the behavior. And then we have already seen that we have 3 dislocation movement mechanisms that is thermally activated dislocation mechanism and then drag control and relativistic effects. So there are as you know that the additional consideration or additional deformation mechanism are twinning.

So, so far we have discussed more about dislocation slip but we have twinning as well and we have phase transformation as well phase transformation. For twinning the BCC and hexagonal close pack HCP metals and alloys show twinning FCC does not show twinning much. So and these tendency of twinning increases with strain rate, I would write epsilon dot strain rate. So if we talk about like iron let us say tungsten, molybdenum then chromium.

And then again if you say like let us say titanium, zirconium. So these are like these all are BCC and these are hexagonal close pack. So these metals or maybe they are alloys they show twinning, they may show significant twinning at high strain rate ok even at low strain rate the twinning can happen but high strain rate the twinning will be more. And also some other materials may undergo phase transformation and that increases with at high stress and strain. So at higher strain rate the phase transformation can happen ok.

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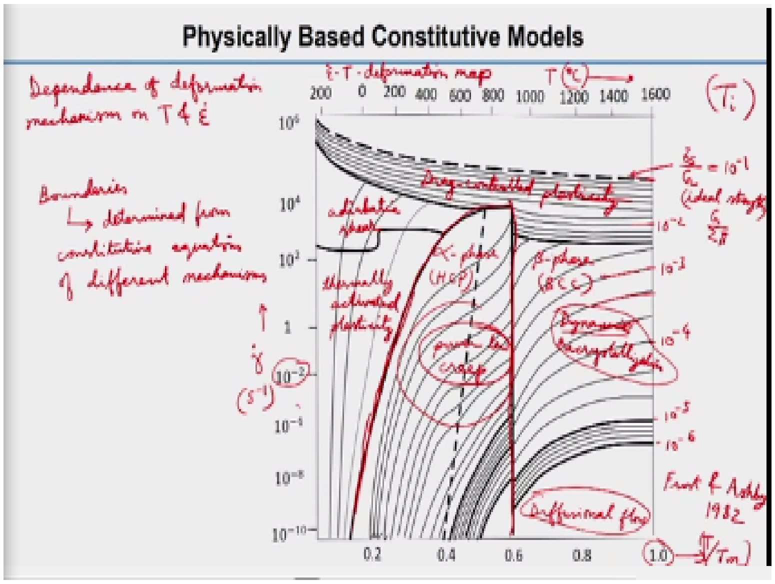

Physically Based Constitutive Models

Dependence of deformation mechanism on T & $\dot{\varepsilon}$

Boundaries
↳ determined from constitutive equation of different mechanisms

So here we got a important figure here, so this is the reference is Frost and Ashley I think 1982 Frost and Ashley 1982. So this is to show you the dependence of deformation mechanism on temperature and strain rate. So here this is actually can call strain rate temperature deformation map, so better we should write that, I just write epsilon dot strain rate temperature deformation map, this is for titanium.

And in the y axis, so Y axis we have gamma dot that is shear strain rate, that is the per second to the power -1 and that is on the y axis or vertical axis. So in the horizontal axis this is called homologous temperature and that is T by Tm temperature divided by the melting temperature T by Tm. And then on top it is temperature in degree Celsius, so what we can see that here this is 1 means that is the melting temperature.

And so what happened here is, so there are different phases by the way, so this portion is this is alpha phase and this is hexagonal close pack. And here at higher temperatures that will be beta phase of titanium that is BCC ok. And so these lines there are lot of lines we have shown, so these represents these lines values will write in terms of sigma s divided by G so G is the shear modulus.

And in these for this line this is equal to 0.1, that means the stress is very high, so sigma s is the stress . Here the stress is very high that is just 1 tenth of the shear modulus, so that is actually the ideal or theoretical strength. We already discussed about that in our materials basics lecture that the ideal strength is something like G twice pi. So that is what I mean it is that in the order of 0.1.

And then there are other lines we can show that here 1 line is probably it is 10 to the power – 2. In this case first one rather we will write 10 to the power – 1, so similarly here is 10 to the power - 3 and then 10 to the power let us say – 4. And then somewhere here it is 10 to the power - 5 and 10 to the power – 6 or it is something like that. So it may not be very accurate please refer to the books diagram, so just to give you an idea about this, so we are drawing.

