Dynamic Behavior of Materials
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Module No # 1
Lecture No # 07
Wave Reflection, Refraction and Interaction

Hello everyone so we have discussed about the wave velocity longitudinal wave velocity and shear wave velocity and even Rayleigh wave velocity in the previous lectures. So today we will discuss about reflection and refraction of wave and also interaction of waves with our boundaries like free round boundary or rigid boundary.

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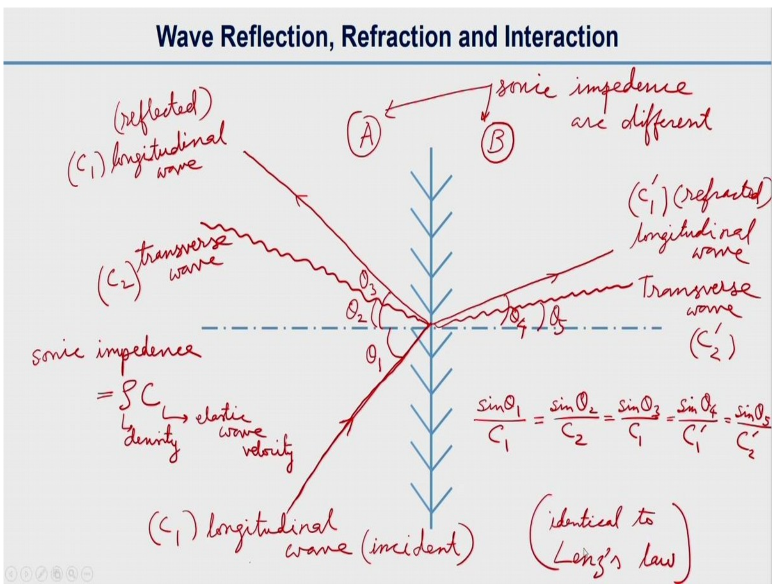

And so we will have two medium here so this is let us say medium 1 and this is medium sorry medium A and medium B and this is the interface in between these two medium and let us assume that one longitudinal wave is incident on the interface longitudinal wave. So what will happen to this wave so like we know the light wave or sound wave so these wave will we know it will reflect and refract. So the refraction will be something like this is longitudinal wave and this is refracted and similarly there will be one reflection.

So this is again longitudinal wave reflected this is the incident wave so apart from this there will be some transverse wave so the longitudinal wave the incident longitudinal wave will generate some transverse wave as well. So these transverse wave in the medium B and is and there will be another wave we have generated in medium A as well. So this is transverse and here again it is transverse so now let us assume that the velocity of the longitudinal wave in medium A is C1 and similarly here also this will be same C1 the reflected one and then the velocity of the transverse wave in medium A is C2.

Similarly here in this case for longitudinal wave the velocity C1 prime because the medium is different this medium is B so this will be C1 prime and C2 prime for transverse wave it is C2 prime. So this medium A and B have different sonic impedance so sonic impedance is defined as Rho multiplied by C so Rho is the density of the medium and C is the wave speed the wave velocity elastic wave velocity. It is nothing but the sound wave velocity elastic wave velocity. So the sonic impedance of A and B are different so the sonic impedance are different for medium A and B.

So now in this case we have we can have this refraction and reflection angles like what we do in our school classes about reflection and refraction of light wave. So this is theta 1 angle and then theta 2 here, then theta 3 here, theta 4, theta 5. So now we have a relationship between the refraction angle and reflection angle with the wave velocity. So we can write sine theta 1 / C1 = sin theta 2 / C2 sin theta 3 / same C1 because both longitudinal wave velocities incident and reflection will be same in medium A.

Then sin theta 4 / C1 prime then sin theta 5 / C2 prime so this is identical to Lenz law in sorry Lenz law in electricity this is identical.

