Dynamic Behavior of Materials
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Module No # 01
Lecture No # 05
Elastic Wave

Hello in a previous lecture we discussed about the different types of deformation mechanism like a Quasi study deformation and dynamic formation.

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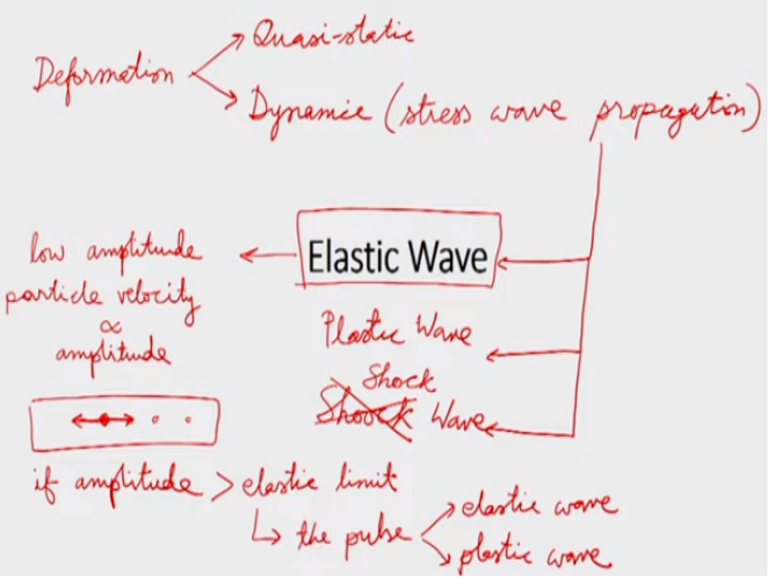

So deformation are of two types one is Quasi study and one is dynamic so today we will discussed about elastic wave and is the types of elastic waves so that comes under the dynamic deformation which is involves which involves stress wave propagation. So stress wave can be classified into 3 categories that we discussed earlier elastic wave, plastic wave and shock wave.

So the in the elastic wave is low amplitude wave it as low amplitude and the particle velocity is linearly proportional to its amplitude particle velocity is proportional to is amplitude. And the material particle as we know this material particles in a body oscillates about its equilibrium position if you assume this is equilibrium position and so its oscillates about the equilibrium position.

And if the amplitude cross the critical limit that is the elastic limit if the amplitude is greater than elastic limit then the pulse will decomposed into elastic and plastic waves plastic waves. So today we will discuss basically elastic wave and its classification sorry so this is a not shock so this should be shock wave sorry about that shock wave. So in a previous lecture we discussed about the elastic wave equation in a string or helical string or any string.

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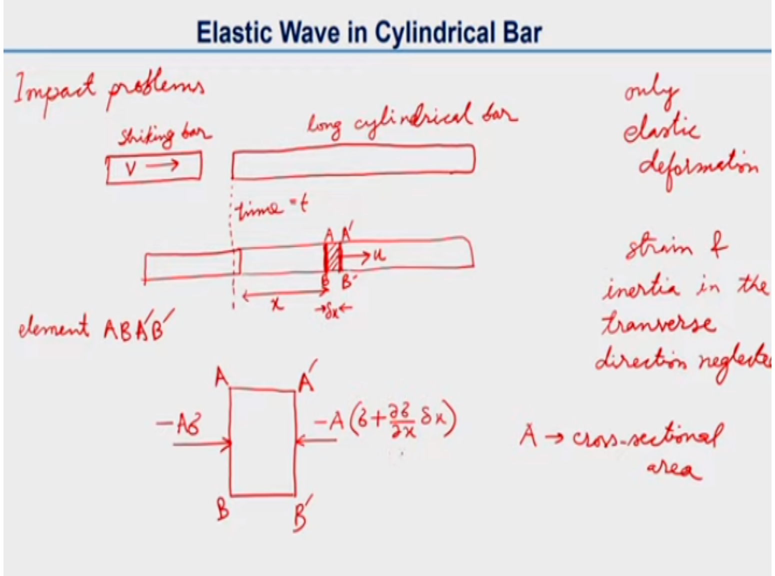

Elastic Wave in Cylindrical Bar

So today we will discuss a elastic wave in impact problems so we will discussed wave in impact problems and so we will take an example of a striking bar impacting on a long cylindrical bar. So this is a striking bar on left hand side and this is long cylindrical body or you can say a bar as a cylinder body. So this striking bar will hit the cylindrical bar long cylindrical bar which are velocity V which is the impact velocity.