So now what will happen is at a high temperature in this region, so this will be diffusional flow at high temperature diffusional flow. And then at high temperature here we get what is called dynamic recrystallization, recrystallization means the nucleation of new grains and the growth of the grain. So what is static recrystallization that happens in heat treatment process you must have read that in your material science course.

But in dynamic recrystallization means it happens with deformations so with deformation grain will nucleate and it they will grow. So now we will see what happens at the low temperatures, so in this region, so the entire region **we** it will have power law creep. So this region is entire region for the creep, so you can see this region, so this is for creep. And then at very low temperatures, so here in this region, so separated by this boundary.

So this is obstacle control plasticity or what we got to know earlier thermally activated dislocation motion thermally activated zone here, we can write thermally activated plasticity or dislocation motion. And if you go to higher strain rate here that will be adiabatic shear, so that we will discuss in another chapter adiabatic shear bend chapter. And then this portion is drag control plasticity, drag control as you know that means phonon drag.

If you go to a higher strain it probably relativistic effects will also be will also come into a picture. So boundaries between these different regions are determined from constitutive equation of different mechanisms, so those boundaries I would write this boundaries between these reasons let us say this boundary or let us some other boundary like this. So these are that are determined from constitutive equations of the different mechanisms different deformation mechanisms.

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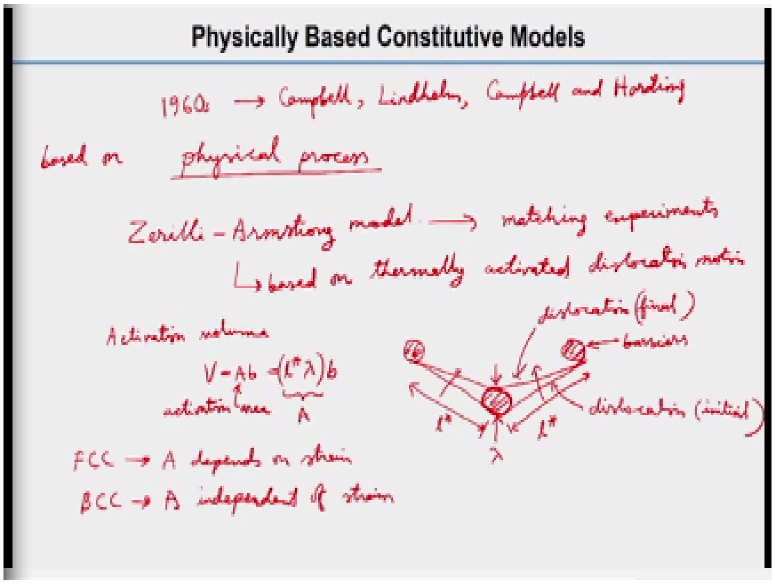

Physically Based Constitutive Models

1960s → Campbell, Lindhalm, Campbell and Harding

based on <u>physical process</u>

Zerilli - Armstrong model → matching experiments
↳ based on thermally activated dislocation motion

Activation volume

$V = \underset{\text{activation area}}{A}b = \underbrace{(l^* \lambda)}_{A} b$

FCC → A depends on strain
BCC → A independent of strain

So there are many physically based constitutive equations it started from 1960s and some of the very early researchers are like Campbell, Lindhalm, Campbell and Harding**.** So these are very early researchers, so who pioneered this work but we will not going to discuss all the models there are many models based on physically based process.

So what we are discussing here is a physical I would right just based on physical processes not based directly on experiment but based on physical processes. So what happens in the and a micro scale what is the physics behind that means how the dislocations move or how the dislocations crosses the barriers. So those things will be taken care of by these equations, so very popular one of the popular one among these Zerilli Armstrong constitutive model.

So let us discuss this little bit not much away, so there are they proposed 2 micro structurally based constitutive equations. That actually shows very excellent match with experimental results that matching with experiment ok. So this A is based on thermally activated dislocation motion**.** So what they found is they actually the basis of this is I will just draw quickly you can check Mark Meyer's book for details.

So let us say these are the some barriers ok, so these are barriers and let us say there is a dislocation line something like this, this is the dislocation line and dislocation line is moving towards this direction. These 3 are barriers the circular structures or barriers ok, so now what is happening here is this is the initial position of the dislocation and then when the dislocation will cross let us say it will cross this middle barrier dislocation will cross.