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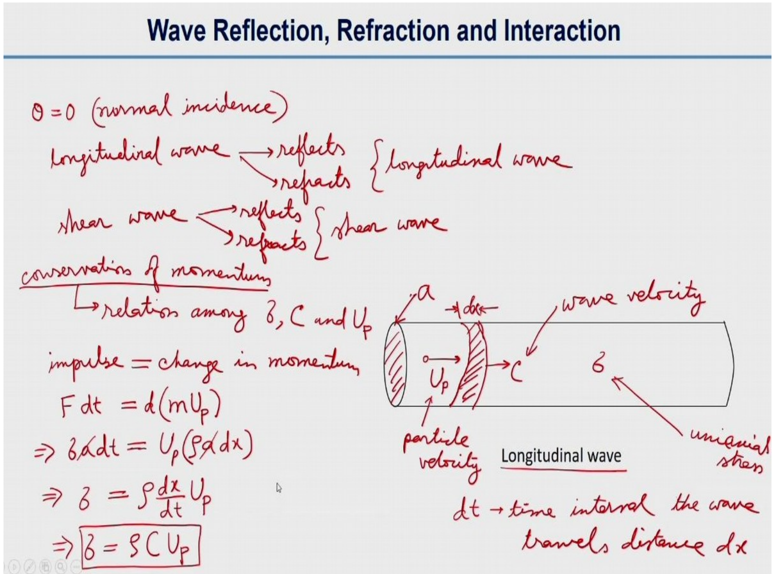

So now we will discuss about a incident wave which is normal to the interface. So now this is a longitudinal we are talking about a longitudinal wave travelling in a cylinder so now if it is a normal incidence. So in case of a normal incidence that means theta = 0 normal incidence. In this case this it will be simpler so longitudinal wave will result in reflection and reflect refraction of longitudinal wave only no transverse wave.

Similarly for shear wave reflection and refraction of shear wave only so now we will consider in this figure we will consider longitudinal wave. Let us assume that a wave as a velocity C and it is a particle velocity Up particle velocity this is particle velocity and this is wave velocity and the stress is let us assume this stress is sigma is the stress uniaxial stress. Now we will use conservation of momentum to get the relation among sigma wave velocity and particle velocity.

So from the conservation of momentum we know that the impulse is equal to change in momentum. So let us assume a small segment dx small segment that link with dx and let us assume that dt is the time interval the wave travels distance dx. So now impulse will be as we know force into dt which is impulse is nothing but the integral of the force. So now this is impulse is F dt which will be equal to the differential of m UP (()) (12:07) in momentum and the F can be written as sigma a dt where a is the cross sectional area.

So this cross sectional area we can have as a and so we can have UP and then dm will be Rho a dx. Now sigma is equal to a will be cancel out so Rho dx / dt multiplied by UP and dx / dt is nothing but the wave velocity because that is the time interval that wave travels distance dx so that means uniaxial stress is equal to Rho C UP. So that is the relation between medial (()) (13:28) stress with wave velocity and particle velocity.

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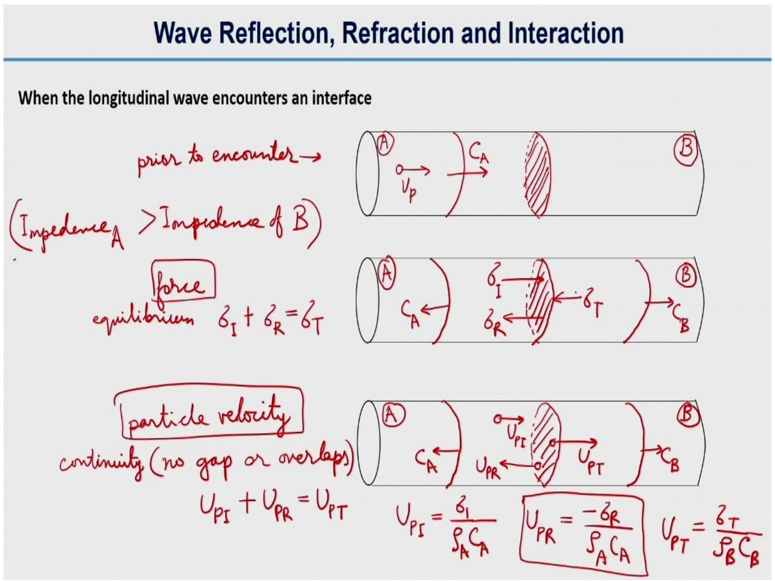

Now let us consider about an interface let us say we have an interface something like this. This is the interface and we have the wave so here let us say we have a medium A in this left side and B in the right side. So the wave velocity is CA that is in medium A and then the particle velocity is UP this first figure is for the case prior to encounter. Now in the second figure we will show the forces or stresses so we have this interface and we can see that incident stress and then reflected stress, transmitted stress.