So we are assuming here only elastic deformation only elastic deformation and no permanent of plastic deformation. So if we impact with the striking bar so this striking bar will hit the long bar and at a time = t so let us assume that the wave front of the disturbance the wave reaches at the point x and that section we call it as the A, B that is the front of the disturbance are wave the section A, B.

And if we take another section A prime B prime at a distance delta x so and then we can consider the small element infinite decimal element which as a displacement eu also one assumptions is that strain and inertia in a transverse direction is neglected. So we will now consider this infinite decimal element AB, A prime, B prime. So this element if we see the forces or we will make it little longer so this is the element AB, A prime, B prime that is the small element in the bar and then we can see the compressive forces A sigma, sigma is the stress multiplied by A area is the cross sectional area of the cylindrical bar.

And deeper into the bar at the section A prime B prime the force will have the additional increment with A multiplied by sigma partial of sigma which respect to x delta x as we know the sigma the stresses multipliable function thus that is partial of sigma which respect to x. So now we will apply the Newton's law of second law.

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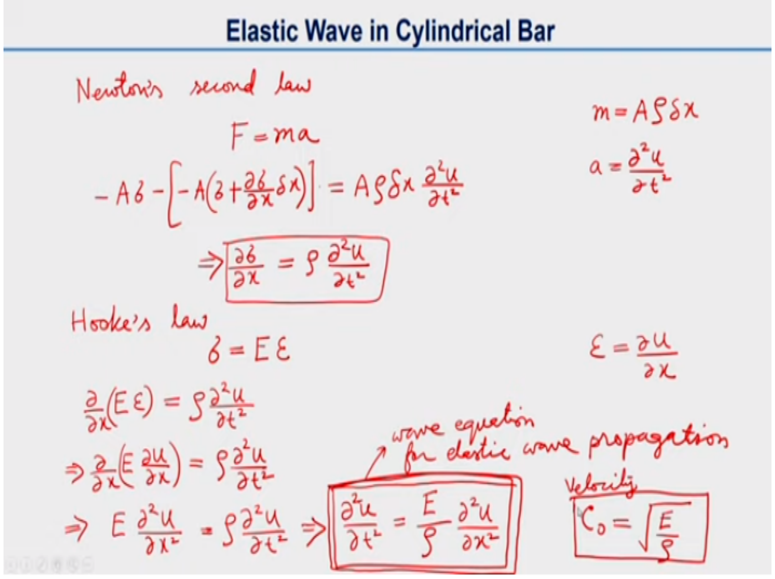

Elastic Wave in Cylindrical Bar

Newton's second law

$F = ma$

$$-A\sigma - \left[-A\left(\sigma + \frac{\partial\sigma}{\partial x}\delta x\right)\right] = A\rho\delta x \frac{\partial^2 u}{\partial t^2}$$

$m = A\rho\delta x$

$a = \frac{\partial^2 u}{\partial t^2}$

$$\Rightarrow \boxed{\frac{\partial\sigma}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}}$$

Hooke's law

$\sigma = E\varepsilon$

$\varepsilon = \frac{\partial u}{\partial x}$

$$\frac{\partial}{\partial x}(E\varepsilon) = \rho \frac{\partial^2 u}{\partial t^2}$$

$$\Rightarrow \frac{\partial}{\partial x}\left(E\frac{\partial u}{\partial x}\right) = \rho\frac{\partial^2 u}{\partial t^2}$$

wave equation for elastic wave propagation

$$\Rightarrow E\frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2} \Rightarrow \boxed{\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 u}{\partial x^2}}$$

Velocity

$$\boxed{C_0 = \sqrt{\frac{E}{\rho}}}$$

So applying Newton's second law on that segment Newton's second law on so that we know that F = ma and the force will be equal to A sigma – A multiplied by sigma + partial of sigma which aspect to x delta x which will be equal to A Rho A the cross section area Rho is the mass density and delta x partial of u second partial of u with respect to t is the second partial of u with respect to t is the exhilaration and mass = cross sectional area multiplied by delta x the thickness of the bar and the mass density Rho.