So I will write the final position, so this is the dislocation initial position I would write initial. And then let us say this is dislocation final position final, final means rather I should say that this is after the crossing the barrier not exactly it is not like it will stop there. But then this width of this barrier is lambda, so that is the distance lambda and the spacing between the barriers is l star, so here also we resume the same spacing, so this is l star.

So now based on that there is one term called activation volume which is equal to V the volume is equal to a multiplied by b, b is the magnitude of the burgers vector. And that will give l star lambda that is A is the area by the way, this activation area and b is the magnitude of the burgers vector. And this looks like the sorry I did not write a b here, so this is the area activation area. So what Zerilli and Armstrong found that that for FCC, so the activation area A depends on strain but for BCC A is independent of strain. So that is an important finding.

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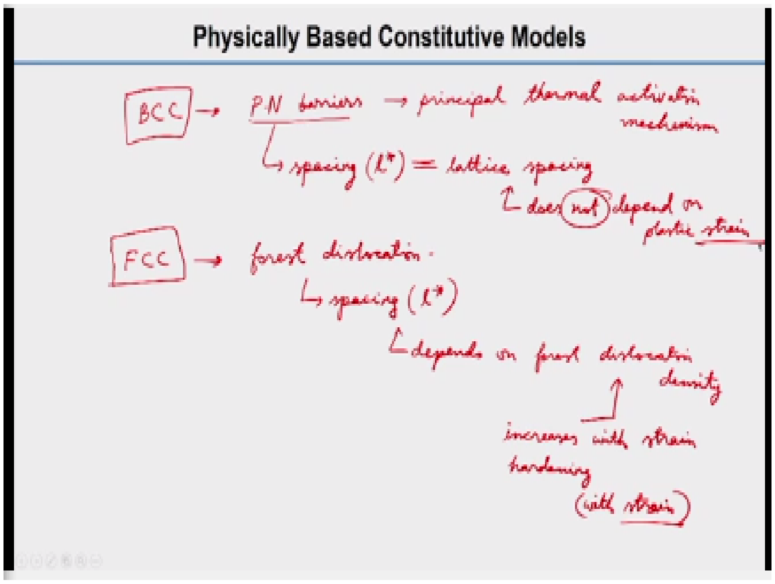

Physically Based Constitutive Models

BCC → PN barriers → principal thermal activation mechanism
↳ spacing (l^*) = lattice spacing
↳ does not depend on plastic strain

FCC → forest dislocation.
↳ spacing (l^*)
↳ depends on forest dislocation density
↑ increases with strain hardening (with strain)

So why it happened is the Zerilli and Armstrong concluded that for BCC metals pulse Nabarro barriers as you know pulse number barriers we discussed about that are the principal thermal activation mechanism. This for PN barriers the spacing that means the l star let us say. so is just the lattice spacing that means equal to the lattice that is actually equal to the lattice spacing or I would just write equal to the lattice spacing.

And that does not depend on plastic strain, it does not depend on strain. On the other hand for FCC as we know that forest dislocations are responsible. Forest dislocations we discussed that in earlier class, these are the principal thermal activation mechanisms. And spacings that l star are depends on forest dislocation density, these density increases with strain hardening or rather we can say that with strain which plastic strain ok. So that is why for FCC the activation area A depends on strain and for BCC it does not depend on strain.

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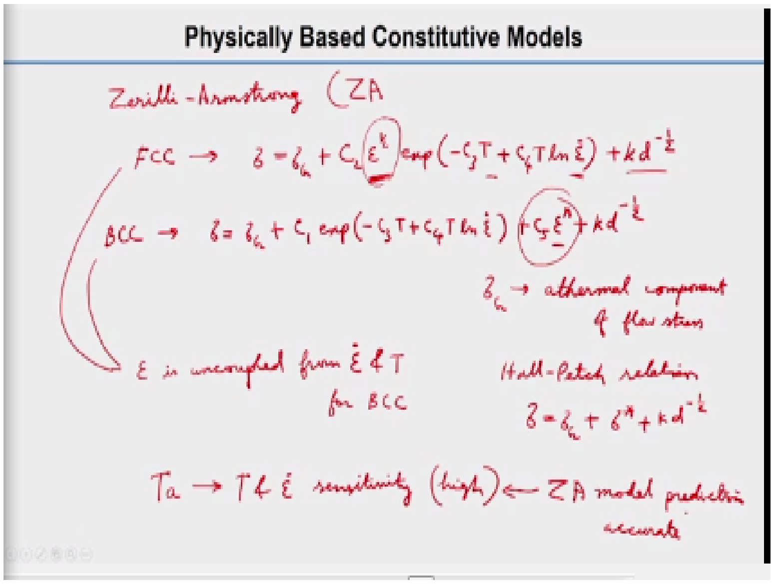