So here the wave velocity will be CB and then the wave reflected with velocity CA so if we apply the equilibrium condition so equilibrium force equilibrium condition sigma I the incident stress plus the reflected one will be equal to the transmitted stress and for the third case we are here we are talking about force and the third image is for particle velocity the same interface in this case also I forgot to write it this is medium A medium B here medium A medium B.

So in this case will show here this is our particle velocity incident then particle velocity reflected and then particle velocity transmitted so this is our wave velocity and this again the reflected wave velocity that is CA. So for continuity that means no gap or overlap of matter overlap no gap or overlap so the condition will be that the particle velocity incident plus particle velocity reflected will be equal to particle velocity transmitted.

So the signs of the in this slide for forces and particle velocities that was set such a manner that the medium A has a higher impedance so impedance the sonic impedance actually. Impedance of A is higher than the impedance of B so it is set like that and as we know our earlier relation that this UP incident will be equal to sigma 1 Rho C Rho AC which is impedance for medium A.

Similarly UPR will be equal to - sigma R / Rho R Rho ACA impedance in the medium 1 and UP transmitted is sigma T Rho BCB impedance in medium B this minus for reflected particle velocity means that if the stress is positive the particle velocity will be negative after reflection so that we can discuss more on this later.

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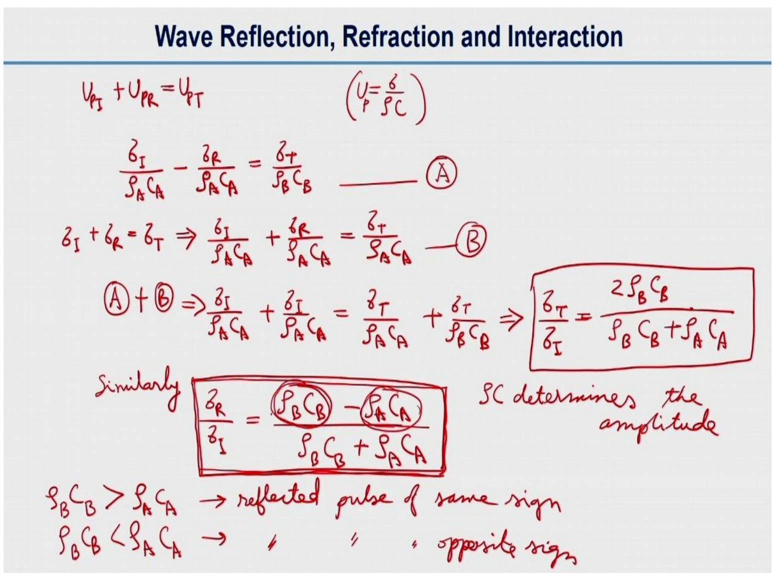

So now from our earlier relations we got a relation UPI + UPR UPT so what we are doing here is we want to know the relation between particle velocity and reflected particle velocity similarly relation between the incident particle velocity and transmitted particle velocity and again we can get some relationship between the stresses. The ratio of reflected that is stress with incident stress and then ratio with the reflected one and sorry transmitted one with incident stress.

So first we had this relation but UPI + UPR = U subscript PT so that is the relation we had in the previous slide and similarly we have the relation between the particle velocity is sigma and CA sorry Rho C. So those three relations we had so from these two relations what we can get is that sigma I Rho ACA - sigma R Rho ACA is equal to sigma transmitted rho BCB.

And again from equilibrium what we got is sigma I + sigma R = sigma T and if we divide by either impedance sonic impedance of the medium A that is by Rho ACA what we will get is sigma I Rho ACA + sigma R Rho ACA = sigma T Rho A CA and then from these two equations A and B so we will combine from these two A and B. What we can get is like if we add them this will give us sigma I Rho A CA sigma I Rho A CA = sigma T Rho A CA + sigma T Rho B CB.

So what we are doing here is we have eliminated the sigma R term so that we can get a relationship between sigma I and sigma T the incident and the transmitted the stresses. So that is why we removed eliminated the terms the reflected stresses. So this will give us the relation between sigma T and sigma I which will be equal to twice Rho B CB / Rho B CB + Rho A CA and similarly we can get sigma R / sigma I = Rho B CB – Rho A CA Rho B CB + Rho A CA.