So now we can simplify this expression so that will give us partial of sigma with respect to x which will be equal to Rho second partial of u which respect to t. So as you we can see the area on the left hand side and right hand side and we will cancel so we can now use the Hook's Law that is for linear elastic solid. Now if we apply Hook's law that is sigma = Young's Modulus multiplied by strain epsilon.

And then from our earlier equation from this equation we can have partial of e epsilon with respect to x = Rho multiplied by second partial of u which respect to t and then we know the strain can be expressed as partial of u with respect to x which is also a multi waveable function and on the displacement is the multi waveable function and the on the displacement (()) (11:34) function and that will give us partial of u with respect to x = Rho second partial of u which respect to t.

And this we finally give us E second partial of u which respect to x Rho second partial of u which respect to t which we can write little differently as second partial of u which respect to t = Young's modulus divided by mass density sorry multiplied by partial of u second partial of u which respect to x. So this is the wave equation for elastic wave propagation so while is striker may impacted a cylindrical thin bar (()) (13:09) so this is the problem we do.

And then finally derive the wave equation for elastic wave propagation which is the double partial of displacement with respect to time is equal to Young's modulus divided by mass density multiplied by double partial of displacement which respect to x. So from the earlier discussion in earlier lectures and also from this equation we can say that the velocity of the wave C naught can be expressed by square root of the ratio of young's Modulus divided by mass density.

So we can have the velocity has the square root of the ratio of this young's modulus and mass density.

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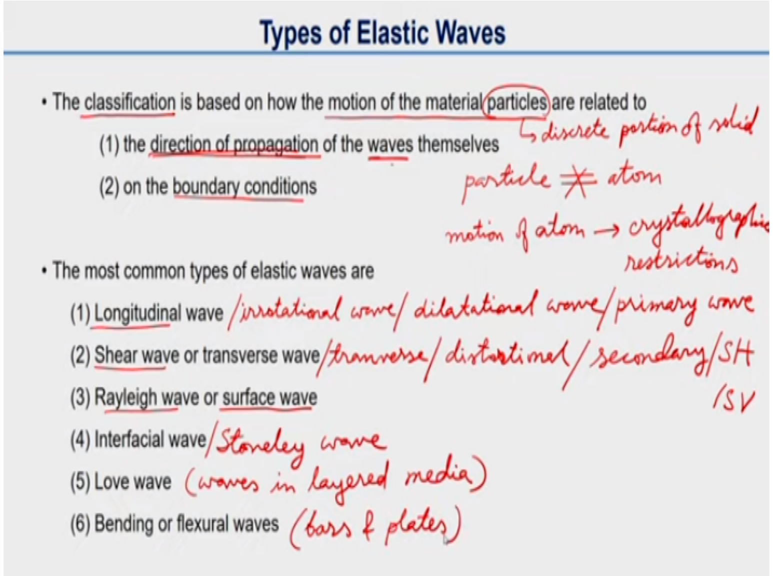

Types of Elastic Waves

- The classification is based on how the motion of the material particles are related to
  - (1) the direction of propagation of the waves themselves
  - (2) on the boundary conditions

↳ discrete portion of solid

particle ≠ atom

motion of atom → crystallographic restrictions

- The most common types of elastic waves are
  - (1) Longitudinal wave /irrotational wave/ dilatational wave/primary wave
  - (2) Shear wave or transverse wave/transverse/ distortional/ secondary/SH /SV
  - (3) Rayleigh wave or surface wave
  - (4) Interfacial wave/Stoneley wave
  - (5) Love wave (waves in layered media)
  - (6) Bending or flexural waves (bars & plates)

Now we will discuss little bit about the types of velocity which we will mostly focused on two or three of them but there are several of them and we will just touch little bit of that those classifications. So basically the classifications is based on how the motion of the material particles are related to the direction of propagation of the wave themselves and on the boundary conditions. So when we call this particle so particle is not atom because the motion of atoms are not straight forward as the motion of the particle because it needs to follow some crystallographic restrictions.