Physically Based Constitutive Models

Zerilli - Armstrong (ZA

FCC → $\sigma = \sigma_G + C_2 \boxed{\varepsilon^{\frac{1}{2}}} \exp(-C_3 T + C_4 T \ln \dot{\varepsilon}) + \underline{kd^{-\frac{1}{2}}}$

BCC → $\sigma = \sigma_G + C_1 \exp(-C_3 T + C_4 T \ln \dot{\varepsilon}) \boxed{+ C_5 \varepsilon^n} + kd^{-\frac{1}{2}}$

σ_G → athermal component of flow stress

ε is uncoupled from $\dot{\varepsilon}$ & T for BCC

Hall-Petch relation

$\sigma = \sigma_G + \sigma^n + kd^{-\frac{1}{2}}$

Ta → T & $\dot{\varepsilon}$ sensitivity (high) ← ZA model prediction accurate

So we are not you know writing the details of the Zerilli Armstrong formulation just I want to write the final the expression if you want we are interested you can check Mark Meyer's book or any other book. So the final expression is look like sigma = sigma G C to epsilon to the power 1/2 exponential - C3 T + C4 T (()) (29:06) of the strain rate + kd – 1/2. So similarly for BCC sigma = sigma G + C1 exponential - C3 T + C4 T logarithm of epsilon dot + C5 epsilon to the power this term is different than the FCC and kd – half.

So here sigma G is the attermal component of flow stress, we discuss in a previous class a attermal component of flow stress. As we know these terms the last terms come from Hall-Petch relation we discuss in the materials basic class. So that is sigma we can write this attermal component and + sigma star + kd to the power – half, so d the diameter of the grain, that we know from the our materials basics lecture. So these are constants C1, C2, C3, C4, so these constants and even C5 there are 5 constants here.

And also we will have that additional this n with epsilon to the power n. So for probably yeah they are different that only difference is here I think in this term for BCC what we can see that here in the for FCC there is epsilon is here the plastic strain. So difference between BCC and FCC is that the plastic strain this epsilon this plastic strain is uncoupled from strain rate and temperature for BCC, you can see the second formula.

So here in the case of FCC it was coupled with temperature and strain rate term. But here it is uncoupled for Zerilli and Armstrong relation is he has very excellent maths with experiments and you can see for tantalum this plot is available in the Meyer's book. Tantalum is showing temperature and epsilon dot sensitivity and the sensitivity is actually high this high. And Zerilli Armstrong model, so this I should write ZA Zerilli Armstrong model prediction accurate, so very close to the experimental result you can get.

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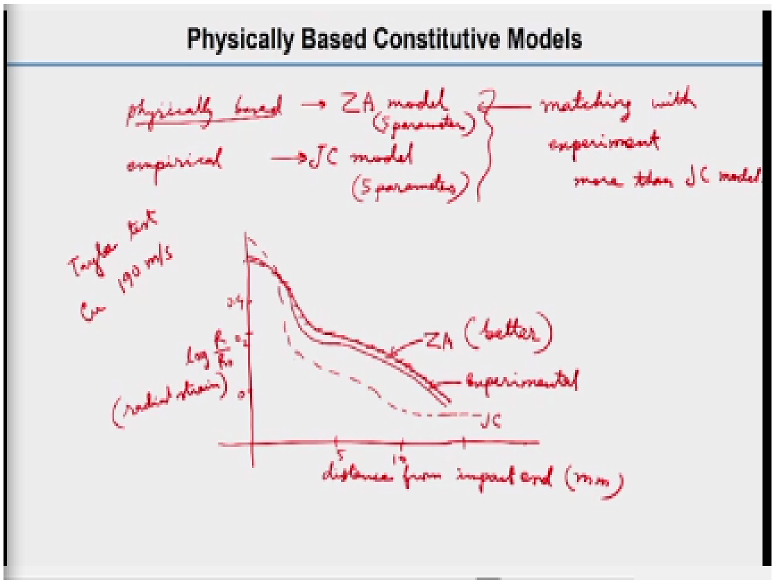

Physically Based Constitutive Models

Physically based → ZA model (5 parameter)
empirical → JC model (5 parameter)
— matching with experiment more than JC model

As we discussed this is a physically based is Zerilli Armstrong model and the empirical relation is our Johnson cook that is JC model. So both of these will require different like parameters here also we need 5 parameters. And even for Zerilli Armstrong also require 5 parameters these are experimental we need to find out from experimental test. So once the parameters are determined, so it is useful to validate this the models with experiments.