So these are the two relations so now in these cases we can see that we can get the stresses the ratio between the stresses of incident and transmitted one an incident and the reflected one. So that means we can say that the sonic impedance Rho C determines even the amplitude of the reflected and transmitted waves. Another interesting fact is if the sonic impedance of material B is greater than the sonic impedance of material A.

The reflected pulse will be of the same sign which you can see from this relation that sigma R / sigma I. So this is if sigma B sorry Rho B CB this term is higher than this term. So then reflected pulses is of same sign but if we the other way around sigma B CB is lower than sorry Rho B CB is lower than Rho A CA then reflected pulse will be of opposite sign. That you can see from now clear from this that will be a negative. So if a Rho A CA is bigger than Rho B CB so then that will be negative.

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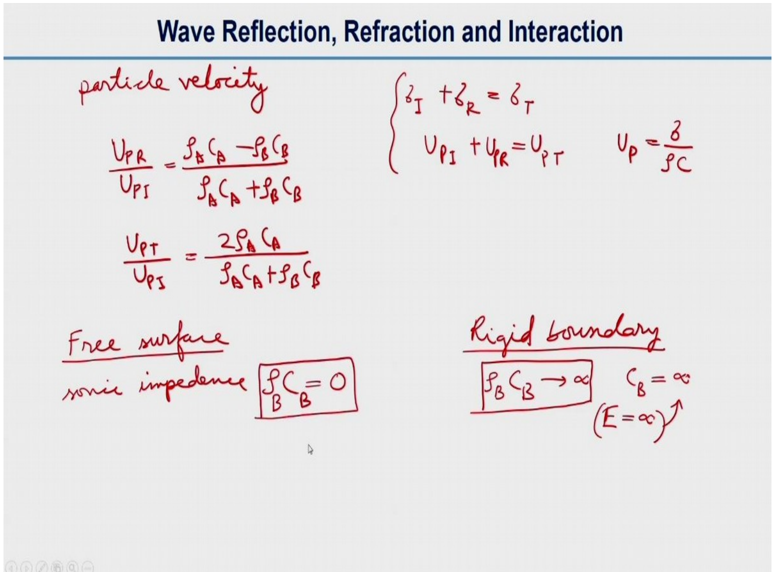

Now what we talked is about the stresses and then we will talk about the particle velocity as well for particle velocity if we use all the relation like what are relations sigma I sigma R = sigma T and then UPI + UPR = UPT and the other relations are all three relations of UP = sigma Rho C there are three relations. So we can with the help of these three relations what we can get is UPR / UPI = Rho A CA Rho B CB then Rho A CA + Rho B CB and then UPT / UPI = Rho A CA + Rho B CB twice Rho A CA.

So there are two cases one is free surface and another is we can call rigid surface or rigid boundary. So whatever interfaces we had in this figure so it can be let us say free surface or it can be a rigid boundary so what are the difference free surface means the sonic impedance or acoustic impedance. Sonic impedance Rho C = 0 and that is actually for the other medium the second medium the Rho B CB will be equal to 0 and for the rigid surface the sonic impedance will be will tend to infinity.

So Rho B CB tend to infinity actually the CB is infinity and because if it is rigid so we assume that the young's modulus is infinity and that will lead to CB infinity okay. So the sonic impedance is infinity for the rigid boundary and it is 0 for the free surface.

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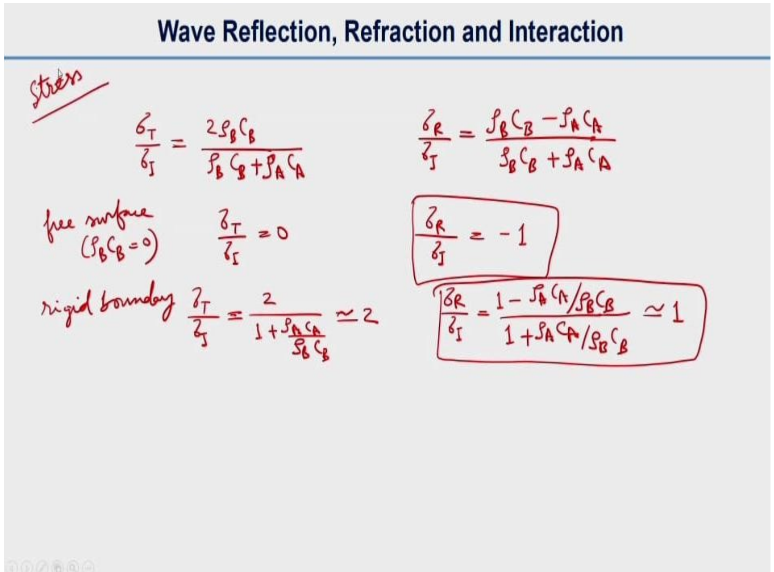