So particle is the descript portions of solid portions of solid but not atom as we can see that the motion of the particles how they orient with the direction of the propagation of the waves themselves will decide what is that wave is whether it is a longitudinal wave or a shear wave. So the longitudinal wave is also known as irrotational wave so for finite infinite and semi-infinite media. So this is called as dilatational wave and also in seismology it is known as primary wave for seismology the wave study is very important for (()) (16:47) and this is called as primary wave.

So we will discuss what is that longitudinal wave is and for simply if we want to say that this is the motion of the particle in this wave is along the direction of propagation of wave and for shear wave which is also called as transverse wave or distortional wave and for seismology this called as secondary wave or SH or SV wave. Surface waves are known as Rayleigh wave and then interfacial waves or it called as stoneley wave.

Love wave that is the waves in layered media bending of flexural waves are found in bars and plates. Okay let us discuss about the longitudinal wave now. The longitudinal waves are the waves which have the particle motion along the motion of the wave.

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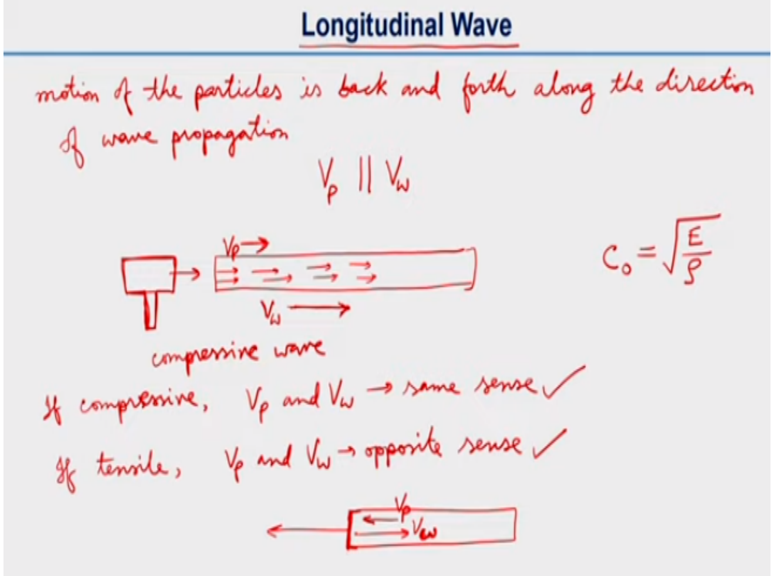

Longitudinal Wave

motion of the particles is back and forth along the direction of wave propagation

$V_p \parallel V_w$

$C_0 = \sqrt{\frac{E}{\rho}}$

compressive wave

If compressive, V_p and V_w → same sense ✓

If tensile, V_p and V_w → opposite sense ✓

So I will write it here the motion of the particles is back and forth along the direction or parallel to the direction of wave propagation. So then it is called a longitudinal wave basically we can call them the velocity of the particle which you represent as Vp is parallel to Vw which is the wave propagation velocity. So let us consider an example so this is a long cylindrical bar we are hitting that bar with a hammer.

So what will happen is that this particles if we hit hitting with the hammer this particle velocity will be along this rod form left to right and similarly as we know this wave propagation is also from left to right this is the compressive wave and the as you know from earlier discussing the velocity here we can express it as square root of the ratio of Young's modulus to mass density.

And if the wave is compressive if the wave is compressive then Vp and Vw will have same sense and I will tell what is that same sense and then if it is tensile the wave is tensile Vp and Vw will have opposite chance. So that means in the previous case as this is a compressive wave so what is happening here is the velocity of the wave propagation all the wave propagation is from left to right direction and even the particle velocity is also from left to right direction.