So these models if we compare them, so what we will find that, so this is based as we know that Zerilli Armstrong model is based on physical processes. So the matching with experiment more than JC model. So we will quickly see a plot that Zerilli Armstrong did for Taylors test, Taylors test is very commonly used for these type of constitutive models Taylors test for copper at 190 meter per second.

And this is a plot logarithm of R by R0 the radial strain the copper the Taylor bar that you can understand the Taylor test means that bar impact test. And then in the horizontal axis it is distance from impact end, so that is in millimeter. So if this is the case let us say I will not draw the scale now, this is in millimeter let us say 5, 10 or something like that, so let us say this is 0, 0.2, 0.4 or something like this. So I will just simply draw the curve try to draw the curve. So the experimental curve is going something like this.

And then the Zerilli Armstrong curve I first say that the Johnson cook curve, so Johnson cook is let us say a dash line which will so this will come like this, so this is let us say Johnson cook model and this is experimental ok. So now for Zerilli Armstrong model this would look like, so this is the Zerilli Armstrong curve ok, so this one is Zerilli Armstrong model. So you can see that the predictions are better for Zerilli Armstrong because this is physically based model as compared to the empirical model of Johnson cook.

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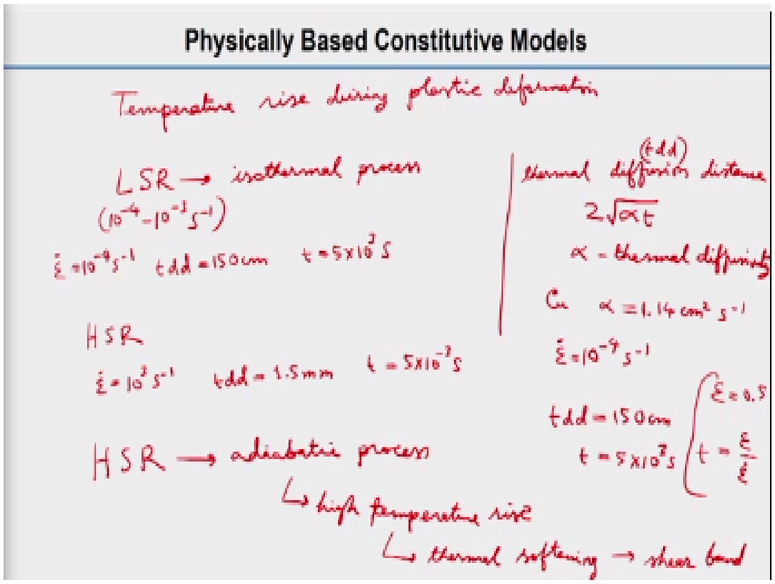

Physically Based Constitutive Models

Temperature rise during plastic deformation

LSR → isothermal process

$(10^{-4}-10^{-3}s^{-1})$

$\dot{\varepsilon}=10^{-4}s^{-1}$ tdd = 150cm $t=5\times10^{3}s$

HSR

$\dot{\varepsilon}=10^{3}s^{-1}$ tdd = 1.5mm $t=5\times10^{-3}s$

HSR → adiabatic process

↳ high temperature rise

↳ thermal softening → shear band

thermal diffusion distance (tdd)

$2\sqrt{\alpha t}$

α - thermal diffusivit

Cu $\alpha=1.14\,cm^2s^{-1}$

$\dot{\varepsilon}=10^{-4}s^{-1}$

tdd = 150cm

$t=5\times10^{3}s$

$\left\{ \begin{array}{l} \varepsilon=0.5 \\ t=\frac{\varepsilon}{\dot{\varepsilon}} \end{array} \right.$

So we will just quickly go through the temperature rise during plastic deformation. So what happened at the low strain rate I will write LSR, at low strain rate we can assume the process as isothermal process because what happens is the strain rate is very low 10 to the power – 4, 10 to the power - 3 per second those test we that used UTM to do the quasi static test.