So if we talk about stresses whatever stress value we got sigma T and sigma I are ratio so which is twice Rho B CB / Rho B CB + Rho A CA and similarly sigma R sigma I is Rho B CB – Rho A CA Rho B CB + Rho A CA. So in case of free surface so as we have talked about Rho B CB will be equal to 0 which will give you sigma T sigma I equal to 0 and then sigma R sigma I will be equal to -1 this will be equal to -1.

For rigid boundary so this will be equal to 2 because sigma T / sigma I which will be equal to we can express as 2 / 1 + Rho A CA Rho B CB if we divide both denominator and numerator by Rho B CB and that will definitely give us equal to 2 and then similarly sigma R sigma I will be if we divide both numerator and denominator by Rho B CB so Rho A CA / Rho B CB 1 – 1 + sorry Rho A CA / Rho B CB so this will be equal to 1.

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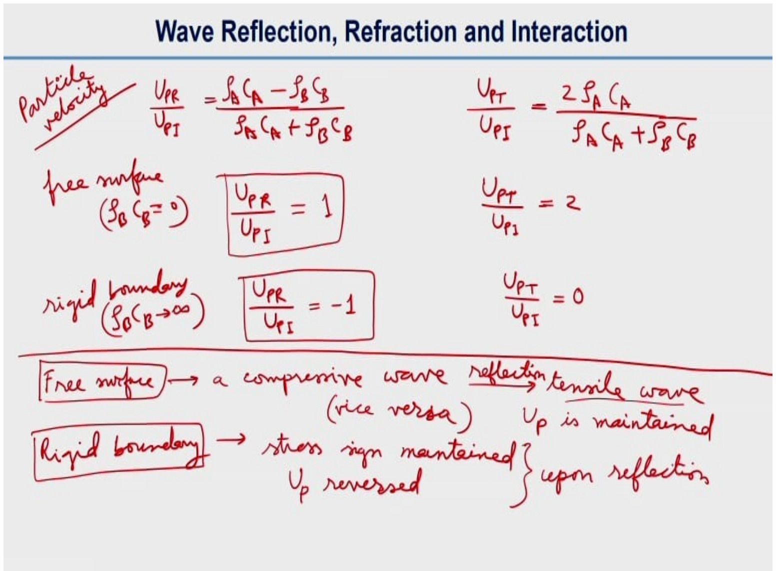

And similarly we have particle velocity our relationship are UPR UPI which is Rho A CA – Rho B CB Rho A CA – + Rho B CB and then UPT UPI which will be equal to twice Rho A CA + Rho B CB. So now if we talk about a free surface where Rho B CB equal to 0. So we can get U PT sorry UPR UPI equal to 1 and UPT UPI will be equal to 2 and for rigid boundary where Rho B CB will be infinity so that for UPR UPI will be equal to -1 and UPT UPI will be equal to 0.

So that means for if it is a free surface a compressive wave will be tensile upon reflection. So that means the stress has changes sign because in this case we have saw that for a free surface the reflected stress is -1 of times the incident stress. So that means the compressive wave after reflection will be a tensile wave but that the sign of UP is maintained UP in this case is maintained.

In case of a rigid boundary the case is opposite by the way in the other free surface case as well if you take a tensile wave the case will be opposite so alright vice versa for if we take a complete tensile wave then after a (()) (36:35) it will be compressive but UP is still maintained and for rigid boundary the stress sign will be maintained if it is compressive even after reflection also it is compressive but UP will be reversed.

So we can see that so here in for rigid boundary UP is reversed and UPR is -1 and in this case here for rigid boundaries sigma R sigma is +1 so that means stress and this is for particle velocity so particle velocity so UP is reverse here we can see -1 UPR / UPI is -1 upon reflection.