So that is why it as same chance and if the wave is tensile that means if you are giving a tensile loading here the velocity of wave will be anyways in this direction because we are the (()) (21:56) starts from this and of the bar so velocity of wave sorry that is this is not Vp the velocity of wave will be in this direction Vw this Vw and velocity of p the particle velocity as we are pulling this or keeping a tensile loading here.

So velocity of particles that is Vp will be in the opposite direction so that is why we wrote here that is at in opposite chance. So that is the difference between the compressive way and tensile wave both are longitudinal wave here and if you want to discuss about shear wave.

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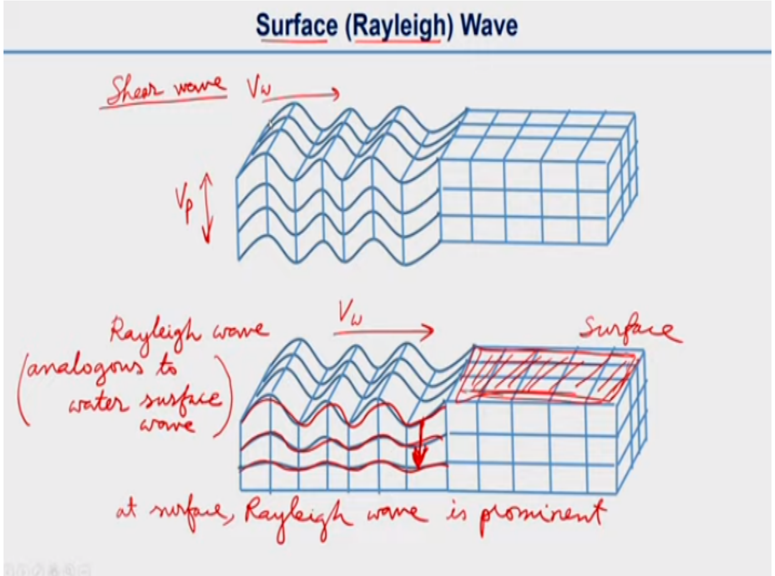

Surface (Rayleigh) Wave

So shear waves is the velocity of particle is perpendicular to the velocity of wave propagation. So in this case there is no resulting change in density resulting change in density and the longitudinal strains epsilon 1 1, epsilon 2 2, or epsilon3 3 are 0. So we will see with the diagram we will explain this with a diagram suppose we have a again we are taking a cylindrical bar. So we are giving some torsion at left hand let us say torque is applied on the left hand and we are keeping a clamp at a few distance at a little distance away from the torque from the application of the torque.

So this portion will develop a you know some torsion here and when release the clamp so when we release the clamp when release the clamp so the wave will propagate in this direction so this is velocity of wave or velocity of wave propagation from left to slide but as you can see that this is the torsion and the particle velocity will be perpendicular to the direction of this axis of this cylindrical bar.

So that is why it is also called as distortional or transverse wave okay so this is all about the shear wave.

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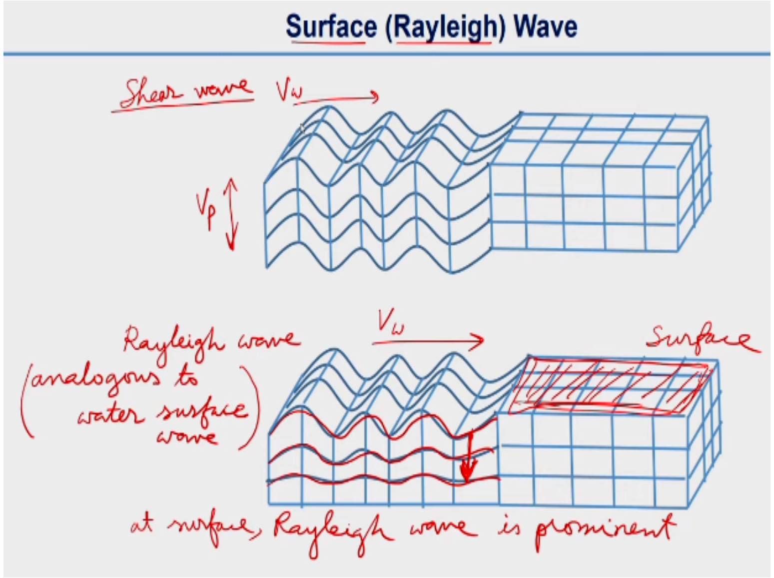