And there is we called diffusion distance thermal diffusion distance, what is that we will see here. Thermal diffusion distance is twice alpha t, so alpha is the thermal diffusivity. So what it means is suppose for a copper specimen copper specimen where our diffusivity parameter is 1.14 centimeter square per second. So if it is the strain rate is 10 to the power - 4 per second then the diffusion distance I would write let us say thermal diffusion tdd I will write tdd.

So the tdd = 150 centimeter and the time required will be let us say 5 into 10 to the power 3 second. So that how we calculate it is like how we calculate this at time is strain divided by strain rate. Suppose in this case let us say epsilon is 0.5 and we get this, so suppose here that is why I am writing let us say if the thermal diffusion distance is 150 centimeter and time is 5 into 10 cube per second.

But if you do the test at high strain rate let us say high strain rate, so here the strain rate was 10 to the power - 4 per second. And then here at high strain it let us say the strain rate is 10 cube per second**.** So then thermal diffusion distance will be 1.5 millimeter corresponding to a time 5 into

10 to the power - 3 second that means 5 millisecond. So that is the difference so that is why the high strain rate is an adiabatic process and the high temperature rise during this process happens, high temperature rise.

So these high temperature rise will lead to thermal softening we will talk about that in the next lecture, that is next not next lecture in the next chapter thermal softening. And that lead to as you know the shear band that we will talk about that the about shear bands.

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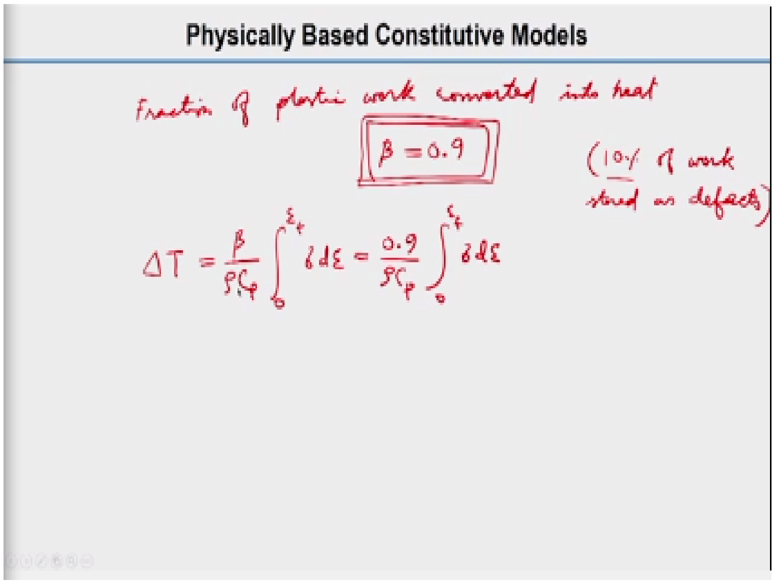

Physically Based Constitutive Models

Fraction of plastic work converted into heat

$\boxed{\beta = 0.9}$

(10% of work stored as defects)

$$\Delta T = \frac{\beta}{\rho C_p}\int_0^{\varepsilon_f} \sigma \, d\varepsilon = \frac{0.9}{\rho C_p}\int_0^{\varepsilon_f} \sigma \, d\varepsilon$$

So these the friction of plastic work converted into heat is considered as 0.9 for most materials or most metals. So that means 10% of work of deformation and is stored as material defects stores as defects. So now I am just to tell you so what if you want to know the temperature rise that will be this beta, so this is we call beta.

So beta by rho C p the specific heat our 0 to epsilon F sigma D epsilon = 0.9 rho CP 0 to epsilon F sigma D Epsilon. So just to show you that this beta is considered as around 90% and rest 10% is work stored as defects. So that you can understand that the heat is too much or like temperature rise will be too much for adiabatic processes, that means that plastic deformation under high strain rate.

So with that we are concluding for today's lecture, so this is end of the chapter that is plastic deformation under high strain rate. So we have discussed about constitutive models, we discussed about the empirical based constitutive models. For example Johnson cook model and we have also discussed about physically based models like Zerilli Armstrong models. And also we discussed elaborately on different dislocation movement mechanisms.

Like thermally activated dislocation motion, drag control dislocation motion and relativistic effects control dislocation motion. So that is all for today, so we will discuss on plastic deformation the shock wave in the next lecture, thank you.