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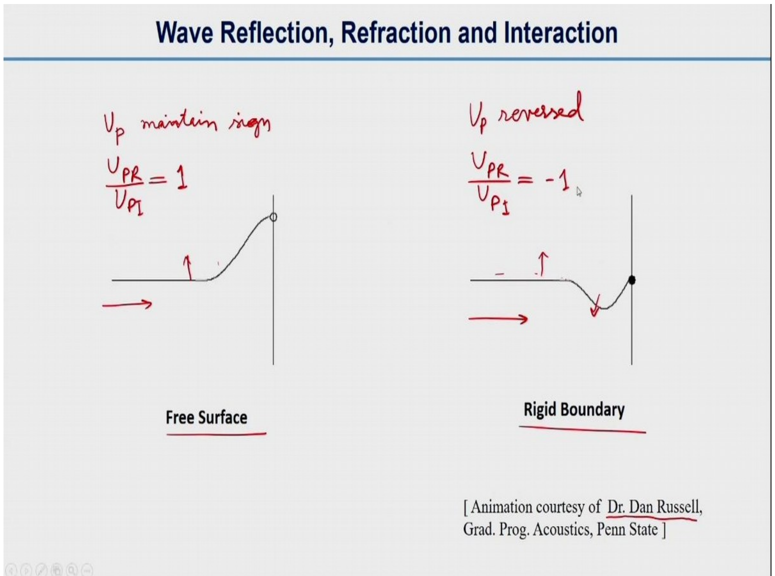

And so this is actually board from Dr. Dan Russell from Penn State and is available online so this is a free surface so the stress is moving from this side and this is a free surface so what is happening here is we are talking about the particle velocity UP under reflection is maintained a sign that means sorry maintain sign that means we wrote UPR UPI and UPI is = 1 so this = 1 this UPR / UPI is = 1 that means the particle velocity is only in the above direction.

If you see the other case this particle velocity when the wave is travelling in this way then particle velocity will be reversed UP reversed sorry reversed so what is happening this is the rigid boundary and there is a free surface so particle velocity if you say. In this rigid boundary case the particular particles are moving above and then after reflection it is actually below this line.

So this is particle velocity is reversed and in this case of the free surface the particle velocity is always under above the line. So that means the particles in this point will move above and I know and here also it is the particle before reflection it was in this direction and there after reflection this is before reflection and after reflection it moves this direction. So that is why that

U PR / UPI will be -1 and so as we discussed in the previous slide **(Video End Time:40:43)** that a compressive wave upon reflection will convert to a tensile wave in case of a free surface.

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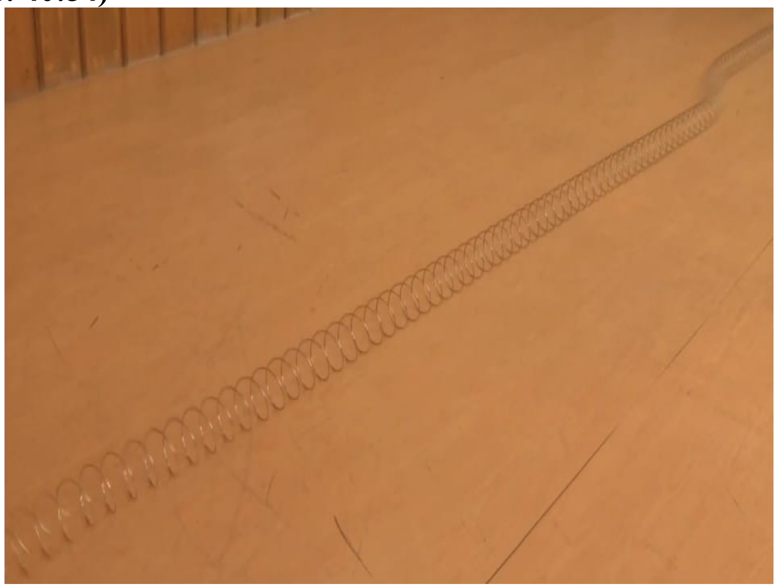

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So now we will see an impractical example then we will see the reflection of waves a boundaries we will see now reflection at the fixed boundary. So wave is reflected at the fixed boundary and it is coming back so you can see the reflection it is coming back to it is the origin. So now we will see a boundary which is not fixed so you can see that reflection is different than the earlier case. So now we will demonstrate the wave interaction so we will have waves from the both the sides so you can see some interference if the waves come from both the side. It has some interference sometimes it can be constructive or sometimes it can be destructive.

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