Surface (Rayleigh) Wave

And we will now discuss about the third wave that is surface wave surface or it is call it as Rayleigh wave as well. So here we have some diagram the first one is nothing but the shear wave that we will discussed in this last slide and the second diagram is surface wave or Rayleigh wave. So we can see the difference now the Rayleigh wave is analogous to water surface wave so when we throw a stone in a pond and water may wave we see that is a surface wave and that is in the water and whatever the surface wave we encounter in solids that is we call it as a Rayleigh wave.

So that is analogous to the water surface wave so one can think that the shear waves and or Rayleigh waves or may be in a similar from this two figures so what happens is if we inside this is let us say the surface of the water sorry the is surface of the solid it can be water bed this is in this case we are taking surface of the solid so this phase is the top phase is the surface of the solid.

So when we go deep inside actually here in this case the wave is propagating from this side and this portion is now undisturbed this right side portion so the wave will propagate this way wave propagation so if we go deep inside the wave is not prominent. So here at the surface at surface Rayleigh wave is prominent and then it will dump down when you go inside you can see this lines so this is very prominent at the surface and then when you go inside it is dumps down and you know shear it is very little wave can be seen in the deep inside.

But for shear wave we can see that its little different and this is the particle velocity here is perpendicular to the direction of wave propagation so wave propagation in the slander bar and semi-infinite body so we will now discuss what is the difference if we take difference objects so if we take a thin long bar or what will happen to the waves as we have seen that the longitudinal wave will propagate in thin bar. But if we take a semi-infinite body so what will happen we will see on this picture here.

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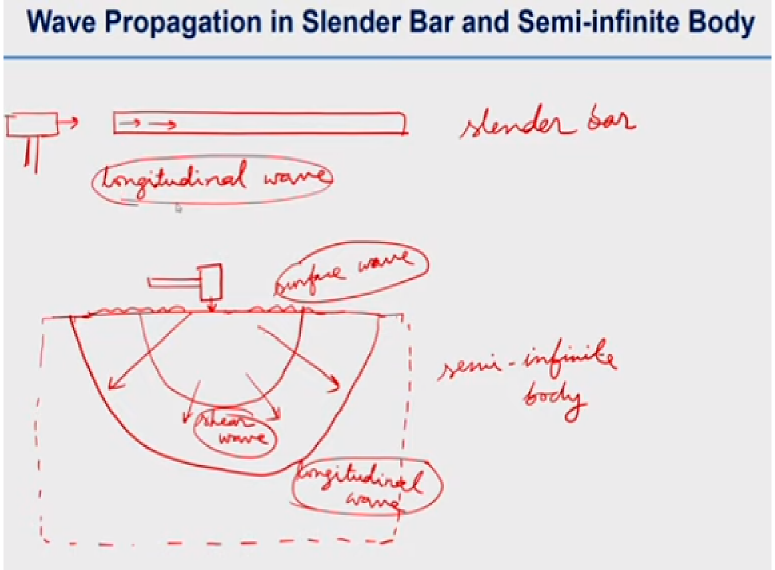

Wave Propagation in Slender Bar and Semi-infinite Body

So as we have already discussed that if you take a hammer and hit with long thin bar so you will see longitudinal wave here that if you take a semi-infinite body and hit with a hammer this is the semi-infinite that means so this is the body semi-infinite so if you hit the hammer in the semi-infinite body so what will happen is one longitudinal wave will propagate and that front of that longitudinal wave will be will look like you know it will propagate in this direction it will look like this.

And then even the shear wave will also propagate but the difference is shear wave the particle velocity will be perpendicular to the propagation of this wave so this is longitudinal wave and there will be shear with but the difference is that the particle velocity will be perpendicular to the wave propagation velocity. And the third wave is this surface wave on the surface wave so this is generally if we sorry this is longitudinal.

So if you take a semi-infinite body so this is a semi-infinite body you can see that majorly three types of waves are present longitudinal wave, shear wave and surface wave and in this case for a thin bar or a slender bar we have only longitudinal wave the other waves are negligible.

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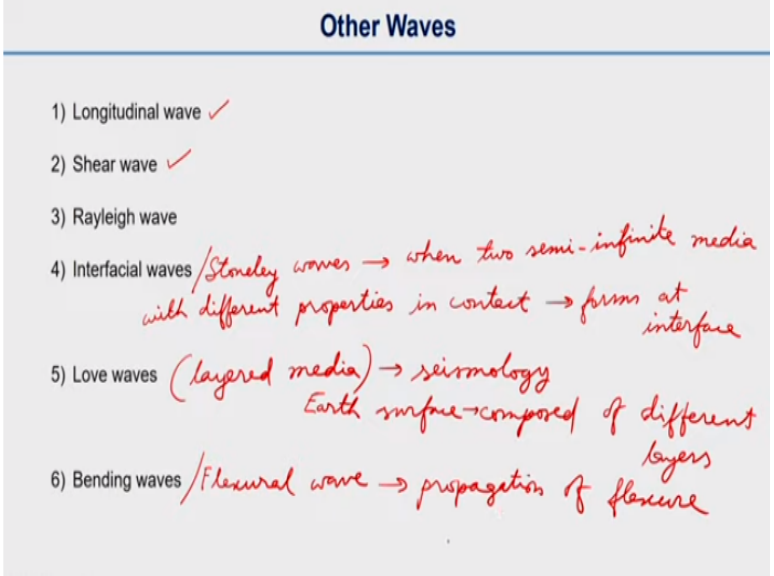

Other Waves

1) Longitudinal wave ✓
2) Shear wave ✓
3) Rayleigh wave
4) Interfacial waves / Stoneley waves → when two semi-infinite media with different properties in contact → forms at interface
5) Love waves (layered media) → seismology
Earth surface → composed of different layers
6) Bending waves / Flexural wave → propagation of flexure

So we can see that we talked about longitudinal wave we talked about shear wave we talked Rayleigh wave and we will talk about now we do not discuss much about the interfacial love wave or bending waves. So interfacial waves or stoneley waves are found when two semi-infinite media with different properties in contact. Then a wave special wave forms at the surface interface forms at interface and then love waves are that is in layered media important in seismology.

As you know that earth surface is composed of different layers so these waves seen in earth surface and seismology and the horizontal component of this displacement is significantly larger than the vertical component and then bending wave or flexural wave the this involves propagation of flexure in1D or 2D configuration.

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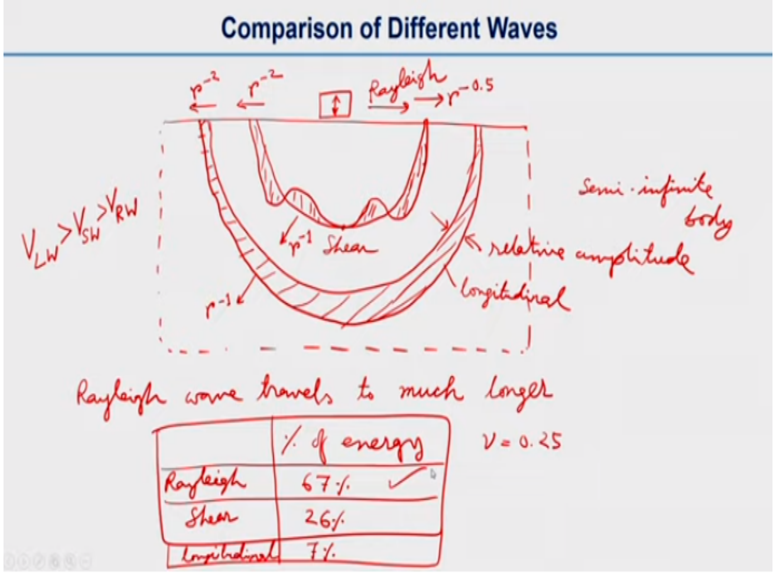

Comparison of Different Waves

Rayleigh wave travels to much longer

	% of energy
Rayleigh	67% ✓
Shear	26%
Longitudinal	7%

$\nu = 0.25$

So comparison of different waves so we want to compare their amplitude and their how the vary in a half space so that is half space again we are taking it is a semi-infinite body so this is let say the source of disturbance so the longitudinal wave this is the front of the longitudinal wave so if we draw the amplitude this is just a schematic just to show the relative amplitude if you draw the amplitude.

So you can see here it will be thin and if you go along this side this will be a little thicker so this is the amplitude for longitudinal wave and the shear wave is little bit complicated so it looks something like this is we are drawing the relative amplitude the shear wave so you can see that at some part so we have 0 amplitude for shear wave and the Rayleigh wave will be under sorry Rayleigh wave will be under surface.

So just it is important to mention that the velocity of the longitudinal wave is the highest fastest and then comes the shear wave and then the surface wave or you can write as Rayleigh wave is the slowest one here. And we will how it decay at the surface the longitudinal wave decay as r to the power -2 it is inversely proportional to the radial distance the square of the radial distance and similarly for shear wave also it is r to the power -2

But in this direction deep inside this is decay r to the power -1 and here also decay r to the power -1 and that means it is slower decay is slower in this direction or deep inside and then in the surface it is faster and also for Rayleigh the Rayleigh wave the decay is very slow r to the power -0.5 which means Rayleigh wave will travels to much longer on the surface so and the this actually as I told you this is the relative amplitude and we will show you another aspect of it.

Thus percentage of energy for these three waves this is important percentage of energy for these three waves are so for Rayleigh wave it is the highest 67% and for shear wave this is 26% and for longitudinal compressive wave it is just 7% all these calculation is for Poisson's ratio of 0.25. So you can see that the Rayleigh wave has the most energy is carrying out by the Rayleigh wave.

So we will demonstrate with the helical spring the different between the transverse and longitudinal wave.

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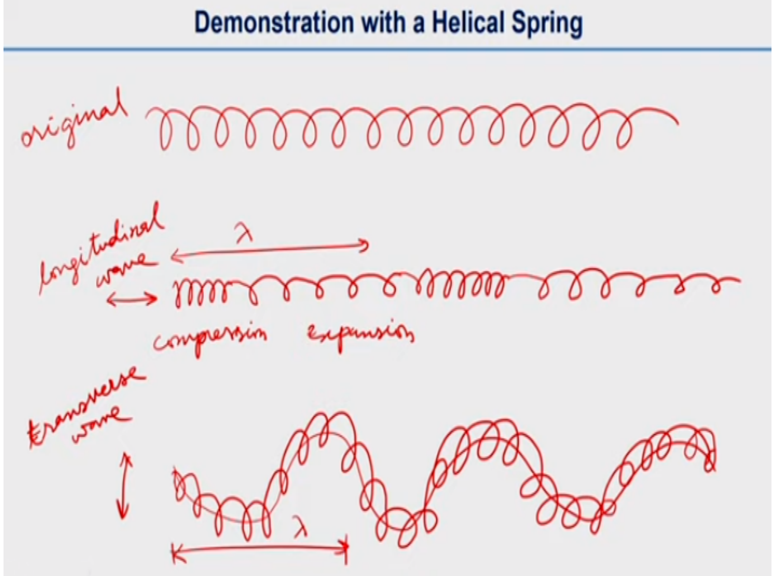

Demonstration with a Helical Spring

We have a helical spring like this this is the original spring and we will demonstrate here in this slide by drawing but in real life experiment we will also follow TA's will show you a demonstration which of this whatever we will discuss here today. So for a longitudinal wave so if we give this home and back and forth so what will happen is so there will be compression and there will be expansion and then again will be compression and there will be an expansion.

So this is compression and expansion and this will give us the wave length lambda similarly for transverse wave or shear wave we can give the motion up and down for this and then what will get is those spring will look like this sorry this diagram is not very clear probably but this spring will look like this so this if you see the wave length lambda is this distance. So this is transverse wave and longitudinal wave anyways we will discuss this with the real experiments and demonstration. So that is the end of this lecture thank you.